Maths Basics and Calculate Iterations

Agenda of this Lecture:

- Maths Basics
- GCD
- LCM
- Iteration Calculation

Let us learn some basic maths concepts, starting with **Sum of natural numbers.**

Question

Sum of first N natural numbers =

Choices

- ☐ N * (N + 1)
- ☑ N * (N + 1) / 2
- ☐ N * (N - 1)
- ☐ N * (N - 1) / 2

Explanation

The sum of the first N natural numbers is equal to $\frac{N(N+1)}{2}$. For example, the sum of the first 6 natural numbers is $\frac{6(6+1)}{2} = \frac{42}{2} = 21$.

Question

How many numbers are there in this range [3,10] ? both corners included

Choices

- ☐ 7
- ☑ 8
- ☐ 9
- ☐ 10

Explanation:

The numbers in this range are: 3, 4, 5, 6, 7, 8, 9, and 10. Therefore, there are 8 numbers in the range [3, 10] (both corners included).

Question

How many numbers are there from [a b] both included

Choices

- ☐ b - a
- ☑ b - a + 1
- ☐ b - a - 1
- ☐ None of them

Explanation:

If we have a range [a, b] where both endpoints are included, we can calculate the number of numbers in that range by taking the absolute difference between b and a and adding 1.

Question

How many numbers are there in this range [4,7] ? both corners included

Choices

- ☐ 2
- ☐ 3
- ☑ 4
- ☐ 5

Explanation:

If we apply the formula here ie b - a + 1, here b = 7, a = 4. Hence b - a + 1 = 4.

Point to remember:-

- Elements in range [a,b). In this a is **included but b is excluded.**
- Elements in range (a,b). In this **both a and b are excluded.**

Geometric Progression

Definition

A sequence of numbers is called a Geometric progression (GP) if the ratio of any two consecutive terms is always the same.

Example 1:

```
2    6    18    54    162
```

The sequence 2, 6, 18, 54, 162 is a GP because ratio of any two consecutive terms in the series (common difference) is same

$$\frac{6}{2} = \frac{18}{6} = \frac{54}{18} = \frac{162}{54} = 3$$

.

Example 2:

```
3    6    12    24    48    96
```

The sequence 3, 6, 12, 24, 48, 96 is a GP because ratio of any two consecutive terms in the series is same(ie 2).

Explanation

In simple terms, A geometric series is a list of numbers where each number, or term, is found by multiplying the previous term by a common ratio r.

The formula for the sum of the nth term of Geometric Progression:

Sum = $a\frac{(r^n-1)}{(r-1)}$

Where,

Sum = Sum of all Geometric Progressions n= number of terms r = Common ratio

Example

Given series: **2,4,8,16,32**

Sum of first five terms in ths series will be given by:

a = 2 , r = 2, n = 5

sum= $a\frac{(r^n-1)}{(r-1)}$ = $2 * \frac{(2^5-1)}{2-1}$ = 2 * (32 - 1) = 62

Break Statement

To stop the iterations of a loop before it actually completes, we use the break statement.

Question

What is the output of the following code?

```
for(int i = 1; i <= 5; i ++ ) {
    SOPln(i);
}
```

Choices

- ☑ 1 2 3 4 5
- □ 1 2 3 4
- □ 1 2
- □ None of the above

Question

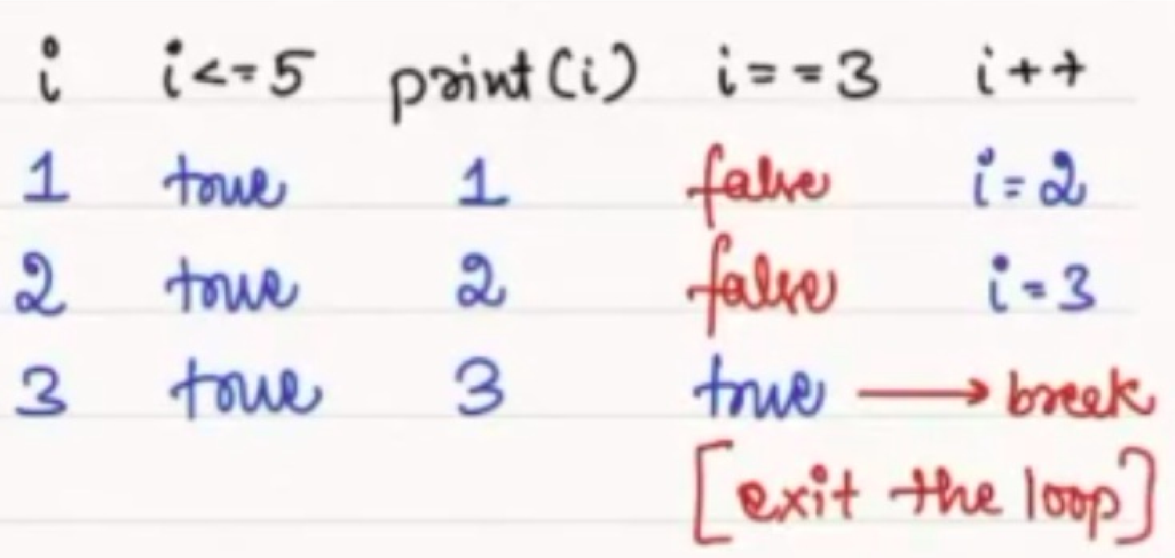
What is the output of the following code?

```
for(int i = 1; i <= 5; i ++ ) {
    SOPln(i);
    if (i == 3) {
        break;
    }
}
```

