Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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3.3.1. Find the closed loop transfer function for the system in Fig. given that

$$G(s) = \frac{1}{s^2 + 2s} \tag{3.3.1.1}$$

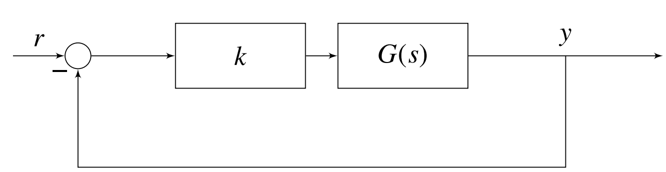

Fig. 3.3.1

Solution: The closed loop transfer function is

$$H(s) = \frac{kG(s)}{1 + kG(s)} \tag{3.3.1.2}$$

$$= \frac{k}{s^2 + 2s + k} \tag{3.3.1.3}$$

after substituting from (3.3.1.1).

3.3.2. Find the step response of the system.

Solution: The characteristics of the poles of transfer function describes the property of the transfer function.

Calculating poles :

$$s^2 + 2s + k = 0 \tag{3.3.2.1}$$

$$s = \frac{-2 \pm \sqrt{(-2)^2 - 4(1)(k)}}{2(1)} \tag{3.3.2.2}$$

$$s = -1 \pm \sqrt{1-k} \tag{3.3.2.3}$$

Calculating step response for a general damping system (3.3.1.3) :

$$X(s) = \frac{1}{s} \tag{3.3.2.4}$$

$$Y(s) = H(s)X(s) \tag{3.3.2.5}$$

$$= \frac{k}{s^2 + 2s + k}\frac{1}{s} \tag{3.3.2.6}$$

$$y(t) = u(t) + \frac{k}{\left(2\sqrt{1-k}\right)\left(-1+\sqrt{1-k}\right)} e^{\left(-1+\sqrt{1-k}\right)t} u(t) + \frac{k}{\left(2\sqrt{1-k}\right)\left(1+\sqrt{1-k}\right)} e^{\left(-1-\sqrt{1-k}\right)t} u(t) \text{ [when } k \neq 1\text{]} \tag{3.3.2.7}$$

$$y(t) = \left(1 - e^{-t} - te^{-t}\right)u(t) \text{ [when } k = 1\text{]} \tag{3.3.2.8}$$

Over Damped System:

For the closed loop transfer function (3.3.1.3) to be Over damped system,poles should be Real and Distinct,this happens when k is less than 1.

Considering $k < 1$ and solving (3.3.2.7)

$$y(t) = \left[1 - e^{-t}\left(\frac{\sinh\left(\sqrt{1-k}\right)t}{\sqrt{1-k}} + \cosh\left(\sqrt{1-k}\right)t\right)\right]u(t) \tag{3.3.2.9}$$

Under Damped System:

For the closed loop transfer function (3.3.1.3) to be Under damped system,poles should be Complex and Conjugate,this happens when k is greater than 1.

Considering $k > 1$ and solving (3.3.2.7)

$$y(t) = \left[1 - e^{-t}\left(\frac{\sin\left(\sqrt{k-1}\right)t}{\sqrt{k-1}} + \cos\left(\sqrt{k-1}\right)t\right)\right]u(t) \tag{3.3.2.10}$$

Critical Damped System:

For the closed loop transfer function (3.3.1.3) to be Critical damped system,poles should be Real at same location,this happens when k is 1.

$$y(t) = \left(1 - e^{-t} - te^{-t}\right)u(t) \tag{3.3.2.11}$$

3.3.3. Find steady state response using the final value theorem.

Solution:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \tag{3.3.3.1}$$

$$= \lim_{s\to 0} s\frac{k}{s^2 + 2s + k}\frac{1}{s} \tag{3.3.3.2}$$

$$= \lim_{s\to 0} \frac{k}{s^2 + 2s + k} \tag{3.3.3.3}$$

$$= \frac{k}{k} = 1 \tag{3.3.3.4}$$

3.3.4. Find the settling time in terms of 'k' .

Solution:

$$0.98 < y(t) < 1.02 \tag{3.3.4.1}$$

$$0.98 < u(t) + \frac{k}{\left(2\sqrt{1-k}\right)\left(-1+\sqrt{1-k}\right)} e^{\left(-1+\sqrt{1-k}\right)t} u(t) + \frac{k}{\left(2\sqrt{1-k}\right)\left(1+\sqrt{1-k}\right)} e^{\left(-1-\sqrt{1-k}\right)t} u(t) < 1.02 \tag{3.3.4.2}$$

On further simplifying (3.3.4.2)

$$-0.02 < \frac{-1}{\sqrt{1-k}} e^{-t} \left[\sinh\left(\sqrt{1-k}\right) t + \sqrt{1-k} \cosh\left(\sqrt{1-k}\right) t < 0.02 \quad (3.3.4.3)$$

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