Assignment - I (CE - III/II)

COMP-304

1) Solve the following LP-Problem using simplex method

$\text{Min } Z = 5x_1 + 3x_2 + 7x_3$

subject to

$x_1 + x_2 + 2x_3 \leq 22$

$3x_1 + 2x_2 + x_3 \leq 26$

$x_1 + x_2 + 2x_3 \leq 18$

$x_1, x_2, x_3 \geq 0$

Soln:

Standard form of above LP-Problem is

$\text{Min } Z = 5x_1 + 3x_2 + 7x_3 + 0.s_1 + 0.s_2 + 0.s_3$

subject to

$x_1 + x_2 + 2x_3 + s_1 = 22$

$3x_1 + 2x_2 + x_3 + s_2 = 26$

$x_1 + x_2 + 2x_3 + s_3 = 18$

$s_1, s_2, s_3, x_1, x_2, x_3 \geq 0$

First iteration:

		C_j	5	3	7	0	0	0	Min. Ratio
C_B	B	X_{Bi}	x_1	x_2	x_3	s_1	s_2	s_3	X_{Bi}/a_{rj}
0	s_1	22	1	1	2	1	0	0	$22/1 = 22$
0	s_2	26	3	(2)	1	0	1	0	$26/2 = 13$ ←
0	s_3	18	1	1	2	0	0	1	$18/1 = 18$
		Z_j	0	0	0	0	0	0	
		$Z_j - C_j$	-5	-3	-7	0	0	0	

↑ pivot column

Since the objective function is of minimization type, pivot column is corresponding to most positive $Z_j - C_j$ row element.

Since $Z_j - c_j \leq 0$ we stop and the optimal solution is attained at $x_1 = 0$, $x_2 = 0$ and $x_3 = 0$.

$$\text{Min } Z = 5 \times 0 + 3 \times 0 + 7 \times 0$$
$$= 0.$$

2) Using Big-M method solve the following LP-Problem

$$\text{Max } Z = x_1 + 2x_2 + 3x_3$$

subject to

$$x_1 - x_2 + x_3 \geq 4$$
$$x_1 + x_2 + 2x_3 \leq 8$$
$$x_1 - x_3 \geq 2$$
$$x_1, x_2, x_3 \geq 0$$

Soln:

Standard form of above LP-Problem is

$$\text{Max } Z = x_1 + 2x_2 + 3x_3 + 0.s_1 + 0.s_2 + 0.s_3 - MA_1 - MA_2$$

subject to

$$x_1 - x_2 + x_3 - s_1 + A_1 = 4$$
$$x_1 + x_2 + 2x_3 + s_2 = 8$$
$$x_1 - x_3 - s_3 + A_2 = 2$$
$$x_1, x_2, x_3, s_1, s_2, s_3, A_1, A_2 \geq 0$$

Iteration-1		C_j	1	2	3	0	0	0	$-M$	$-M$	Min. Ratio
C_B	B	X_{B_i}	x_1	x_2	x_3	s_1	s_2	s_3	A_1	A_2	X_B/x_1
$-M$	A_1	4	1	-1	1	-1	0	0	1	0	$4/1 = 4$
0	S_2	8	1	1	2	0	1	0	0	0	$8/1 = 8$
$-M$	A_2	2	(1)	0	-1	0	0	-1	0	1	$2/1 = 2 \rightarrow$
		Z_j	$-2M$	M	0	M	0	M	$-M$	$-M$	
		$Z_j - C_j$	$-2M-1 \uparrow$	$M-2$	-3	M	0	M	0	0	

Since the objective function is of ~~maximi~~ maximization type, pivot column corresponds to most negative $Z_j - C_j$.

Here the most negative $Z_j - C_j = -2M-1$ and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 2 and its row index is 3. so, the outgoing basis variable is A_2.

$\therefore$ The pivot element is 1.

