CONTROL SYSTEMS

Presentation

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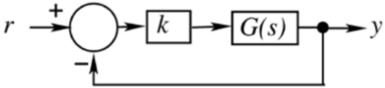
In the feedback system given below $G(s) = \frac{1}{s^2+2s}$.
The step response of the closed-loop system should have minimum settling time and have no overshoot.



The required value of gain k to achieve this is ___________

Solution

SettlingTime: The time required for the transient's damped oscillations to reach and stay within 2% of the steady-state value.

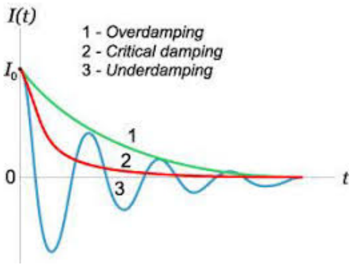
Overshoot:The amount that the waveform overshoots the steady state, or final, value at the peak time, expressed as a percentage of the steady-state value.

The Transfer function of the negative unity feedback system is given by $\frac{G(s)}{1+G(s)H(s)}$ (Where G(s) is the open-loop gain of the system and H(s) is feedback gain)

In the given Question G(s) = k x G(s) and H(s) = 1 .So, Transfer function of the whole feedback system is $\frac{kG(s)}{1+kG(s)}$

By substituting G(s)=$\frac{1}{s^2+2s}$ in the above equation we get

Closed loop Transfer function = $\frac{k}{s^2+2s+K}$



By observing the above figure,minimum settling time is obtained for Critical Damped System.

Also, Critically Damped System doesn't overshoot.

Transfer function of the Critical Damped System is given by $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ (Where, $\zeta = 1 for Critical Damped System$)
By comparing Obtained Transfer function $\frac{k}{s^2+2s+K}$ and the general transfer function of Critical Damped System $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

We get $\omega_n^2 = K$
$\omega_n = \sqrt{k}$

$2\zeta\omega_n s = 2s$
$\zeta = \frac{1}{\omega_n}$
$\zeta = \frac{1}{\sqrt{K}}$

As, $\zeta = 1$

$\frac{1}{\sqrt{K}}$ =1

K=1

Therefore, The value of K is 1.

The closed loop Transfer function is $\frac{1}{s^2+2s+1}$

$$\frac{1}{s^2+2s+1} = \frac{1}{(s+1)^2}$$

Laplace Transforms:

$\frac{1}{(s)} \longleftrightarrow u(t)$

$\frac{1}{(s+1)} \longleftrightarrow e^{-t}u(t)$

$\frac{1}{(s+1)^2} \longleftrightarrow te^{-t}u(t)$

In time domain $\frac{1}{(s+1)^2}$ is equivalent to $te^{-t}u(t)$

Plot of Transfer function in time domain is: