Quantum Computing

Guide:
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Schmidt decomposition

- The Schmidt decomposition is essentially a restatement of the singular value decomposition in a different context.
- It refers to a particular way of expressing a vector in the tensor product of two inner product spaces.

Theorem

Let H_1 and H_2 be Hilbert spaces of dimensions n and m respectively. Assume $n \geq m$. For any vector w in the tensor product $H_1 \otimes H_2$, there exist orthonormal sets $\{u_1, \ldots, u_m\} \subset H_1$ and $\{v_1, \ldots, v_m\} \subset H_2$ such that $w = \sum_{i=1}^{m} \alpha_i u_i \otimes v_i$, where the scalars α_i are real, non-negative, and unique up to re-ordering.

Proof

The Schmidt decomposition is essentially a restatement of the singular value decomposition in a different context. Fix orthonormal bases $\{e_1, \ldots, e_n\} \subset H_1$ and $\{f_1, \ldots, f_m\} \subset H_2$. We can identify an elementary tensor $e_i \otimes f_j$ with the matrix $e_i f_j^\mathsf{T}$, where f_j^T is the transpose of f_j. A general element of the tensor product

$$w = \sum_{1 \le i \le n, 1 \le j \le m} \beta_{ij} e_i \otimes f_j$$

can then be viewed as the $n \times m$ matrix

$$M_w = (\beta_{ij}).$$

By the singular value decomposition, there exist an $n \times n$ unitary U, $m \times m$ unitary V, and a positive semidefinite diagonal $m \times m$ matrix Σ such that

$$M_w = U \begin{bmatrix} \Sigma \\ 0 \end{bmatrix} V^*.$$

Write $U = \begin{bmatrix} U_1 & U_2 \end{bmatrix}$ where U_1 is $n \times m$ and we have

$$M_w = U_1 \Sigma V^*.$$

Let $\{u_1, \ldots, u_m\}$ be the m column vectors of U_1, $\{v_1, \ldots, v_m\}$ the column vectors of $\overline{V}$, and $\alpha_1, \ldots, \alpha_m$ the diagonal elements of Σ. The previous expression is then

$$M_w = \sum_{k=1}^{m} \alpha_k u_k v_k^{\mathsf{T}},$$

Then

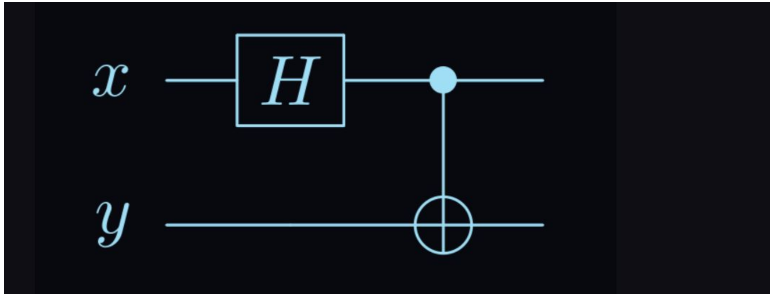
$$w = \sum_{k=1}^{m} \alpha_k u_k \otimes v_k,$$

which proves the claim.

Superdense coding

- Superdense coding also known as dense coding.

- Superdense coding is a quantum communication protocol that allows for a sender to send two classical bits of information to another user by utilizing only one qubit.
- This algorithm takes advantage of something called a “bell circuit”.A bell circuit is a combination of a Hadmard gate going into the control bit of the controlled not gate.
- Superdense coding is the underlying principle of secure quantum secret coding. The necessity of having both qubits to decode the information being sent eliminates the risk of eavesdroppers intercepting messages.