Choices

- ☑ 1 2 3
- □ 1 2 3 4
- □ 1 2
- □ None of the above

Dry Run:



i	i<=5	print(i)	i==3	i++
1	true	1	false	i=2
2	true	2	false	i=3
3	true	3	true →	break
				[exit the loop]

Question

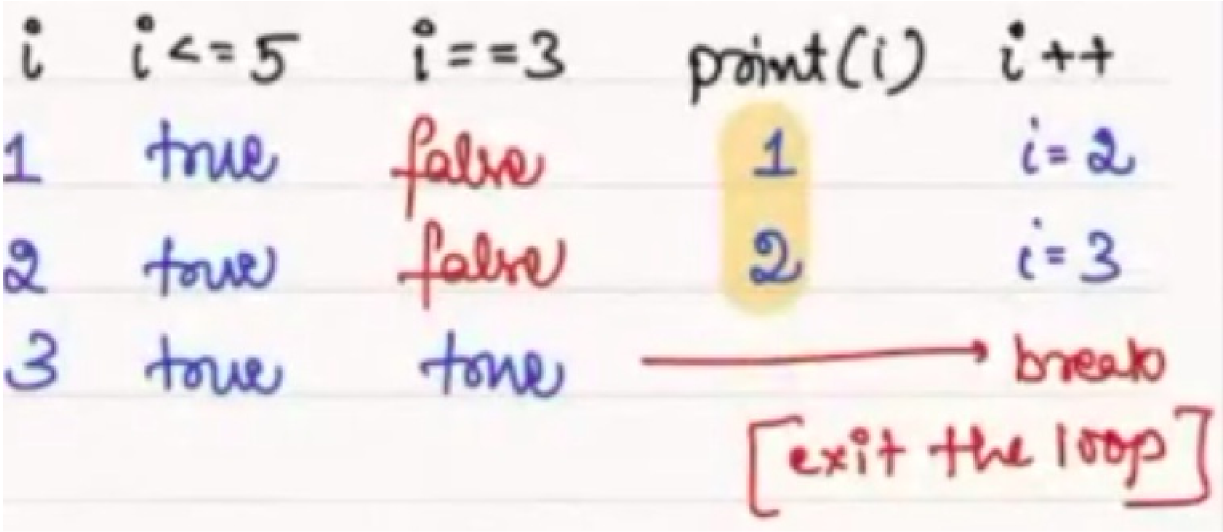
What is the output of the following code?

```
for(int i = 1; i <= 5; i ++ ) {
    if (i == 3) {
        break;
    }
    SOPln(i);
}
```