Iteration-2		C_j	1	2	3	0	0	0	$-M$	Min. Ratio
C_B	B	X_{B_i}	x_1	x_2	x_3	s_1	s_2	s_3	A_1	X_{Bi}/x_3
$-M$	A_1	2	0	-1	(2)	-1	0	1	1	$2/2=1 \rightarrow$
0	s_2	6	0	1	3	0	1	1	0	$6/3=2$
1	x_1	2	1	0	-1	0	0	-1	0	—
		Z_j	1	M	$-2M-1$	M	0	$-M-1$	$-M$	
		Z_j-C_j	0	$M-2$	$-2M-4 \uparrow$	M	0	$-M-1$	0	

Most negative $Z_j - C_j = -2M-4$ and the corresponding column index is 3. so, the entering variable is x_3

Minimum ratio = 1 and its row index is 1. so, the outgoing basis variable is A_1

$\therefore$ The pivot element is 2.

Iteration-3		C_j	1	2	3	0	0	0	Min. Ratio
C_B	B	X_{B_i}	x_1	x_2	x_3	s_1	s_2	s_3	X_{Bi}/x_2
3	x_3	1	0	-0.5	1	-0.5	0	0.5	—
0	s_2	3	0	(2.5)	0	1.5	1	-0.5	$3/2.5=1.2 \rightarrow$
1	x_1	3	1	-0.5	0	-0.5	0	-0.5	—
		Z_j	1	-2	3	-2	0	1	
		Z_j-C_j	0	$-4 \uparrow$	0	-2	0	1	

Most negative $Z_j - C_j = -4$ and its column index is 2.
So, the entering variable is x_2.

Minimum ratio is 1.2 and its row index is 2. So the outgoing variable is S_2.
$\therefore$ Pivot element is 2.5.

Iteration-4		C_j	1	2	3	0	0	0	Mini Ratio
C_B	B	X_{B_i}	x_1	x_2	x_3	s_1	s_2	s_3	X_{B_i}/a_{rj}
3	x_3	1.6	0	0	1	-0.2	0.2	0.4	
2	x_2	1.2	0	1	0	0.6	0.4	-0.2	
1	x_1	3.6	1	0	0	-0.2	0.2	-0.6	
		Z_j	1	2	3	0.4	1.6	0.2	
		$Z_j - C_j$	0	0	0	0.4	1.6	0.2	

Since all $Z_j - C_j \geq 0$, we stop and this is the optimality criteria.

~~Max Z~~ Hence, optimal solution is attained with value of variables as:
$x_1 = 3.6$, $x_2 = 1.2$ and $x_3 = 1.6$.
where,

$$\text{Max } Z = 3.6 + 2(1.2) + 3(1.6)$$
$$= 3.6 + 2.4 + 4.8$$
$$= 10.8.$$

3) Find the dual of the following

$\text{Min } Z = 3x_1 - 2x_2 + 4x_3$

subject to

$2x_1 + 5x_2 + 4x_3 \geq 7$

$6x_1 + x_2 + 3x_3 \geq 4$

$7x_1 - 2x_2 - 3x_3 \leq 10$

$x_1 - 2x_2 + 5x_3 \geq 3$

$4x_1 + 7x_2 - 2x_3 \geq 2$

and $x_1, x_2, x_3 \geq 0$;

~~Solu~~ State also the solution of primal from the solution of dual.

Soln: Since the objective function of the above LP problem is of minimization, the direction of each constraint has to be changed to $\geq$ type by multiplying both sides by -1.

$\text{Min } Z = 3x_1 - 2x_2 + 4x_3$

subject to

$2x_1 + 5x_2 + 4x_3 \geq 7$

$6x_1 + x_2 + 3x_3 \geq 4$

$-7x_1 + 2x_2 + 3x_3 \geq -10$

$x_1 - 2x_2 + 5x_3 \geq 3$

$4x_1 + 7x_2 - 2x_3 \geq 2$

$x_1, x_2, x_3 \geq 0$

The standard form of above LP-Problem is

$$\text{Min } Z = 3x_1 - 2x_2 + 4x_3 + 0.s_1 + 0.s_2 + 0.s_3 + 0.s_4 + 0.s_5$$

subject to

$$2x_1 + 5x_2 + 4x_3 - s_1 = 7$$
$$6x_1 + x_2 + 3x_3 - s_2 = 4$$
$$-7x_1 + 2x_2 + 3x_3 - s_3 = -10$$
$$x_1 - 2x_2 + 5x_3 - s_4 = 3$$
$$4x_1 + 7x_2 - 2x_3 - s_5 = 2$$
$$s_1, s_2, s_3, s_4, s_5, x_1, x_2, x_3 \geq 0$$

