

MATHEMATICS

Secondary ONE

Year 2021



Name: *Suggested Solution* () Class:

Unit 9B Geometrical Constructions (Chapter 11)

Topical Enduring Understanding

- Geometry is a branch of mathematics that studies the size, shape and position of 2 dimensional shapes and 3 dimensional figures where these shapes and figures are formed by a collection of points.
- Everything is built from points, lines and planes and follows rules.
- In geometry, one explores spatial sense and geometric reasoning and develop problem solving skills.
- Geometrical **diagrams** help us visualise spatial relationships from the given information, and in turn obtain new information.
- Angle properties of a polygon is **invariant** regardless how the shape or size is changed.
- Geometry can be found in art, architecture, engineering, robotics, land surveys, astronomy, sculptures, space, nature, sports, machines, cars, etc. It is used daily by architects, engineers, architects, physicists and land surveyors.

Topical Enduring Questions

- What are the basic parts of any construction or description in geometry?
- How do you find the distance (i.e. length) and the midpoint of a line segment?
- What are angle bisectors, perpendicular lines and how are they useful?

Key Points

- Construction of simple geometrical figures (triangles and quadrilaterals) from given data using mathematical instruments like compasses, ruler, set squares and protractors, where appropriate
- Relationship between any point on the perpendicular bisector and two end points of a line segment
- Relationship between any point on the angle bisector to the two sides of the angle

Textbook: *Think! Mathematics New Syllabus Mathematics 1B (8th edition)* Chapter 11

Online resource: Student Learning Space (learning.moe.edu.sg)

Pre-requisites (Primary Mathematics syllabus, implementation starting with 2013 Primary 1 cohort)

- Properties of rectangle and square, triangle, parallelogram, rhombus and trapezium
- Construct perpendicular and parallel lines using set squares and protractors
- Sketch and draw triangles and special quadrilaterals according to given angles and lengths using ruler, protractor and set squares.

Diagrams help us visualise the given information so that we can think of a solution.
Notations help to convey ideas in a *concise* and *precise* manner. e.g. Pentagon *ABCDE* means that its vertices must be in this order: *A, B, C, D* and *E*. However, it does not matter whether the order is in the clockwise or clockwise direction

BIG IDEA

Lesson 1A Perpendicular Bisector

Estimated duration: 15 min

At the end of the activity, you should be able to construct a perpendicular bisector

- As you construct the perpendicular, think of the relationship between any point on the perpendicular bisector and the two end points of a line segment

- Construct a **perpendicular bisector** for a line segment AB .
 - Mark the points where the 2 pairs of arcs intersect as P and Q .
 - Join P and Q with a straight line
- The line PQ intersects AB at the point M . Mark the point and label it M .