Choices

- ☑ 1 2
- □ 1 2 3 4
- □ 1 2 4 5
- □ None of the above

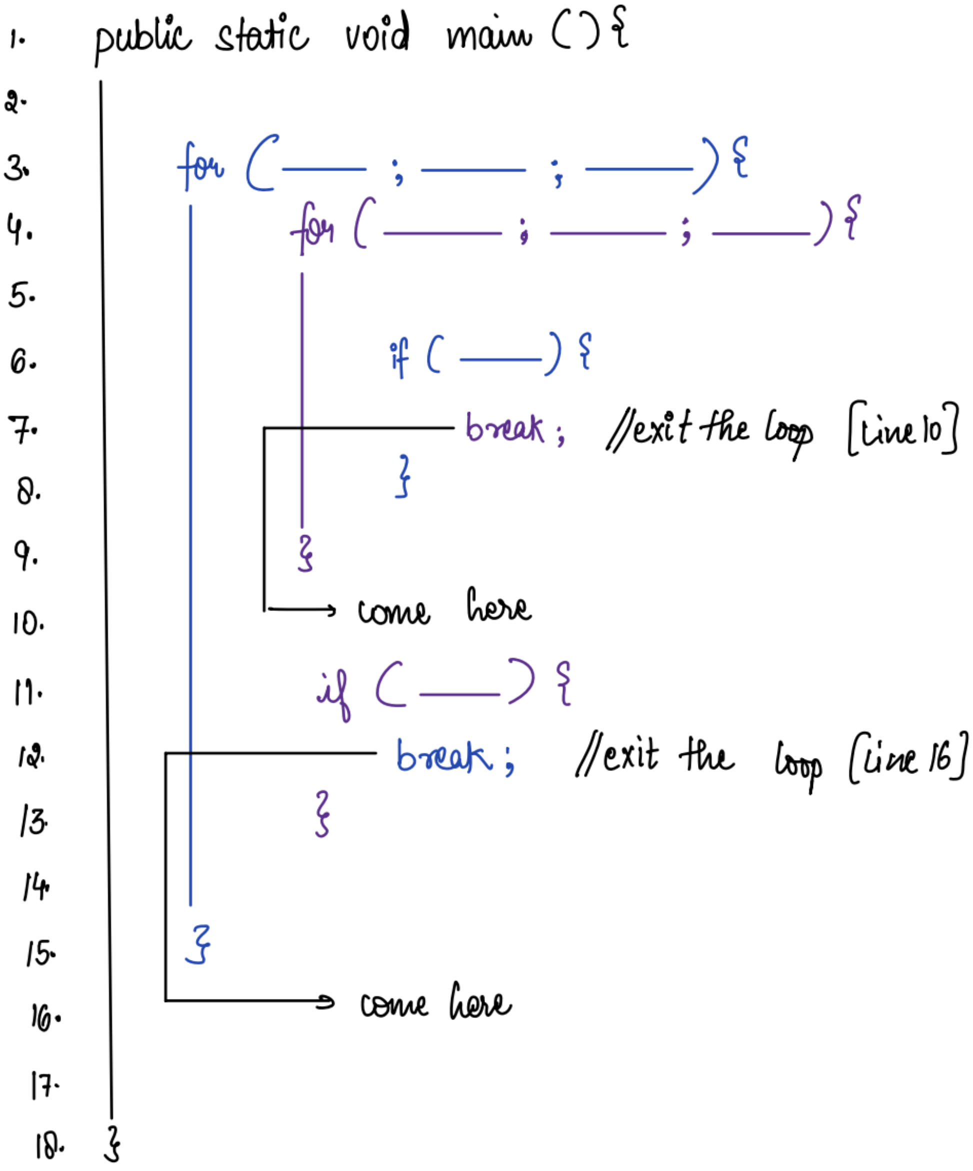
Dry Run:



i	i<=5	i==3	print(i)	i++
1	true	false	1	i=2
2	true	false	2	i=3
3	true	true	→	break
				[exit the loop]

> Explain break statement in case of nested loops.

break in nested loops



```
1.  public static void main (){
2.
3.      for (——— ; ——— ; ———){
4.          for (——— ; ——— ; ———){
5.
6.              if (———){
7.                  break; //exit the loop [line 10]
8.              }
9.          }
10.         come here
11.         if (———){
12.             break; //exit the loop [line 16]
13.         }
14.
15.     }
16.     come here
17.
18. }
```

Given two numbers A and B, find their GCD.

Note: GCD of A and B means the greatest **positive** integer that divides both A and B.

Examples:

```
 A = 24
B = 36

Factors of A = 1, 2, 3, 4, 6, 8, 12, 24
Factors of B = 1, 2, 3, 4, 6, 9, 12, 18, 36
Common Factors: 1, 2, 3, 4, 6, 12

GCD = 12

A = 5
B = 10

Factors of A = 1, 5
Factors of B = 1, 2, 5, 10
Common Factors: 1, 5

GCD = 5

A = 12
B = 18

Factors of A = 1, 2, 3, 4, 6, 12
Factors of B = 1, 2, 3, 6, 9, 18
Common Factors: 1, 2, 3, 6

GCD = 6
```

> Take more examples if required.

Using examples, explain the following observations:

