Computational Methods HW- 9

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https://github.com/AasimZahoor/Comp_methods.git

Question 1.

Given
This problem has 4 different tasks and a bonus task towards at the end. Here we are given a file *SN-m-tot-V2.0* which has sunspot numbers observed at different years starting from 1749. From observation it is known that there is a yearly cycle and a cycle of 11 years.

Task-1

This task asks us to elaborate on the AR model I will be using. The model I have chosen has noise, μ (average) dependence, and has terms which account for dependence on previous value, yearly cycle and the 11 year cycle. Mathematically, my model can be written as;

$$X_t = c + \phi_1 X_{t-1} + \phi_{12} X_{t-12} + \phi_{132} X_{t-132} + N(0, \sigma_z)$$

So my AR model has a p value of 3. Here, c accounts for μ, the ϕ values account for different dependencies and N accounts for noise.

$$\text{Model} = c + \phi_1 X_{t-1} + \phi_{12} X_{t-12} + \phi_{132} X_{t-132}$$
$$\text{Noise} = N(0, \sigma_z)$$
$$\text{AR model} = c + \phi_1 X_{t-1} + \phi_{12} X_{t-12} + \phi_{132} X_{t-132} + N(0, \sigma_z)$$

Task-2

For this task I made three functions.

- **lnprior(theta)**:

  This function returns prior for the given values of theta. My function favors phi values <=1.

  Parameters-
  theta : Array of parameters.

  Returns : Value of log-prior

- **lnlike(theta,data)**:

  This function returns loglikelihood for given parameters of the model. The model is described in this part of the code.

  Parameters-
  theta : Array of parameters
  data : data

  Returns-
  loglikelihood of the model with the given parameters.

- **lnprob(theta, data)**:
  This function gives the probability of the model with given parameters. This function calls the previous two functions

  Parameters-
  theta : Array of parameters
  data : data

  Returns-
  Probability of the model with the given parameters.

So after defining the functions I used the emcee package to find the parameter values. Using those parameter values I found the fit values(for model and AR model) and noise. The equation I used is:

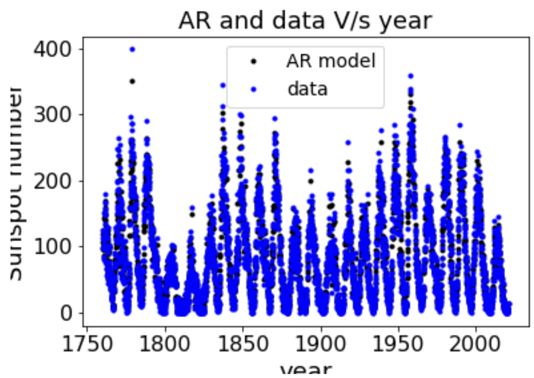
$$\text{Model} = c + \phi_1 X_{t-1} + \phi_{12} X_{t-12} + \phi_{132} X_{t-132}$$

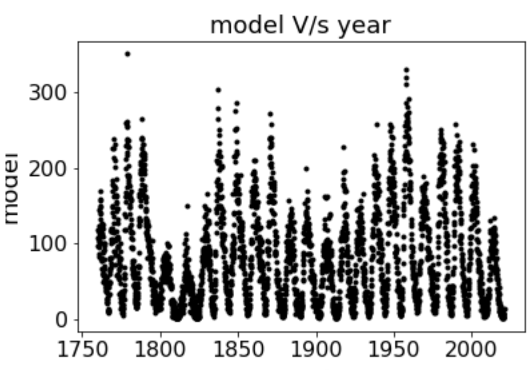
$$\text{Noise} = N(0, \sigma_z)$$

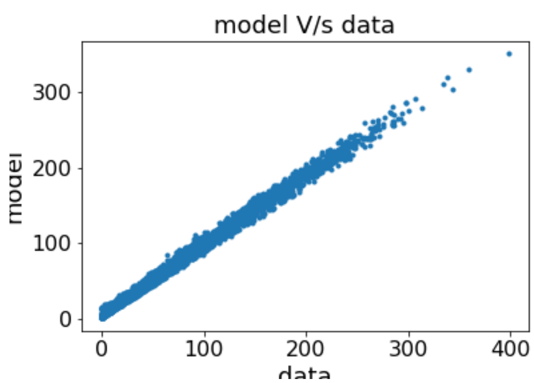
$$\text{AR model} = c + \phi_1 X_{t-1} + \phi_{12} X_{t-12} + \phi_{132} X_{t-132} + N(0, \sigma_z)$$

