Department of Computer Science and Engineering
Islamic University of Technology (IUT)
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Analysis Report on Multi Server Queuing System (MSQS)

CSE 4550: Simulation Modeling and Performance Evaluation

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Section:	**2B**
Semester:	**5th (Winter)**
Academic Year:	**2021 – 2022**

Date of Submission: 22th October, 2022

TITLE

Simulation Modeling and Performance Evaluation of a Multi Server Queueing System (SSQS).

PROBLEM STATEMENT

A multi-server queueing system can be considered as an extension of the Simple Server Queuing System (SSQS) model but with multiple Servers. In this experiment, we will be analyzing three different MSQS scenarios –

Scenario 1

The first scenario consists of a single queue and two servers. Upon arrival of a customer, the customer has to wait in a queue, and whenever a server is idle, the customer standing in the front of the queue can enter the server and take service. After completing service, the customer leaves the server and the customer standing in front of the queue enters the server.

So, in short, the conditions for the first scenario are –

- *i)* *One queue, Two Servers*
- *ii)* *Customer joins the queue upon arrival, when one of the servers is idle the customer at the front of the queue enters the server, after taking service it departs and the process continues.*

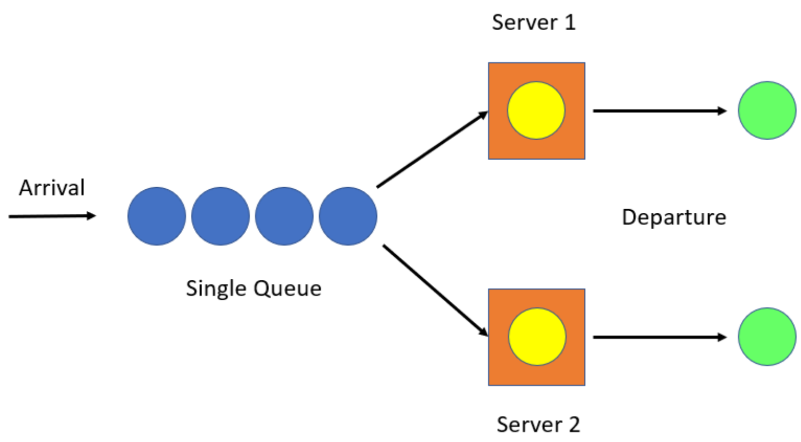

Fig : Scenario 1 (One queue, Two Servers)

Scenario 2

The second scenario consists of two queues and two servers. Each server has its own queue. Upon arrival of a customer, the customer chooses the shortest queue and joins the queue, and whenever a server is idle, the customer standing in the front of the queue can enter the server and take service. After completing service, the customer leaves the server and the customer standing in front of the queue enters the server.

So, in short, the conditions for the second scenario are –

- *i)* *Two queues, Two Servers (each queue associated with a server)*
- *ii)* *Customer joins the shortest queue upon arrival, when one of the servers is idle the customer at the front of the queue enters the server, after taking service it departs and the process continues.*
- *iii)* *No jockeying or switching queues is allowed.*

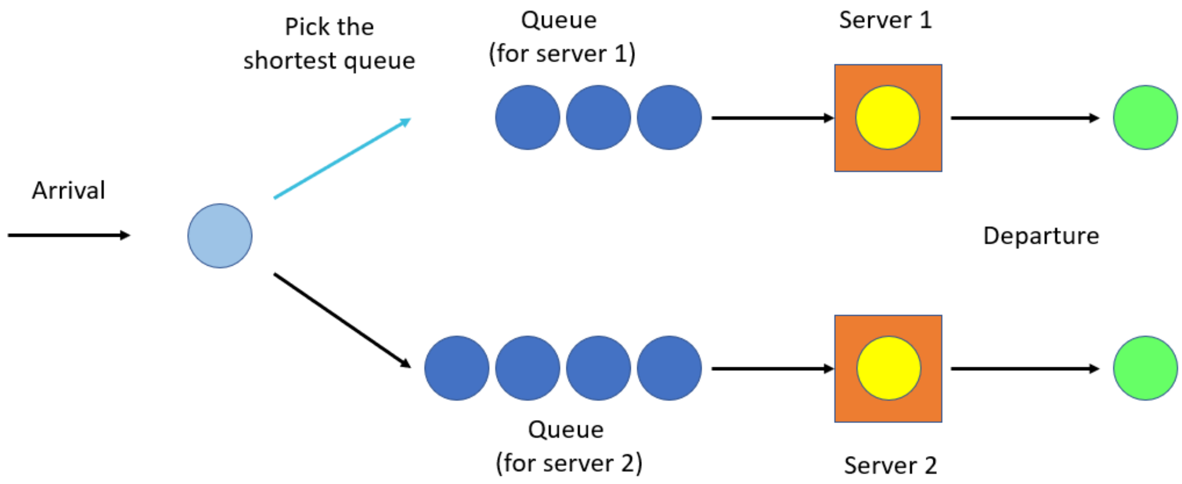

Fig : Scenario 2 (Two queues, Two Servers)