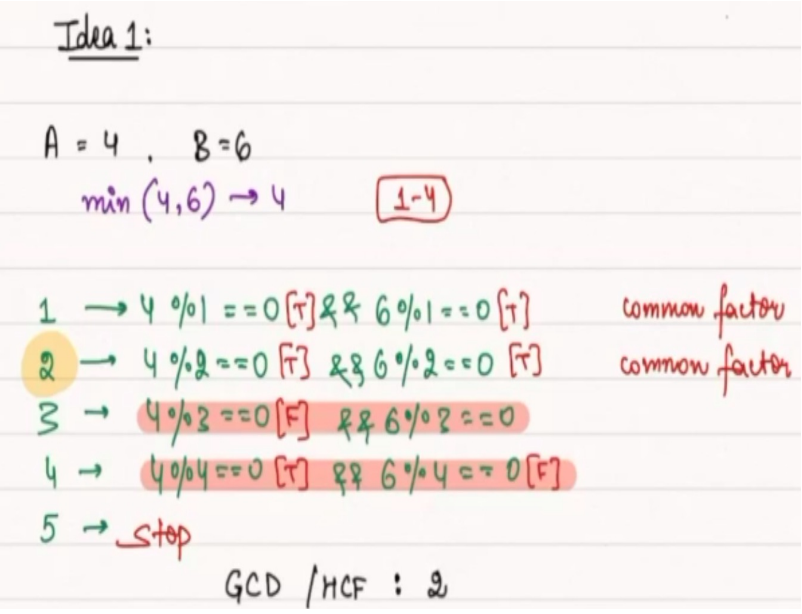
Observation 1

Minimum number that can possibly divide A and B = 1. Maximum number that can possibly divide A and B = MIN(A, B).

Observation 2

The GCD lies in the range of 1 to MIN(A, B).

Idea 1



Idea 1:

A = 4 , B = 6

min (4,6) → 4   1-4

1 → 4 %1 ==0 [T] && 6 %1 ==0 [T]   common factor

2 → 4 %2 ==0 [T] && 6 %2 ==0 [T]   common factor

3 → 4 %3 ==0 [F] && 6 %3 ==0

4 → 4 %4 ==0 [T] && 6 %4 == 0 [F]

5 → stop

GCD / HCF : 2

Check all possible candidates from 1 to MIN(A, B).

Code:

```
 int A = scn.nextInt();
int B = scn.nextInt();

int min = 0;
if (A < B) {
    min = A;
}
else min = B;

int gcd = 0;
for(int i = 1; i <= min; i ++ ) {
    if (A % i == 0 && B % i == 0) {
        gcd = i;
    }
}

SOPln(gcd);
```

> Show dry run with A = 8, B = 6.

Question

What is the HCF/GCD of 7 and 12?

Choices

- ☑ 1
- □ 2
- □ 7
- □ None of the above

Question

What is the HCF/GCD of 10 and 15?

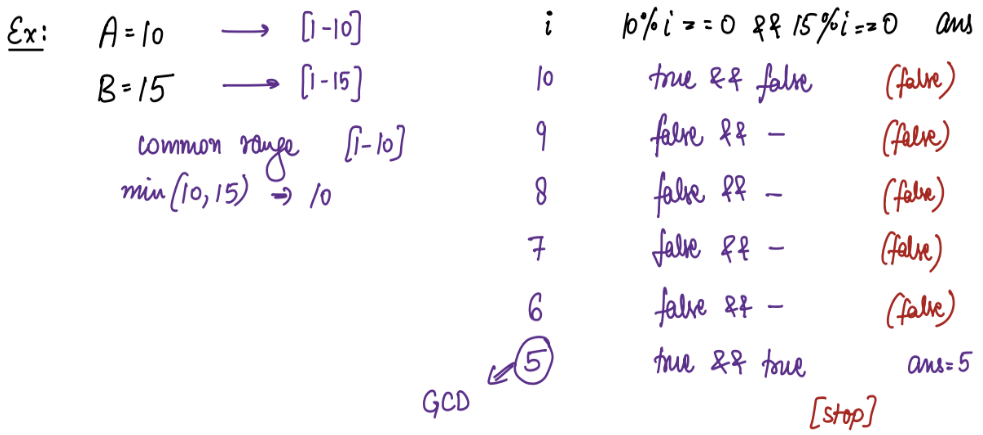
Choices

- □ 10
- □ 1
- □ 3
- ☑ 5

Idea 2 for Calculating GCD

Go from MIN(A, B) to 1 and whenever you find first factor of both A and B, break and print that factor.

Dry run for A=10 and B=15



Ex: A=10 → [1-10]

B=15 → [1-15]

common range [1-10]

min(10,15) → 10

i	10%i==0 && 15%i==0	ans
10	true && false	(false)
9	false && –	(false)
8	false && –	(false)
7	false && –	(false)
6	false && –	(false)
5	true && true	ans=5

[stop]

GCD

> Show examples.

```
 public static void main(String[] args) {
    Scanner sc = new Scanner(System.in);
    int a = sc.nextInt();
    int b = sc.nextInt();

    int min = 0;
    if(a < b){
        min = a;
    }
    else{
        min = b;
    }

    int gcd = 0;
    for (int i = min; i >= 1; i -- ) {
        if (a % i == 0 && b % i == 0) {
            gcd = i;
            break;
        }
    }

    SOPln(gcd);
}
```

Question: Given two numbers, find their LCM.

```
 A = 4 -> 4, 8, 12, 16, 20, 24, 28, ...
B = 5 -> 5, 10, 15, 20, 25, 30, ...

LCM = 20
```

```
 A = 6 -> 6, 12, 18, 24, 30, 36, 42, ...
B = 7 -> 7, 14, 21, 28, 35, 42, ...

LCM = 42
```

```
 A = 2 -> 2, 4, 6, 8, 10, 12, 14, ...
B = 9 -> 9, 18, 27, 36, 45, 54, ...

LCM = 18
```

> Show more examples if required.