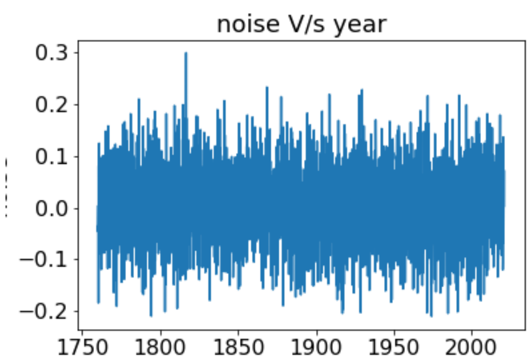
Using the above equations I plotted these graphs and found the following best fit parameters.

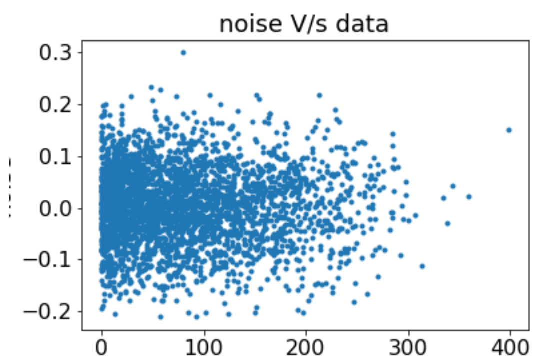
Plots for task 2







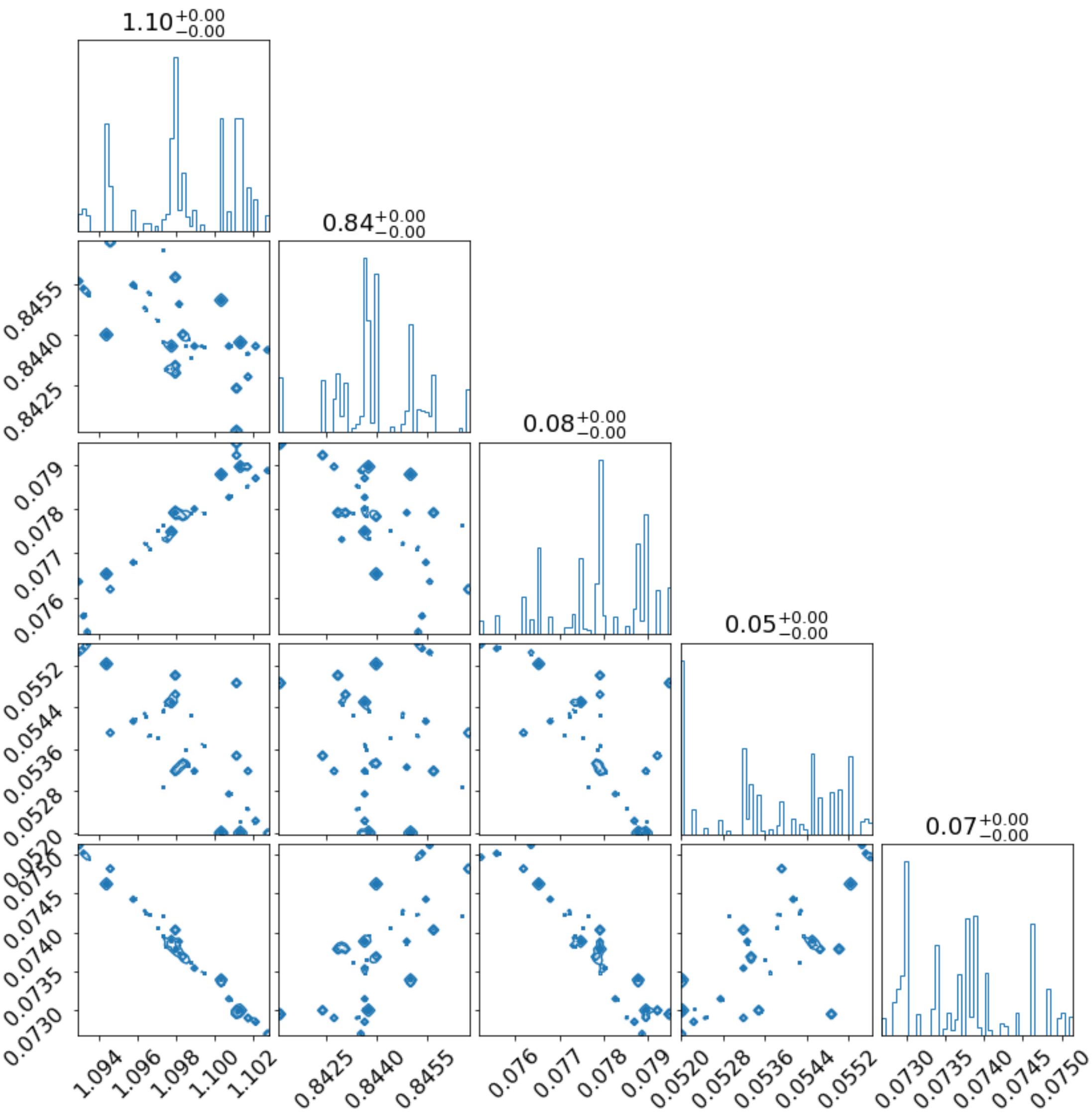





```
In [18]: runcell('----------------------------------------plotting the model +noise
values---------------------------------', '/Users/aj3008/Desktop/MS_3rd_Sem/
Comp_methods_in_AST/Comp_methods/Hw_9/pb1.py')
c= 1.098479015918067 phi= 0.8439781586629621 phi12= 0.07793689909117908 phi132=
0.05369646245989251 sigmaz= 0.07371053832247128

In [19]: runcell(3, '/Users/aj3008/Desktop/MS_3rd_Sem/Comp_methods_in_AST/Comp_methods/
Hw_9/pb1.py')
<Figure size 432x288 with 0 Axes>

In [20]:
```



