Diffeomorphic Temporal Alignment Nets

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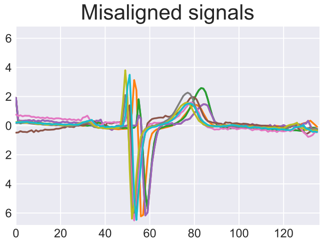
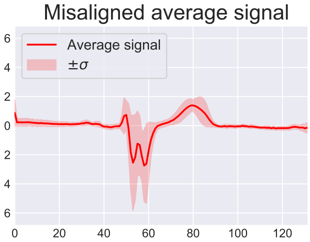
Outline

1. Nonlinear Misalignment
2. Preliminaries
3. Diffeomorphic Temporal Alignment Nets
4. Experiments and Results
5. Conclusion

Problem Formulation - Time-Series Joint Alignment

- Time-series data often presents a significant amount of nonlinear misalignment.
- This may include simple phase-shift between similar instances, artifacts due to sampling methods and/or stretching and squeezing of different regions of the signal.
- Even trivial task such as computing the arithmetic mean (i.e. average signal) is hard under nonlinear misalignment.





Problem Formulation - Time-Series Joint Alignment

- Let $(\boldsymbol{U}_i)_{i=1}^N$ be a set of N time-series observations. The nonlinear misalignment could be written as:

$$(\boldsymbol{U}_i)_{i=1}^N = (\boldsymbol{V}_i \circ W_i)_{i=1}^N \tag{1}$$

- Where $\boldsymbol{U}_i$ is the i^{th} misaligned signal, $\boldsymbol{V}_i$ is the i^{th} latent aligned signal and W_i is a latent warp of the domain of $\boldsymbol{V}_i$.
- For technical reasons, the misalignment is usually viewed in terms of $T_i \triangleq W_i^{-1}$, the inverse warp of W_i, implicitly suggesting W_i is invertible.
- It is also typically assumed that $(T_i)_{i=1}^N$ belong to some nominal family of warps, parametrized by $\boldsymbol{\theta}$:

$$(\boldsymbol{V}_i)_{i=1}^N = (\boldsymbol{U}_i \circ T^{\boldsymbol{\theta}_i})_{i=1}^N\,, \quad T_i = T^{\boldsymbol{\theta}_i} \in \mathcal{T}\ \forall i \in (1,\ldots,N)\,. \tag{2}$$

Problem Formulation - Time-Series Joint Alignment

- We argue that this problem should be seen as a learning one, mostly due to the need for generalization, and define:

Definition (the joint-alignment problem)

Given the observations $(\boldsymbol{U}_i)_{i=1}^{N}$, infer the latent $(T^{\boldsymbol{\theta}_i})_{i=1}^{N} \subset \mathcal{T}$.

- To solve the joint alignment problem one must define a:
  - Warp family - $\mathcal{T}$
  - Model capable of generalizing the learned alignment.
  - Metric measuring the misalignment.
- We will define each in following slides.

Preliminaries - Spatial/Temporal Transformer Nets

- Introduced by Jaderberg et al. (2015) , the Spatial Transformer Nets (STN) is an end-to-end differential module that allows for certain invariances to spatial warps.
- Paramerized by any differential transformation family (*i.e.* Affine), the STN Learns input-depended spatial transformation to allow for robuts spatial invariance in deep neural networks.

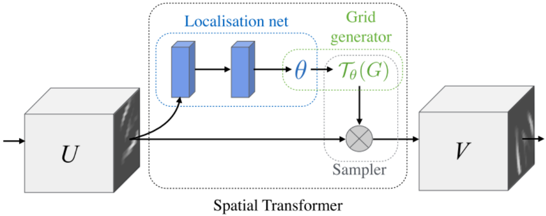


Figure taken from: Jaderberg et al. NIPS (2015)

Preliminaries - Spatial/Temporal Transformer Nets

- Temporal Transformer Nets (TTN), the time-series analog of STN.
- In more detail, let $\boldsymbol{U}$ denote the input of the TT layer. Its output consists of $\boldsymbol{\theta} = f_{\mathrm{loc}}(\boldsymbol{U})$ and $\boldsymbol{V} = \boldsymbol{U} \circ T^{\boldsymbol{\theta}}$ (the latter, *i.e.*, the warped signal, is what is being passed downstream the TTN), where $T^{\boldsymbol{\theta}} \in \mathcal{T}$ is a 1D warp parameterized by $\boldsymbol{\theta}$. The function $f_{\mathrm{loc}} : \boldsymbol{U} \mapsto \boldsymbol{\theta}$ is itself a neural net called the localization net.