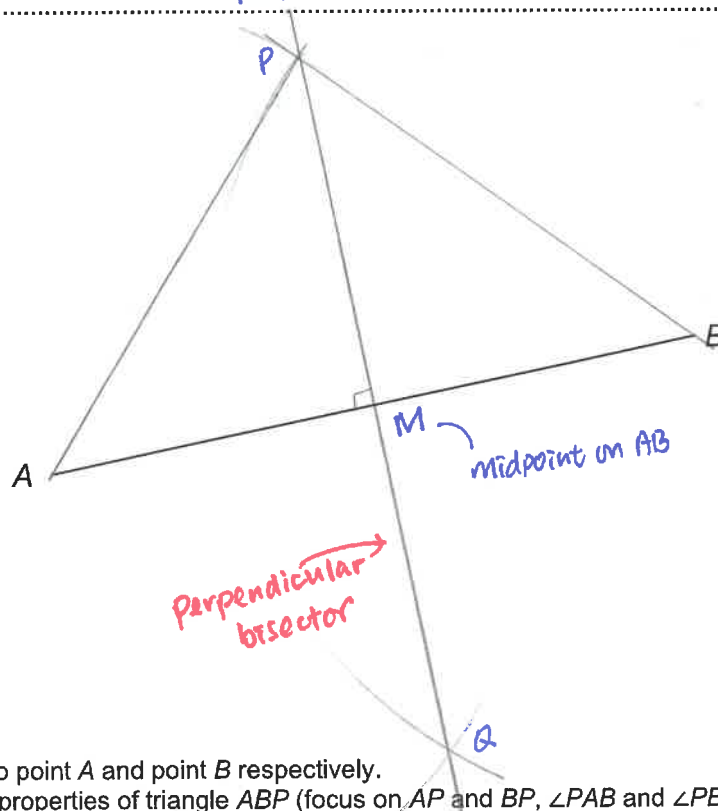
<https://youtu.be/MINxOtH9qhM>

- Measure the length of AM and BM .

$AM = 4.3$ cm; $BM = 4.3$ cm

- Measure the angles where the 2 lines intersect. What do you notice about the relationship between these 2 lines?

90° . The 2 lines are perpendicular to each other



- Join point P to point A and point B respectively. Describe the properties of triangle ABP (focus on AP and BP , $\angle PAB$ and $\angle PBA$)

$AP = BP$, $\angle PAB = \angle PBA$ $\therefore \triangle ABP$ is an isosceles triangle

- Join any point along PQ to point A and point B respectively. How would you describe the distance between the point and A and B respectively?

It is always the same distance from points A and B

OR Any point on the perpendicular bisector is always EQUIDISTANT from points A and B
(i.e. the 2 ends of the line segment)

Lesson 1B Angle Bisector

Estimated duration: 15 min

At the end of the activity, you should be able to construct an angle bisector

- As you construct the angle bisector, try to draw out the relationship between any point on the angle bisector to the two sides of the angle

- Construct an **angle bisector** for $\angle ABC$ below. (Note: $\angle ABC = 48^\circ$)
 - Mark 2 arcs on the arms BA and BC . Label them P and Q respectively. (Note: $BP = BQ$)
 - Using these 2 points, mark 2 arcs such that they will intersect at X .
 - Join point B to point X .

- Measure $\angle ABX$ and $\angle CBX$.

$\angle ABX = 24^\circ$

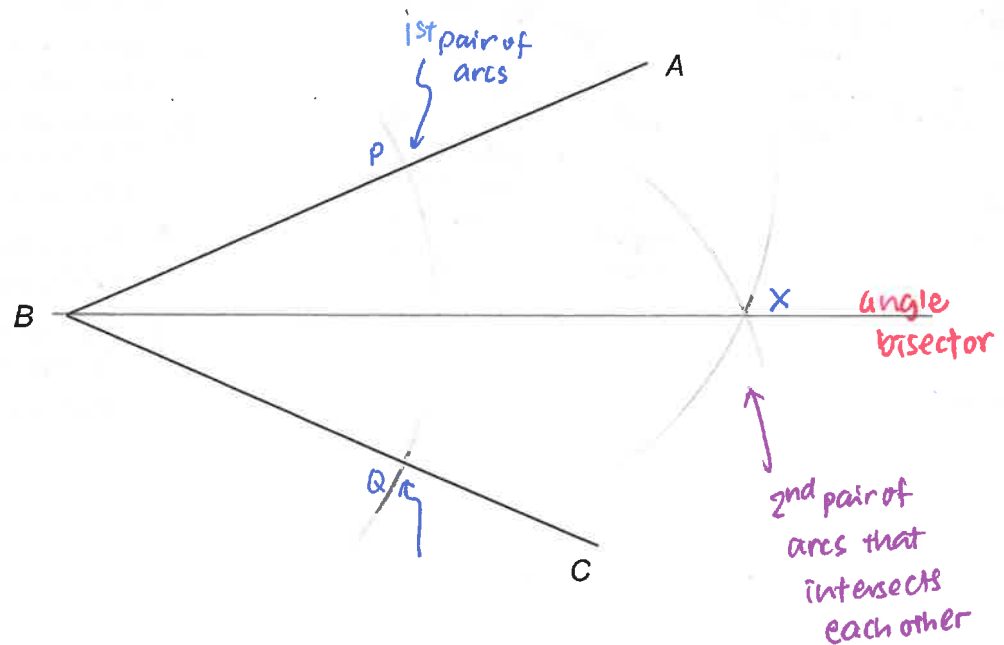
$\angle CBX = 24^\circ$

What do you notice about these 2 angles?

