Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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3.3.1. Find the closed loop transfer function for the system in Fig. given that

$$G(s) = \frac{1}{s^2 + 2s} \tag{3.3.1.1}$$

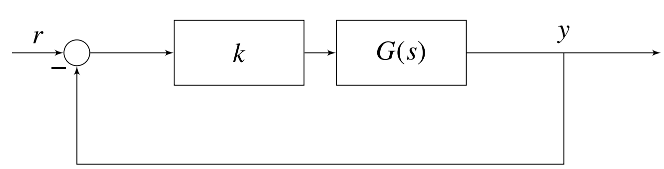

Fig. 3.3.1

Solution: The closed loop transfer function is

$$H(s) = \frac{kG(s)}{1 + kG(s)} \tag{3.3.1.2}$$
$$= \frac{k}{s^2 + 2s + k} \tag{3.3.1.3}$$

after substituting from (3.3.1.1).

3.3.2. Find the step response of the system.

Solution: The characteristics of the poles of transfer function describes the property of the transfer function.
Calculating poles :

$$s^2 + 2s + k = 0 \tag{3.3.2.1}$$
$$s = \frac{-2 \pm \sqrt{(-2)^2 - 4(1)(k)}}{2(1)} \tag{3.3.2.2}$$
$$s = -1 \pm \sqrt{1-k} \tag{3.3.2.3}$$

Over Damped System:
For the closed loop transfer function (3.3.1.3) to be Over damped system,poles should be Real and Distinct.
As poles are Real and Distinct k should be less than 1
Considering k=0.5

$$H(s) = \frac{0.5}{s^2 + 2s + 0.5} \tag{3.3.2.4}$$

after substituting from (3.3.1.3). From (3.3.2.3)

$$s = -1 \pm \sqrt{0.5} \tag{3.3.2.5}$$

Poles are at $-1 + \frac{1}{\sqrt{2}}, -1 - \frac{1}{\sqrt{2}}$
Calculating Step response

$$X(s) = \frac{1}{s} \tag{3.3.2.6}$$
$$Y(s) = H(s)X(s) \tag{3.3.2.7}$$
$$= \frac{0.5}{s^2 + 2s + 0.5}\frac{1}{s} \tag{3.3.2.8}$$

$$y(t) = \left(1 - e^{-t}\cosh\left(\frac{t}{\sqrt{2}}\right) - \sqrt{2}e^{-t}\sinh\left(\frac{t}{\sqrt{2}}\right)\right)u(t) \tag{3.3.2.9}$$

Under Damped System:
For the closed loop transfer function (3.3.1.3) to be Under damped system,poles should be Complex and Conjugate.
As poles are Complex and Conjugate k should be greater than 1
Considering k=2

$$H(s) = \frac{2}{s^2 + 2s + 2} \tag{3.3.2.10}$$

after substituting from (3.3.1.3). From (3.3.2.3)

$$s = -1 \pm \iota \tag{3.3.2.11}$$

Poles are at -1+ι,-1-ι
Calculating Step response

$$X(s) = \frac{1}{s} \tag{3.3.2.12}$$
$$Y(s) = H(s)X(s) \tag{3.3.2.13}$$
$$= \frac{2}{s^2 + 2s + 2}\frac{1}{s} \tag{3.3.2.14}$$
$$y(t) = \left(1 - e^{-t}\cos t - e^{-t}\sin t\right)u(t) \tag{3.3.2.15}$$

Critical Damped System:
For the closed loop transfer function (3.3.1.3) to be Critical damped system,poles should be Real at same location.
As poles are Real and equal k should be 1

$$H(s) = \frac{1}{s^2 + 2s + 1} \tag{3.3.2.16}$$

after substituting from (3.3.1.3). From (3.3.2.3)

$$s = -1 \tag{3.3.2.17}$$

Pole is at -1 and it is a second order pole.
Calculating Step response

$$X(s) = \frac{1}{s} \tag{3.3.2.18}$$
$$Y(s) = H(s)X(s) \tag{3.3.2.19}$$
$$= \frac{1}{s^2 + 2s + 1}\frac{1}{s} \tag{3.3.2.20}$$
$$y(t) = \left(1 - e^{-t} - te^{-t}\right)u(t) \tag{3.3.2.21}$$

3.3.3. Find k in such that the step response of the closed-loop system has minimum settling time and have no overshoot.

Solution: Settling time is defined as the time required for the transient's damped oscillations to reach and stay within ±2% of the steady-state value.

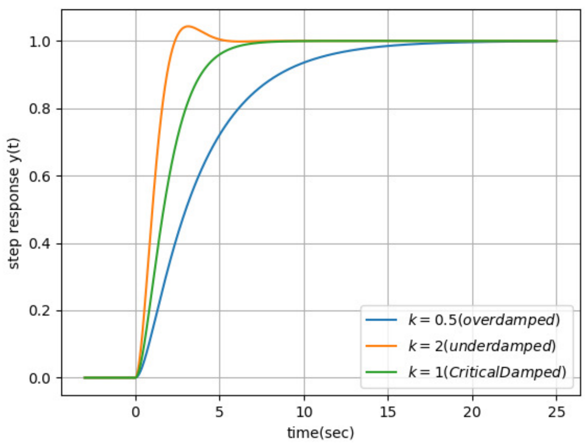

Fig. 3.3.3: Plot of step response of systems

Python Code for above plot is:

codes/ee18btech11035_4.py

Above plot justifies that critical damped system ($k = 1$) has minimum settling time and also doesn't overshoot.
Calculating Settling time

$$1 - e^{-t} - te^{-t} = 0.98 \tag{3.3.3.1}$$

Solving (3.3.3.1) graphically

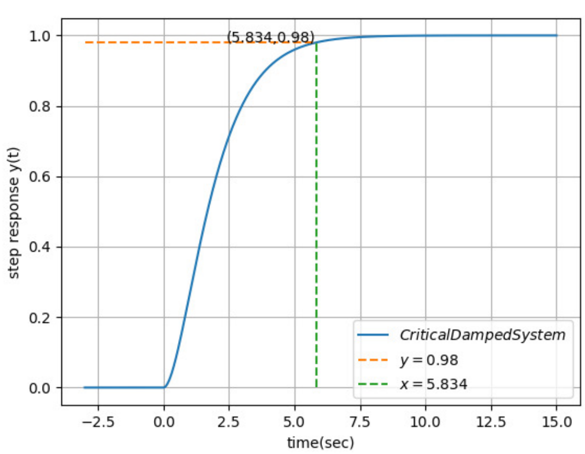

Fig. 3.3.3

Python Code for above plot is:

codes/ee18btech11035_5.py

Therefore,Settling time is 5.834sec.

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