The dual of above primal LP-Problem is

$$\text{Max } Z_y = 7y_1 + 4y_2 - 10y_3 + 3y_4 + 2y_5$$

subject to

$$2y_1 + 6y_2 - 7y_3 + y_4 + 4y_5 \leq 3$$
$$5y_1 + y_2 + 2y_3 - 2y_4 + 7y_5 \leq -2$$
$$4y_1 + 3y_2 + 3y_3 + 5y_4 - 2y_5 \leq 4$$
$$y_1, y_2, y_3, y_4, y_5 \geq 0.$$

Here $b_2 = -2 < 0$,
so multiply this constraint by -1 to make $b_2 > 0$.

$$\Rightarrow -5y_1 - y_2 - 2y_3 + 2y_4 - 7y_5 \geq 2$$

Standard form of above dual LP-Problem is

$$\text{Max } Z_y = 7y_1 + 4y_2 - 10y_3 + 3y_4 + 2y_5 + 0.s_1 + 0.s_2 + 0.s_3 - MA_1$$

subject to

$$2y_1 + 6y_2 - 7y_3 + y_4 + 4y_5 + s_1 = 3$$
$$-5y_1 - y_2 - 2y_3 + 2y_4 - 7y_5 - s_2 + A_1 = 2$$
$$4y_1 + 3y_2 + 3y_3 + 5y_4 - 2y_5 + s_3 = 4$$
$$y_1, y_2, y_3, y_4, y_5, s_1, s_2, s_3, A_1 \geq 0.$$

Iteration 1		C_j	7	4	-10	3	2	0	0	0	-M	
C_B	B	Y_{B_i}	y_1	y_2	y_3	y_4	y_5	S_1	S_2	S_3	A_1	Min. Ratio Y_{B_i}/y_4
0	S_1	3	2	6	-7	1	4	1	0	0	0	$3/1 = 3$
-M	A_1	2	-5	-1	-2	2	-7	0	-1	0	1	$2/2 = 1$
0	S_3	4	4	3	3	(5)	-2	0	0	1	0	$4/5 = 0.8 \rightarrow$
		Z_j	5M	M	2M	-2M	7M	0	M	0	-M	
		$Z_j - C_j$	5M-7	M-4	2M+10	-2M-3 ↑	7M-2	0	M	0	0	

Since the objective function of above dual LP-problem is of maximization type, the pivot column corresponds to most negative $Z_j - C_j$.

Here the most negative $Z_j - C_j$ is $-2M-3$ and its column index is 4. So the entering variable is y_4.

Minimum ratio is 0.8 and its row index is 3.

So the outgoing variable is S_3.

∴ The pivot element is 5.

Iteration-2		C_j	7	4	-10	3	2	0	0	0	-M	
C_B	B	Y_{B_i}	y_1	y_2	y_3	y_4	y_5	S_1	S_2	S_3	A_1	Min. Ratio Y_{B_i}/y
0	S_1	2.2	1.2	5.4	-7.6	0	4.4	1	0	-0.2	0	
-M	A_1	0.4	-6.6	-2.2	-3.2	0	-6.2	0	-1	-0.4	1	
3	y_4	0.8	0.8	0.6	0.6	1	-0.4	0	0	0.2	0	
		Z_j	6.6M+2.4	2.2M+1.8	3.2M+1.8	3	6.2M-1.2	0	M	0.4M+0.6	-M	
		$Z_j - C_j$	6.6M-4.6	2.2M-2.2	3.2M+11.8	0	6.2M-3.2	0	M	0.4M+0.6	0	

Since all $Z_j - C_j \geq 0$, we stop and the optimal solution is attained.

$$\text{Max } Z_y = 7\times 0 + 4\times 0 - 10\times 0 + 3\times 0.8 + 2\times 0$$
$$= 2.4$$

at $y_1 = 0$, $y_2 = 0$, $y_3 = 0$, $y_4 = 0.8$ and $y_5 = 0$.

But this solution is not feasible because the final solution violates the 2nd constraint $-5y_1 - y_2 - 2y_3 + 2y_4 - 7y_5 \geq 2$ and the artificial variable A_1 appears in the basis with positive value 0.4.

