Control Systems

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Abstract—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Signal Flow Graph

1.1 Mason's Gain Formula

1.2 Matrix Formula

2 Bode Plot

2.1 Introduction

2.2 Example

3 Second order System

3.1 Damping

3.2 Example

3.3 Settling Time

3.3.1. Find the closed loop transfer function for the system in Fig. given that

$$G(s) = \frac{1}{s^2 + 2s} \qquad (3.3.1.1)$$

Fig. 3.3.1

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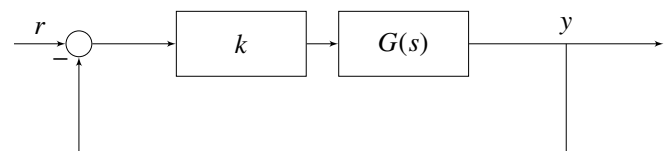
Solution: The closed loop transfer function is

$$H(s) = \frac{kG(s)}{1 + kG(s)} \qquad (3.3.1.2)$$

$$= \frac{k}{s^2 + 2s + k} \qquad (3.3.1.3)$$

after substituting from (3.3.1.1).

3.3.2. Find the step response of the system.

Solution: The characteristics of the poles of transfer function describes the property of the transfer function.

Calculating poles :

$$s^2 + 2s + k = 0 \qquad (3.3.2.1)$$

$$s = \frac{-2 \pm \sqrt{(-2)^2 - 4(1)(k)}}{2(1)} \qquad (3.3.2.2)$$

$$s = -1 \pm \sqrt{1 - k} \qquad (3.3.2.3)$$

Over Damped System:
For the closed loop transfer function (3.3.1.3) to be Over damped system,poles should be Real and Distinct.

As poles are Real and Distinct k should be less than 1

Considering k=0.5

$$H(s) = \frac{0.5}{s^2 + 2s + 0.5} \qquad (3.3.2.4)$$

after substituting from (3.3.1.3). From (3.3.2.3)

$$s = -1 \pm \sqrt{0.5} \qquad (3.3.2.5)$$

Poles are at $-1 + \frac{1}{\sqrt{2}}, -1 - \frac{1}{\sqrt{2}}$
Calculating Step response

$$X(s) = \frac{1}{s} \qquad (3.3.2.6)$$

$$Y(s) = H(s)X(s) \qquad (3.3.2.7)$$

$$= \frac{0.5}{s^2 + 2s + 0.5} \frac{1}{s} \qquad (3.3.2.8)$$

$$y(t)$$
$$= \left(1 - e^{-t}\cosh\left(\frac{t}{\sqrt{2}}\right) - \sqrt{2}e^{-t}\sinh\left(\frac{t}{\sqrt{2}}\right)\right)u(t) \qquad (3.3.2.9)$$

Under Damped System:
For the closed loop transfer function (3.3.1.3) to be Under damped system,poles should be

Complex and Conjugate.

As poles are Complex and Conjugate k should be greater than 1

Considering k=2

$$H(s) = \frac{2}{s^2 + 2s + 2} \qquad (3.3.2.10)$$

after substituting from (3.3.1.3). From (3.3.2.3)

$$s = -1 \pm \iota \qquad (3.3.2.11)$$

Poles are at -1+ι,-1-ι
Calculating Step response

$$X(s) = \frac{1}{s} \qquad (3.3.2.12)$$

$$Y(s) = H(s)X(s) \qquad (3.3.2.13)$$

$$= \frac{2}{s^2 + 2s + 2} \frac{1}{s} \qquad (3.3.2.14)$$

$$y(t) = (1 - e^{-t}\cos t - e^{-t}\sin t)u(t) \qquad (3.3.2.15)$$

Critical Damped System:
For the closed loop transfer function (3.3.1.3) to be Critical damped system,poles should be Real at same location.

As poles are Real and equal k should be 1

$$H(s) = \frac{1}{s^2 + 2s + 1} \qquad (3.3.2.16)$$

after substituting from (3.3.1.3). From (3.3.2.3)

$$s = -1 \qquad (3.3.2.17)$$

Pole is at -1 and it is a second order pole.
Calculating Step response

$$X(s) = \frac{1}{s} \qquad (3.3.2.18)$$

$$Y(s) = H(s)X(s) \qquad (3.3.2.19)$$

$$= \frac{1}{s^2 + 2s + 1} \frac{1}{s} \qquad (3.3.2.20)$$

$$y(t) = (1 - e^{-t} - te^{-t})u(t) \qquad (3.3.2.21)$$

3.3.3. Find k in such that the step response of the closed-loop system has minimum settling time and have no overshoot.

Solution: Settling time is defined as the time required for the transient's damped oscillations to reach and stay within ±2% of the steady-state value.
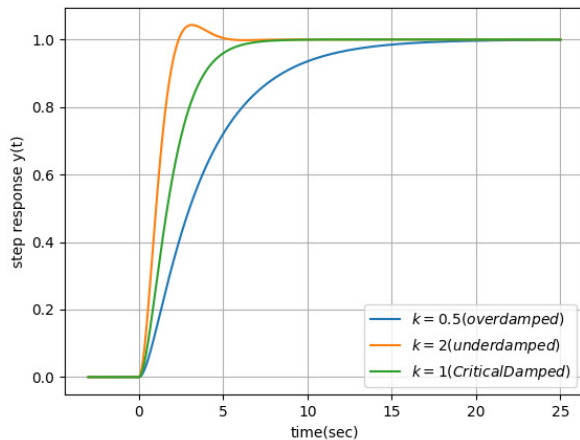
Fig. 3.3.3: Plot of step response of systems

Python Code for above plot is:

codes/ee18btech11035_4.py

Above plot justifies that critical damped system ($k = 1$) has minimum settling time and also doesn't overshoot.
Calculating Settling time

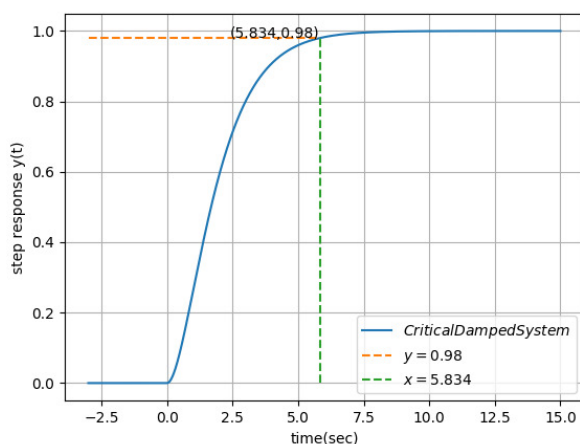$$1 - e^{-t} - te^{-t} = 0.98 \qquad (3.3.3.1)$$

Solving (3.3.3.1) graphically

Fig. 3.3.3

Python Code for above plot is:

codes/ee18btech11035_5.py

Therefore,Settling time is 0.584sec.