$1.10^{+0.00}_{-0.00}$
$0.84^{+0.00}_{-0.00}$
$0.08^{+0.00}_{-0.00}$
$0.05^{+0.00}_{-0.00}$
$0.07^{+0.00}_{-0.00}$
0.8425
0.8440
0.8455
0.076
0.077
0.078
0.079
0.0520
0.0528
0.0536
0.0544
0.0552
0.0730
0.0735
0.0740
0.0745
0.0750
1.094
1.096
1.098
1.100
1.102

Task-3

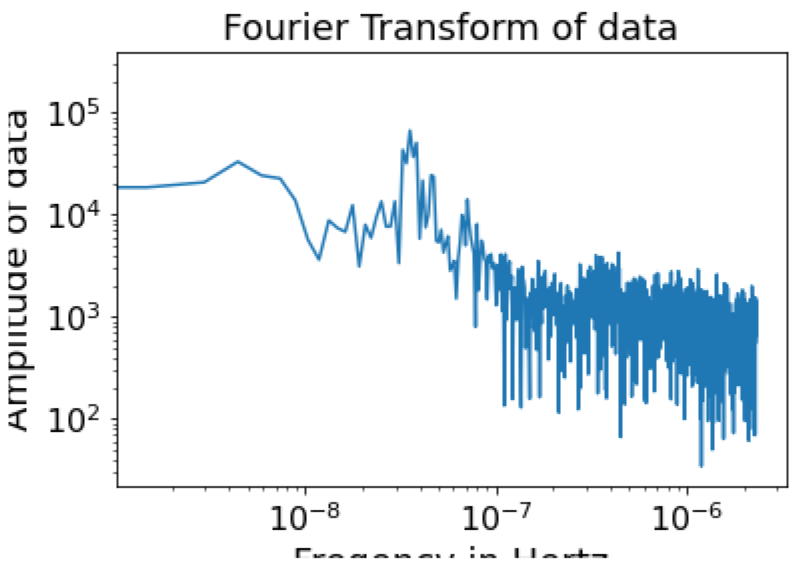
Here we are asked to plot the spectrum. I used np.fft.fft to find the fourier transform. Using the fourier transform I calculated the Amplitude. The frequency was calculated using a frequency function described below:

- **freq(time)**:
  This finds the frequency for the fourier transform using the time at which data was taken.

