Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Signal Flow Graph

1.1 Mason's Gain Formula

1.2 Matrix Formula

2 Bode Plot

2.1 Introduction

2.2 Example

3 Second order System

3.1 Daming

3.2 Example

3.1. In the Feedback System given below

$$G(s) = \frac{1}{s^2 + 2s} \tag{3.1.1}$$

The step response of the closed-loop system should have minimum settling time and have no overshoot

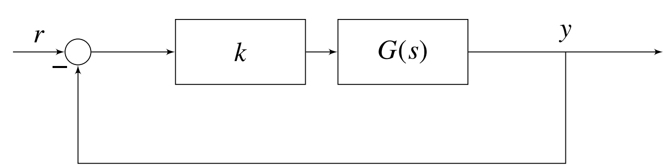

Fig. 3.1: Block Diagram of given question

3.2. The required value of gain 'k' to achieve this is

Solution:

$Settling time$: The time required for the transient's damped oscillations to reach and stay within 2% of the steady-state value.

Overshoot : The amount that the waveform overshoots the steady state, or final, value at the peak time, expressed as a percentage of the steady-state value.

Calculating the transfer function of the system

$$H(s) = \frac{kG(s)}{1 + kG(s)} \tag{3.2.1}$$

From (3.1.1) Substituting G(s) in (3.2.1)

Our Transfer Function becomes

$$H(s) = \frac{k * \frac{1}{s^2+2s}}{1 + k * \frac{1}{s^2+2s}} \tag{3.2.2}$$

On further simplifying we get,

$$H(s) = \frac{k}{s^2 + 2s + k} \tag{3.2.3}$$

For the output to have minimum settling time and also doesn't have overshoot,the system function should also have minimum settling time and also doesn't have overshoot.

Now, observing the Transfer function of different types systems in time domain. The system which has minimum settling time and also doesn't overshoot is critical damped system.

So,when unit step is given as input for critical damped system the output of the system has minimum settling time also the output doesn't overshoot.

We know that damping ratio(ζ) of a critical damped system is 1

By comparing the obtained transfer function (3.2.3) with general transfer function of a second order system
We get,

$$\omega_n^2 = k, \omega_n = \sqrt{k} \tag{3.2.4}$$

$$2\zeta\omega_n s = 2s, \zeta\omega_n = 1 \tag{3.2.5}$$

From (3.2.4) and (3.2.5)

$$\zeta\sqrt{k} = 1 \tag{3.2.6}$$

As $\zeta = 1$ eq(3.2.6) becomes

$$\sqrt{k} = 1, k = 1 \tag{3.2.7}$$

Therefore, Transfer function is

$$H(s) = \frac{1}{s^2 + 2s + 1} = \frac{1}{(s+1)^2} \tag{3.2.8}$$

3.3. Calculating the output when input is unit step and Verifying that output doesn't overshoot by plotting the output.
Solution:

$$X(s) = \frac{1}{s} \tag{3.3.1}$$

$$Y(s) = H(s) * X(s) \tag{3.3.2}$$

$$Y(s) = \frac{1}{(s+1)^2} * \frac{1}{s} \tag{3.3.3}$$

Converting Y(s) into partial fraction
We get,

$$Y(s) = \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2} \tag{3.3.4}$$

Calculating y(t) by applying the inverse laplace transform for equation (3.3.4)

$$\mathscr{L}^{-1}(Y(s)) = y(t) \tag{3.3.5}$$

$$\mathscr{L}^{-1}(\frac{1}{s}) = u(t) \tag{3.3.6}$$

$$\mathscr{L}^{-1}(\frac{1}{s+1}) = e^{-t}u(t) \tag{3.3.7}$$

$$\mathscr{L}^{-1}(\frac{1}{(s+1)^2}) = te^{-t}u(t) \tag{3.3.8}$$

Therefore,

$$y(t) = (1 - e^{-t} - te^{-t})u(t) \tag{3.3.9}$$

Plot of y(t):

Python code for the plot y(t) is

```
codes/ee18btech11035_3.py
```

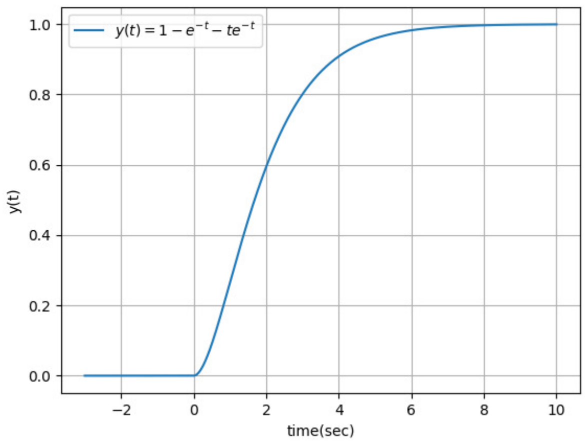

Fig. 3.3: Plot of y(t)

By observing the above plot we can justify that y(t) doesn't overshoot.

4 Routh Hurwitz Criterion

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

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