Solution for primal LP-problem is:

$x_1 = |Z_j - C_j$ element corresponding $S_1| = |0| = 0$

$x_2 = |Z_j - C_j$ element corresponding $S_2| = |M| = M$

$x_3 = |Z_j - C_j$ element corresponding $S_3| = |0.4M + 0.6| = 0.4M + 0.6$.

$$\text{Min } Z_x = 3\times 0 - 2\times M + 4(0.4M + 0.6)$$
$$= 0 - 2M + 1.6M + 2.4$$
$$\simeq -\cancel{2M} + \cancel{2M} + 2.4 \quad [\because M \text{ is very large undesirable coefficient}].$$
$$= 2.4$$

4) LG Company has been a producer of picture tubes for television sets and certain printed circuits for radios. The company has just expanded into full scale production and marketing of AM and AM-FM radios. It has built a new plant that can operate for 48 hours per week. Production of an AM radio will require 2 hours and AM-FM radio will require 3 hours. Each AM radio will contribute Rs. 40 to profits while an AM-FM radio will cotribute Rs. 80 to profits. The marketing department, after extensive research, has determined that a maximum of 15 AM radios and 10 AM-FM radios can be sold each week.

(i) Set up the mathematical model for the production mix of AM-FM radios.

(ii) Find from the graphical method, the number of AM and AM-FM radios that maximize the total profit.

Soln:

Let the decision variables be

x_1 = number of units of AM radio to be produced

x_2 = number of units of AM-FM radio to be produced.

Then, LP model of the given problem is:

Maximize (total profit) $Z = 40x_1 + 80x_2$

subject to constraints

Plant: $2x_1 + 3x_2 \leq 48$ — (i)

AM radio: $x_1 \leq 15$ — (ii)

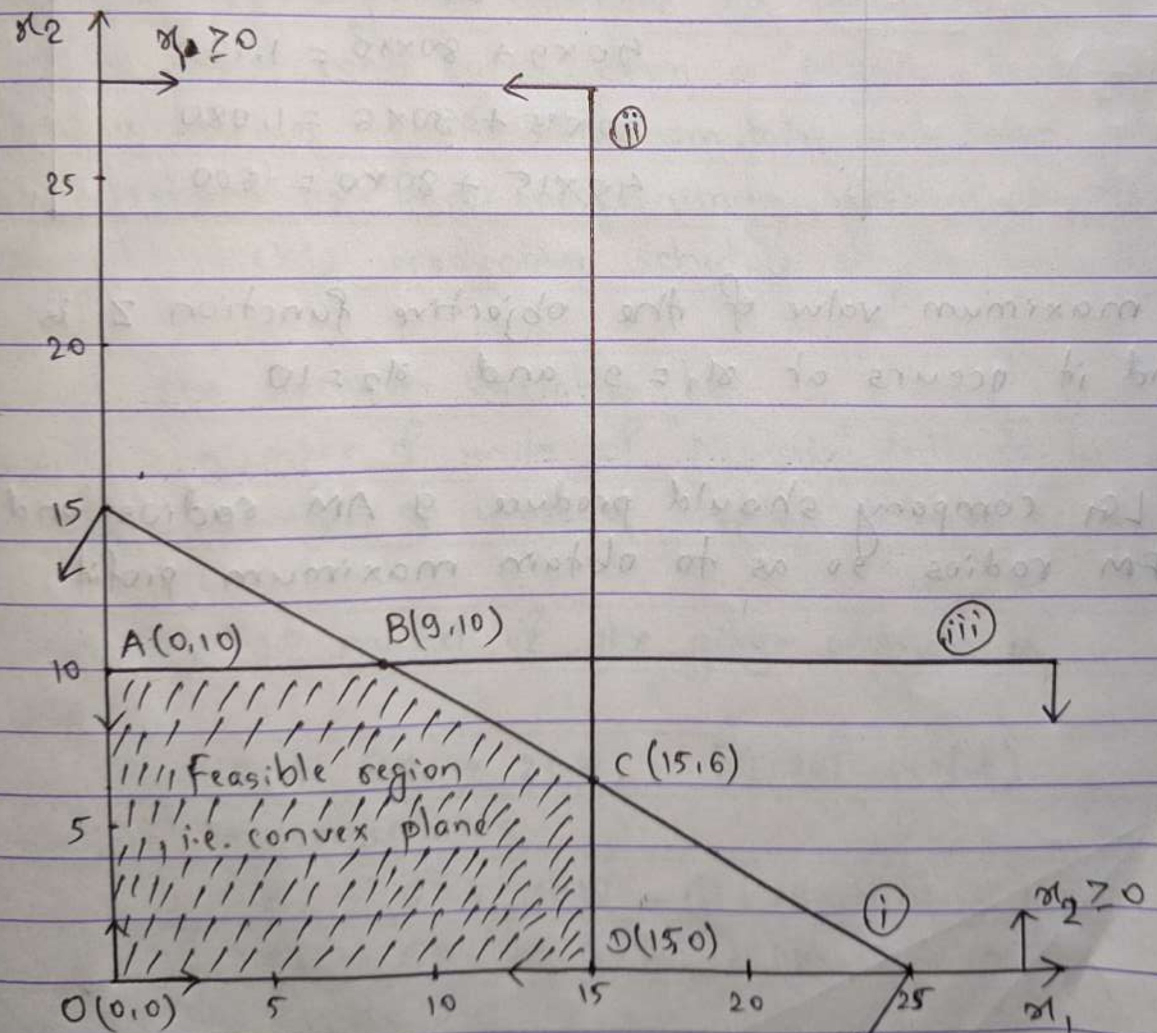
AM-FM radio: $x_2 \leq 10$ — (iii)

