Diffeomorphic Temporal Alignment Nets

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Problem Formulation

Goal:

- Statistical analysis of time-series data.

Problem:

- Nonlinear temporal misalignment confounds statistical analysis.

Most traditional methods for alignment usually:

- are computationally expensive;
- do not scale well with N (# of signals) and/or L (signal length);
- lack generalization abilities;
- cannot handle multiple classes;
- are based on pairwise alignment.

Our proposed solution:

- Diffeomorphic Temporal Alignment Nets (DTAN).

(a) Train (b) Test

Figure 1: DTAN generalizes joint alignment from train to test set of the ECGFiveDays dataset [1].

CPAB Diffeomorphisms

- We model the nonlinear time warps using CPAB transformations [3, 4] which combine expressiveness and efficiency.
- CPA = Continuous Piecewise-Affine (velocity fields)
- CPAB = CPA-Based (transformations)

$$\mathcal{T} \triangleq \{ T^{\theta} : x \mapsto \phi^{\theta}(x;1) \text{ s.t. } \phi^{\theta}(x;t) = x + \int_0^t v^{\theta}(\phi^{\theta}(x;\tau))\, \mathrm{d}\tau \text{ where } v^{\theta} \in \mathcal{V} \}$$

where $\mathcal{V}$ is the a space of CPA velocity fields.

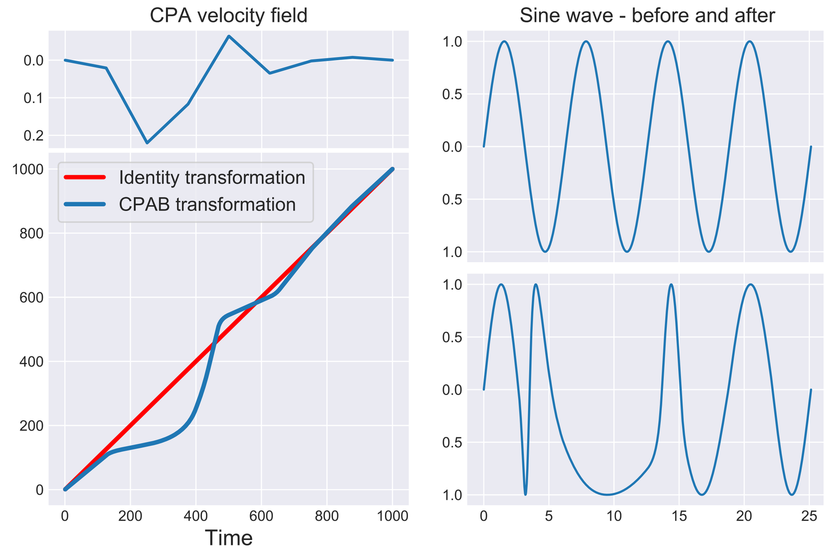


Figure 2: A CPAB warp and a CPA velocity field

Diffeomorphic Temporal Alignment Nets (DTAN)