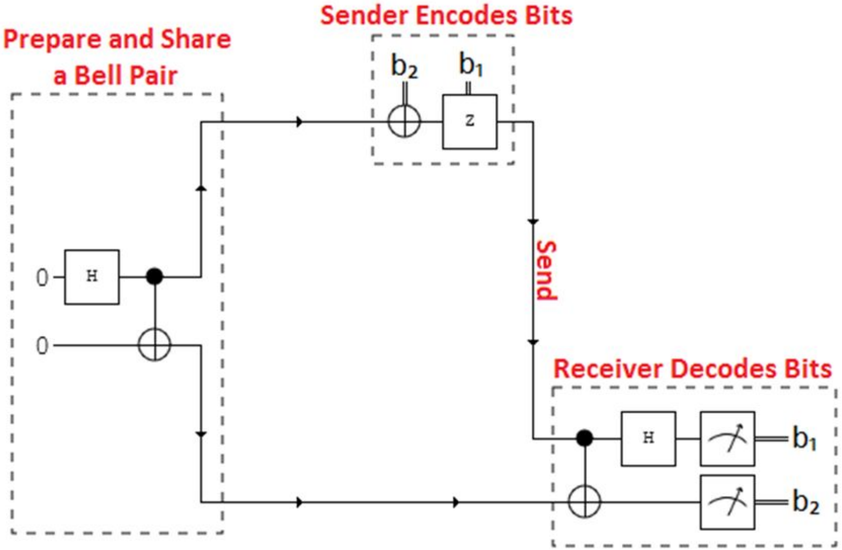


A two qubit bell circuit will have an equal probability of getting either 00 or 11 as result

In the superdense coding algorithm have three different users.

1) The first users shares the two outputs of a two qubit bell state with the receivers and the sender.
2) The second user who is the sender,encode the qubit.
3) The final receivers those qubits and decodes them.

Circuit diagram showing the Superdense coding protocol



The control bit is entangled with the not gate.so when we set the control bit into a super position it also changes the target bit to a superposition as well,because they are entangled

When the sender and receiver share a Bell state, two classical bits can be packed into one qubit. In the diagram, lines carry qubits, while the doubled lines carry classic bits. The variables b_1 and b_2 are classic boolean, and the zeroes at the left hand side represent the pure quantum state |0>

Step 1: Preparing the Bell pair

First a bell pair consisting of 2 qubits is prepared. Where q0 is the senders qubit and q1 is the receivers qubit. To do this q0 is put in to a superposition of states using a hadamard gate.
Then a CNOT operation is performed with q0 being the control and q1 being the target.

Step 2: Encode the information on to q0

- Next the sender has to encode the information they want to send on to q0 by applying certain operations to it.
- If they want to send 00 then they perform no operation.
- If they want to send 01 then they perform a Pauli-Z operation where q1s state is flipped.
- If they want to send 10 then they apply a Pauli-X gate.
- If they want to send 11 then apply a Pauli-Z gate followed by a Pauli-X gate

Step 3: Receiver decodes the information

Next q0 is sent and the receiver has to decode the qubit. This is done by applying a CNOT where the received q0 is the control and q1 is the target. Then a hadamard gate is applied to q0.

Teleportation

Teleportation is the transfer of physical objects from one place to another distant place without transferring the physical particles that constitute the original object.

It involves dematerializing an object at one point and sending the details of that objects precise atomic configuration to another location where it will be reconstructed.

Steps involved in telepotation technology

- Scanning the object completely.
- Dis-assembling the object and sending all the information about the object.
- Reassembling the object from the information which was send.

Applications :

- Human and object teleportation : it would help us to teleport human as well as associated objects instantly without crossing physical distance.
- Quantum computer : computer that has data transmission rates many times faster thantodays most powerful computers.
- Suspended animation : by creating a copy many years after the information was stored.
- Superdense coding : in this two quantum bits can be transmitted for the price of one.

- Backup copies : to create a copy recently stored information if theoriginal being involved in a mishap.

Advantages:

- Teleportation allows a more natural form of conversation as compared to video conferencing.people achieves a sense of presence that cannot be gained from any other technology.
- It can save our organization time and money and enhance our internal and external communication network.
- It would reduce environment degradation ie.pollution,global warming.
- It would help in sustainable development of the mankind.

Thank You…