Feedback Circuits

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0.0.1. A dc amplifier having a single-pole response with pole frequency 10Hz and unity-gain frequency of 1MHz is operated in a loop whose frequency-independent feedback factor is 0.01. Find the low-frequency gain, the 3-dB frequency, and the unity-gain frequency of the closed-loop amplifier. By what factor does the pole shift?
Solution: The open-loop gain of the amplifier is

$$G(s) = \frac{A_O}{1 + \frac{s}{\omega_p}} = \frac{A_O}{1 + \frac{s}{2\pi.10}} \tag{0.0.1.1}$$

Given that unity gain frequency is 1MHz
Replacing s with $\jmath\omega$ in this equation.

$$\left| \frac{A_O}{1 + \frac{\jmath.2\pi.10^6}{2\pi.10}} \right| = 1 \tag{0.0.1.2}$$

$$|A_O| = \left| 1 + \jmath \frac{2\pi.10^6}{2\pi.10} \right| \tag{0.0.1.3}$$

$$A_O \approx 10^5 \tag{0.0.1.4}$$

0.0.2. Draw the bode plots of open-loop circuit
Solution:

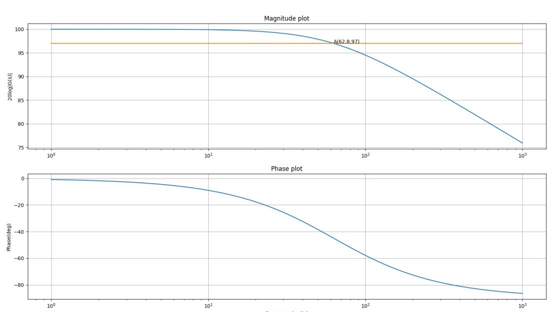

Fig. 0.0.2: Bode plots of Open-loop Transfer Function

Python code for above plot is

```
codes/ee18btech11035_bode1.py
```

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0.0.3. Design a Circuit for G(s)
Solution: Designing G(s) Using Op-amp with DC gain of 10^5 and a pole at 10Hz.

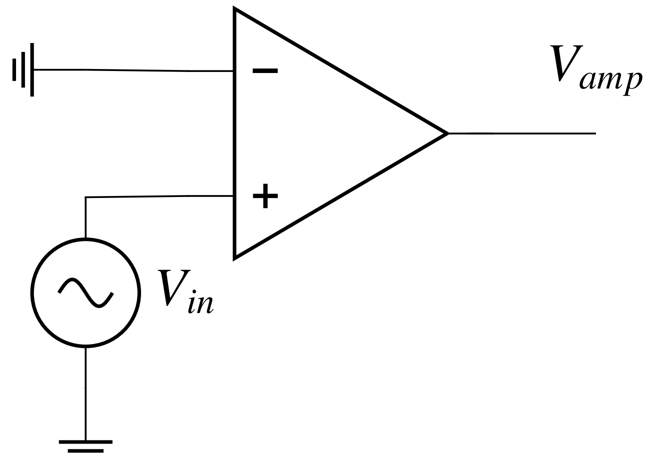

Fig. 0.0.3

0.0.4. Verify the gain using spice
Solution: Netlist file for simulation:

```
spice/ee18btech11035_spice1.net
```

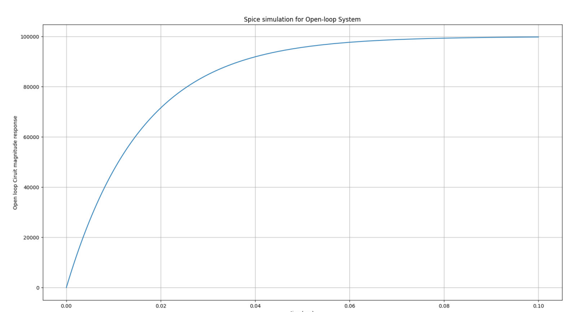

Fig. 0.0.4: Spice Simulation of Open-loop Transfer Function

Python code for above plot:

```
codes/ee18btech11035_spice1.py
```

0.0.5. Verification of step response of open-loop transfer function through python
Solution:

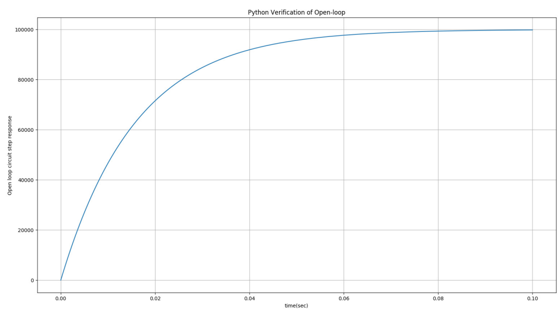

Fig. 0.0.5: Python verification of Open-loop Transfer Function

Python code for above verification is

```
codes/ee18btech11035_pythonverify1.py
```