and $x_1, x_2 \geq 0$.

Changing ~~ca~~ inequalities (i), (ii) and (iii) into equations, we have

$$2x_1 + 3x_2 = 48$$
$$x_1 = 15$$
$$x_2 = 10$$

Now, plotting above equations and then using inequality condition of each constraint to make the feasible region, ~~we~~ we get



(0,0) in inequalities (i), (ii) and (iii) give true-meaning, therefore all the closed-half planes contain origin.

Vertices of convex polygon are :
A (0,10), B (9,10), C (15,6) and D (15,0)

The objective function values at each vertex are :

Coordinates (x_1, x_2)	Objective Function Value $Z = 40x_1 + 80x_2$
(0,10)	$40 \times 0 + 80 \times 10 = 800$
(9,10)	$40 \times 9 + 80 \times 10 = 1,160$
(15,6)	$40 \times 15 + 80 \times 6 = 1,080$
(15,0)	$40 \times 15 + 80 \times 0 = 600$

Since the maximum value of the objective function Z is 1,160 and it occurs at $x_1 = 9$ and $x_2 = 10$

Therefore LG company should produce 9 AM radios and 10 AM-FM radios so as to obtain maximum profit.

5) A manufacturer of baby dolls make two types of dolls. One is sold under the brand name "Monila" and the other under ~~S.T.Z.~~ "Suzie". These two dolls are processed on two machines - M and N. The processing time for each "Monila" is 3 hours and 7 hours on the machines M and N respectively and that for each "Suzie" is 6 hours and 6 hours on machines M and N respectively. There is 18 hours of time available per day on machine M and 30 hours on machine B. The profit contribution from a "Monila" is Rs. 8 and that from a "Suzie" is Rs. 20, Formulate and solve graphically this problem as linear programming problem to determine the optimal weekly production schedule of the two dolls.

Soln:

Let the decision variables be

x_1 = number of units of "Monila" doll to be produced.

x_2 = number of units of "Suzie" doll to be produced.

