Homework 4

Aasim Zahoor

September 24, 2020

1 Problems

Link `https://github.com/AasimZahoor/Comp_methods.git`

Note: To solve these problems I had to make a modified RK−4method which is in Hw_4 directory as RK.py. The reason is because we have to set dp/dr=0

Problem 1

I have defined one function(**func(k1,k2)**) in the code which returns an array where the first element is dP/dr (non relativistic hydrostatic equation)and second one is dM/dr. The variables of the returned functions are P, M_{enc} and r. The arguments of this function are:

$k1 = G * u_e/l^{3/5}$

$k2 = 4 * pi * u_e/l^{3/5}$

G= Gravitational constant,

c= speed of light,

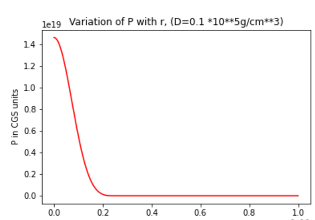
l= K(the constant multiplied to rho in the relation between P and rho)

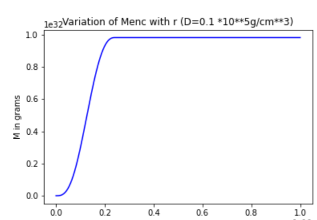
$u_e = 2$.

Note: Units given as arguments should be in CGS units.

Approach for getting M_{enc}V/s radii plots

We have been asked to solve non relativistic hydrostatic equation for given density range and plot M_{enc} V/s r. I have chosen the density values to be $[10^4, 5 * 10^4, 10^5, 5 * 10^5, 10^6]$. The max radius is 10^{10} cm and the step size is 10^6cm.





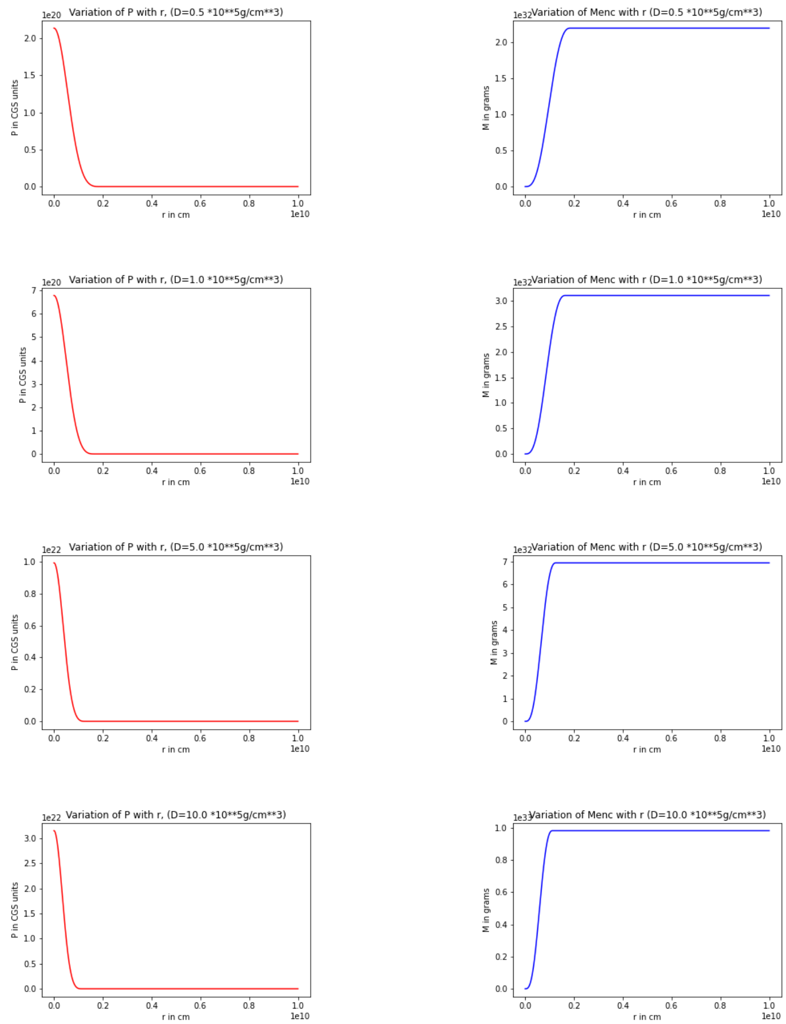


Figure 1a: It can be observed that as the density increased we have smaller radii.

Approach for getting M of the star v/s r of the star graph

I started with $1 * 10^4 g/cm^3$ and kept on adding $5 * 10^4$ till I reached $99 * 10^4 g/cm^3$. This gave me 33 data points for density. For each density I the $RK-4$ to get Mass and radius. Then I plotted them to get this plot.

