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[Tutorial-4]

1. $T(n) = 3T(n/2) + n^2$

$\Rightarrow T(n) = aT(n/b) + f(n)$

$a \geq 1, b > 1$

On comparing

$a = 3, b = 2, f(n) = n^2$

Now,

$c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$\therefore f(n) \gg n^c$

$\therefore T(n) = \theta(n^2)$

2. $T(n) = 4T(n/2) + n^2$

$\Rightarrow a \geq 1, b > 1$

$a = 4, b = 2, f(n) = n^2$

$c = \log_2 4 = 2$

$\therefore n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \theta(n^2 \log_2 n)$

3. $T(n) = T(n/2) + 2^n$

$\Rightarrow a = 1, b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \theta(2^n)$

4. $T(n) = 2^n T(n/2) + n^n$

$\Rightarrow a = 2^n, b = 2, f(n) = n^n$

$c = \log_b a = \log_2 2^n = n$

$n^c \Rightarrow n^n$

$\therefore f(n) = n^c$

$\therefore T(n) = \theta(n^2 \log_2 n)$

5. $T(n) = 16T(n/4) + n$

$\Rightarrow a = 16, b = 4, f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2$

$n^c = n^2$

$f(n) = n^2$

$f(n) < n^c$

$T(n) = \theta(n^2)$

6. $T(n) = 2T(n/2) + n \log_2 n$

$\Rightarrow a = 2, b = 2, f(n) = n \log_2 n$

$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$

since, $n \log_2 n > n$

$\therefore f(n) \gg n^c$

$\therefore T(n) = \theta(n \log_2 n)$

7. $T(n) = 2T(n/2) + n/\log n$

$\Rightarrow a = 2, b = 2, f(n) = n/\log n$

$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$

since, $\frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n)$

8. $T(n) = 2T(n/4) + n^{0.51}$

$\Rightarrow a = 2, b = 4, f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$\therefore n^c = n^{0.5}$

since, $n^{0.5} < n^{0.51}$

$f(n) \gg n^c$

$\therefore T(n) = \theta(n^{0.51})$

9. $T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$

$\Rightarrow a = 0.5, b = 2$

Since, acc. to Master theorem $a \geq 1$, but here a is 0.5 so we cannot apply master theorem.

11. $4T(n/2) + \log n$

$\Rightarrow a = 4, b = 2, f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$f(n) = \log n$

Since $\log n < n^2$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n^c)$

$= \Theta(n^2)$

13. $T(n) = 3T(n/2) + n$

Sol^n $a = 3, b = 2, f(n) = n$

$c = \log_b a = \log_2 3 = 1.5849$

$\therefore n^c = n^{1.5849}$

$\therefore n < n^{1.5849}$

$\Rightarrow f(n) = n^c$

$\therefore T(n) = \Theta(n^{1.5849})$

15. $T(n) = 4T(n/2) + n$

$\Rightarrow a = 4, b = 2, c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$\therefore cn < n^2$ (for any constant)

$\therefore f(n) \leq n^c$

$\therefore T(n) = \Theta(n^2)$

10. $T(n) = 16T(n/4) + n!$

$\Rightarrow a = 16, b = 4, f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

Now, $n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \Theta(n!)$

12. $T(n) = \text{sqrt}(n)\,T(n/2) + \log n$

$\Rightarrow a = \sqrt{n}, b = 2,$

$\therefore c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2}\log_2 n$

$\therefore \frac{1}{2}\log_2 n < \log(n)$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(f(n))$

$= \Theta(\log n)$

14. $T(n) = 3T(n/3) + \text{sqrt}(n)$

$\Rightarrow a = 3, b = 3, c = \log_b a = \log_3 3 = 1$

$\therefore n^c = n^1 = n$

As $\text{sqrt}(n) < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n)$

16. $T(n) = 3T(n/4) + n\log n$

$\Rightarrow a = 3, b = 4, f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$\therefore n^{0.792} < n\log n$

$\therefore T(n) = \Theta(n \log n)$

17. $T(n) = 3T(n/3) + n/2$

$\Rightarrow a=3, b=3, c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

$\therefore n^c = n^1 = n$

As $n/2 < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n)$

18. $T(n) = 6T(n/3) + n^2 \log n$

$\Rightarrow a=6, b=3, c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

As $n^{1.6309} < n^2 \log n$

$\therefore T(n) = \theta(n^2 \log n)$

19. $T(n) = 4T(n/2) + n/\log n$

$\Rightarrow a=4, b=2, f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\therefore \frac{n}{\log n} < n^2$

$\therefore T(n) = \theta(n^2)$

20. $T(n) = 64T(n/8) - n^2 \log n$

$\Rightarrow a=64, b=8$

$c = \log_b a = \log_8 64 = \log_8 (8)^2$

$c = 2$

$\therefore n^c = n^2$

$\therefore n^2 \log n > n^2$

$\therefore T(n) = \theta(n^2 \log n)$

21. $T(n) = 7T(n/3) + n^2$

$\Rightarrow a=7, b=3, f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$\Rightarrow n^{1.7712} < n^2$

$\therefore T(n) = \theta(n^2)$

22. $T(n) = T(n/2) + n(2 - \cos n)$

$\Rightarrow a=1, b=2, c = \log_b a = \log_2 1 = 0$

$\therefore n^c = n^0 = 1$

$\therefore n(2 - \cos n) > n^c$

$\therefore T(n) = \theta(n(2 - \cos n))$