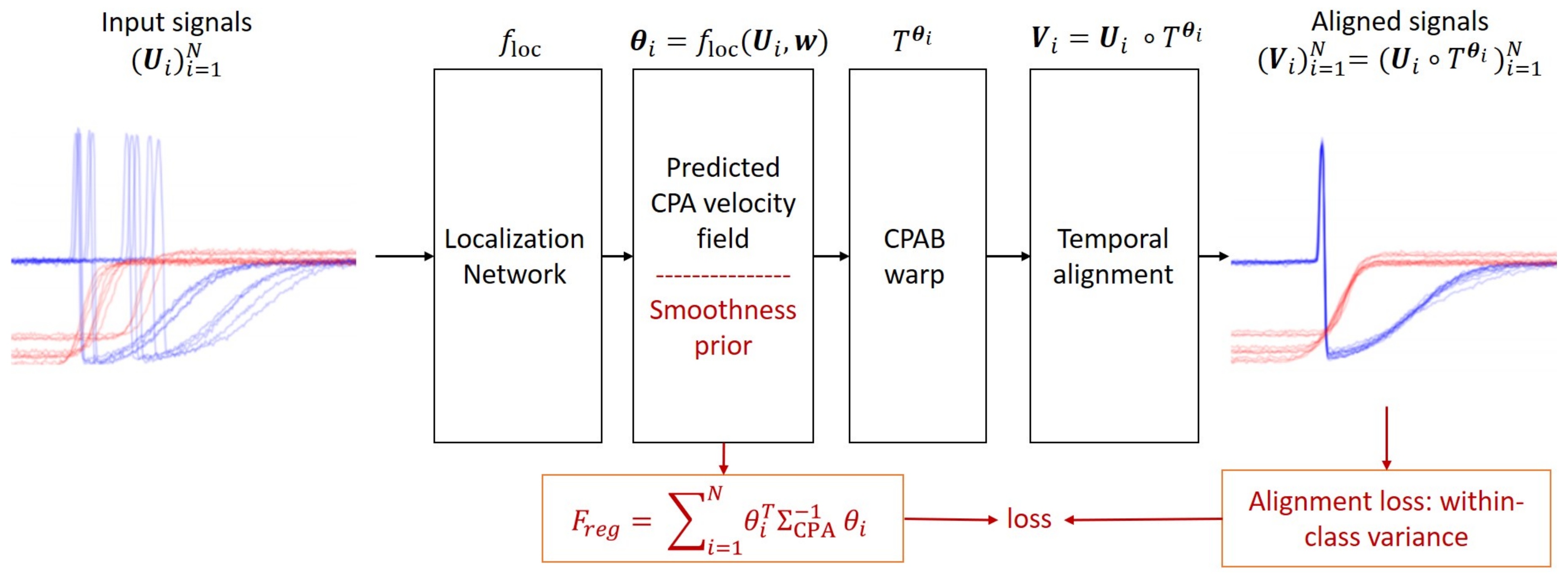


Figure 3: The model is based on Temporal Transformer Nets [5] together with CPAB diffeomorphisms [3, 4].

Loss Function

- The sum of the within-class variances of the aligned signals:

$$F_{\text{data}}\left(\boldsymbol{w}, (\boldsymbol{U}_i)_{i=1}^N\right) \triangleq \sum_{k=1}^{K} \frac{1}{N_k} \sum_{i:z_i=k} \left\| \boldsymbol{V}_i(\boldsymbol{U}_i; \boldsymbol{w}) - \frac{1}{N_k} \sum_{j:z_j=k} \boldsymbol{V}_j(\boldsymbol{U}_j; \boldsymbol{w}) \right\|_{\ell_2}^2$$

- A smoothness prior on the warps:

$$F_{\text{reg}}(\boldsymbol{w}, (\boldsymbol{U}_i)_{i=1}^N) = \sum\nolimits_{i=1}^{N} (\boldsymbol{\theta}_i(\boldsymbol{w}, \boldsymbol{U}_i))^T \Sigma_{\text{CPA}}^{-1} \boldsymbol{\theta}_i(\boldsymbol{w}, \boldsymbol{U}_i)$$

- The full loss function is:

$$F(\boldsymbol{w}, (\boldsymbol{U}_i)_{i=1}^N) = F_{\text{data}}(\boldsymbol{w}, (\boldsymbol{U}_i)_{i=1}^N) + F_{\text{reg}}(\boldsymbol{w}, (\boldsymbol{U}_i)_{i=1}^N)$$

