

Control Systems

G V V Sharma*

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*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

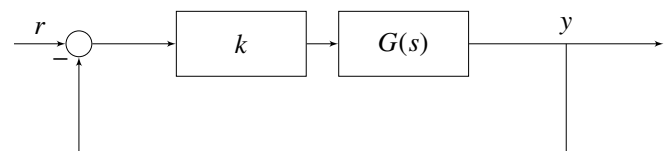


Fig. 3.3.1

Solution: The closed loop transfer function is

$$H(s) = \frac{kG(s)}{1 + kG(s)} \quad (3.3.1.2)$$

$$= \frac{k}{s^2 + 2s + k} \quad (3.3.1.3)$$

after substituting from (3.3.1.1).

3.3.2. Find the step response of the system.

Solution: The characteristics of the poles of transfer function describes the property of the transfer function.

Calculating poles :

$$s^2 + 2s + k = 0 \quad (3.3.2.1)$$

$$s = \frac{-2 \pm \sqrt{(-2)^2 - 4(1)(k)}}{2(1)} \quad (3.3.2.2)$$

$$s = -1 \pm \sqrt{1-k} \quad (3.3.2.3)$$

Calculating step response for a general damping system (3.3.1.3) :

$$X(s) = \frac{1}{s} \quad (3.3.2.4)$$

$$Y(s) = H(s)X(s) \quad (3.3.2.5)$$

$$= \frac{k}{s^2 + 2s + k} \frac{1}{s} \quad (3.3.2.6)$$

$$\begin{aligned} \Rightarrow y(t) = & \left[1 + \frac{k}{(2\sqrt{1-k})(-1 + \sqrt{1-k})} e^{(-1+\sqrt{1-k})t} \right. \\ & \left. + \frac{k}{(2\sqrt{1-k})(1 + \sqrt{1-k})} e^{(-1-\sqrt{1-k})t} \right] u(t) \end{aligned}$$

[when $k \neq 1$] (3.3.2.7)

$$y(t) = (1 - e^{-t} - te^{-t}) u(t) \text{ [when } k = 1] \quad (3.3.2.8)$$

Over Damped System:

For the closed loop transfer function (3.3.1.3) to be Over damped system, poles should be Real and Distinct, this happens when k is less than 1.

Considering $k < 1$ and solving (3.3.2.7)

$$\begin{aligned} \Rightarrow y(t) = & \left[1 - e^{-t} \left\{ \frac{\sinh(\sqrt{1-k})t}{\sqrt{1-k}} \right. \right. \\ & \left. \left. + \cosh(\sqrt{1-k})t \right\} \right] u(t) \end{aligned} \quad (3.3.2.9)$$

Under Damped System:

For the closed loop transfer function (3.3.1.3) to be Under damped system, poles should be Complex and Conjugate, this happens when k is greater than 1.

Considering $k > 1$ and solving (3.3.2.7)

$$\begin{aligned} \Rightarrow y(t) = & \left[1 - e^{-t} \left\{ \frac{\sin(\sqrt{k-1})t}{\sqrt{k-1}} \right. \right. \\ & \left. \left. + \cos(\sqrt{k-1})t \right\} \right] u(t) \end{aligned} \quad (3.3.2.10)$$

Critical Damped System:

For the closed loop transfer function (3.3.1.3) to be Critical damped system, poles should be Real at same location, this happens when k is 1.

$$y(t) = (1 - e^{-t} - te^{-t}) u(t) \quad (3.3.2.11)$$

3.3.3. Find steady state response using the final value theorem.

Solution:

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} sY(s) \quad (3.3.3.1)$$

$$= \lim_{s \rightarrow 0} s \frac{k}{s^2 + 2s + k} \frac{1}{s} \quad (3.3.3.2)$$

$$= \lim_{s \rightarrow 0} \frac{k}{s^2 + 2s + k} \quad (3.3.3.3)$$

$$= \frac{k}{k} = 1 \quad (3.3.3.4)$$

3.3.4. Find the settling time in terms of 'k'.

Solution:

$$0.98 < y(t) < 1.02 \quad (3.3.4.1)$$

$$\begin{aligned} \Rightarrow 0.98 < & \left[1 + \frac{k}{(2\sqrt{1-k})(-1+\sqrt{1-k})} e^{(-1+\sqrt{1-k})t} \right. \\ & \left. + \frac{k}{(2\sqrt{1-k})(1+\sqrt{1-k})} e^{(-1-\sqrt{1-k})t} \right] u(t) < 1.02 \\ & \text{[when } k \neq 1 \text{]} \quad (3.3.4.2) \end{aligned}$$

On further simplifying (3.3.4.2)

$$\begin{aligned} \Rightarrow -0.02 < & \left[-e^{-t} \left\{ \frac{\sinh(\sqrt{1-k})t}{\sqrt{1-k}} \right. \right. \\ & \left. \left. + \cosh(\sqrt{1-k})t \right\} \right] u(t) < 0.02 \quad (3.3.4.3) \end{aligned}$$

4 ROUTH HURWITZ CRITERION

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

4.4 Example

5 STATE-SPACE MODEL

5.1 Controllability and Observability

5.2 Second Order System

5.3 Example

5.4 Example

5.5 Example

6 NYQUIST PLOT

6.1 Introduction

6.2 Example

7 COMPENSATORS

7.1 Phase Lead

7.2 Example

8 GAIN MARGIN

8.1 Introduction

8.2 Example

9 PHASE MARGIN

10 OSCILLATOR

10.1 Introduction

10.2 Example