  Parameters:
  time : time at which each sunspot value was taken(in days).

  Returns:
  frequency in Hertz.

Results of task-3

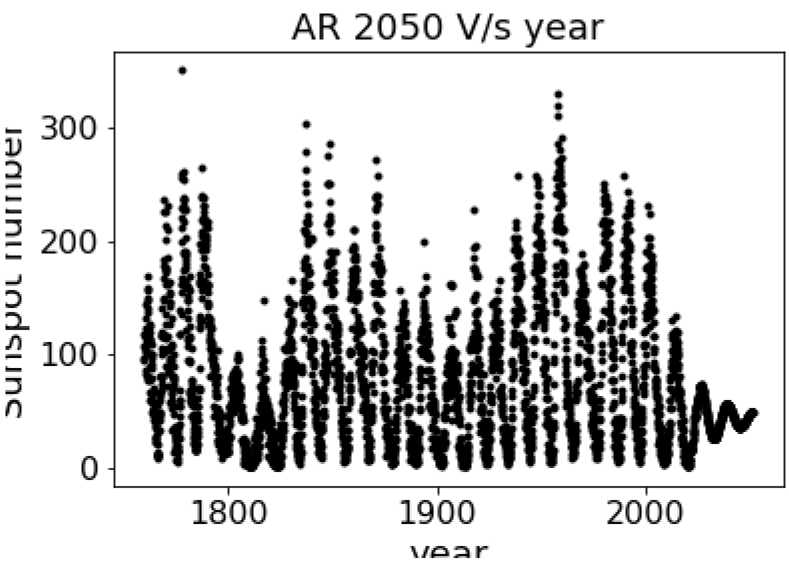


From the plot it we can see there are two peaks. I think the two peak represent the yearly periodicity and 11 year periodicity. I imagine the yearly periodicity to be a wave enveloped by the 11 year periodicity wave.

Task-4

We are asked to plot till year 2050. To do that I first made a new time array which goes from 1749 - 2050. Then using the below equation and parameter values found earlier I found the sunspot values at different values. I fell my model is good for predicting this far in time.

Plot



Bonus

This problem asks us to find AIC for three models, including the model I used for earlier tasks , values using the equation;

$$\text{AIC} = 2k - 2\ln(L_{max})$$

Here, I used the following three AR models.Mathematically;

$$\text{Model-1} = c + \phi_1 X_{t-1} + \phi_{12} X_{t-12} + \phi_{132} X_{t-132} + N(0, \sigma_z)$$

This model accounts for previous value dependence, yearly cycle and 11 year cycle. k value is 5.

$$\text{Model-2} = c + \phi_1 X_{t-1} + \phi_{12} X_{t-12} + N(0, \sigma_z)$$

This model accounts for previous value dependence and yearly cycle. k value is 4.

$$\text{Model-3} = c + \phi_1 X_{t-1} + N(0, \sigma_z)$$

This model accounts for jusr previous value dependence. k value is 3.

```
In [21]: runcell(1, '/Users/aj3008/Desktop/MS_3rd_Sem/Comp_methods_in_AST/Comp_methods/
Hw_9/Bonus.py')
100%|██████████| 100/100 [00:18<00:00,  5.52it/s]
100%|██████████| 100/100 [00:18<00:00,  5.40it/s]
100%|██████████| 100/100 [00:18<00:00,  5.35it/s]
model 1 and 2 comparison 4.128818819509455
model 1 and 3 comparison 4.5381489151464764
model 2 and 3 comparison 1.099139757284301

In [22]:
```

Comparison was done by taking the ratio of AIC values.
I think it makes sense that model 1 is the best since it accounts for all dependencies and cycles. Model 2 accounts for yearly cycle so it is somewhat better than model 3.