Scenario 3

The third scenario is similar to that of the second, it consists of two queues and two servers. Each server has its own queue. Upon arrival of a customer, the customer chooses the shortest queue and joins the queue, and whenever a server is idle, the customer standing in the front of the queue can enter the server and take service. After completing service, the customer leaves the server and the customer standing in front of the queue enters the server. But in this case the customers waiting in the queue can switch queues if the other one is shorter, so jockeying is allowed.

So, in short, the conditions for the third scenario are –

- *i)* *Two queues, Two Servers (each queue associated with a server)*
- *ii)* *Customer joins the shortest queue upon arrival, when one of the servers is idle the customer at the front of the queue enters the server, after taking service it departs and the process continues.*
- *iii)* *Jockeying or switching queues is allowed.*

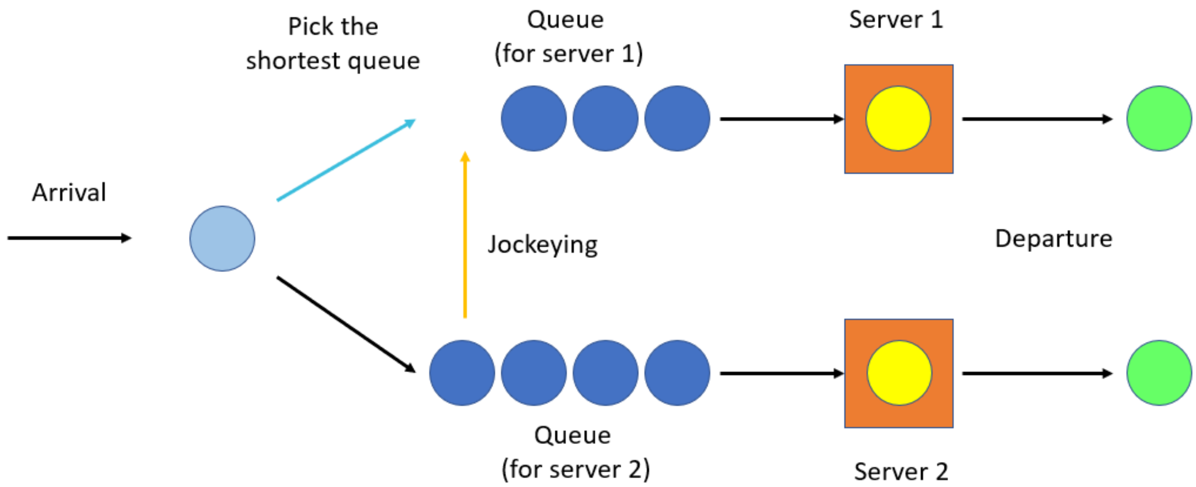

Fig : Scenario 3 (Two queues, Two Servers, Jockeying allowed)

GOALS AND OBJECTIVES

The goals and objectives of this simulation are –

1) Determine Traffic Intensity
2) Determine Queuing Delay
3) Determine Average Queue Length
4) Determine Average System Delay

STATE VARIABLES

The state variables of the Single Server Queuing System are –
Queue Length at time t, q(t) and
Status of the server at time t, x(t)

STATE EQUATIONS

$$x(t^+) = \begin{cases} x(\mathrm{t}) == 0\,?\,1 : x(t) & \text{if arrival occurs at time } t \\ q(\mathrm{t}) > 0\,?\,\mathrm{x(t)} : 0 & \text{if departure occurs at time } t \\ x(t) & \text{otherwise} \end{cases}$$

$$q(t^+) = \begin{cases} x(t) > 0\,?\,q(t)+1 : q(t) & \text{if arrival occurs at time } t \\ \max(0, q(t)-1) & \text{if departure occurs at time } t \\ q(t) & \text{otherwise} \end{cases}$$

STATE SPACE

For Server Status, X_x = {0, 1},
For Queue Length, X_q ={ 0,1,… }

X = {{0,0}, {1,0}, {1,1}, {1,2}, {1,3},.. }

OUTPUT VARIABLES

The output variables are –

i) Traffic Intensity
ii) Average Queuing Delay
iii) Average System Time or Delay (Average Queuing Delay and Average System Delay)
iv) Average Queue Length