Then the LP model of the given problem is

$$\text{Max } Z = 8x_1 + 20x_2 \quad \text{(Total profit)}$$

subject to constraints

$$3x_1 + 6x_2 \le 18 \text{ — (i) (Machine M)}$$

$$7x_1 + 6x_2 \le 30 \text{ — (ii) (Machine N)}$$

$$x_1, x_2 \ge 0$$

Changing inequalities (i) and (ii) into linear equations, we have

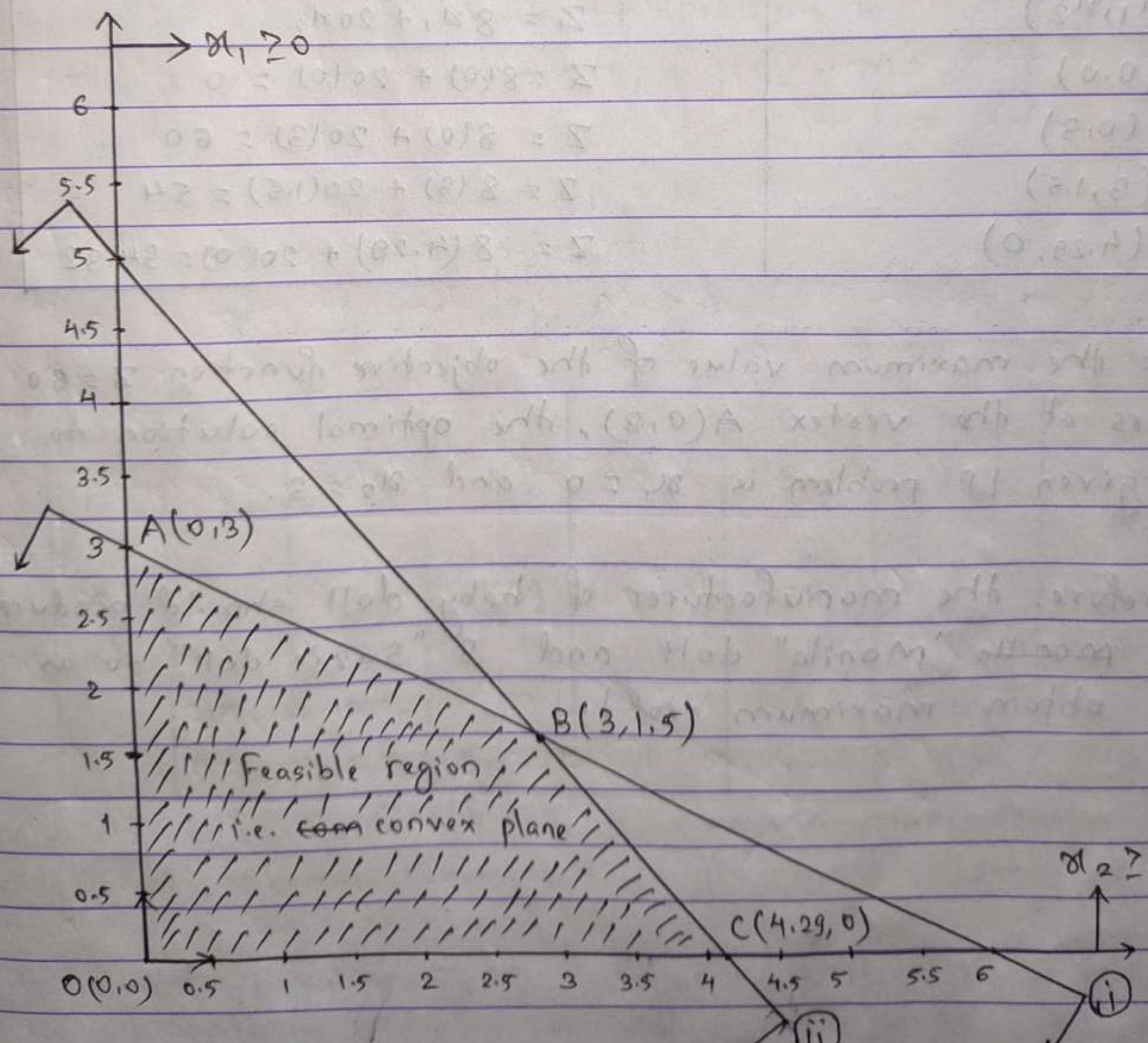
$$3x_1 + 6x_2 = 18$$

$$\Rightarrow \frac{x_1}{6} + \frac{x_2}{3} = 1$$

and $7x_1 + 6x_2 = 30$

$$\Rightarrow \frac{x_1}{\left(\frac{30}{7}\right)} + \frac{x_2}{5} = 1$$

Now plotting above equations and then using inequality condition of each constraint to make the feasible region, we get.



(0,0) in inequalities (i) and (ii) give true meaning, therefore all the closed-half planes contain origin O(0,0).

