

Control Systems

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1 POLAR PLOT

1.1 Introduction

1.2 Example

1.3 Example

1.4 Example

1.5 Example

1.6 Example

1.7 Example

2 BODE PLOT

2.1 Gain and Phase Margin

3 PID CONTROLLER

3.1 Introduction

4 NYQUIST PLOT

4.0.1. Sketch the Nyquist plot for a closed loop system having open-loop transfer function

$$G(s)H(s) = \frac{2e^{-s\tau}}{s(1+s)(1+0.5s)} \quad (4.0.1.1)$$

Determine the maximum value of τ for the system to be stable.

4.0.2. Find $\text{Re}\{G(j\omega)H(j\omega)\}$ and $\text{Im}\{G(j\omega)H(j\omega)\}$.

Solution: From (4.0.1.1),

$$\begin{aligned} \Rightarrow \text{Re}\{G(j\omega)H(j\omega)\} &= \\ &-4 \left[\frac{3\omega^2 \cos(\omega\tau) - (\omega^3 - 2\omega) \sin(\omega\tau)}{(3\omega^2)^2 + (\omega^3 - 2\omega)^2} \right] \end{aligned} \quad (4.0.2.1)$$

$$\begin{aligned} \Rightarrow \text{Im}\{G(j\omega)H(j\omega)\} &= \\ &4 \left[\frac{(\omega^3 - 2\omega) \cos(\omega\tau) + 3\omega^2 \sin(\omega\tau)}{(3\omega^2)^2 + (\omega^3 - 2\omega)^2} \right] \end{aligned} \quad (4.0.2.2)$$

Abstract—The objective of this manual is to introduce control system design at an elementary level.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/ketan/codes>

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4.0.3. Determine the maximum value of τ for the system to be stable.

Solution: Determining the stability of closed

loop transfer function using Nyquist stability Criterion.

$$Z = P + N \quad (4.0.3.1)$$

Poles of open loop transfer function are on left half of s-plane. Therefore, $P = 0$

To ensure that the system is stable N should be 0

For maximum value of τ for stability, the nyquist plot cuts the real axis at $-1+j0$.

$$G(s)H(s) = -1 + j0 \quad (4.0.3.2)$$

$$\text{Im}\{G(j\omega)H(j\omega)\} = 0 \quad (4.0.3.3)$$

$$\text{Re}\{G(j\omega)H(j\omega)\} = -1 \quad (4.0.3.4)$$

From (4.0.2.2) and (4.0.3.3)

$$\Rightarrow \tan(\omega\tau) = \frac{-(\omega^3 - 2\omega)}{3\omega^2} \quad (4.0.3.5)$$

From (4.0.2.1) and (4.0.3.4) and substituting $\tan(\omega\tau) = \frac{-(\omega^3 - 2\omega)}{3\omega^2}$

$$\Rightarrow \omega^6 + 5\omega^4 + 4\omega^2 - 16 = 0 \quad (4.0.3.6)$$

Solving (4.0.3.6) graphically.

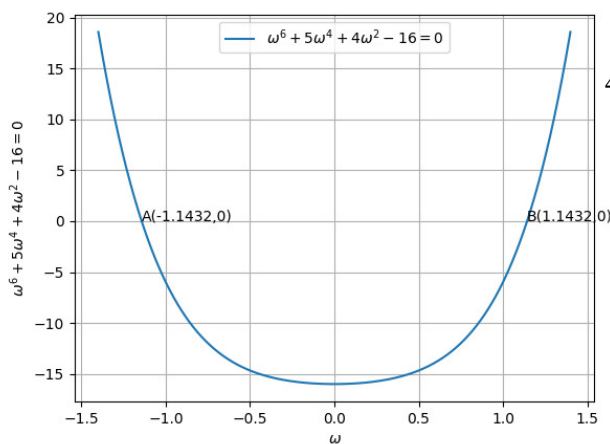


Fig. 4.0.3

Python code for the above plot is

```
codes/ee18btech11035_1.py
```

$\omega = 1.1432, -1.1432$ (As, ω is positive)
Therefore, $\omega = 1.1432$

Substituting ω in (4.0.3.5)

$$\tan(1.1432\tau) = 0.2021 \quad (4.0.3.7)$$

$$\tau = 0.1744 \quad (4.0.3.8)$$

4.0.4. Sketch the Nyquist plot.

Solution: The following python code generates the Nyquist plot.

```
codes/ee18btech11035_2.py
```

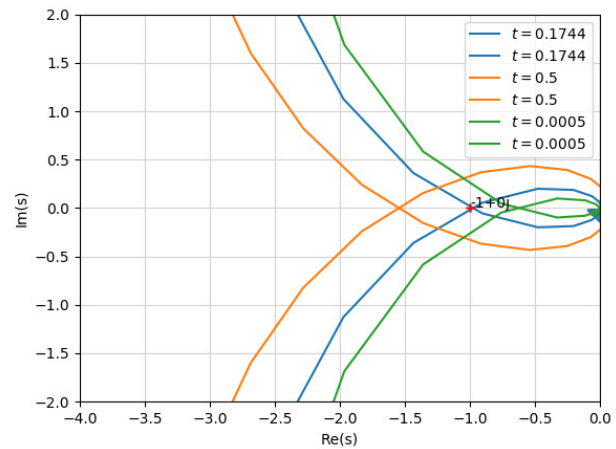


Fig. 4.0.4: Nyquist plot for variable τ

From the above figure (4.0.4) $\tau \leq 0.1744$ for a stable system.

4.0.5. Stability Criterion as varying τ

Solution:

τ	P	N	Z	Description
0.1744	0	1	1	System is Marginally stable
0.5	0	0	0	System is unstable
0.0005	0	0	0	System is stable

TABLE 4.0.5

Therefore, $\tau_{max} = 0.1744$