OUTPUT EQUATIONS

Traffic Intensity, $T_i = \frac{Arrival\ Mean}{Departure\ Mean}$

Average service time, $\bar{s} = \frac{1}{n}\sum_{i=1}^{n} s_i$

Average queue delay, $\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$

Average System Time, $\bar{w} = \frac{1}{n}\sum_{i=1}^{n} w_i$

$$= \frac{1}{n}\sum_{i=1}^{n}(d_i + s_i)$$
$$= \bar{d} + \bar{s}$$

Average Queue Length, $\bar{q} = \frac{Queue\ Product}{Last\ Event\ Time}$ [where Queue Product $= \sum Queue\ Length \times Duration\ of\ last\ event$]

EVENTS

i) Arrival, a

ii) Departure, d

iii) Jockeying, j (for scenario 3)

CODE

Scenario 1 : https://github.com/Aashnan-Rahman/CSE-4550---Simulation-Lab/Lab_03/MSQS/Scenario_1
Scenario 2 : https://github.com/Aashnan-Rahman/CSE-4550---Simulation-Lab/Lab_03/MSQS/Scenario_2
Scenario 3 : https://github.com/Aashnan-Rahman/CSE-4550---Simulation-Lab/Lab_03/MSQS/Scenario_3

OUTPUT DATA

For Scenario 1,

Arrival Mean	Traffic Intensity	Average Queue Delay	Average Queue Length	Average Service Delay
5	0.5	33.3128	6.30545	42.9968
6	0.6	13.892	2.29007	23.9141
7	0.7	13.1594	1.9114	23.4442
8	0.8	5.58747	0.688667	15.4139
9	0.9	5.50073	0.640744	15.8792
10	1.0	3.58279	0.356297	13.7359

For Scenario 2,

Server 1

Arrival Mean	**Traffic Intensity**	**Average Queue Delay**	**Average Queue Length**	**Average Service Delay**
5	0.5	27.4533	2.32086	37.3922
6	0.6	20.442	1.45983	30.6053
7	0.7	12.8145	0.741542	22.6463
8	0.8	6.92251	0.316656	16.795
9	0.9	5.57177	0.197815	15.3449
10	1.0	5.0277	0.15409	14.7668

Server 2

Arrival Mean	**Traffic Intensity**	**Average Queue Delay**	**Average Queue Length**	**Average Service Delay**
5	0.5	31.769	2.7159	42.0111
6	0.6	24.2485	1.95134	35.3053
7	0.7	15.0962	1.13195	26.0593
8	0.8	9.28331	0.68553	19.7031
9	0.9	7.95919	0.525818	18.6805
10	1.0	6.98427	0.458734	17.9879

For Scenario 3,

Server 1

Arrival Mean	**Traffic Intensity**	**Average Queue Delay**	**Average Queue Length**	**Average Service Delay**
5	0.5	19.3052	1.7709	28.6008
6	0.6	12.8701	1.10969	22.4816
7	0.7	9.51602	0.736547	19.1117
8	0.8	6.03249	0.434747	15.3836
9	0.9	5.33757	0.355141	15.006
10	1.0	4.82926	0.29813	14.7921

Server 2

Arrival Mean	**Traffic Intensity**	**Average Queue Delay**	**Average Queue Length**	**Average Service Delay**
5	0.5	19.2543	1.55471	28.735
6	0.6	13.0945	0.957208	22.77
7	0.7	9.66004	0.58542	19.7441
8	0.8	6.18064	0.326961	16.3796
9	0.9	5.78934	0.265497	16.2003
10	1.0	4.81173	0.199506	14.8547

OUTPUT GRAPHS

For Scenario 1,
The behavior of average queue delay, average service delay and average queue lengths (y-axis) with time (x-axis) were illustrated.

For Scenario 2 and 3,
The comparison of the behavior of average queue delay, average service delay and average queue lengths (y-axis) of the two servers and associated queues with time (x-axis) were shown.