To build intuition, ask the following questions.

Question 1

Minimum number that can be divisible by A and B?

Answer: MAX(A, B)

Explanation: Let X < MAX(A, B). Then either X < A or X < B. It implies either X is not divisible by A or it is not divisible by B. Hence, it means X is not divisible by at least one of A and B. Thus, any number less than MAX(A, B) cannot be divisible by both.

Maximum number that must be divisible by A and B

Answer: A * B

Explanation: A * B is divisible by both A and B clearly. Also, it is possible that any number till A * B is not divisible by both A and B. However, as we touch the minimum level of A * B, it is divisible by both A and B.

Question 2

What is the range of LCM of two numbers A and B?

Expected: MAX(A, B) to A * B

```
 int A = scn.nextInt();
int B = scn.nextInt();

int max = 0;
if (A > B) {
    max = A;
}
else{
    max = B;
}

int lcm = 0;
for(int i = max; i <= A * B; i ++ ) {
    if (i % A == 0 && i % B == 0) {
        lcm = i;
        break;
    }
}

SOPln(lcm);
```

Question

What is the LCM of 6 and 9?

Choices

- ☑ 18
- □ 9
- □ 6
- □ 1

Relation between GCD and LCM

```
 GCD * LCM = A * B
```

Question

How many iterations will be there in this loop

```
for (i = 1; i <= 100; i++) {
  S = S + i;
}
```

Choices

- ☐ 99
- ☑ 100
- ☐ 98
- ☐ 101

Explanation:

The loop will iterate 100 times. Starting from i = 1, the loop will continue as long as i is less than or equal to 100, incrementing i by 1 in each iteration. Therefore, the loop will execute a total of 100 iterations.

Question

How many iterations will be there in this loop

```
for (i = 3; i <= 50; i++) {
  S = S + i;
}
```

Choices

- ☐ 47
- ☑ 48
- ☐ 49
- ☐ 50

Explanation:

The loop will iterate 48 times. Starting from i=3, the loop will continue as long as i is less than or equal to 50, incrementing i by 1 in each iteration. Therefore, the loop will execute a total of 48 iterations as 50 - 3 + 1 = 48.

Question

How many iterations will be there in this loop

```
for (i = 1; i <= N; i++) {
  S = S + i;
}
```

Choices

- ☐ N^2
- ☑ N
- ☐ N / 2
- ☐ logN

Explanation:

The number of iterations in this loop depends on the value of N. If N is a positive integer, the loop will iterate N times. Starting from i = 1, the loop will continue as long as i is less than or equal to N, incrementing i by 1 in each iteration.

Question

How many iterations will be there in this loop Given N > 0

```
for (i = 0; i < N; i++) {
  S = S + i;
}
```

Choices

- ☐ N - 1
- ☑ N
- ☐ N / 2
- ☐ logN

Explanation:

The loop will iterate N times. Starting from i = 0, the loop will continue as long as i is less than N, incrementing i by 1 in each iteration.

Question

How many iterations are made by the code below?

```
for (i = 1; i <= N; i++) {
  print(i);
}
for (j = 1; j <= M; j++) {
  print(j);
}
```

Choices

- ☐ N
- ☐ M
- ☐ N * M
- ☑ N + M
- ☐ 2N

Explanation:

The code consists of two separate loops. The first loop iterates N times, and the second loop iterates M times. Therefore, the total number of iterations made by the code is N + M.

Question

How many iterations will the following code make?

```
for (i = 1; i <= 2^N; i++) {
  System.out.println("Hi");
}
```

Choices

- ☐ NlogN
- ☑ 2^N
- ☐ N^2
- ☐ N
- ☐ Infinite

Explanation:

The code will make 2^N iterations.

