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Parameter Estimation of a Gravitational Wave Source

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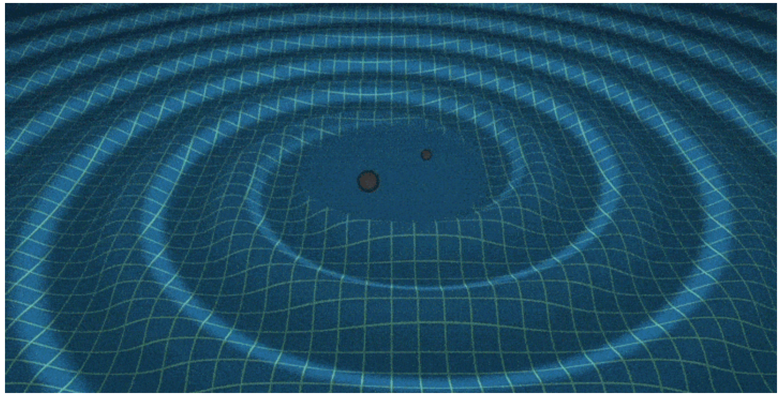
https://github.com/AasimZahoor/Comp_project.git

What are Gravitational Waves?



- Gravitational waves are disturbances in the curvature of spacetime
- As a gravitational wave passes an observer, that observer will find spacetime distorted by the effects of strain.

Gravitational Wave Sources

- Every moving object with mass produces gravitational waves.
- We can only detect Binary Black Hole (BBH) or Binary Neutron Star (BNS) systems using current equipment.
- The first gravitational wave **GW150914** detected in 2015 was produced by a Binary Black Hole system.

What do we mean by Parameter Estimation?

- A binary system has 8 intrinsic parameters and 9 extrinsic parameters.
- 8 Intrinsic parameters are mass m_1, m_2, s_{1x}, s_{1y}, s_{1z}, s_{2x}, s_{2y}, s_{2z}.
- 9 extrinsic parameters are due to position relative to the Binary system such as the distance and are usually not important.
- We study the gravitational wave to recover these parameters and this is called parameter estimation.

How do we model Gravitational Waves?

1. Post-Newtonian Theory

Post-Newtonian (PN) formalism is an approximation to GR in slow-motion, weak field regime.

2. Numerical Relativity

Solving Einstein's equations numerically to study the dynamics of the binary system and hence the gravitational wave. Most accurate but computationally expensive.

3. Effective-One-Body

Approximation to GR. Binary system is reduced to a test particle with the reduced mass μ moving in an effective Kerr (Rotating blackhole) background spacetime.

Schmidt, 2020
Gravitational Waves From Binary Black Hole Mergers: Modeling and Observations

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Einstein's Field Equation

10 non linear partial differential equations solved over multiple times. Not an easy task!

4. Phenomenological Waveforms

Instead of focusing on the dynamics of gravitational wave source, we model the gravitational wave directly.

5. Numerical Relativity Surrogates

Interpolate between the different Numerical Relativity solutions. Most accurate after the Numerical Relativity solutions.

Schmidt, 2020
Gravitational Waves From Binary Black Hole Mergers: Modeling and Observations

My Approach

- Post Newtonian Theory.
- Others computationally expensive and wouldn’t be possible to do on one tiny laptop.
- Even in Post Newtonian Theory one lower orders taken.
- Using Synthetic data

Approach contd

- 3 Assumptions:

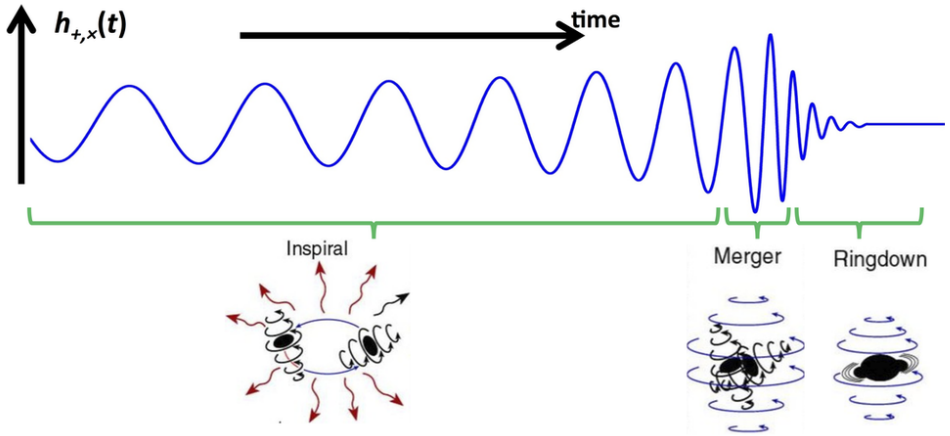
1. Weak field: Leads to linearized field equations.

$$\text{metric} = \text{flat space} + \text{strain}$$

$$g_{ab} = n_{ab} + h_{ab}$$

Approach contd

- 2. Can only use inspiral phase to infer parameters.