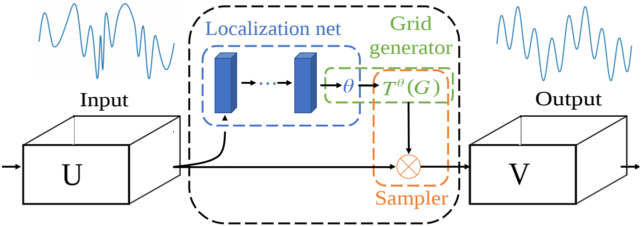


Figure taken from: Skafte et al. CVPR (2018)

Preliminaries - Diffeomorphisms

- As previously mentioned, $\mathcal{T}$ needs to be specified. In the context of time warping, diffeomorphisms are a natural choice (Mumford & Desolneux, 2010).

Definition (Diffeomorphisms)

A diffeomorphism is a differentiable invertible map with a differentiable inverse.

- Unfortunately, most representations of highly-expressive diffeomorphisms are complicated and computationally expensive.
- In our case, since the proposed method explicitly incorporates them in a DL architecture, it is even more important to drastically reduce the computational difficulties.

Preliminaries - CPAB

- We now define the warp family to be incorporated in our model.
- Following Skafte et al. (CVPR, 2018) incorporation of diffeomorphisms in STNs, the CPAB warps that had been proposed by Freifeld et al. (ICCV, 2015; PAMI, 2017) and are also used in this work.
- While any diffeomorphism could be integrated into DTAN, we chose CPAB warps since they:
  1. Are highly expressive and efficient.
  2. Offer an efficient and highly-accurate way to evaluate $x \mapsto \nabla_{\boldsymbol{\theta}} T^{\boldsymbol{\theta}}(x)$ (*i.e.* CPAB gradient).

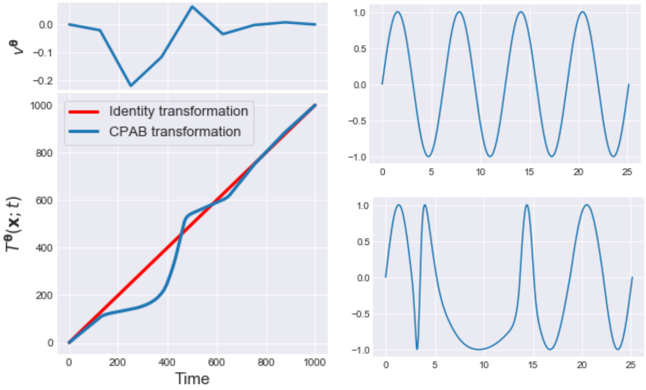
Preliminaries - CPAB

- The name CPAB, short for CPA-Based, is due to the fact that these warps are based on Continuous Piecewise-Affine (CPA) velocity fields. The term “piecewise” is w.r.t. a partition, denoted by Ω, of the signal's domain into subintervals.
- Let $\mathcal{V}$ denote the linear space of CPA velocity fields w.r.t. such a fixed Ω, let $d = \dim(\mathcal{V})$, and let $v^{\boldsymbol{\theta}} : \Omega \to \mathbb{R}$, a velocity field parametrized by $\boldsymbol{\theta} \in \mathbb{R}^d$, denote the generic element of $\mathcal{V}$, where $\boldsymbol{\theta}$ stands for the coefficient w.r.t. some basis of $\mathcal{V}$ is

$$\mathcal{T} \triangleq \{T^{\boldsymbol{\theta}} : x \mapsto \phi^{\boldsymbol{\theta}}(x;1) \text{ s.t. } \phi^{\boldsymbol{\theta}}(x;t) = x + \int_0^t v^{\boldsymbol{\theta}}(\phi^{\boldsymbol{\theta}}(x;\tau))\,\mathrm{d}\tau\}; \quad (3)$$

Preliminaries - CPAB

- Left: An illustration of a CPAB warp (relative to the identity transformation) with its corresponding CPA velocity field (above). Right: a sine wave before (top) and after (bottom) being warped by the presented CPAB transformation.



Diffeomorphic Temporal Alignment Nets

- As mentioned before, to solve the joint alignment problem one must define a:
  - Warp family - $\mathcal{T}$
  - Model capable of generalizing the learned alignment.
  - Metric measuring the misalignment.
- Thus, we set:
  - Warp family - CPAB.
  - Model - Temporal Transformer Nets (TTN).
  - Metric measuring the misalignment - least squares (to be define below).

Loss Function

- Definition 1 requires the specification of $\mathcal{T}$ and a loss function for estimating $(T^{\boldsymbol{\theta}_i})_{i=1}^{N}$.
- Let $\boldsymbol{U}_i$ denote an input signal, let $\boldsymbol{\theta}_i = f_{\text{loc}}(\boldsymbol{U}_i, \boldsymbol{w})$ denote the corresponding output of the localization net $f_{\text{loc}}(\cdot, \boldsymbol{w})$ of weights $\boldsymbol{w}$, and let $\boldsymbol{V}_i$ denote the result of warping $\boldsymbol{U}_i$ by $T^{\boldsymbol{\theta}_i} \in \mathcal{T}$; *i.e.*, $\boldsymbol{V}_i = \boldsymbol{U}_i \circ T^{\boldsymbol{\theta}_i}$.

