

## How to compute TF-IDF

Suppose we are looking for documents using the query  $Q$  and our database is composed of the documents  $D_1$ ,  $D_2$ , and  $D_3$ .

- $Q$ : The cat.
- $D_1$ : The cat is on the mat.
- $D_2$ : My dog and cat are the best.
- $D_3$ : The locals are playing.

**TF(word, document) = “number of occurrences of the word in the document” / “number of words in the document”**

TF scores of the words “the” and “cat” (i.e. the query words) with respect to the documents  $D_1$ ,  $D_2$ , and  $D_3$ .

$$\begin{aligned} \text{TF}(\text{“the”}, D_1) &= 2/6 = 0.33 \\ \text{TF}(\text{“the”}, D_2) &= 1/7 = 0.14 \\ \text{TF}(\text{“the”}, D_3) &= 1/4 = 0.25 \\ \text{TF}(\text{“cat”}, D_1) &= 1/6 = 0.17 \\ \text{TF}(\text{“cat”}, D_2) &= 1/7 = 0.14 \\ \text{TF}(\text{“cat”}, D_3) &= 0/4 = 0 \end{aligned}$$

IDF can be calculated by taking the total number of documents, dividing it by the number of documents that contain a word, and calculating the logarithm.

If the word is very common and appears in many documents, this number will approach 0. Otherwise, it will approach 1.

**IDF(word) =  $\log(\text{number of documents} / \text{number of documents that contain the word})$**

Let's compute the IDF scores of the words "the" and "cat".

$$\text{IDF}(\text{"the"}) = \log(3/3) = \log(1) = 0$$

$$\text{IDF}(\text{"cat"}) = \log(3/2) = 0.18$$

Multiplying TF and IDF gives the TF-IDF score of a word in a document. The higher the score, the more relevant that word is in that particular document.

$$\text{TF-IDF}(\text{word, document}) = \text{TF}(\text{word, document}) * \text{IDF}(\text{word})$$

Let's compute the TF-IDF scores of the words "the" and "cat".

$$\text{TF-IDF}(\text{"the", D1}) = 0.33 * 0 = 0$$

$$\text{TF-IDF}(\text{"the", D2}) = 0.14 * 0 = 0$$

$$\text{TF-IDF}(\text{"the", D3}) = 0.25 * 0 = 0$$

$$\text{TF-IDF}(\text{"cat", D1}) = 0.17 * 0.18 = 0.0306$$

$$\text{TF-IDF}(\text{"cat", D2}) = 0.14 * 0.18 = 0.0252$$

$$\text{TF-IDF}(\text{"cat", D3}) = 0 * 0 = 0$$

The next step is to use a ranking function to order the documents according to the TF-IDF scores of their words.

We can use the average TF-IDF word scores over each document to get the ranking of  $D_1$ ,  $D_2$ , and  $D_3$  with respect to the query  $Q$ .

$$\text{Average TF-IDF of } D_1 = (0 + 0.0306) / 2 = 0.0153$$

$$\text{Average TF-IDF of } D_2 = (0 + 0.0252) / 2 = 0.0126$$

$$\text{Average TF-IDF of } D_3 = (0 + 0) / 2 = 0$$

The word "the" does not contribute to the TF-IDF scores of each document. This is because "the" appears in all of the documents and thus it is considered a not-relevant word.