Scientific Computing
Assigment 3

Andres Alam Sanchez Torres

November 10, 2023

Problem 1

$$
\begin{aligned}
00100101_2 &= 1 \cdot 2^0 + 0 \cdot 2^1 + 1 \cdot 2^2 + 0 \cdot 2^3 + 0 \cdot 2^4 + 1 \cdot 2^5 + 0 \cdot 2^6 + 0 \cdot 2^7 \\
&= 1 + 4 + 32 \\
&= 37 \\
\text{Two's Complement: } & 11011010_2 + 1_2 \\
& 11011011_2
\end{aligned}
$$

$$
\begin{aligned}
01110101_2 &= 1 \cdot 2^0 + 0 \cdot 2^1 + 1 \cdot 2^2 + 0 \cdot 2^3 + 1 \cdot 2^4 + 1 \cdot 2^5 + 1 \cdot 2^6 + 0 \cdot 2^7 \\
&= 1 + 4 + 16 + 32 + 64 \\
&= 117 \\
\text{Two's Complement: } & 10001010_2 + 1_2 \\
& 10001011_2
\end{aligned}
$$

With the same procedure:

$$
\begin{aligned}
0101100010010110_2 &= 22678 \\
\text{Two's Complement: } & 1010011101101010_2
\end{aligned}
$$

$$
\begin{aligned}
0110100110100110_2 &= 27046 \\
\text{Two's Complement: } & 1001011001011010_2
\end{aligned}
$$

Problem 2

1. How many different bit patterns can be represented using 63 bits?
   2^{63}

2. If we want to store any integer x where $0 \leq x \leq 25$. What is the smallest number of bits we can use?
   $\lceil log_2(25) \rceil = 5$
   To elaborate further, $25 = 11001_2$, so we need at least 5 digits to represent 25 in binary.

3. If we represent colours as red-green-blue (RGB) triples, where we use 4 bits for red, 4 bits for green, and 4 bits for blue. How many different colours can we represent?
   As a whole, a color will be represented by 12 bits, which can represent 2^{12} colors.

4. If we represent a decimal number (like 13579) by concatenating the binary representations of each decimal digit. How many bits would we need for a number which contains n digits?

In general,

$$\sum_{i=0}^{n} \begin{cases} d_i \in \{0,1\} & 1 \\ \text{other} & \lceil log_2(d_i) \rceil \end{cases}$$

Where d_i is the ith digit of the decimal number. If we use a fixed set for each digit we will need 4 bits per digit (since the maximum value for a digit, 9, requires such bits). Then, we will need $4 \cdot n$ bits.