- 3. No spin

$$\overline{S_1} = 0 \,,\ \overline{S_2} = 0$$

My Model

M= m1+m2
μ= m1*m2/(m1+m2)
r = distance at which the wave is detected
R = orbital separation

The strain (h) produced by the gravitational wave is given by:

$$h = {}^{\mu M}/_{rR} \qquad \text{Eq 1}$$

The distance R between the two objects changes according to this equation:

$$\frac{dR}{dt} = \mu M^2/R^3 \qquad \text{Eq 2}$$

The noise is Gaussian of zero mean:

Noise = *N*(0, sigma)

Equations from: M. Celoria *et al*
"Lecture notes on black hole binary astrophysics"

Code

Likelihood function

$$L = \frac{1}{\sqrt{2\pi\sigma^2}} exp^{-\frac{1}{2}(\frac{data-model}{\sigma})^2} \qquad \text{Eq 3}$$

Monte Carlo Markov Chain algorithm

If(r>=1)

$$x_t = y$$

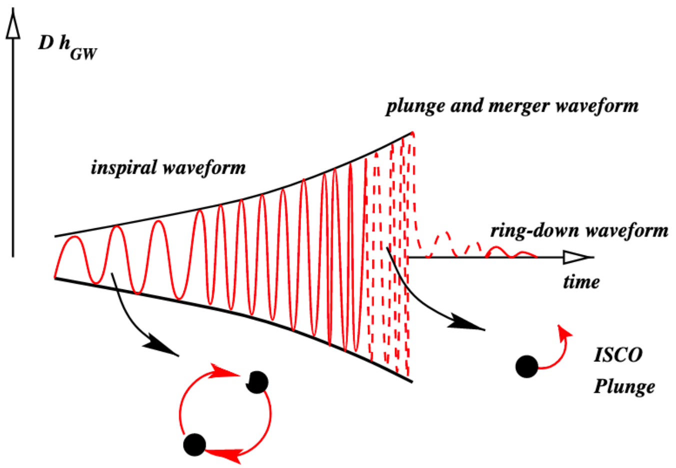
If(r<1)

$$x_t = y \quad \text{if} \quad U(0,1) <= r \qquad \text{OR} \qquad x_t = x_t \quad \text{if} \quad U(0,1) > r$$

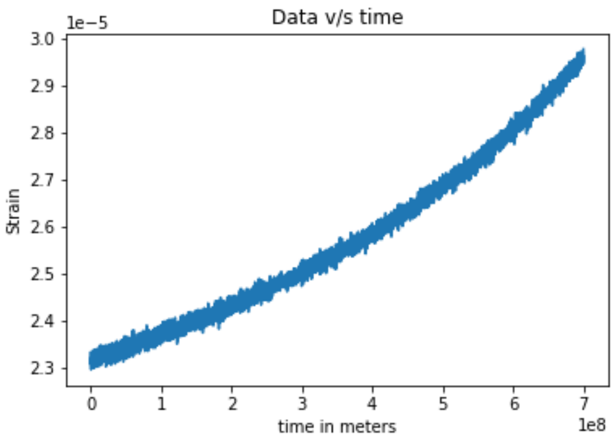
Here,

$$r = \frac{likelihood(model, y)}{likelihood(model, x_t)}$$

Data



Expected data



Synthetic data

Negative h values omitted

Synthetic Data is for a Binary Black Hole system with m_1=35 M_{sun} and m_2 =30 M_{sun}
Error = $N(0,10^{-7})$ (sigma is 0.1 times the value of smallest h)

Results

The initial values were
m_1= 40 M_{sun},
m_2 =40 M_{sun}
Sigma =2*10-7 (kept fixed)
Number of iterations= 1000

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the value of parameters is [35.262679199930616, 29.744067538466243, 2e-07]

In [84]:
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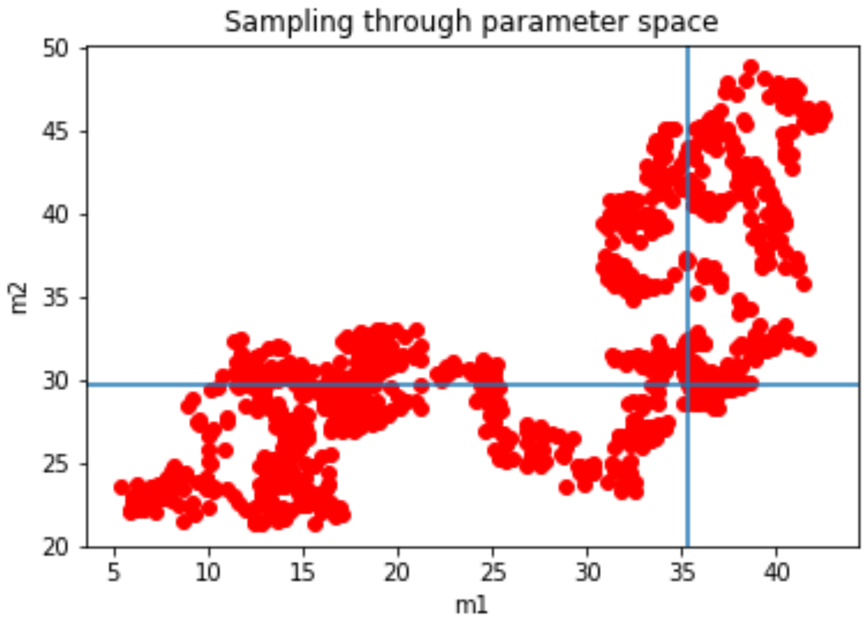

The plot shows the different values my MCMC went through. The blue lines represent the inferred value of parameters

Potential Improvement

- Make MCMC faster by reducing the number of times it checks the likelihood ratio.
- Add priors
- Increase the number of iterations.

Thank you