

Feedback Circuits

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0.0.1. A dc amplifier having a single-pole response with pole frequency 10Hz and unity-gain frequency of 1MHz is operated in a loop whose frequency-independent feedback factor is 0.01. Find the low-frequency gain, the 3-dB frequency, and the unity-gain frequency of the closed-loop amplifier. By what factor does the pole shift?

Solution: The open-loop gain of the amplifier is

$$G(s) = \frac{A_o}{1 + \frac{s}{\omega_p}} = \frac{A_o}{1 + \frac{s}{2\pi \cdot 10}} \quad (0.0.1.1)$$

Given that unity gain frequency is 1MHz
Replacing s with $j\omega$ in this equation.

$$\left| \frac{A_o}{1 + \frac{j \cdot 2\pi \cdot 10^6}{2\pi \cdot 10}} \right| = 1 \quad (0.0.1.2)$$

$$|A_o| = \left| 1 + j \frac{2\pi \cdot 10^6}{2\pi \cdot 10} \right| \quad (0.0.1.3)$$

$$A_o \approx 10^5 \quad (0.0.1.4)$$

0.0.2. Draw the bode plots of open-loop circuit

Solution:

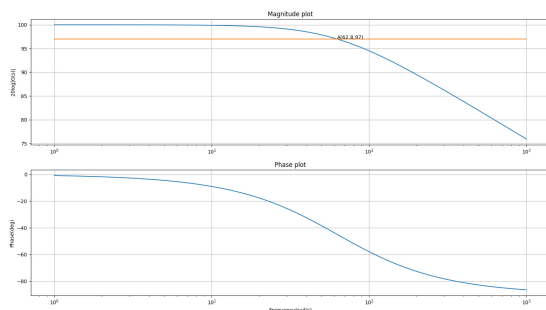


Fig. 0.0.2: Bode plots of Open-loop Transfer Function

Python code for above plot is

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codes/ee18btech11035_bode1.py

0.0.3. Design a Circuit for $G(s)$

Solution: Designing $G(s)$ Using Op-amp with DC gain of 10^5 and a pole at 10Hz.

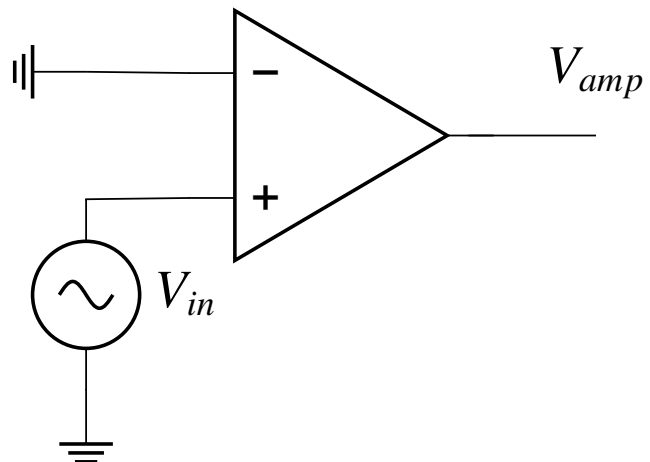


Fig. 0.0.3

0.0.4. Verify the gain using spice

Solution: Netlist file for simulation:

spice/ee18btech11035_spice1.net

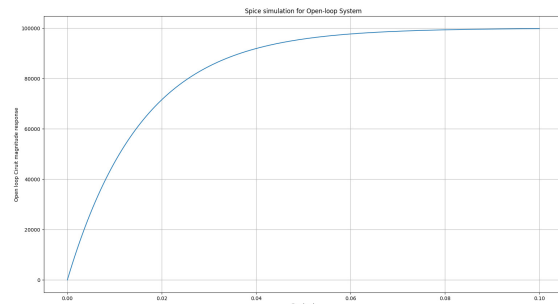


Fig. 0.0.4: Spice Simulation of Open-loop Transfer Function

Python code for above plot:

codes/ee18btech11035_spice1.py

0.0.5. Verification of step response of open-loop transfer function through python

Solution:

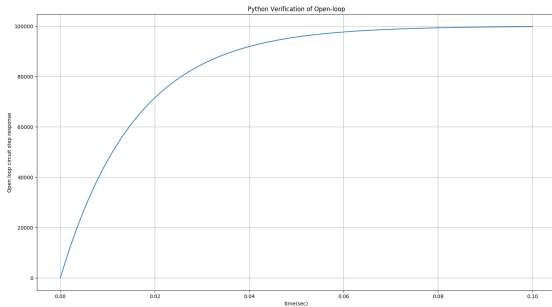


Fig. 0.0.5: Python verification of Open-loop Transfer Function

Python code for above verification is

codes/ee18btech11035_pythonverify1.py

0.0.6. Add a feedback for above circuit with a feedback factor of 0.01 and calculate closed-loop transfer function

Solution: $G(s) = \frac{10^5}{1 + \frac{s}{2\pi \cdot 10}}$ and $H(s) = 0.01$

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} \quad (0.0.6.1)$$

$$T = \frac{\frac{A_O}{1 + A_O H(s)}}{1 + \frac{s}{20\pi(1 + A_O H(s))}} \quad (0.0.6.2)$$

$$T(s) = \frac{99.90}{1 + \frac{s}{2\pi \cdot 10010}} \quad (0.0.6.3)$$

Block diagram representation of the amplifier

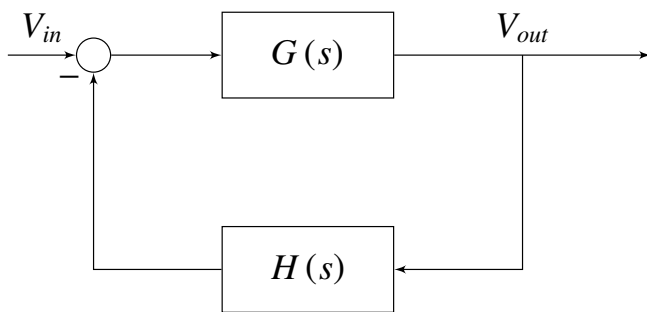


Fig. 0.0.6

0.0.7. Find the low frequency gain of the closed-loop amplifier.

Solution: As frequency is low, substituting

$$s = 0$$

$$T(0) = \frac{G(0)}{1 + G(0)H(0)} \quad (0.0.7.1)$$

$$= \frac{10^5}{1 + 10^5 \cdot (0.01)} \quad (0.0.7.2)$$

$$= 99.900 \quad (0.0.7.3)$$

0.0.8. Find the 3-dB frequency of the closed loop amplifier

Solution:

$$T = \frac{\frac{A_O}{1 + A_O H(s)}}{1 + \frac{s}{20\pi(1 + A_O H(s))}} \quad (0.0.8.1)$$

$$\text{3-dB frequency} = 20\pi \cdot (1 + A_O H(s)) \quad (0.0.8.2)$$

$$= 20\pi \cdot (1 + 10^5 \cdot (0.01)) \quad (0.0.8.3)$$

$$= 62862.8 \text{ rad/s} = 10.01 \text{ kHz} \quad (0.0.8.4)$$

0.0.9. Find the unity gain frequency of the closed loop amplifier

Solution: Unity-gain frequency of the closed loop amplifier is obtained as follows

$$|T| = 1 \quad (0.0.9.1)$$

$$\frac{A_O}{1 + A_O H(s)} = \left| 1 + \frac{j\omega}{20\pi(1 + A_O H(s))} \right| \quad (0.0.9.2)$$

$$99.900 = \left| 1 + \frac{j\omega}{62862.8} \right| \quad (0.0.9.3)$$

$$\omega = 6279.649 \text{ Krad/s} = 999.94 \text{ kHz} \quad (0.0.9.4)$$

0.0.10. By what factor does the pole shift?

Solution: Open-loop pole is 10Hz and the Closed-loop pole is $20\pi \cdot (1 + A_O H(s))$

$$\text{Pole-shift Factor} = \frac{20\pi(1 + A_O H(s))}{20\pi} \quad (0.0.10.1)$$

$$= 1 + A_O H(s) = 1001 \quad (0.0.10.2)$$

0.0.11. Design the feedback with $H(s) = 0.01$

Solution: Designing Feedback circuit:

As, Feedback is a constant value designing it by using simple voltage divider circuit

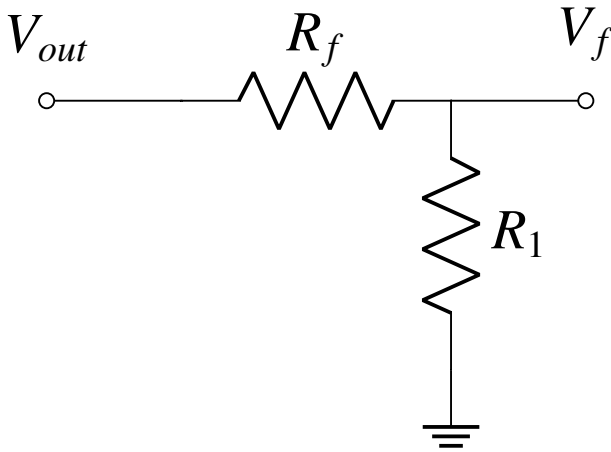


Fig. 0.0.11

$$\frac{V_f}{V_{out}} = \frac{R_1}{R_1 + R_f} = H(s) = 0.01 \quad (0.0.11.1)0.0.13.$$