Scenario 1

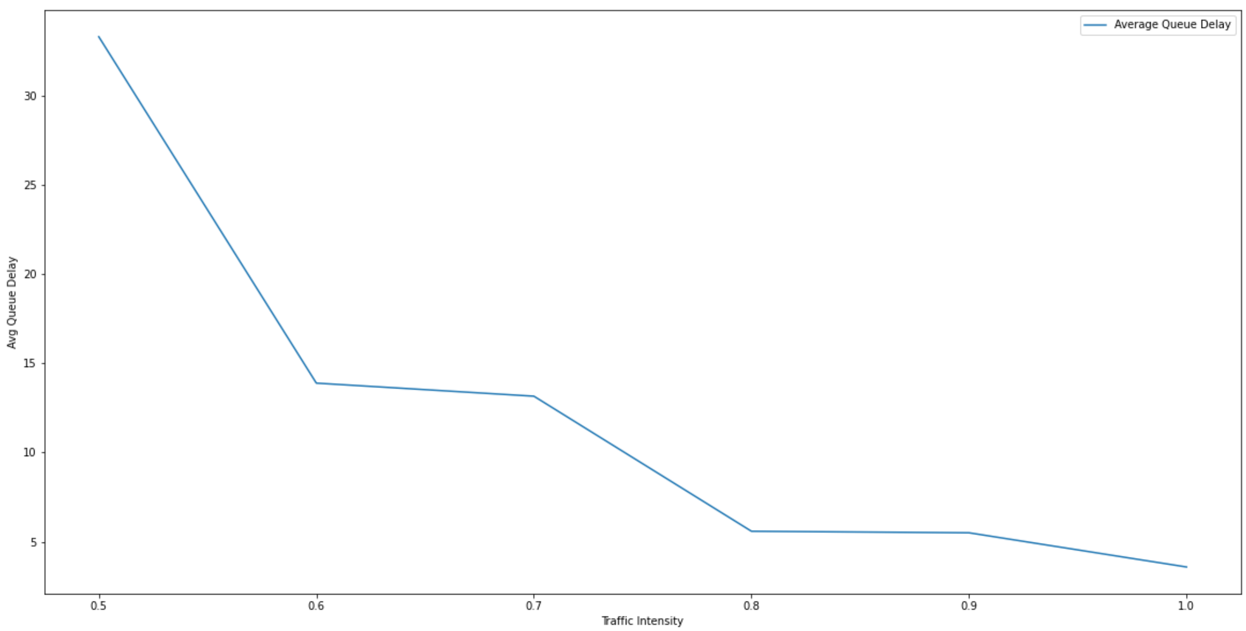

Fig : Average Queue Delay vs Traffic Intensity

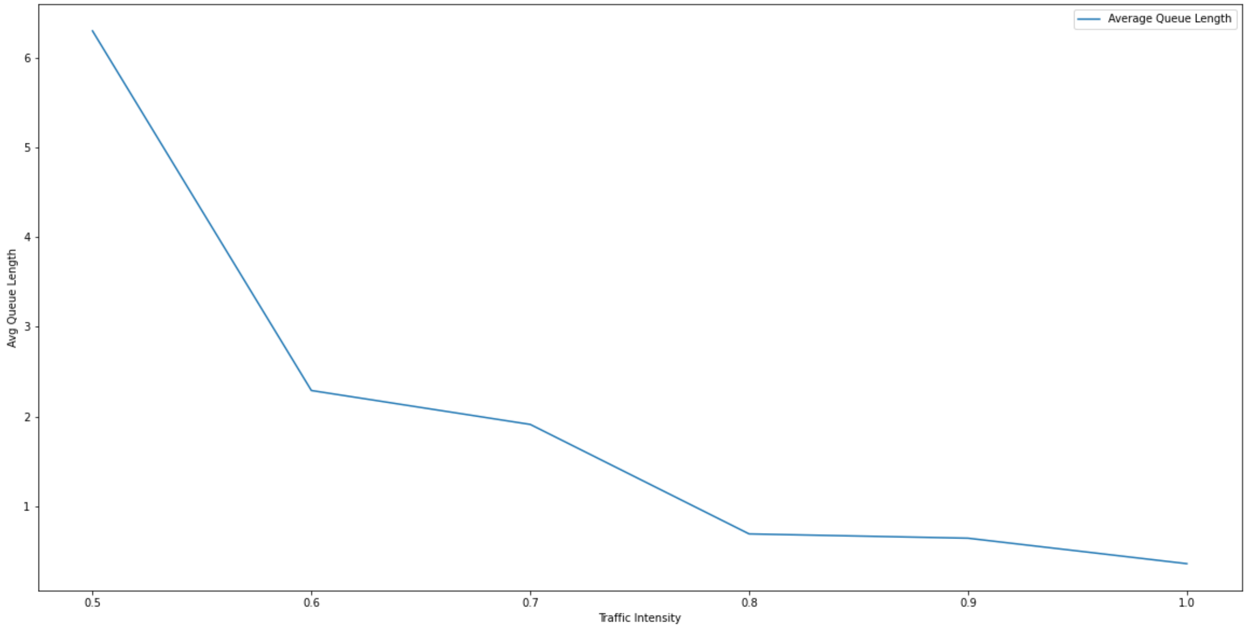

Fig : Average Queue Length vs Traffic Intensity

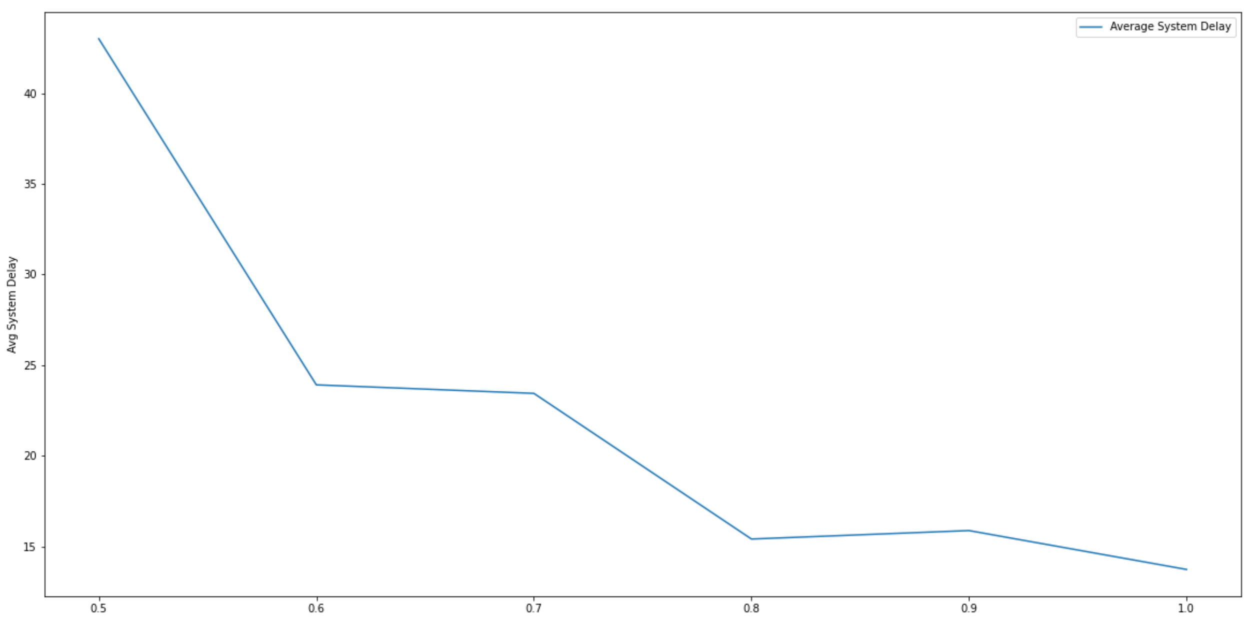

Fig : Average Service Delay vs Traffic Intensity

Scenario 2

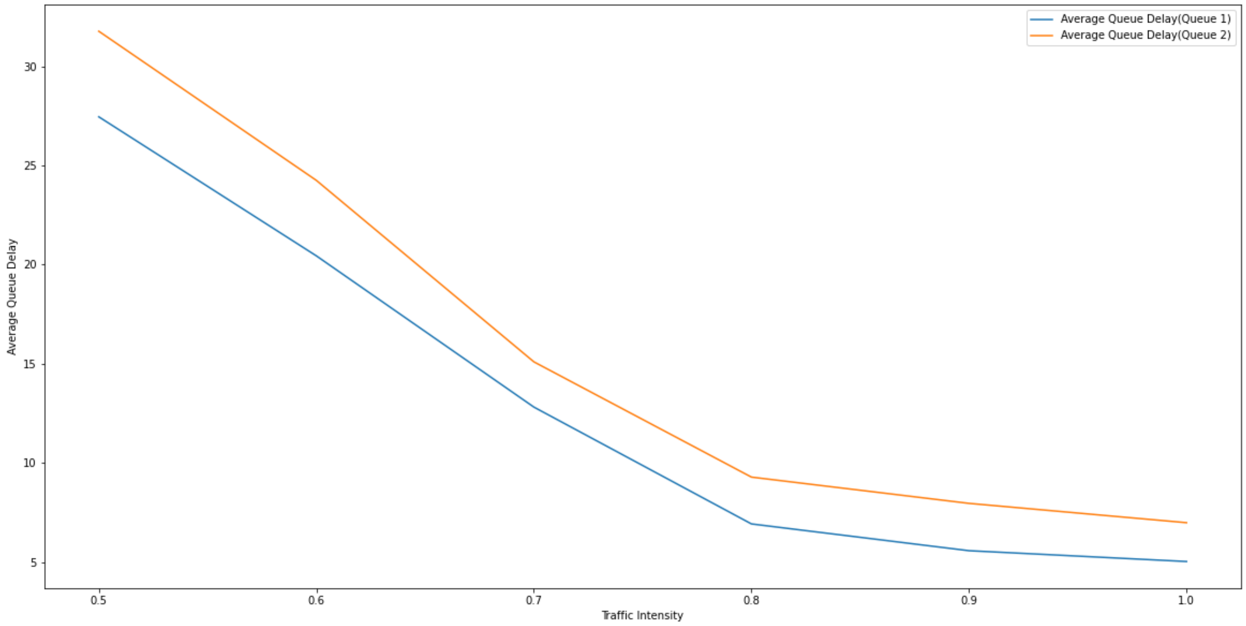

Fig : Comparison of Average Queue Delay vs Traffic Intensity

