Control Systems

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Contents

Abstract—**This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/feedback/codes
```

1 Feedback Voltage Amplifier: Series-Shunt

2 Feedback Current Amplifier: Shunt-Series

3 Feedback Current Amplifier: Shunt-Series

3.0.1. A dc amplifier having a single-pole response with pole frequency 10Hz and unity-gain frequency of 1MHz is operated in a loop whose frequency-independent feedback factor is 0.01. Find the low-frequency gain, the 3-dB frequency, and the unity-gain frequency of the closed-loop amplifier. By what factor does the pole shift?
Solution: The open-loop gain of the amplifier is

$$A(s) = \frac{A_O}{1 + \frac{s}{\omega_p}} = \frac{A_O}{1 + \frac{s}{2\pi.10} } \tag{3.0.1.1}$$

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Given that unity gain frequency is 1MHz
Replacing s with $\jmath\omega$ in this equation.

$$\left| \frac{A_O}{1 + \frac{\jmath .2\pi.10^6}{2\pi.10}} \right| = 1 \tag{3.0.1.2}$$

$$|A_O| = \left| 1 + \frac{2\pi.10^6}{2\pi.10} \right| \tag{3.0.1.3}$$

$$A_O \approx 10^5 \tag{3.0.1.4}$$

Block diagram for amplifier is

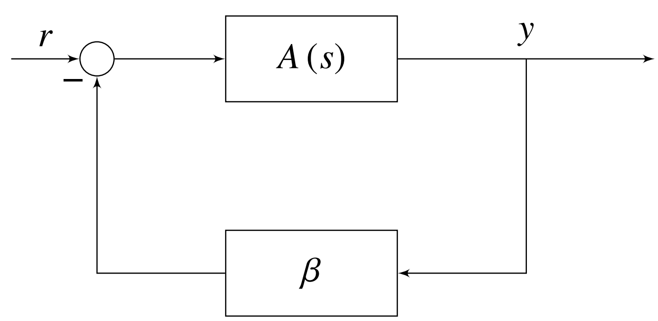

Fig. 3.0.1

Where $A(s) = \frac{10^5}{1+\frac{s}{10}}$

3.0.2. Find the low frequency gain of the closed-loop amplifier.
Solution: Given that frequency-independent feedback factor of amplifier is 0.01

$$\beta = 0.01 \tag{3.0.2.1}$$

Closed loop gain of amplifier is given by

$$T(s) = \frac{A(s)}{1 + A(s)\beta} \tag{3.0.2.2}$$

As frequency is low,substituting $s = 0$

$$T(0) = \frac{A(0)}{1 + A(0)\beta} \tag{3.0.2.3}$$

$$= \frac{10^5}{1 + 10^5.(0.01)} \tag{3.0.2.4}$$

$$= 99.900 \tag{3.0.2.5}$$

3.0.3. Find the 3-dB frequency of the closed loop amplifier
Solution: Substituting A(s) and β in (3.0.2.2)

$$T = \frac{\frac{A_O}{1+A_0\beta}}{1 + \frac{s}{20\pi(1+A_O\beta)}} \tag{3.0.3.1}$$

$$\text{3-dB frequency} = \omega\,(1 + A_O\beta) \tag{3.0.3.2}$$

$$= 20\pi.\left(1 + 10^5.\,(0.01)\right) \tag{3.0.3.3}$$

$$= 62862.8 rad/s = 10.01 kHz \tag{3.0.3.4}$$

3.0.4. Find the unity gain frequency of the closed loop amplifier
Solution: Unity-gain frequency of the closed loop amplifier is obtained as follows

$$|T| = 1 \tag{3.0.4.1}$$

$$\frac{A_O}{1 + A_O\beta} = \left|1 + \frac{\jmath\omega}{20\pi\,(1 + A_O\beta)}\right| \tag{3.0.4.2}$$

$$99.900 = \left|1 + \frac{\jmath\omega}{62862.8}\right| \tag{3.0.4.3}$$

$$\omega = 6279.649 Krad/s = 999.94 kHz \tag{3.0.4.4}$$

3.0.5. By what factor does the pole shift?
Solution: Open-loop pole is ω and the Closed-loop pole is $\omega\,(1 + A_O\beta)$

$$\text{Pole-shift Factor} = \frac{\omega\,(1 + A_O\beta)}{\omega} \tag{3.0.5.1}$$

$$= 1 + A_O\beta = 1001 \tag{3.0.5.2}$$