Choosing R_1 as 10Ω and R_f as 990Ω

Overall Circuit is as follows:

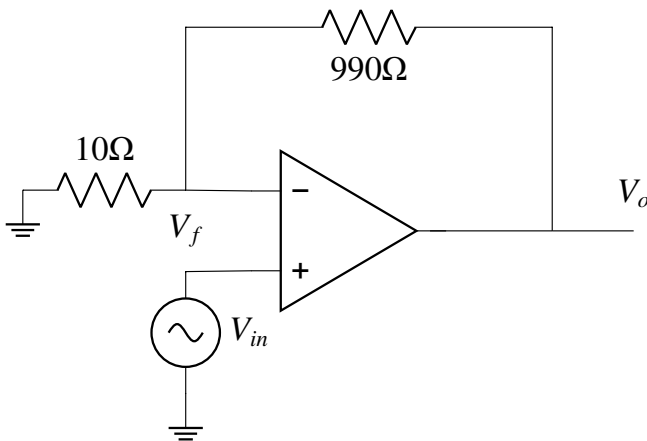


Fig. 0.0.11

Circuit Element	Parameter Value
Op-amp Gain	10^5
Op-amp pole	$10Hz$
R_1	10Ω
R_f	990Ω

TABLE 0.0.11

0.0.12. Draw the Bode plots of closed-loop circuit
Solution:

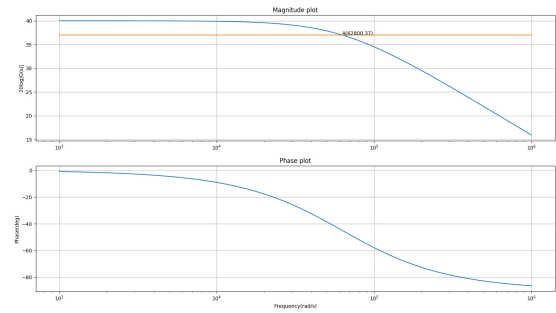


Fig. 0.0.12: Bode plots of Closed-loop Transfer Function

Python code for above plot is

codes/ee18btech11035_bode2.py

0.0.13. Verify the gain of closed loop Circuit using spice

Solution: Netlist file for simulation:

spice/ee18btech11035_spice2.net

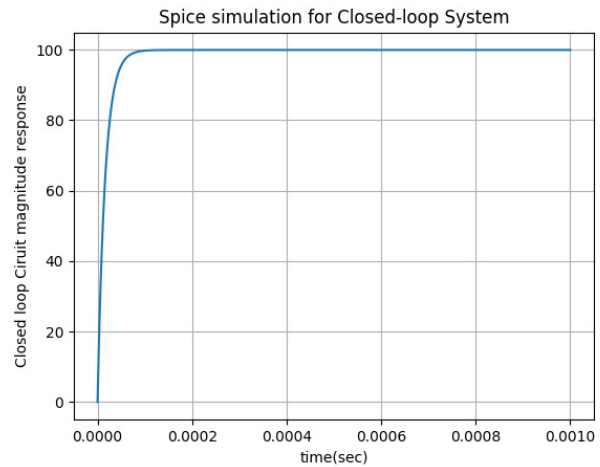


Fig. 0.0.13: Spice simulation of Closed-loop Transfer Function

Python code for above plot:

codes/ee18btech11035_spice2.py

0.0.14. Verification of step response of open-loop transfer function through python

Solution:

Python code for above verification is

codes/ee18btech11035_pythonverify2.py

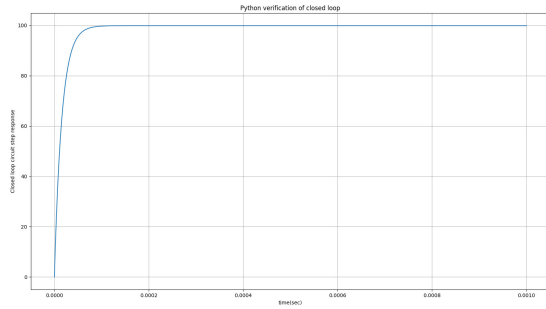


Fig. 0.0.14: Python verification of closed-loop Transfer Function

0.0.15. Tabulating Gain Bandwidth and Gain bandwidth product

Solution:

	$G(s)$	$T(s)$
Gain	10^5	99.9
Band-width	20π	$20\pi.1001$
Gain band-width product	$2\pi.10^6$	$2\pi.10^6$

TABLE 0.0.15

Therefore, By using feedback we can get desired Gain of an amplifier while maintaining constant Gain Bandwidth product.