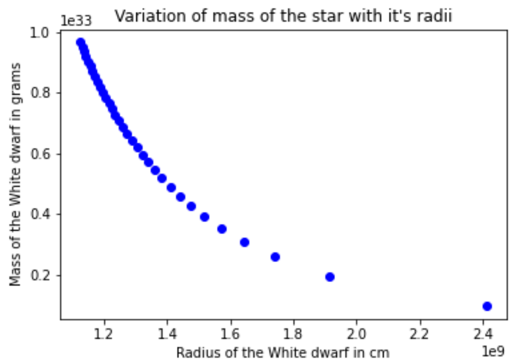


Figure 1b: M decreases as r increases

Problem 2

I have defined one function(**func(G,c,l)**) in the code which returns an array where the first element is dP/dr (TOV) and second one is dM/dr. The variables of the returned functions are P, M_{enc} and r. The arguments of this function are:

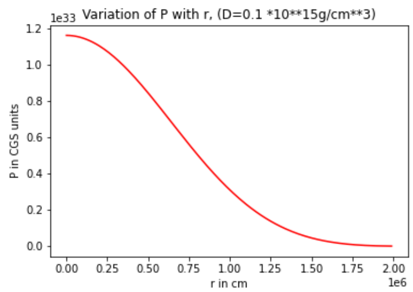
G= Gravitational constant,

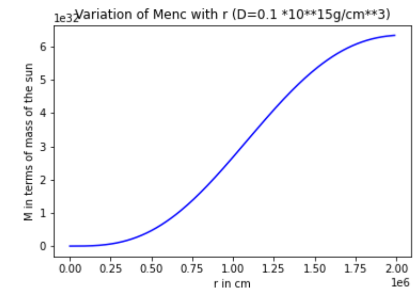
c= speed of light,

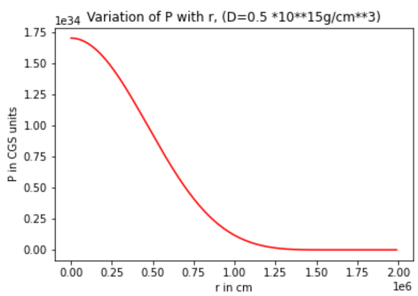
l= K(the constant multiplied to rho in the relation between P and rho). *Note: Units given as arguments should be in CGS units.*

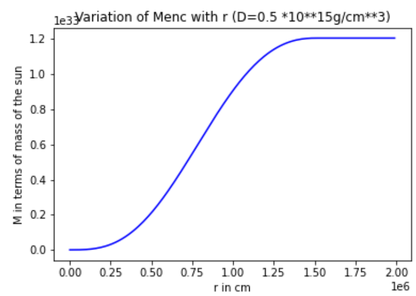
Approach for getting M_{enc}V/s radii plots

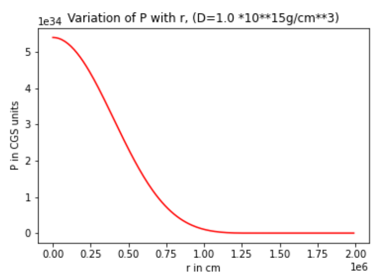
We have been asked to solve TOV equation for given density range and plot M_{enc} V/s r. I have chosen the density values to be $[10^{14}, 5*10^{14}, 10^{15}, 5*10^{15}, 10^{16}]$. The max radius is 20km and the step size is 10 m.