Question

How many iterations will be there in this loop

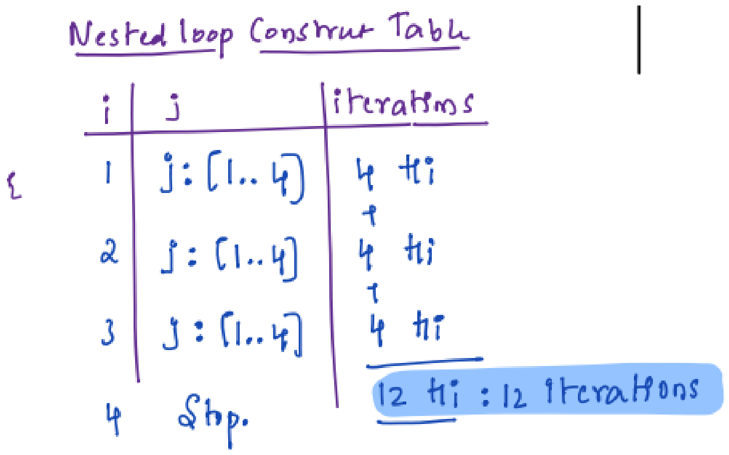
```
for (i = 1; i <= 3; i++) {
  for (j = 1; j <= 4; j++) {
    print("Hi");
  }
}
```

Choices

- ☐ 10
- ☐ 11
- ☑ 12
- ☐ 13

Explanation:

The number of iterations in the loop will be 12. This is because the outer loop will run 3 times, and the inner loop will run 4 times each time the outer loop runs. So, the total number of iterations is 3 * 4 = 12.



Nested loop Construct Table

i	j	iterations
1	j : [1..4]	4 hi +
2	j : [1..4]	4 hi +
3	j : [1..4]	4 hi
4	Stop.	12 hi : 12 iterations

Here is a breakdown of the loop:

- The outer loop runs 3 times, from i = 1 to i = 3.
- Each time the outer loop runs, the inner loop runs 4 times, from j = 1 to j = 4.

- So, the total number of iterations is 3 * 4 = 12.

Question

How many iterations will be there in this loop

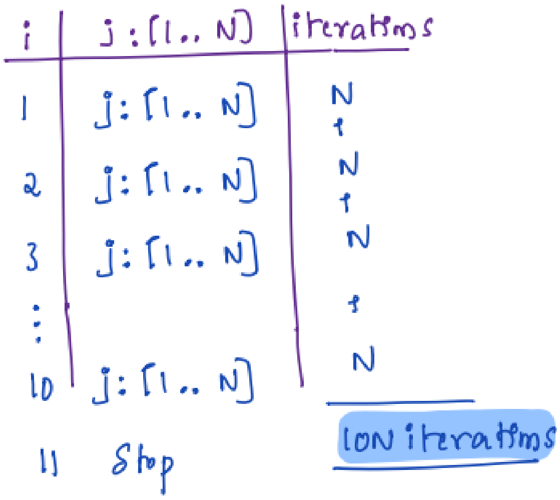
```
for (i = 1; i <= 10; i++) {
  for (j = 1; j <= N; j++) {
    print("Hi");
  }
}
```

Choices

- ☐ N
- ☑ 10N
- ☐ 10logN
- ☐ 10sqrt(N)

Explanation:

The number of iterations in the loop will be 10 * N, where N is the upper bound of the inner loop. This is because the outer loop will run 10 times, and the inner loop will run N times each time the outer loop runs. So, the total number of iterations is 10 * N.



i	j : [1.. N]	iterations
1	j: [1.. N]	N +
2	j: [1.. N]	N +
3	j: [1.. N]	N
⋮		+
10	j: [1.. N]	N
11	Stop	10N iterations

Here is a breakdown of the loop:

- The outer loop runs 10 times, from i = 1 to i = 10.
- Each time the outer loop runs, the inner loop runs N times, from j = 1 to j = N.
- So, the total number of iterations is 10 * N.

Question

How many iterations will be there in this loop

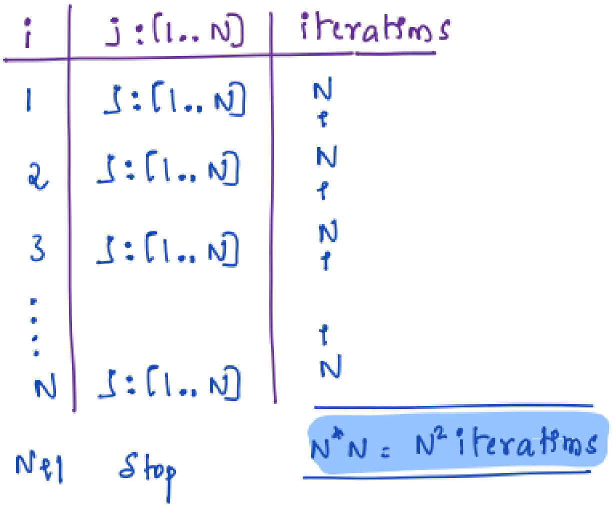
```
for (i = 1; i <= N; i++) {
  for (j = 1; j <= N; j++) {
    print("Hi");
  }
}
```

