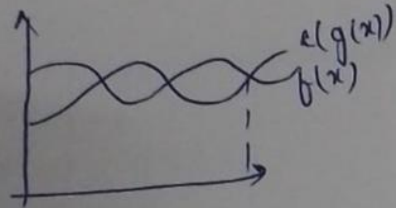
1. Asymptotic notations are used to represent the complexities for algorithms for asymptotic analysis

⇒ These notations are mathematical tools to represent complexities.

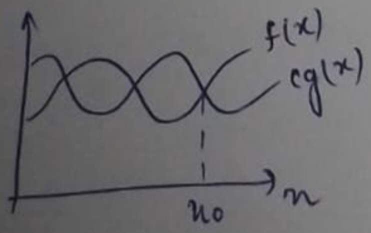
<u>Big O notation</u> ⇒ Gives the upper bound for a $f(x)$ with a const. factor.

$$f(x) = O(g(x))$$
$$\text{if } f(x) \leq O(g(x))$$
$$\text{for } c>0 \text{ and } n \geq n_0.$$



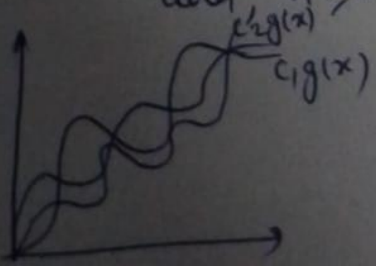
<u>Big Omega Notation</u> ⇒ Gives the lower bound for a f^n $p(x)$ within a const. factor.

$$f(x) = \Omega(g(x))$$
$$\text{if } f(x) \geq g(x)$$
$$f(x) \text{ for } c>0 \text{ and } n \geq n_0$$



<u>Big Theta Notation</u> ⇒ Gives bound for a fun^n $F(x)$ within a const factor.

$$f(x) = O(g(x))$$
$$\text{if } c_1 g(x) = f(x) \leq c_2 g(x)$$
$$c_1 > c_2 > 0$$
$$\text{and } n \geq n_0$$



2. T.C for → P (i = 1 to n)

$i = i + 2$

$i = 1 \quad 2 \quad 4 \quad 8 \; --- \; n$

$2^0 \quad 2^1 \quad 2^2 \quad 2^3 \; --- \; 2^k$

$G.P = a r^{k-1}$

$n = 1 \cdot 2^{k-1}$

$n = \frac{2^k}{2} \Rightarrow 2n = 2^k$

$\log 2n = k \log 2$

$\log 2 + \log n = k \log 2$

$\boxed{\log n = k}$

$\therefore T(n) = O(\log(x))$

3. $T(n) = 3T(n-1), \; n>0$ otherwise

$T(0) = 1$

$T(1) = 3T(0)$

$= 3$

$T(2) = 3T(1)$

$= 9$

$T(3) = 3T(2) = 27$

$T(n) = 3^n$

$= O(3^n)$

4. $T(n) = 2T(n-1) - 1, \; n>0$ otherwise —— (1)

Let $n = n-1$

$T(n-1) = 2T(n-2) - 1$

$= 2T(n-2) - 1$

put $T(n-1)$ in (1)

$T(n) = 4T(n-2) - 3$ —— (2)

put $n = n-2$

$T(n-2) = 2T(n-3) - 1$

put in (2)

$T(n) = 8T(n-3) - 5$

$(n-k) = 1$

$k = n-1$

$T(n) = 2^{n-1} T(n-n+1) - 5 = 2^{n-1} T(1) - 5$

$\boxed{T(n) = O(2^n)}$

5. while (s <= n)
{ i++;
s = s+i;
printf("#");
}

$i = 1 = i{+}{+},\ i = 2$

$s = 3$

$i = 3$

$s = 6$

$i = 4$

$s = 10$

$i = 5$

$s = 15$

$i = 2 \quad 3 \quad 4 \quad 5 \longrightarrow$

$s+1+2 \quad s+1+2+3 \quad s+1+2+3+4$

$s = s+1+2+3+4 - - - k$

$s(k) = k(k+1)/2 \leq n$

$k^2 + \frac{k}{2} \leq n$

$k^2 \leq n \Rightarrow k \leq \sqrt{n}$

$\boxed{T(n) = O(\sqrt{n})}$

6. void funⁿ (int n)
{ int i, count = 0;
for (i=0; i*i <= n; i++)
{ count ++;
}
}