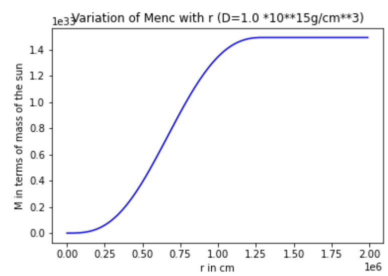












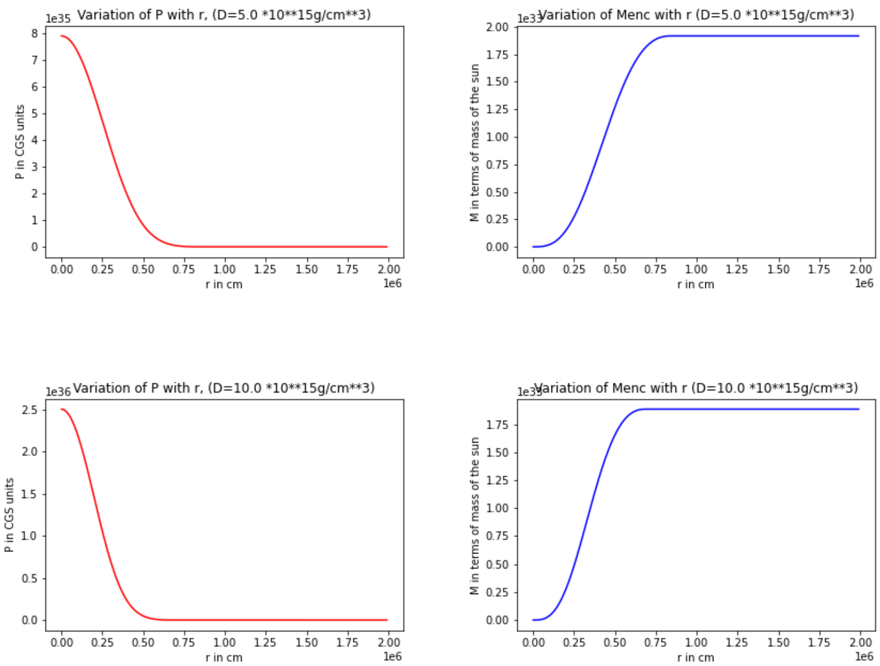


Figure 2: It is observed maximum mass in reached at lesser radius as density is increased.

Approach for getting M of the star v/s r of the star graph

I started with $1 * 10^{14} g/cm^3$ and kept on adding $5 * 10^{14}$ till I reached $99 * 10^{14} g/cm^3$. This gave me 33 data points for density. For each density and called the $RK-4$ to get Mass and radius. Then I plotted them to get this plot.

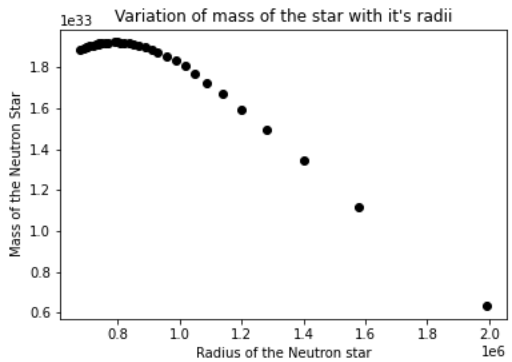


Figure 1b: M first increases then decreases as r increases

Problem 3

In the code for Problem 3 I have defined one function. It is:

- **func(G,c,l)**

  This function returns an array where the first element is dP/dr (TOV) and second one is dM/dr. The variables of the returned functions are P, M_{enc} and r. The arguments of this function are:

  G= Gravitational constant,

  c= speed of light,

  l= K(the constant multiplied to rho in the relation between P and rho). *Note: Units given as arguments should be in CGS units.*

  ### Approach

  We have been asked to find mass of the Neutron star given radius and using the TOV equation.
  (I did this earlier and I wanted to keep it just to show what assuming a density gives you as the answer.) I approached this problem by assuming density to be $10^{(}16)g/cm^3$ and then using the TOV equation and dM_{enc}/dr and RK-4 solver to find the values of M and P at different R. Then I found the maximum mass in the returned mass array. I made the code run till $r = 13.02km$ with a step size of 100 cm. Here are the graphs and output:

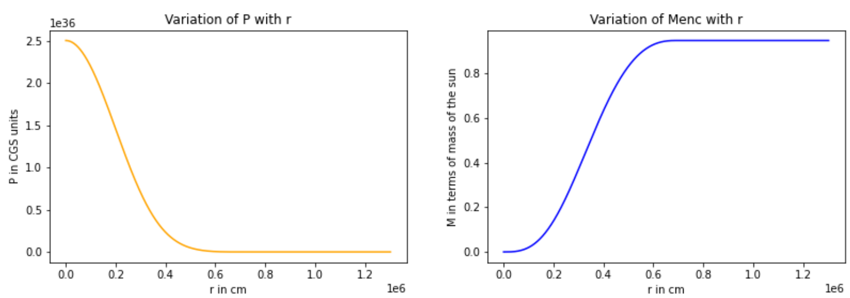
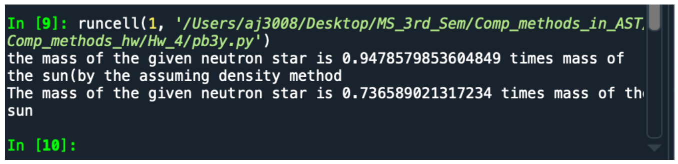


Figure : The graphs for problem 3

Here is what i did to get 0.7 times mass of the sun as answer. I imported the M and R array from problem 2, found the two R points between which 13.02km lies and then interpolated(linear) between those two points to get the mass of a star at radii 13.02km.



```
In [9]: runcell(1, '/Users/aj3008/Desktop/MS_3rd_Sem/Comp_methods_in_AST/
Comp_methods_hw/Hw_4/pb3y.py')
the mass of the given neutron star is 0.9478579853604849 times mass of
the sun(by the assuming density method
The mass of the given neutron star is 0.736589021317234 times mass of th
sun

In [10]:
```

Figure 3: The output for the code for problem 3.