Loss Function

- As the variance of the observed $(\boldsymbol{U}_i)_{i=1}^N$ is (at least partially) explained by the latent warps, $(T^{\boldsymbol{\theta}_i})_{i=1}^N$, we seek to minimize the empirical variance of the warped signals, $(\boldsymbol{V}_i)_{i=1}^N$. In other words, our data term in this setting is

$$F_{\text{data}}\left(\boldsymbol{w}, (\boldsymbol{U}_i)_{i=1}^N\right) \triangleq \tfrac{1}{N}\sum\nolimits_{i=1}^{N}\left\|\boldsymbol{V}_i(\boldsymbol{U}_i;\boldsymbol{w}) - \tfrac{1}{N}\sum\nolimits_{j=1}^{N}\boldsymbol{V}_j(\boldsymbol{U}_j;\boldsymbol{w})\right\|_{\ell_2}^2 \tag{4}$$

- For multi-class problems, our data term is the sum of the within-class variances:

$$F_{\text{data}}\left(\boldsymbol{w}, (\boldsymbol{U}_i)_{i=1}^N\right) \triangleq \sum\nolimits_{k=1}^{K}\tfrac{1}{N_k}\sum\nolimits_{i:z_i=k}\left\|\boldsymbol{V}_i\left(\boldsymbol{U}_i;\boldsymbol{w}\right) - \tfrac{1}{N_k}\sum\nolimits_{j:z_j=k}\boldsymbol{V}_j(\boldsymbol{U}_j;\boldsymbol{w})\right\|_{\ell_2}^2$$

- where K is the number of classes, z_i takes values in $\{1,\dots,K\}$.

Loss Function - Regularization

- In both the single- and multi-class cases, we also use a regularization term on the warp

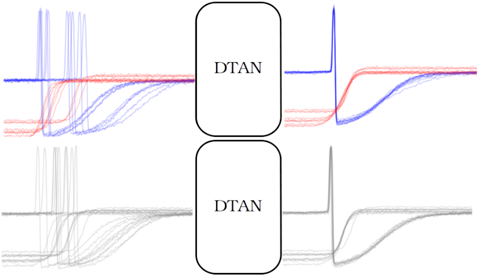
$$F_{\text{reg}}(\boldsymbol{w}, (\boldsymbol{U}_i)_{i=1}^N) = \sum_{i=1}^N (\boldsymbol{\theta}_i(\boldsymbol{w}, \boldsymbol{U}_i))^T \boldsymbol{\Sigma}_{\text{CPA}}^{-1} \boldsymbol{\theta}_i(\boldsymbol{w}, \boldsymbol{U}_i) \tag{5}$$

- Where $\boldsymbol{\Sigma}_{\text{CPA}}$ is a CPA covariance matrix (proposed by Freifeld et al. (ICCV, 2015; PAMI, 2017) associated with a zero-mean Gaussian smoothness prior over CPA fields.
- $\boldsymbol{\Sigma}_{\text{CPA}}$ has two parameters: λ_{var}, which controls the overall variance, and λ_{smooth}, which controls the smoothness of the field.
- A small λ_{var} favors small warps (*i.e.*, close to the identity); similarly, the larger λ_{smooth} is, the more it favors CPA velocity fields that are almost purely affine.
- Our loss function, to be minimized over $\boldsymbol{w}$, is

$$F(\boldsymbol{w}, (\boldsymbol{U}_i)_{i=1}^N) = F_{\text{data}}(\boldsymbol{w}, (\boldsymbol{U}_i)_{i=1}^N) + F_{\text{reg}}(\boldsymbol{w}, (\boldsymbol{U}_i)_{i=1}^N) \,. \tag{6}$$

Diffeomorphic Temporal Alignment Nets

- Diffeomorphic Temporal Alignment Nets (DTAN) Allows for time-series nonlinear joint alignment in an input-dependent manner.
- In a single-class case, the method is unsupervised: the ground-truth alignments are unknown.
- In the multi-class case, it is semi-supervised in the sense that class labels (but not the ground-truth alignments) are used during learning.
- During test, however, the class labels are unknown.



Recurrents DTANs

- We also propose a recurrent varition of DTAN, R-DTAN.
- inspired by, how Lin et al. (CVPR, 2017) used a recurrent net with affine 2D warps, we propose the iteratively warp the input signal by DTAN, using the same localization net.
- By enforcing a zero-boundary condition ($\boldsymbol{U}[0] = \boldsymbol{V}[0], \boldsymbol{U}[n] = \boldsymbol{V}[n]$, where n is the signal's length), we can propagate the signal itself, thus avoiding the need for inverse composition.
- This allows for increasing the expressiveness of the CPAB diffeomorphisms without increasing the number of trainable weights or the partition Ω.

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