0.0.6. Add a feedback for above circuit with a feedback factor of 0.01 an calculate closed-loop transfer function
Solution: $G(s) = \frac{10^5}{1+\frac{s}{2\pi.10}}$ and $H(s) = 0.01$

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} \tag{0.0.6.1}$$

$$T = \frac{\frac{A_O}{1+A_0H(s)}}{1 + \frac{s}{20\pi(1+A_OH(s))}} \tag{0.0.6.2}$$

$$T(s) = \frac{99.90}{1 + \frac{s}{2\pi.10010}} \tag{0.0.6.3}$$

Block diagram representation of the amplifier

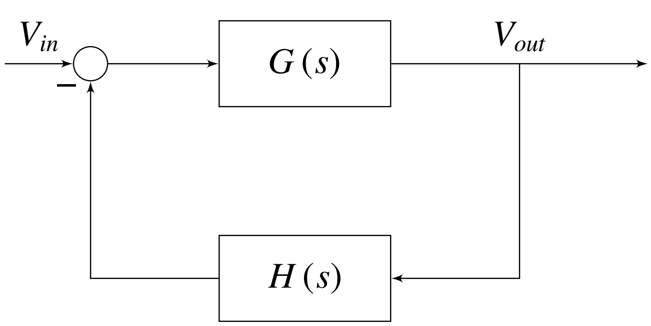

Fig. 0.0.6

0.0.7. Find the low frequency gain of the closed-loop amplifier.
Solution: As frequency is low,substituting $s = 0$

$$T(0) = \frac{G(0)}{1 + G(0)H(0)} \tag{0.0.7.1}$$
$$= \frac{10^5}{1 + 10^5.(0.01)} \tag{0.0.7.2}$$
$$= 99.900 \tag{0.0.7.3}$$

0.0.8. Find the 3-dB frequency of the closed loop amplifier
Solution:

$$T = \frac{\frac{A_O}{1+A_0H(s)}}{1 + \frac{s}{20\pi(1+A_OH(s))}} \tag{0.0.8.1}$$

$$\text{3-dB frequency} = 20\pi.(1 + A_OH(s)) \tag{0.0.8.2}$$
$$= 20\pi.\left(1 + 10^5.(0.01)\right) \tag{0.0.8.3}$$
$$= 62862.8 rad/s = 10.01 kHz \tag{0.0.8.4}$$

0.0.9. Find the unity gain frequency of the closed loop amplifier
Solution: Unity-gain frequency of the closed loop amplifier is obtained as follows

$$|T| = 1 \tag{0.0.9.1}$$
$$\frac{A_O}{1 + A_OH(s)} = \left|1 + \frac{\jmath\omega}{20\pi(1 + A_OH(s))}\right| \tag{0.0.9.2}$$
$$99.900 = \left|1 + \frac{\jmath\omega}{62862.8}\right| \tag{0.0.9.3}$$
$$\omega = 6279.649 Krad/s = 999.94 kHz \tag{0.0.9.4}$$

0.0.10. By what factor does the pole shift?
Solution: Open-loop pole is 10Hz and the Closed-loop pole is $20\pi.(1 + A_OH(s))$

$$\text{Pole-shift Factor} = \frac{20\pi(1 + A_OH(s))}{20\pi} \tag{0.0.10.1}$$
$$= 1 + A_OH(s) = 1001 \tag{0.0.10.2}$$

0.0.11. Design the feedback with $H(s) = 0.01$
Solution: Designing Feedback circuit:
As,Feedback is a constant value designing it by using simple voltage divider circuit

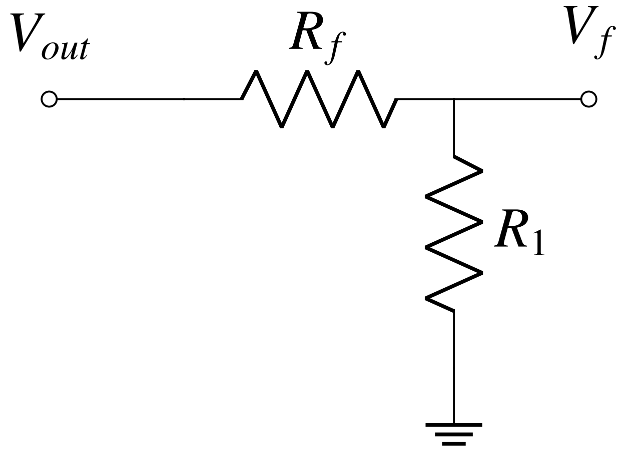

Fig. 0.0.11

$$\frac{V_f}{V_{out}} = \frac{R_1}{R_1 + R_f} = H(s) = 0.01 \qquad (0.0.11.1)$$

Choosing R_1 as 10Ω and R_f as 990Ω

Overall Circuit is as follows:

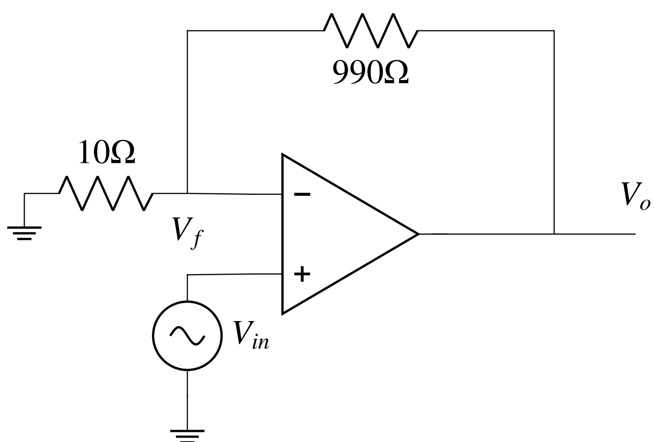

Fig. 0.0.11

Circuit Element	Parameter Value
Op-amp Gain	10^5
Op-amp pole	$10Hz$
R_1	10Ω
R_f	990Ω

TABLE 0.0.11

0.0.12. Draw the Bode plots of closed-loop circuit
Solution:

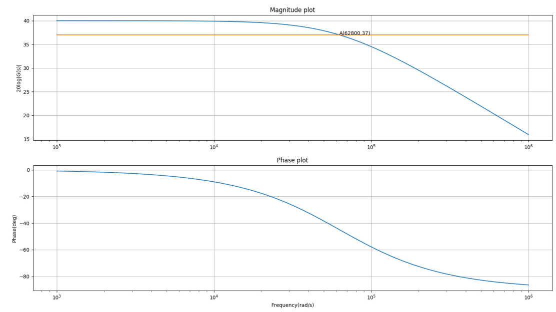

Fig. 0.0.12: Bode plots of Closed-loop Transfer Function

Python code for above plot is

codes/ee18btech11035_bode2.py

0.0.13. Verify the gain of closed loop Circuit using spice
Solution: Follow the Instructions for SPICE simulation:

spice/README.md

Netlist file for simulation:

spice/ee18btech11035_spice2.net

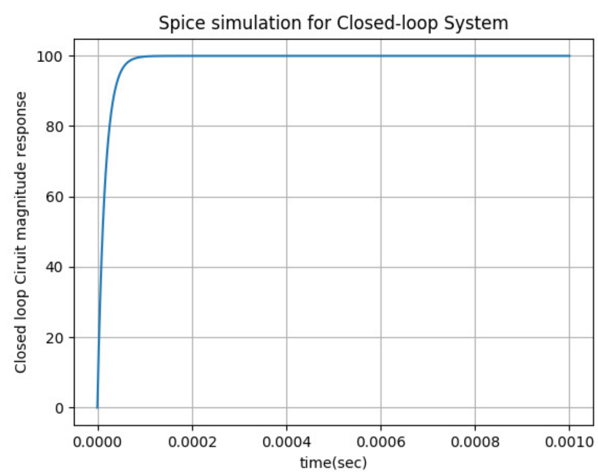

Fig. 0.0.13: Spice simulation of Closed-loop Transfer Function

Pyhton code for above plot:

codes/ee18btech11035_spice2.py

0.0.14. Verification of step response of open-loop transfer function through python
Solution:

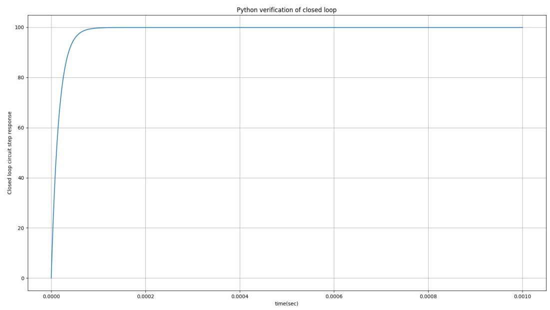

Fig. 0.0.14: Python verification of closed-loop Transfer Function

Python code for above verification is

codes/ee18btech11035_pythonverify2.py

0.0.15. Tabulating Gain Bandwidth and Gain bandwidth product
Solution:

	$G(s)$	$T(s)$
Gain	10^5	99.9
Bandwidth	20π	$20\pi.1001$
Gain bandwidth product	$2\pi.10^6$	$2\pi.10^6$

TABLE 0.0.15

Therefore,By using feedback we can get desired Gain of an amplifier while maintaining constant Gain Bandwidth product.