Choices

- ☐ 2N
- ☑ N * N
- ☐ logN
- ☐ sqrt(N)

Explanation:

The number of iterations in the loop will be N^2, where N is the upper bound of both loops. This is because the outer loop will run N times, and the inner loop will run N times each time the outer loop runs. So, the total number of iterations is $N * N = N^2$.



i	j:[1..N]	iterations
1	j:[1..N]	N +
2	j:[1..N]	N +
3	j:[1..N]	N +
⋮		+ N
N	j:[1..N]	
N+1	Stop	N*N = N² iterations

Here is a breakdown of the loop:

- The outer loop runs N times, from i = 1 to i = N.
- Each time the outer loop runs, the inner loop runs N times, from j = 1 to j = N.
- So, the total number of iterations is $N * N = N^2$.

Question

How many iterations will be there in this loop

```
for (i = 0; i < N; i++) {
  for (j = 0; j <= i; j++) {
    Print("HI");
  }
}
```

Choices

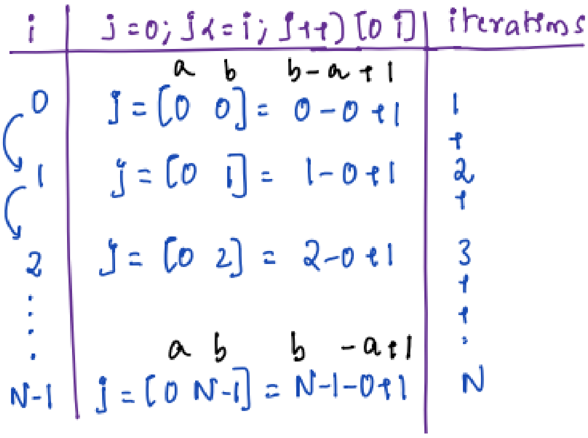
- ☐ N * N
- ☐ Infinite
- ☑ N(N + 1) / 2
- ☐ None of them

Explanation:

The number of iterations in the loop will be $\frac{N(N+1)}{2}$, where N is the upper bound of the outer loop. This is because the inner loop will run from j = 0 to j = i, and each time the inner loop runs, it prints "HI", so the number of times the inner loop runs is i + 1. So, the total number of iterations is N * (i + 1), which can be simplified to

$$\frac{N(N+1)}{2}$$

.



i	j=0; j<=i; j++) [0 i]	iterations
	a b b-a+1	
0	j=[0 0]= 0-0+1	1
		+
1	j=[0 1]= 1-0+1	2
		+
2	j=[0 2] = 2-0+1	3
⋮		+
		+
	a b b -a+1	:
N-1	j=[0 N-1] = N-1-0+1	N

Here is a breakdown of the loop:

- The outer loop runs N times, from i = 0 to i = N - 1.
- Each time the outer loop runs, the inner loop runs i+1 times, from j = 0 to j = i.
- So, the total number of iterations is N * (i + 1).

Example

How many iterations will this make?

```
void func(int N){
  for(int i = 1; i < n; i++){
    for(int j = 1;j <= i; j++){
      print("Hi");
    }
  }
}
```

The code you provided will print "Hi" a total of $\frac{N(N+1)}{2}$ times, where N is the number passed to the func() function. For example, if N is 3, then the code will print "Hi" 9 times.