Vertices of convex polygon are O(0,0), A(0,3), B(3,1.5) and C(4.29,0)

The objective function values at each vertex are:

Coordinates (x_1, x_2)	Objective function value $Z = 8x_1 + 20x_2$
O (0,0)	$Z = 8(0) + 20(0) = 0$
A (0,3)	$Z = 8(0) + 20(3) = 60$
B (3,1.5)	$Z = 8(3) + 20(1.5) = 54$
C (4.29, 0)	$Z = 8(4.29) + 20(0) = 34.32$

Since the maximum value of the objective function $Z = 60$ occurs at the vertex A(0,3), the optimal solution to the given LP problem is $x_1 = 0$ and $x_2 = 3$.

Therefore, the manufacturer of baby doll should produce 0 ~~Monilla~~ "Monila" doll and 3 "Suzie doll" so as to obtain maximum profit.

6) Show by Simplex method that the following LP-problem has unbounded solution:

Max $Z = 3x_1 + 6x_2$

subject to

$3x_1 + 4x_2 \geq 12$

$-2x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0$

Soln:

Standard form of above LP problem is

Max $Z = 3x_1 + 6x_2 + 0.S_1 + 0.S_2 - MA_1$

subject to

$3x_1 + 4x_2 - S_1 + A_1 = 12$

$-2x_1 + x_2 + S_2 = 4$

$x_1, x_2, S_1, S_2, A_1 \geq 0$

Iteration-1		C_j	3	6	0	0	-M	
C_B	B	X_{Bi}	x_1	x_2	S_1	S_2	A_1	Min. Ratio X_{Bi}/x_2
-M	A_1	12	3	(4)	-1	0	1	$12/4 = 3 \rightarrow$
0	S_2	4	-2	1	0	1	0	$4/1 = 4$
		Z_j	-3M	-4M	M	0	-M	
		$Z_j - C_j$	-3M-3	$-4M-6\uparrow$	M	0	0	

Since the objective function is of ~~maxima~~ maximization type, the pivot column corresponds to the most negative $Z_j - C_j$

The most negative $Z_j - C_j = -4M-6$ and its column index is 2. So the entering variable is x_2.

The minimum ratio is 3 and its row index is 1. So, the outgoing basis variable is A_1

∴ The pivot element is 4.

Iteration - 2		C_j	3	6	0	0	
C_B	B	X_{B_i}	x_1	x_2	S_1	S_2	Min. Ratio X_{B_i}/x_1
6	x_2	3	0.75	1	-0.25	0	—
0	S_2	1	-2.75	0	(0.25)	1	$1/0.25 = 4 \rightarrow$
		Z_j	4.5	6	-1.5	0	
		$Z_j - C_j$	1.5	0	$-1.5 \uparrow$	0	

Most negative $Z_j - C_j = -1.5$ and its column index is 3. So the entering variable is S_1.
Minimum ratio is 4 and its row index is 2. So, the ~~Mini~~ outgoing basis variable is S_2.
∴ The pivot element is 0.25

Iteration - 3		C_j	3	6	0	0	
C_B	B	X_{B_i}	x_1	x_2	S_1	S_2	Min. Ratio X_{B_i}/x_1
6	x_2	4	-2	1	0	1	—
0	S_1	4	-11	0	1	4	—
		Z_j	-12	6	0	6	
		$Z_j - C_j$	$-15 \uparrow$	0	0	0	

Most negative $Z_j - C_j$ is -15 and its column index is 1. So x_1 should enter into the basis.

But all the coefficients in the x_1 column i.e. pivot column are negative or zero. So x_1 cannot be entered into the basis.

Hence the solution to the given LP problem is unbounded.

7) Show by simplex method that the following LP-problem has infeasible solution :

$$\text{Max } Z = 3x_1 + 2x_2$$

subject to

$$2x_1 + x_2 \le 2$$

$$3x_1 + 4x_2 \ge 12$$

$$x_1, x_2 \ge 0$$

Soln:

Standard form of above LP-problem is

$$\text{Max } Z = 3x_1 + 2x_2 + 0.S_1 + 0.S_2 - MA_1$$

subject to

$$2x_1 + x_2 + S_1 = 2$$

$$3x_1 + 4x_2 - S_2 + MA_1 = 12$$

$$x_1, x_2, S_1, S_2, A_1 \ge 0$$

Iteration-1		C_j	3	2	0	0	-M	
C_B	B	X_{Bi}	x_1	x_2	S_1	S_2	A_1	Min-Ratio X_{Bi}/x
0	S_1	2	2	(1)	1	0	0	$2/1 = 2 \rightarrow$
-M	A_1	12	3	4	0	-1	1	$12/4 = 3$
		Z_j	-3M	-4M	0	M	-M	
		$Z_j - C_j$	-3M-3	-4M-2 ↑	0	M	0	

