# Control Systems

G V V Sharma*

Contents




***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**


Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```


*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.



## 1 Polar Plot

### *1.1 Introduction*

### *1.2 Example*

### *1.3 Example*

### *1.4 Example*

### *1.5 Example*

### *1.6 Example*

### *1.7 Example*

## 2 Bode Plot

### *2.1 Gain and Phase Margin*

## 3 PID Controller

### *3.1 Introduction*

## 4 Nyquist Plot

4.0.1. Sketch the Nyquist plot for a closed loop system having open-loop transfer function

$$G(s)H(s) = \frac{2e^{-s\tau}}{s(1+s)(1+0.5s)} \tag{4.0.1.1}$$

Determine the maximum value of $\tau$ for the system to be stable.

4.0.2. Find $\text{Re}\{G(\jmath\omega)H(\jmath\omega)\}$ and $\text{Im}\{G(\jmath\omega)H(\jmath\omega)\}$.
**Solution:** From (4.0.1.1),

$$\implies \text{Re}\{G(\jmath\omega)H(\jmath\omega)\} = -4\left[\frac{3\omega^2\cos(\omega\tau) - \left(\omega^3 - 2\omega\right)\sin(\omega\tau)}{(3\omega^2)^2 + (\omega^3 - 2\omega)^2}\right] \tag{4.0.2.1}$$

$$\implies \text{Im}\{G(\jmath\omega)H(\jmath\omega)\} = 4\left[\frac{\left(\omega^3 - 2\omega\right)\cos(\omega\tau) + 3\omega^2\sin(\omega\tau)}{(3\omega^2)^2 + (\omega^3 - 2\omega)^2}\right] \tag{4.0.2.2}$$

4.0.3. Determine the maximum value of $\tau$ for the system to be stable.
**Solution:** Determining the stability of closed

loop transfer function using Nyquist stability Criterion.

$$Z = P + N \tag{4.0.3.1}$$

Poles of open loop transfer function are on left half of s-plane.Therefore,$P = 0$
To ensure that the system is stable $N$ should be 0
For maximum value of $\tau$ for stability ,the nyquist plot cuts the real axis at -1+j0.

$$G(s)H(s) = -1 + j0 \tag{4.0.3.2}$$
$$\text{Im}\{G(\jmath\omega)H(\jmath\omega)\} = 0 \tag{4.0.3.3}$$
$$\text{Re}\{G(\jmath\omega)H(\jmath\omega)\} = -1 \tag{4.0.3.4}$$

From (4.0.2.2) and (4.0.3.3)

$$\implies \tan(\omega\tau) = \frac{-\left(\omega^3 - 2\omega\right)}{3\omega^2} \tag{4.0.3.5}$$

From (4.0.2.1) and (4.0.3.4) and substituting $\tan(\omega\tau) = \frac{-(\omega^3-2\omega)}{3\omega^2}$

$$\implies \omega^6 + 5\omega^4 + 4\omega^2 - 16 = 0 \tag{4.0.3.6}$$

Solving (4.0.3.6) graphically.

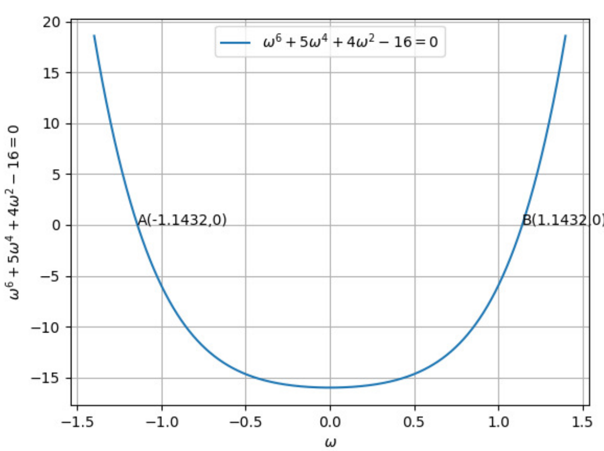


Fig. 4.0.3

Python code for the above plot is

codes/ee18btech11035_1.py

$\omega$ = 1.1432,-1.1432 ($As, \omega$ is positive)
Therefore,$\omega$ = 1.1432

Substituting $\omega$ in (4.0.3.5)

$$\tan(1.1432\tau) = 0.2021 \tag{4.0.3.7}$$
$$\tau = 0.1744 \tag{4.0.3.8}$$

4.0.4. Sketch the Nyquist plot.
**Solution:** The following python code generates the Nyquist plot.

codes/ee18btech11035_2.py

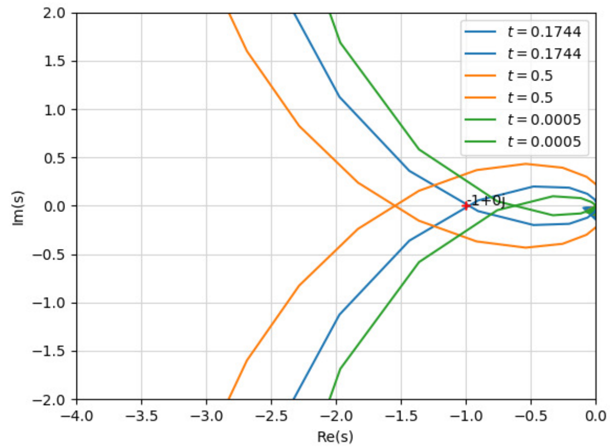


Fig. 4.0.4: Nyquist plot for variable $\tau$

From the above figure (4.0.4) $\tau \leq 0.1744$ for a stable system.

4.0.5. Stability Criterion as varying $\tau$
**Solution:**

| $\tau$ | **P** | **N** | **Z** | **Description** |
|---|---|---|---|---|
| 0.1744 | 0 | 1 | 1 | System is Marginally stable |
| 0.5 | 0 | 0 | 0 | System is unstable |
| 0.0005 | 0 | 0 | 0 | System is stable |

TABLE 4.0.5

Therefore,$\tau_{max} = 0.1744$
.