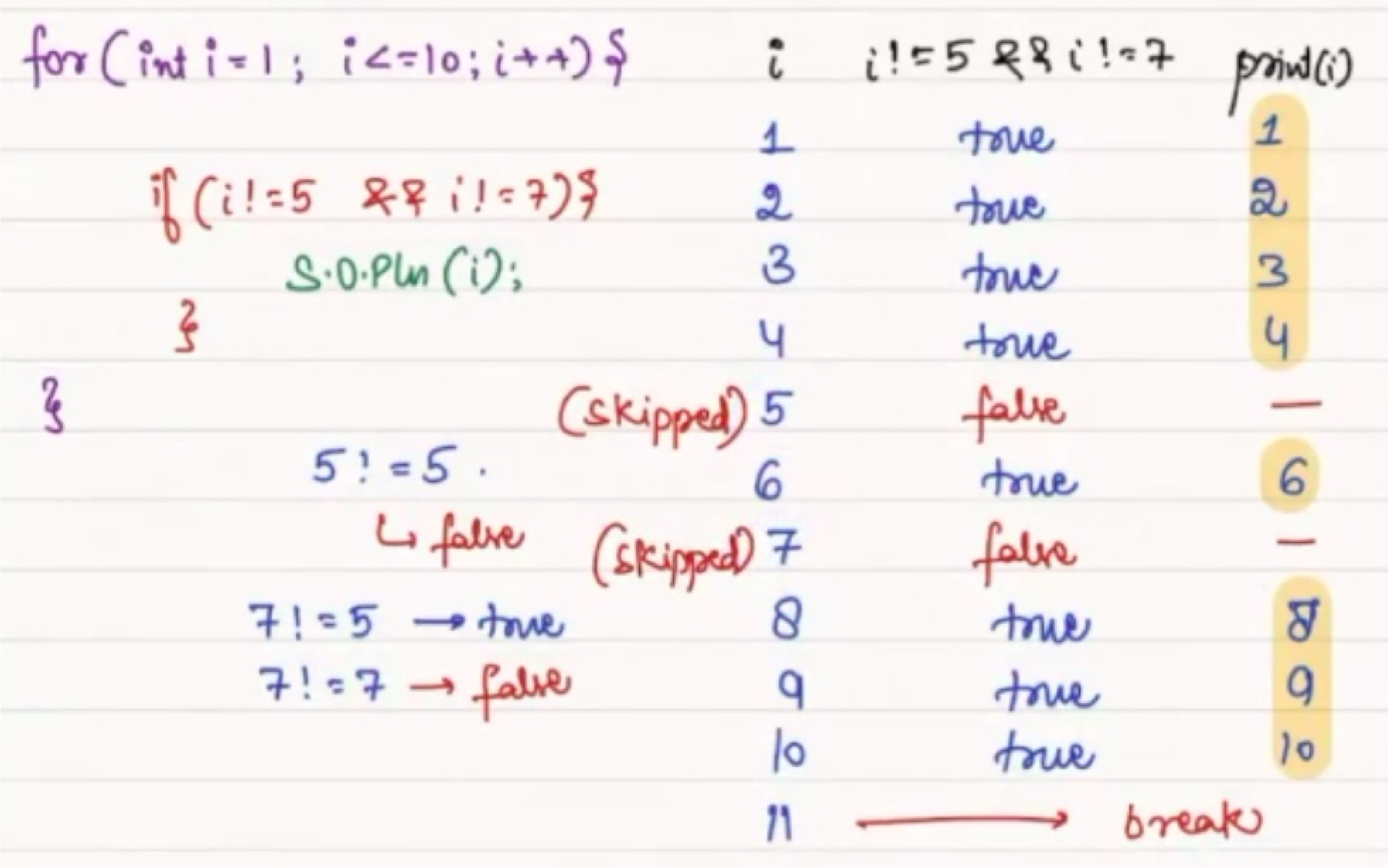
Here is a table of the iterations for N = 3:

i	j	Iterations
1	1	1
1	2	2
2	1	3
2	2	4

Build intuition for continue Statement

Print all numbers from 1 to 10 except 5 and 7.

```
for(int i = 1; i <= 10; i ++ ) {
     if (i != 5 && i != 7) {
         SOPln(i);
     }
}
```



```
for(int i=1; i<=10; i++){

    if(i!=5 && i!=7){
        S.O.Pln(i);
    }
}
```

5!=5.
↳ false

7!=5 → true
7!=7 → false

i	i!=5 && i!=7	print(i)
1	true	1
2	true	2
3	true	3
4	true	4
(skipped) 5	false	—
6	true	6
(skipped) 7	false	—
8	true	8
9	true	9
10	true	10
11	⟶	break

Observation: In the above code, we are effectively skipping two iterations of the for loop.

For this purpose, we have a statement called the continue statement.

Same example using continue:

```
for(int i = 1; i <= 10; i ++ ) {
    if (i == 5 || i == 7) {
        continue;
    }
    SOPln(i);
}
```

Question

Determine the output of the following code:

```
for(int i = 0; i <= 5; i ++ ){
    if(i == 3){
        continue;
    }
    System.out.println(i + " ");
}
```

Choices

- ☐ 0 1 2 3 4 5
- ☐ 0 1 2
- ☑ 0 1 2 4 5

Explanation:

i	i<=5	i==3	println(i)	i++
0	true	false	0	i=1
1	true	false	1	i=2
2	true	false	2	i=3
3	true	true	→	i=4
4	true	false	4	i=5
5	true	false	5	i=6
6	false	→	stop	

Question

Determine the output of the following code:

```
public static void main(String args[]) {
    for(int i = 1; i <= 10; i ++ ) {
        if(i == 4 && i == 6) {
            continue;
        }
        System.out.println(i);
    }
}
```

Choices

- ☐ Print all numbers except 4 and 6
- ☑ Print all numbers
- ☐ Error