The most negative $Z_j - C_j = -4M-2$ and its column index is 2.
So the entering variable is x_2

Minimum ratio is 2 and its row index is 1.
So the outgoing basis variable is S_1

$\therefore$ The pivot element is 1.

Iteration - 2		C_j	3	2	0	0	$-M$	
C_B	B	X_{B_i}	x_1	x_2	S_1	S_2	A_1	Min. Ratio
2	x_2	2	2	1	1	0	0	
$-M$	A_1	4	-5	0	-4	-1	1	
		Z_j	$5M+4$	2	$4M+2$	M	$-M$	
		$Z_j - C_j$	$5M+1$	0	$4M+2$	M	0	

Since all $Z_j - C_j \geq 0$, we stop and the optimal solution is attained at $x_1 = 0$ and $x_2 = 2$.

$$\text{Max } Z = 3 \times (0) + 2 \times (2)$$
$$= 4$$

But this solution is infeasible because the final optimal solution violates the 2nd constraint $3x_1 + 4x_2 \geq 12$ since $3(0) + 4(2) = 8 \leq 12$.

and the artificial variable A_1 appears in the basis with positive value 4.

8) Table below is optimal solution of following LP-problem then with the help of table answer questions given below:

$$\text{Max } Z = 3x_1 + 2x_2 + x_3$$

subject to

$$4x_1 + x_2 + x_3 = 30$$

$$2x_1 + 3x_2 + x_3 \leq 60$$

$$x_1 + 2x_2 + 3x_3 \leq 40$$

$$x_1, x_2, x_3 \geq 0$$

		C_j	3	2	1	0	0
C_B	B	X_{B_i}	x_1	x_2	x_3	S_2	S_3
3	x_1	3	x_1	0	$1/5$	$-1/10$	0
2	x_2	18	0	1	$1/5$	$2/5$	0
0	S_3	1	0	0	$12/5$	$-7/10$	1
		Z_j	3	2	1	$1/2$	0
		$Z_j - C_j$	0	0	0	$1/2$	0

(i) State basic and non-basic variables

⇒ Basic variables : x_1, x_2, S_3

Non-basic variables : x_3, S_2

(ii) State solutions

⇒ Solutions are : $x_1 = 3$, $x_2 = 2$ and $x_3 = 0$.

$$\text{Max } Z = 3 \times 3 + 2 \times 2 + 0$$

$$= 13.$$

(iii) What is the nature of solution in above table?

⇒ The nature of solution in above table is non-degenerate because none of the basic variables has its value zero in x_{B_i} column.

(iv) State does problem ~~pro~~ possess alternative solution? If yes give reason and if not give reason.

⇒ Yes, the above problem possesses alternative solution because ~~one non~~ there exists one non-basic variable x_3 whose corresponding $Z_j - C_j$ row element is zero.

(v) State new objective function value when second resource is increased by unit amount.

⇒ Current Max $Z = 3 \times 3 + 2 \times 2 + 0$

$= 13$

The element of Z_j-row corresponding to ~~S_3 i.e.~~ S_2 i.e. ~~at optimal solutions~~ second resource is $\frac{1}{2}$. which is the dual price of the given problem.

Therefore, new objective function value when second resource is increased by ~~b~~ unit amount $= 13 + \frac{1}{2}$

$= \frac{27}{2}$.

vi) State new objective function value when s_2 is forced into solution.

⇒ The $Z_j - C_j$ row element corresponding to non-basic variable s_2 is $\frac{1}{2}$, which is the reduced cost for it.

Therefore new objective function value when s_2 is forced into solution $= 13 - \frac{1}{2}$

$= \frac{25}{2}$.

vii) What does $s_3 = 1$ indicate?

⇒ s_3 is a slack variable used in third constraint and $s_3 = 1$ indicates that third resource has surplus of 1 unit while producing x_3 product.