They are equal. The line that bisects this angle is known as the angle bisector



<https://youtu.be/0IVPGjEttFw>



Lesson 1C Parallel Lines

Estimated duration: 15 min **REFER TO THE SLIDES**

At the end of the activity, you should be able to draw a pair of parallel lines

Paul and Pauline are tasked to construct a pair of parallel lines that are 5 cm apart.

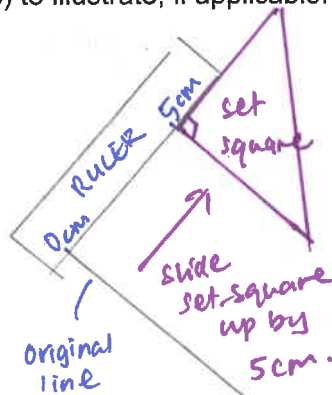
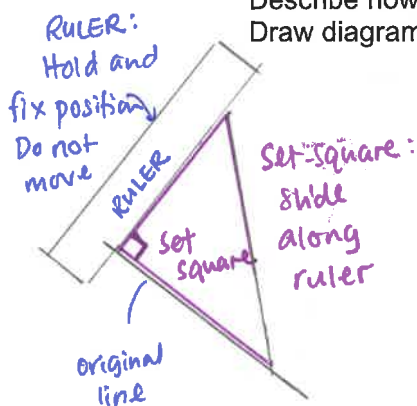
A ruler and a set-square will do the job!



I only need a ruler and a pair of compasses to construct!

Source of image: <https://www.dreamstime.com/>

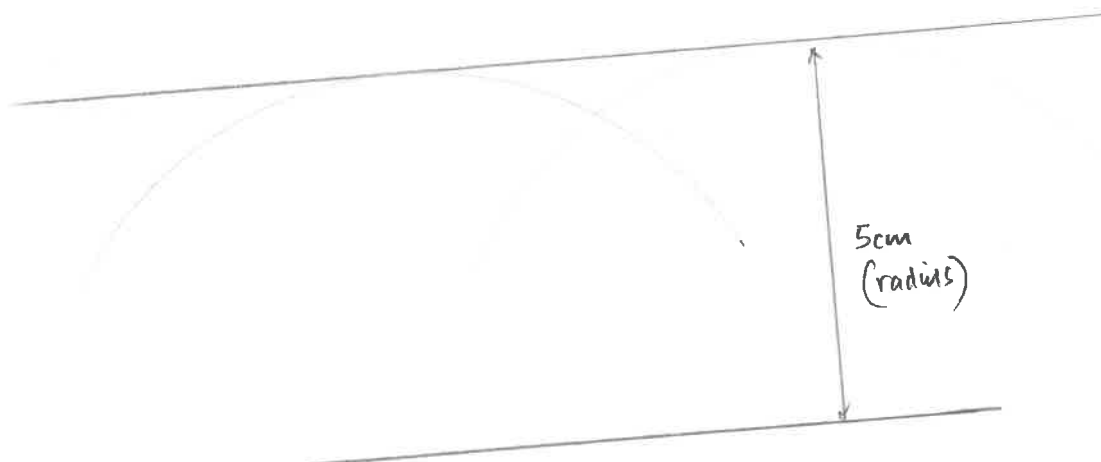
Describe how Paul and Pauline would construct the pair of parallel lines.
Draw diagram(s) to illustrate, if applicable.



Method 2: Use a compass

1. Stretch the arm of the compass to 5 cm.
2. Choose any 2 points on the line (not too close)
3. Mark arcs and identify the furthest point on the arc from the lines.
4. Join these 2 points with a straight line
5. This line will be parallel to the original line and the lines will be 5 cm apart.

Construct a pair of parallel lines that are 5 cm apart using any of the above methods.



