Comp Methods Homework 8

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https://github.com/AasimZahoor/Comp_methods.git

Question 1.

Given
The problem asks to find Mass M and separation R of the system WD 0727+482, a known double WD system. This is to be done by finding the Fourier transform of the given strain data and using the equations given in the Homework sheet. We are asked to use the Cooley-Tukey method of Fourier Transform.

Approach
To find out the Fourier transform of the given data I first defined three functions. They are:

- **DFT(x)**:
  This is the brute force Fourier transform

  Parameters:
  x : data(in time space)(strain).

  Returns:
  data in frequency space

- **FFT(x)**:
  This is the Fast Fourier Transform based on Cooley Tukey method.

  Parameters:
  x : data (in time space)(strain).

  Returns:
  data in frequency space.

- **freq(time)**:
  This finds the frequency for the fourier transform using the time at which data was taken.

  Parameters:
  time : time at which each strain value was taken(in days).

  Returns:
  frequency in Hertz.

I started by calling the FFT function and using the strain as the input data.The FFT is based on Cooley Tukey method which can be writen mathematically:

$$H_k = \sum_{m=0}^{N/2-1} h_{2m} w^{2km} + w^k \sum_{m=0}^{N/2-1} h_{2m+1} w^{2km}$$

Here,

$$h_n \text{ is the nth strain value}$$

$$w = e^{2\pi i/N}$$

Also, we exploit the symmetry which is:

$$H_{k+N/2} = \sum_{m=0}^{N/2-1} h_{2m} w^{2km} - w^k \sum_{m=0}^{N/2-1} h_{2m+1} w^{2km}$$

Since the sums themselves are DFTs we can continue dividing them further into even and odd terms. In my code, the DFT terms are divided into even and odd terms till N=128 is reached. At N=128, the brute force DFT function is called and it calculates the Fourier transform of those 128 terms. Using symmetry equation we calculate the Fourier transform of the remaining terms. This goes on till the Fourier transform of the entire data set is found.

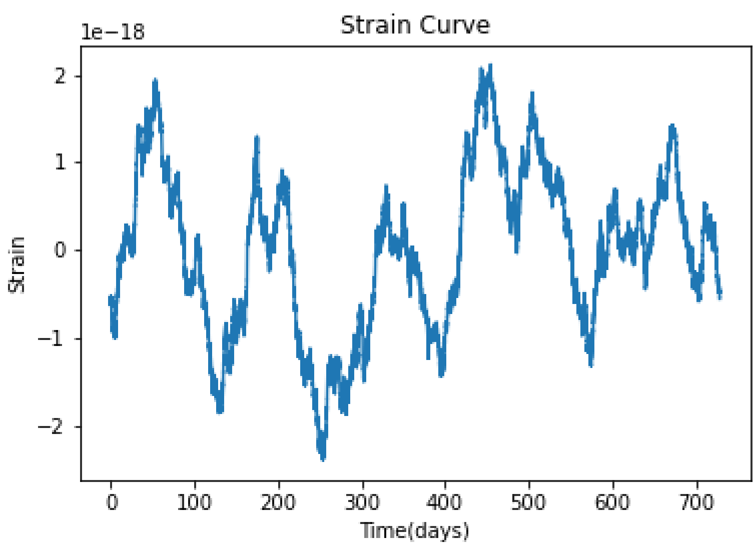
After finding the Fourier transform I calculated the amplitudes and frequency. Frequency is calculated using the frequency function where we find sample frequency by averaging over time differences and taking inverse of that. Then looking at the plot gives us a range where the peak lies, so we make the code search for a high amplitude in that range. This gives us vales of h and f_{GW}.

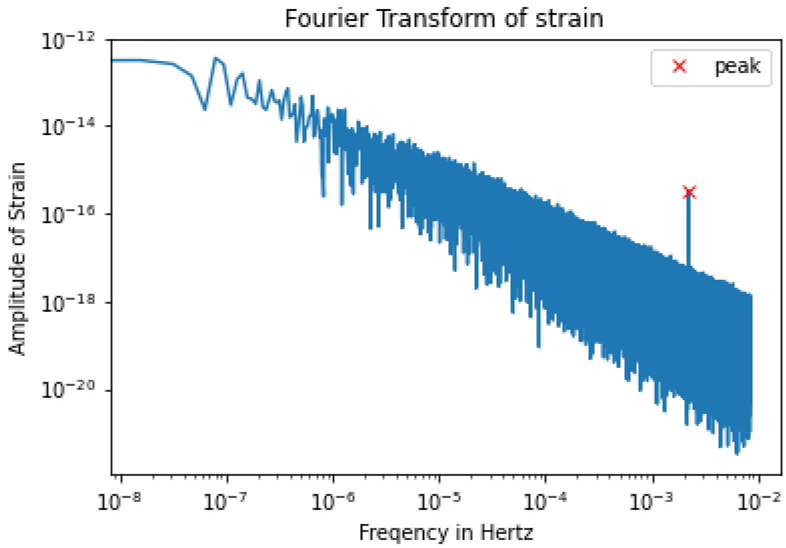
Using the values of h and f_{GW}, and the below equations I found M and R.

$$h = 2.6 * 10^{-21} * M^2 * 12 * R^{-1}$$

$$f_{GW} = 10^{-4} * M^{1/2} * R^{-3/2}$$

Results:





The red point confirms visually that the point I am considering represents the peak. This is a log-log plot.

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In [25]: runcell('\\#%%', '/Users/aj3008/Desktop/MS_3rd_Sem/Comp_methods_in_AST/
Comp_methods/Hw_8/pb1.py')
h= 6.404443898907851e-22 f_Gw= 0.0022037506103515627 Hertz
R is 0.1046183601328835 radius of sun
M is 0.5560945208533652 mass of sun
```