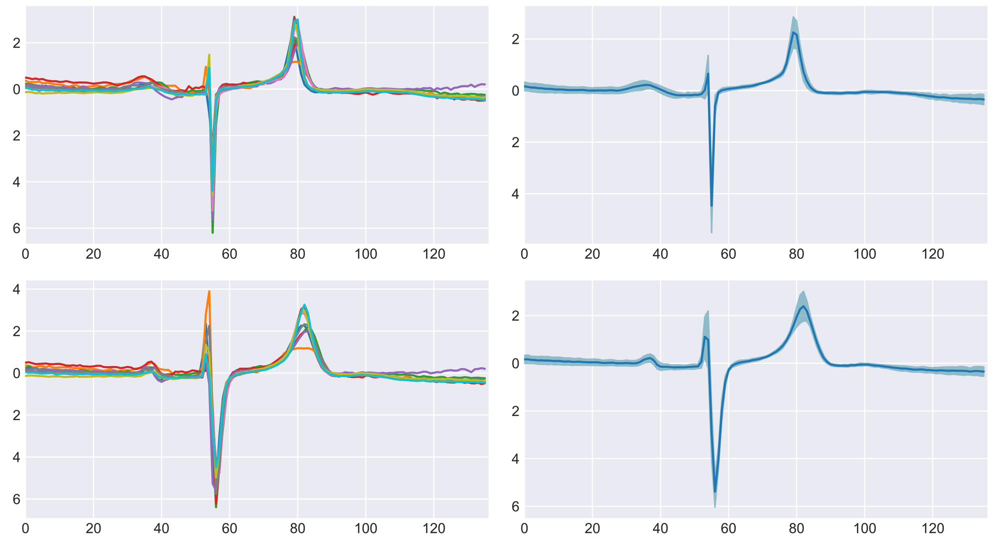


Figure 4: Top: with no prior. Bottom: with a prior

DTAN-CNN

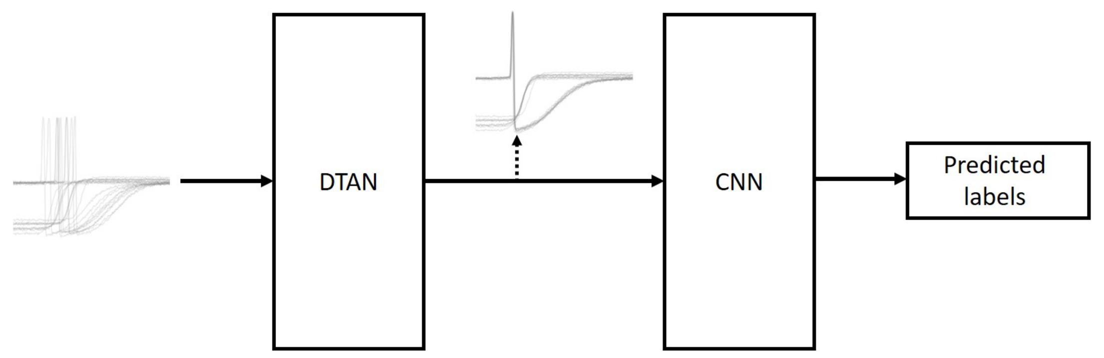


Figure 5: The DTAN-CNN. A trained DTAN is used to align the data which enters the classification net.

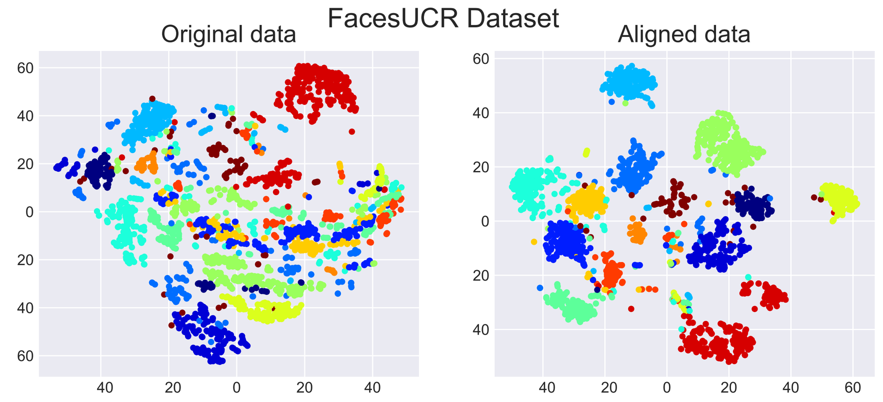
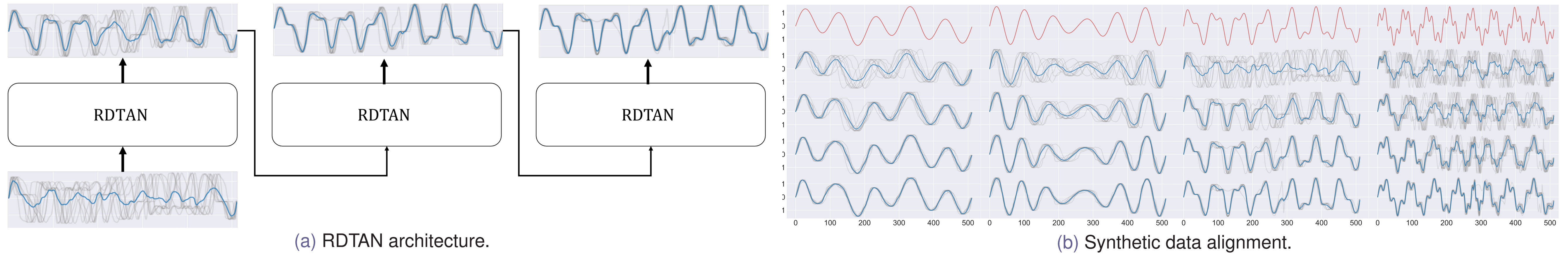


Figure 6: A t-SNE visualization of the FacesUCR dataset from the UCR archive before and after alignment.

Recurrent DTAN (RDTAN)



(a) RDTAN architecture. (b) Synthetic data alignment.

Figure 7: Synthetic data was created by sampling random warps and perturbing the original signals (Red). Columns correspond to different classes.

Experiments

Data:

- The UCR time-series classification archive [1]. Includes 85 real-world datasets.
- Compared with the following time-series averaging/alignment methods: Euclidean averaging, DTW Barycetner Averaging (DBA) [6] and SoftDTW [2].

