# Control Systems

G V V Sharma*

Contents



*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. 

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

## 1 Signal Flow Graph

### *1.1 Mason's Gain Formula*

### *1.2 Matrix Formula*

## 2 Bode Plot

### *2.1 Introduction*

### *2.2 Example*

## 3 Second order System

### *3.1 Damping*

### *3.2 Example*

### *3.3 Settling Time*

3.3.1. Find the closed loop transfer function for the system in Fig. given that

$$G(s) = \frac{1}{s^2 + 2s} \tag{3.3.1.1}$$

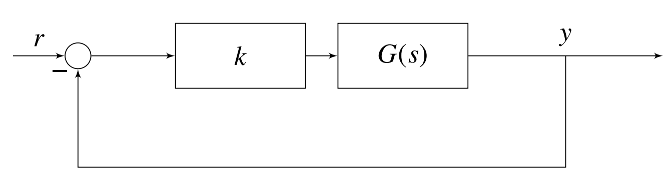


Fig. 3.3.1

**Solution:** The closed loop transfer function is

$$H(s) = \frac{kG(s)}{1 + kG(s)} \tag{3.3.1.2}$$
$$= \frac{k}{s^2 + 2s + k} \tag{3.3.1.3}$$

after substituting from (3.3.1.1).

3.3.2. Find the step response of the system.
**Solution:** From (3.3.1.3), the step response is

$$Y(s) = \frac{k}{s^2 + 2s + k}\frac{1}{s} \tag{3.3.2.1}$$

$$\implies y(t) = \Bigg[1 + \frac{k}{\left(2\sqrt{1-k}\right)\left(-1+\sqrt{1-k}\right)}e^{\left(-1+\sqrt{1-k}\right)t} + \frac{k}{\left(2\sqrt{1-k}\right)\left(1+\sqrt{1-k}\right)}e^{\left(-1-\sqrt{1-k}\right)t}\Bigg]u(t) \quad k \neq 1 \tag{3.3.2.2}$$

and

$$y(t) = \left(1 - e^{-t} - te^{-t}\right)u(t) \quad k = 1 \tag{3.3.2.3}$$

3.3.3. Find the steady state step response of the system usng the final value theorem.
**Solution:** From (3.3.2.1),

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \tag{3.3.3.1}$$
$$= 1 \tag{3.3.3.2}$$

3.3.4. Find the step response for an overdamped system.
**Solution:** For an overdamped system, $k < 1$,

$$\implies y(t) = \left[1 - e^{-t}\left\{\frac{\sinh\left(\sqrt{1-k}\right)t}{\sqrt{1-k}} + \cosh\left(\sqrt{1-k}\right)t\right\}\right]u(t) \tag{3.3.4.1}$$

3.3.5. Find the step response for an underdamped system.
**Solution:** In this case, $k > 1$.

$$\implies y(t) = \left[1 - e^{-t}\left\{\frac{\sin\left(\sqrt{k-1}\right)t}{\sqrt{k-1}} + \cos\left(\sqrt{k-1}\right)t\right\}\right]u(t) \tag{3.3.5.1}$$

3.3.6. Find the step response for a critically damped system.
**Solution:** For $k = 1$,

$$y(t) = \left(1 - e^{-t} - te^{-t}\right)u(t) \tag{3.3.6.1}$$

3.3.7. The settling time $t_s$ is defined as the first instant where

$$|y(t_s) - y_{ss}| \leq 0.02 \tag{3.3.7.1}$$

where $y_{ss}$ is the steady state value of $y(t)$. Find $k$ for which the settling time is minimum.
**Solution:** Considering Critical Damped System,where $k = 1$,
Plot of step response:

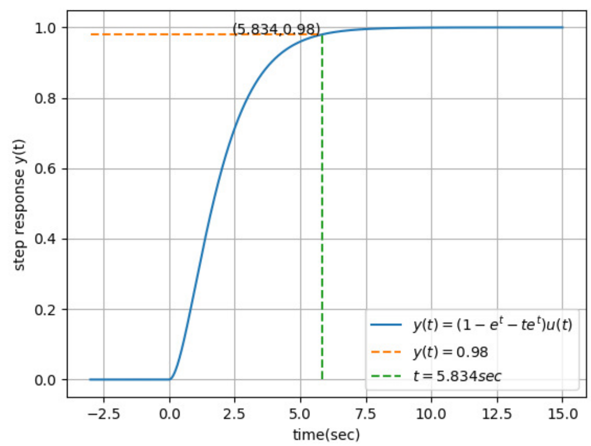


Fig. 3.3.7: Step response of Critical damped system

Python code for above plot is

```
codes/ee18btech11035_1.py
```

Settling time is 5.834sec.
Considering Under Damped System,where $k > 1$ Plot of step response :

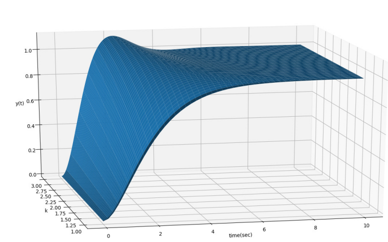

Fig. 3.3.7: Step response of Under damped system

Python code for above plot is

codes/ee18btech11035_2.py

There is an overshoot for every value of $k > 1$.
Considering Over Damped System,where $k < 1$
Plot of step response :

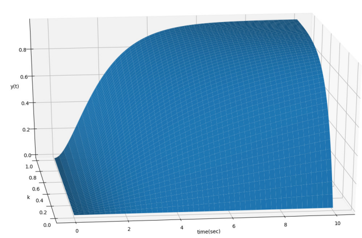

Fig. 3.3.7: Step response of Over damped system

Python code for above plot is

codes/ee18btech11035_3.py

As the value of $k$ is increasing the settling time is increasing.The lowest settling time obtained for a Over damped system is greater than obtained for critical damped case.
Therefore, when $k$ is 1 minimum settling time for step response of the given system (3.3.1.3) is obtained.

## 4 Routh Hurwitz Criterion

### *4.1 Routh Array*

### *4.2 Marginal Stability*

### *4.3 Stability*

### *4.4 Example*

## 5 State-Space Model

### *5.1 Controllability and Observability*

### *5.2 Second Order System*

### *5.3 Example*

### *5.4 Example*

### *5.5 Example*

## 6 Nyquist Plot

### *6.1 Introduction*

### *6.2 Example*

## 7 Compensators

### *7.1 Phase Lead*

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## 8 Gain Margin

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### *8.2 Example*

## 9 Phase Margin

## 10 Oscillator

### *10.1 Introduction*

### *10.2 Example*