$i = 1 \quad 2 \quad 3 \quad 4 \; - \; -$

$i^2 = 1 \quad 4 \quad 9 \quad 16 - - k^2$

$k^2 \leq n \Rightarrow \boxed{T(n) = O(\sqrt{n})}$

. Void funⁿ (int n)
{ int i, j, k, cnt = 0;
for(i = n/2 ; i <= n; i++) $\rightarrow T(n/2)$
{
for(j=1 ; j <= n; j = j*2) $\rightarrow \log n$
{
for (k=1 ; k <= n; k = k+2) $\rightarrow \log n$
{ count ++;
}

$T(n) = T(n/2) * \log n * \log n$

$= \frac{n}{2} * \log n^2$

$= O(n \log n)$

8. fun"(int n)

```
{ if(n==1)
     return;
  for(i=1 to n) → n
  { for(j=1 to n) → n
    { printf("K");
    }
  }
  fun(n-3)
```

$T(n) = n * n * [T(n-3)] = n^2 + T(n-3)$

$= O(n^2)$

9. void fun"(int n)

```
{ for(i=1 to n)
  { for(j=1; j>=n; j=j+1)
    { print("*");
    }
} }
```

$i=1, j=1,2,3,4 --- n$

$i=2, j=1,3,5,7 --- n/2$

$i=3, j=1,4,7,11 --- n/3$

$n + \frac{n}{2} + \frac{n}{3} --- \frac{n}{n} = \log n$

$T(n) = n * \log n \Rightarrow O(n \log n)$

10. n^K and c^n

$n=1$

$n^K = 1^K . c^n = c$

$n=2$

$n^K = 2^K, c^n c c^2$

$n=K, n^K - K^K, c^n = c^K$

∴ we can say that, for any value of $n>0$

$n^K \geq c^n$

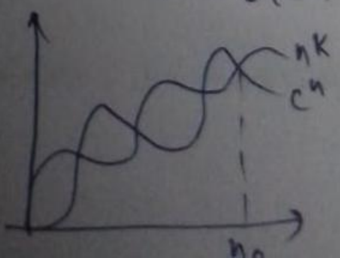
let, $n^K = f(n) . c^n = \log(n)$

$f(x) \geq \log(n)$

$c_0 > 0, n_0 \geq n_0$

∴ $f(x) = O(\log(n))$

$n^K = O(c^n)$

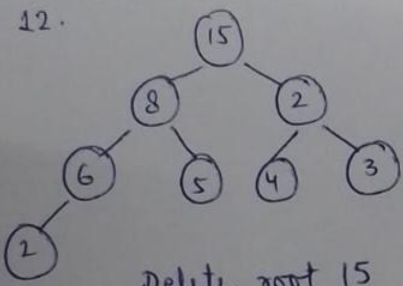


11. Extract Min:-

```
int extract_min( vector<int> &heap)
{ if (heap.empty())
    { return -1;        → O(1)
    }
  swap (heap(0), heap.back()); → O(1)
int minelement = heap.back();
heap.popback(); → O(1)
heapify (heap); → O(log n)
return min_element;
}
```

$T(n) = O(\log(n))$

12.



Delete root 15

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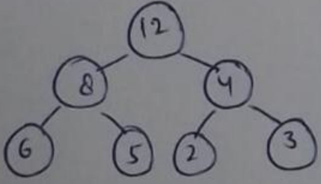

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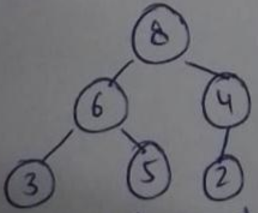
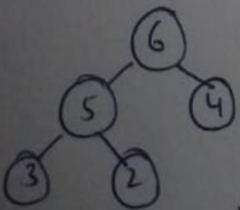


Delete root 8



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