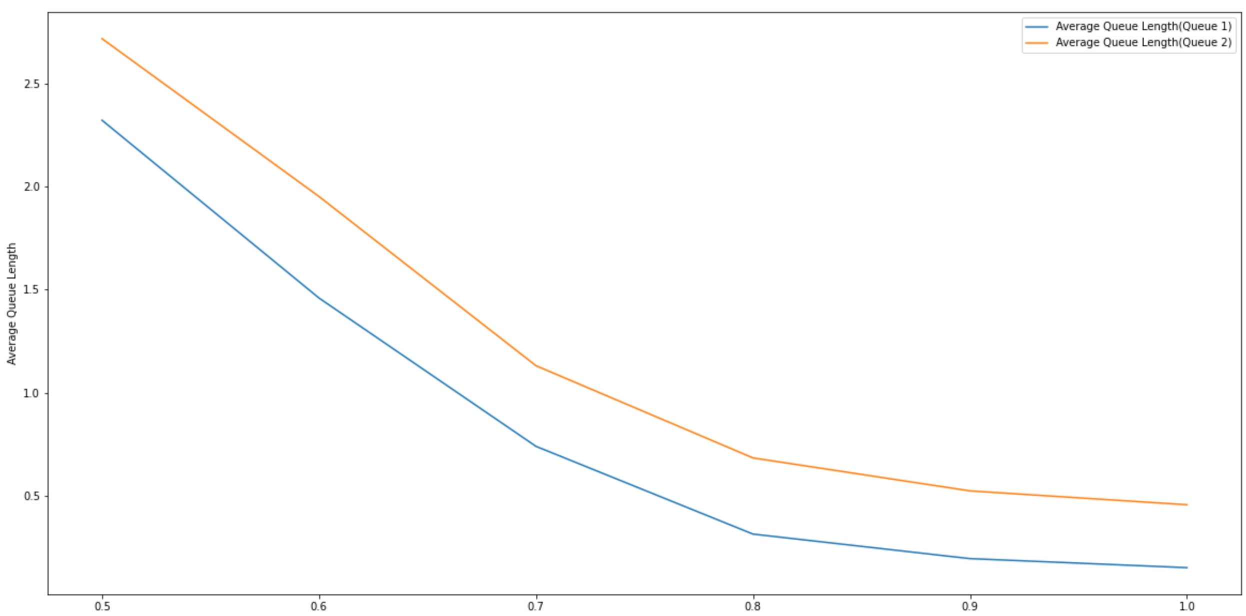

Fig : Comparison of Average Queue Length vs Traffic Intensity

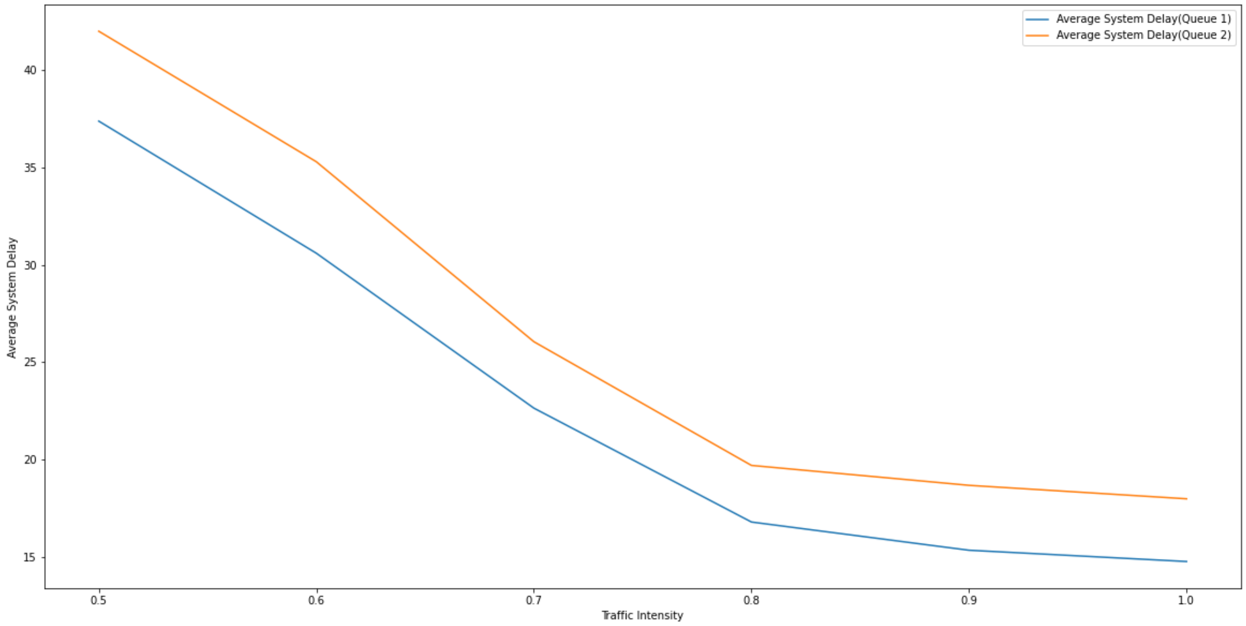

Fig : Comparison of Average Service Delay vs Traffic Intensity

<u>Scenario 3</u>

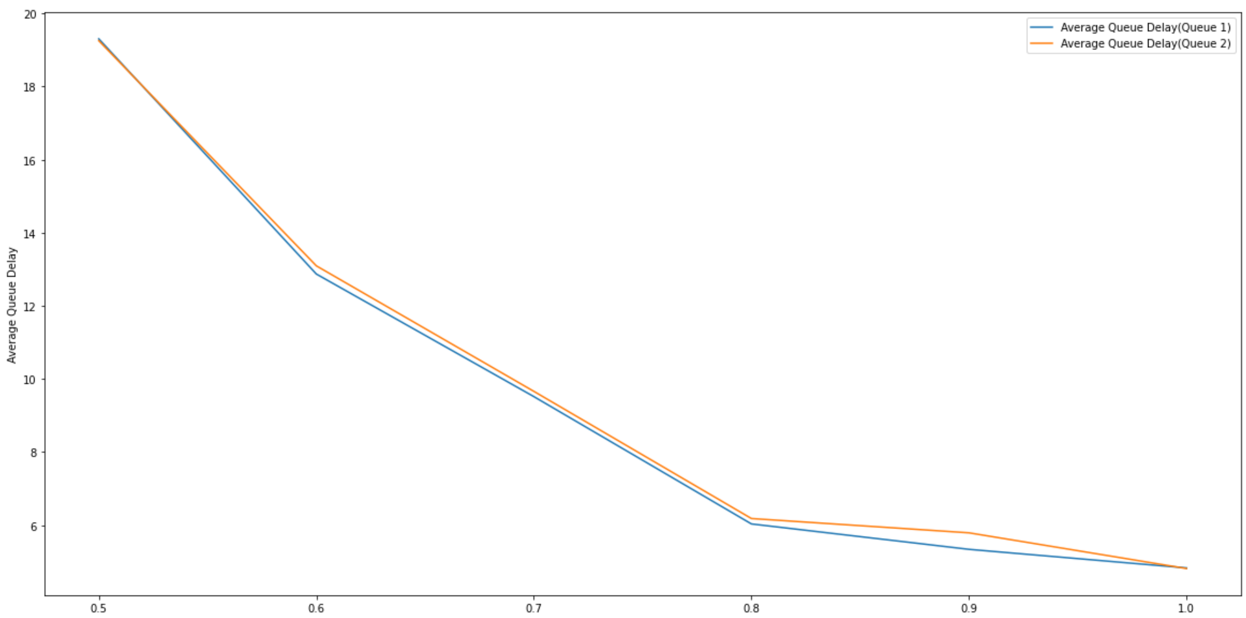

Fig : Comparison of Average Queue Delay vs Traffic Intensity

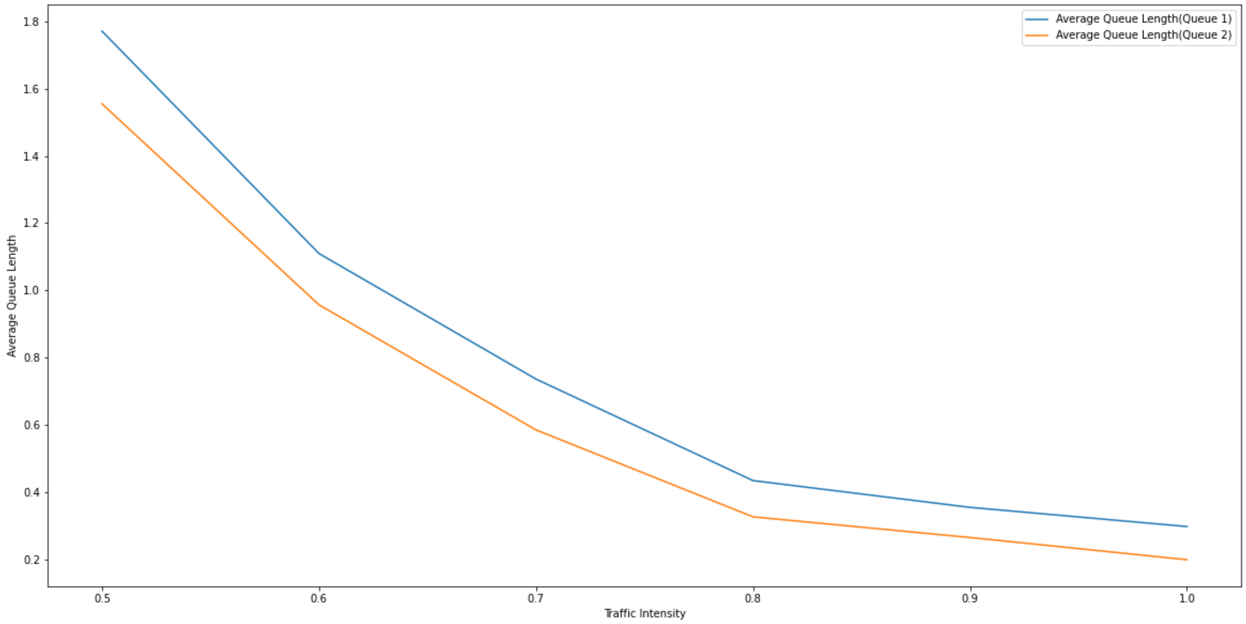

