

# Control Systems

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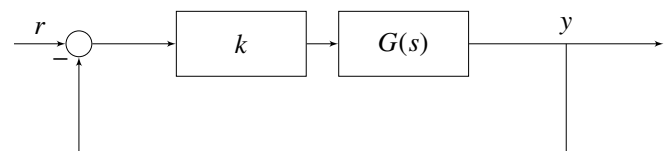


Fig. 3.3.1

**Solution:** The closed loop transfer function is

$$H(s) = \frac{kG(s)}{1 + kG(s)} \quad (3.3.1.2)$$

$$= \frac{k}{s^2 + 2s + k} \quad (3.3.1.3)$$

after substituting from (3.3.1.1).

3.3.2. Find the step response of the system.

**Solution:** The characteristics of the poles of transfer function describes the property of the transfer function.

Calculating poles :

$$s^2 + 2s + k = 0 \quad (3.3.2.1)$$

$$s = \frac{-2 \pm \sqrt{(-2)^2 - 4(1)(k)}}{2(1)} \quad (3.3.2.2)$$

$$s = -1 \pm \sqrt{1 - k} \quad (3.3.2.3)$$

Over Damped System:

For the closed loop transfer function (3.3.1.3) to be Over damped system, poles should be Real and Distinct.

As poles are Real and Distinct  $k$  should be less than 1

Considering  $k=0.5$

$$H(s) = \frac{0.5}{s^2 + 2s + 0.5} \quad (3.3.2.4)$$

after substituting from (3.3.1.3). From (3.3.2.3)

$$s = -1 \pm \sqrt{0.5} \quad (3.3.2.5)$$

Poles are at  $-1 + \frac{1}{\sqrt{2}}, -1 - \frac{1}{\sqrt{2}}$

Calculating Step response

$$X(s) = \frac{1}{s} \quad (3.3.2.6)$$

$$Y(s) = H(s)X(s) \quad (3.3.2.7)$$

$$= \frac{0.5}{s^2 + 2s + 0.5} \frac{1}{s} \quad (3.3.2.8)$$

$y(t)$

$$= \left( 1 - e^{-t} \cosh\left(\frac{t}{\sqrt{2}}\right) - \sqrt{2}e^{-t} \sinh\left(\frac{t}{\sqrt{2}}\right) \right) u(t) \quad (3.3.2.9)$$

Under Damped System:

For the closed loop transfer function (3.3.1.3) to be Under damped system, poles should be

Complex and Conjugate.

As poles are Complex and Conjugate  $k$  should be greater than 1

Considering  $k=2$

$$H(s) = \frac{2}{s^2 + 2s + 2} \quad (3.3.2.10)$$

after substituting from (3.3.1.3). From (3.3.2.3)

$$s = -1 \pm i \quad (3.3.2.11)$$

Poles are at  $-1+i, -1-i$

Calculating Step response

$$X(s) = \frac{1}{s} \quad (3.3.2.12)$$

$$Y(s) = H(s)X(s) \quad (3.3.2.13)$$

$$= \frac{2}{s^2 + 2s + 2} \frac{1}{s} \quad (3.3.2.14)$$

$$y(t) = (1 - e^{-t} \cos t - e^{-t} \sin t) u(t) \quad (3.3.2.15)$$

Critical Damped System:

For the closed loop transfer function (3.3.1.3) to be Critical damped system, poles should be Real at same location.

As poles are Real and equal  $k$  should be 1

$$H(s) = \frac{1}{s^2 + 2s + 1} \quad (3.3.2.16)$$

after substituting from (3.3.1.3). From (3.3.2.3)

$$s = -1 \quad (3.3.2.17)$$

Pole is at  $-1$  and it is a second order pole.

Calculating Step response

$$X(s) = \frac{1}{s} \quad (3.3.2.18)$$

$$Y(s) = H(s)X(s) \quad (3.3.2.19)$$

$$= \frac{1}{s^2 + 2s + 1} \frac{1}{s} \quad (3.3.2.20)$$

$$y(t) = (1 - e^{-t} - te^{-t}) u(t) \quad (3.3.2.21)$$

3.3.3. Find  $k$  in such that the step response of the closed-loop system has minimum settling time and have no overshoot.

**Solution:** Settling time is defined as the time required for the transient's damped oscillations to reach and stay within  $\pm 2\%$  of the steady-state value. Above plot justifies that critical damped system ( $k = 1$ ) has minimum

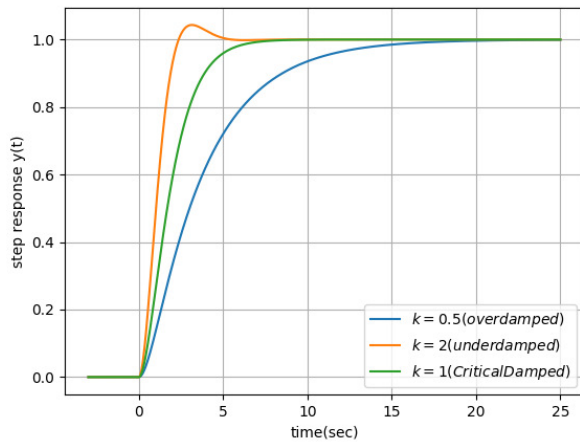


Fig. 3.3.3: Plot of step response of systems

settling time and also doesn't overshoot.  
Calculating Settling time

$$1 - e^{-t} - te^{-t} = 0.98 \quad (3.3.3.1)$$

Solving (3.3.3.1) graphically

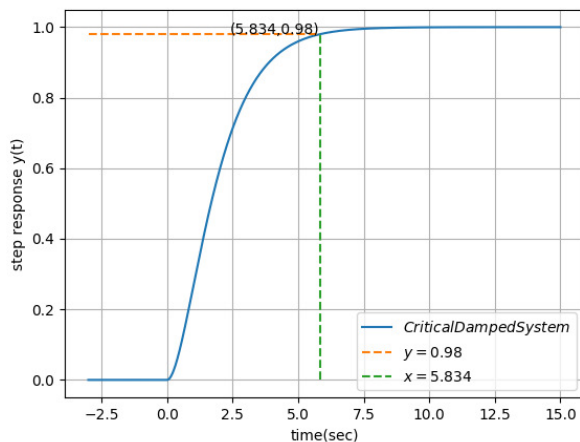


Fig. 3.3.3

Therefore, Settling time is 0.584sec.

## 4 ROUTH HURWITZ CRITERION

### 4.1 Routh Array

### 4.2 Marginal Stability

### 4.3 Stability

### 4.4 Example

## 5 STATE-SPACE MODEL

### 5.1 Controllability and Observability

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## 8 GAIN MARGIN

### 8.1 Introduction

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## 9 PHASE MARGIN

## 10 OSCILLATOR

### 10.1 Introduction

### 10.2 Example