Assignment on Travelling Salesman Problem using "Branch and Bound" method and Heuristic method

Let's denotes the cities from **1 to *n*** and city 1 be the start-city of the salesperson. Also let's assume that **c (*i*, *j*)** is the visiting cost from any city *i* to any other city *j*. The systematic way of solving this problem is mentioned below:

a) Heuristic Algorithm for TSP:

1. First, find out all (n -1)! Possible solutions, where n is the number of cities.
2. Next, determine the minimum cost by finding out the cost of everyone of these (n -1)! Solutions.
3. Finally, keep the one with the minimum cost.

A salesman has to visit five cities A.B, C, D and E. The distances (in hundred kilometers) between the five cities are shown in Table 1. If the salesman starts from city A and has to come back to city A, which route should he select so that total distance traveled become minimum?

		To City				
		A	**B**	**C**	**D**	**E**
From City	**A**	**-**	**1**	**6**	**8**	**4**
	B	**7**	**-**	**8**	**5**	**6**
	C	**6**	**8**	**-**	**9**	**7**
	D	**8**	**5**	**9**	**-**	**8**
	E	**4**	**6**	**7**	**8**	**-**

b) Branch and Bound Algorithm for TSP:

Hints are given below:

- Definition: Find a tour of minimum cost starting from a node S going through other nodes only once and returning to the starting point S.

- Definitions:

  - ✓ A row(column) is said to be reduced iff it contains at least one zero and all remaining entries are non-negative.

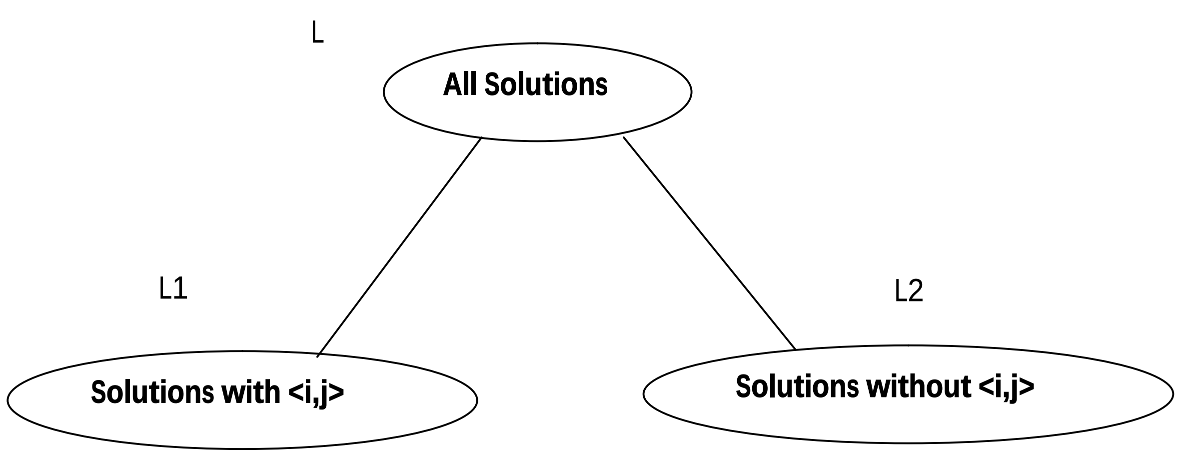
  - ✓ A matrix is reduced iff every row and column is reduced.

- **<u>Branching</u>**:

  - ✓ Each node splits the remaining solutions into two groups: those that include a particular edge and those that exclude that edge

  - ✓ Each node has a lower bound.

  - ✓ Example: Given a graph G=(V,E), let <i,j> $\in$ E,



- **<u>Bounding</u>**: How to compute the cost of each node?

  - ✓ Subtract of a constant from any row and any column does not change the optimal solution (The path).

  - ✓ The cost of the path changes but not the path itself.

  - ✓ Let A be the cost matrix of a G=(V,E).

  - ✓ The cost of each node in the search tree is computed as follows:

    - Let R be a node in the tree and A(R) its reduced matrix
    - The cost of the child (R), S:
      - Set row i and column j to infinity
      - Set A(j,1) to infinity
      - Reduced S and let RCL be the reduced cost.
      - C(S) = C(R) + RCL+A(i,j)

  - ✓ Get the reduced matrix A' of A and let L be the value subtracted from A.
  - ✓ L: represents the lower bound of the path solution
  - ✓ The cost of the path is exactly reduced by L.

- What to determine the branching edge?

  - ✓ The rule favors a solution through left subtree rather than right subtree, i.e., the matrix is reduced by a dimension.

  - ✓ Note that the right subtree only sets the branching edge to infinity.

  - ✓ Pick the edge that causes the greatest increase in the lower bound of the right subtree, i.e., the lower bound of the root of the right subtree is greater.