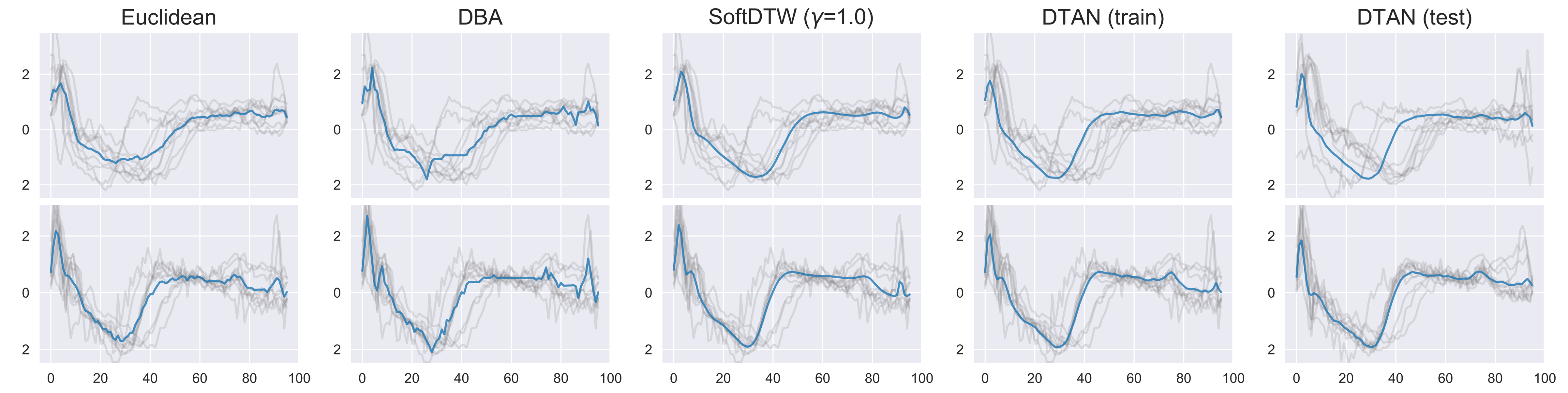


Figure 8: Time-series alignment/averaging comparison. Row: different classes of the ECG200 dataset. Columns: different time-series alignment/averaging methods.

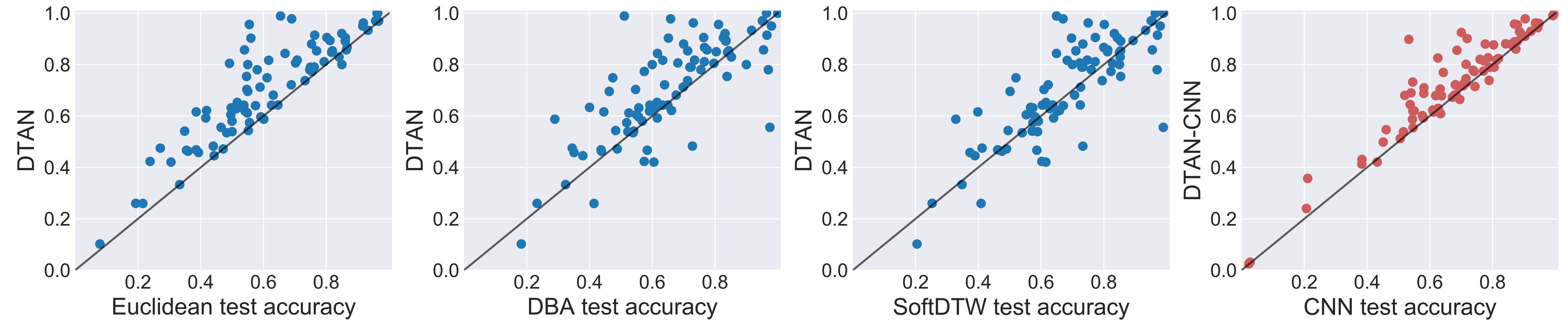


Figure 9: The 3 leftmost panels (Blue): Nearest Centroid test classification results. Each dot corresponds to one dataset of the UCR archive. DTAN's test accuracy is compared with: Euclidean (DTAN was better or no worse in 93% of the datasets), DBA (77%) and SoftDTW (62%). The rightmost panel (Red): DTAN-CNN is compared with CNN (87%).

References

[1] Y. Chen et al. The UCR time-series classification archive, 2015.

[2] M. Cuturi and M. Blondel. Soft-dtw: a differentiable loss function for time-series. In *ICML*, 2017.

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[4] O. Freifeld et al. Transformations based on continuous piecewise-affine velocity fields. In *IEEE TPAMI*, 2017.

[5] M. Jaderberg et al. Spatial transformer networks. In *NeurIPS*, 2015.

[6] F. Petitjean et al. Dynamic time warping averaging of time series allows faster and more accurate classification. In *Data Mining (ICDM)*, 2014.

Code