Fig : Comparison of Average Queue Length vs Traffic Intensity

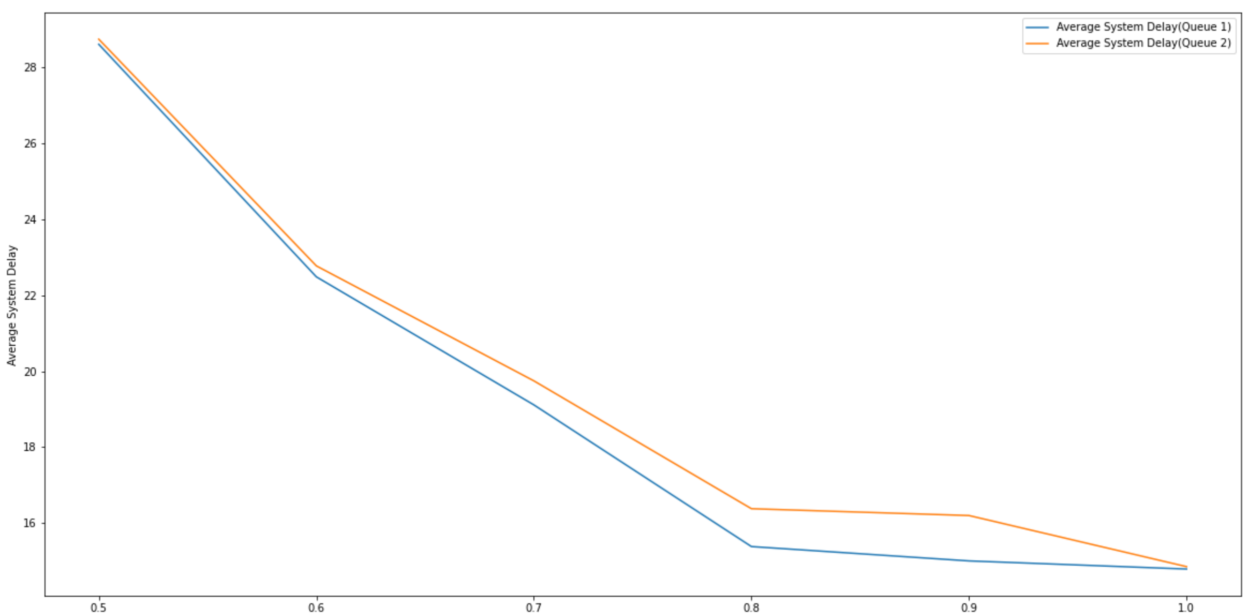

Fig : Comparison of Average Service Length vs Traffic Intensity

[The graphs were generated using matplotlib.pyplot and pandas library of python]

RESULT ANALYSIS

For scenario 1, as traffic intensity increases and reaches to 1, the average service and queuing delay decreases and the average queue length decreases as well. Here we can see an inverse relationship between average service, queuing delay decreases and the average queue length and traffic intensity.

For scenario 2 and 3, as traffic intensity increases and reaches to 1, the average service and queuing delay decreases and the average queue length decreases too. But here we can see a comparison between the behaviors of the two servers. Both the scenarios exhibit similar characteristics as that of scenario 1, but we can see some changes as customer can choose the shortest queue on arrival and for scenario 3, jockeying is allowed. Due to jockeying we can observe some changes in the graphs of scenario 2 and scenario 3.

To conclude, the obtained results and graphs were similar to what was expected thus our the result of our experiment was satisfactory.