Lesson 2 Triangles

At the end of the lesson, you should be able to construct triangles, given the following info:

(a) 3 sides (b) 2 sides and 1 angle (c) 1 side and 2 angles (d) 3 angles

We are going to construct triangles with different sets of information given.

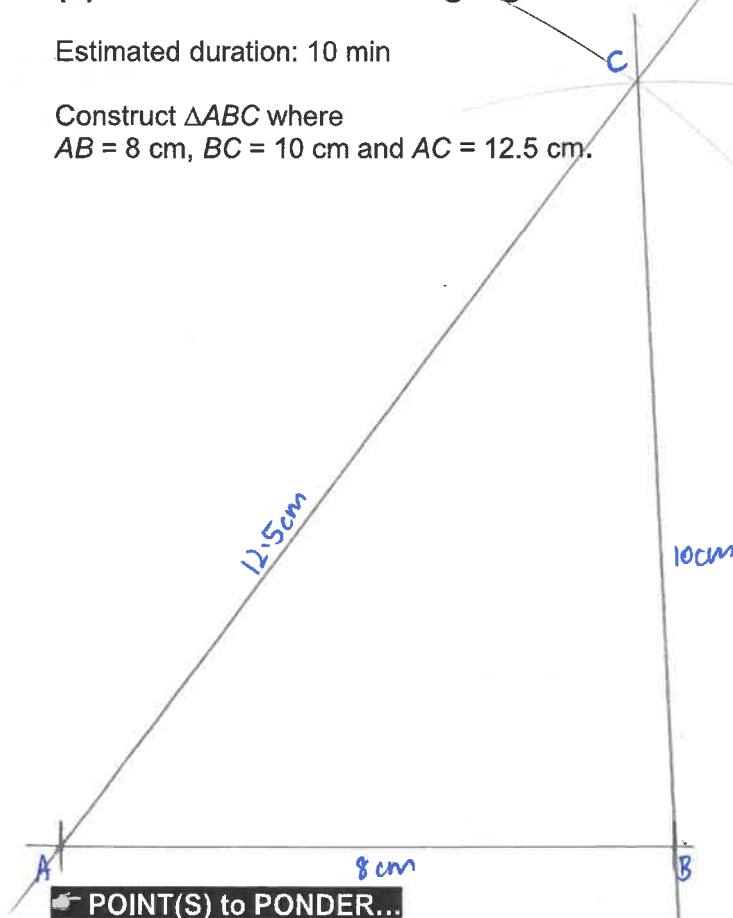
Strategy **BEFORE** constructing the shape, sketch and label it with the information given. This will help us better visualise how the final work looks like.

1. Draw a triangle and label the vertices first.
2. Indicate the length(s)/ angle(s) that are given in the question.
3. Identify the side (given the length) that is 'linked' to most information.
4. Where possible, mark out where the arcs will be in the diagram.
5. Label the steps with (1), (2), (3)....

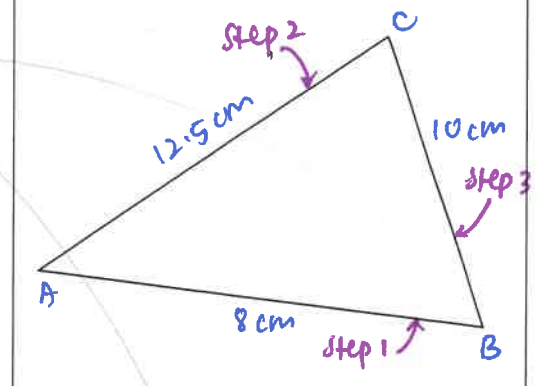
(a) Construction of Triangle given 3 sides

Estimated duration: 10 min

Construct $\triangle ABC$ where
 $AB = 8$ cm, $BC = 10$ cm and $AC = 12.5$ cm.



Sketch – label the triangle with the given information.



step 2 &
step 3:
order can
switch

<https://youtu.be/BNdiOgnpEnM>



POINT(S) to PONDER...

Can you construct a triangle of sides 1 cm, 3 cm and 5 cm?

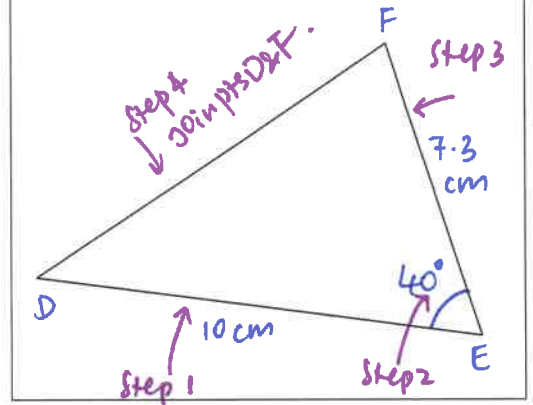
- If yes, how does it look like?
- If no, explain why not.

(b) Construction of Triangle given 2 sides and 1 angle

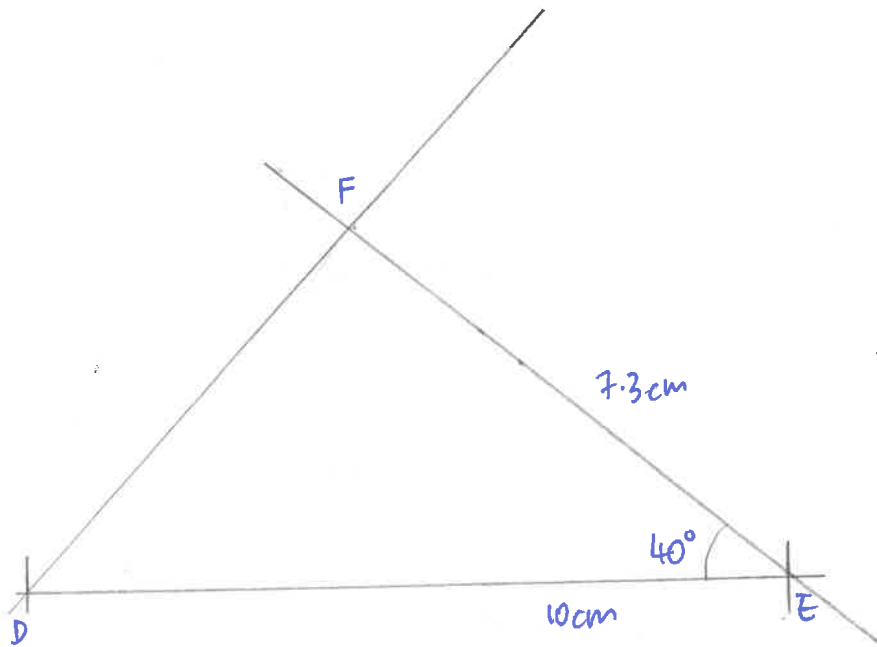
Estimated duration: 10 min

Construct $\triangle DEF$ where
 $DE = 10\text{ cm}$, $EF = 7.3\text{ cm}$ and $\angle DEF = 40^\circ$.

Sketch – label the triangle with the given information.



<https://youtu.be/JHi7VKDJMOs>

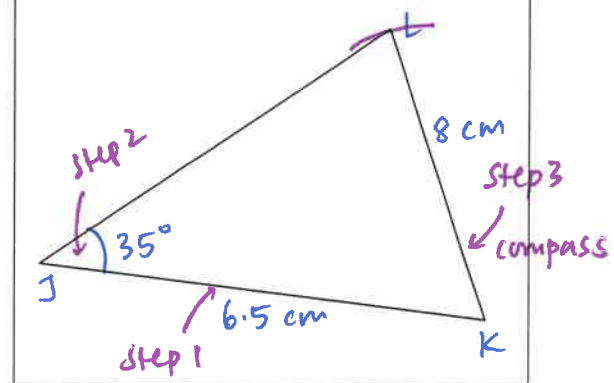


(c) Construction of Triangle given 2 sides and 1 angle

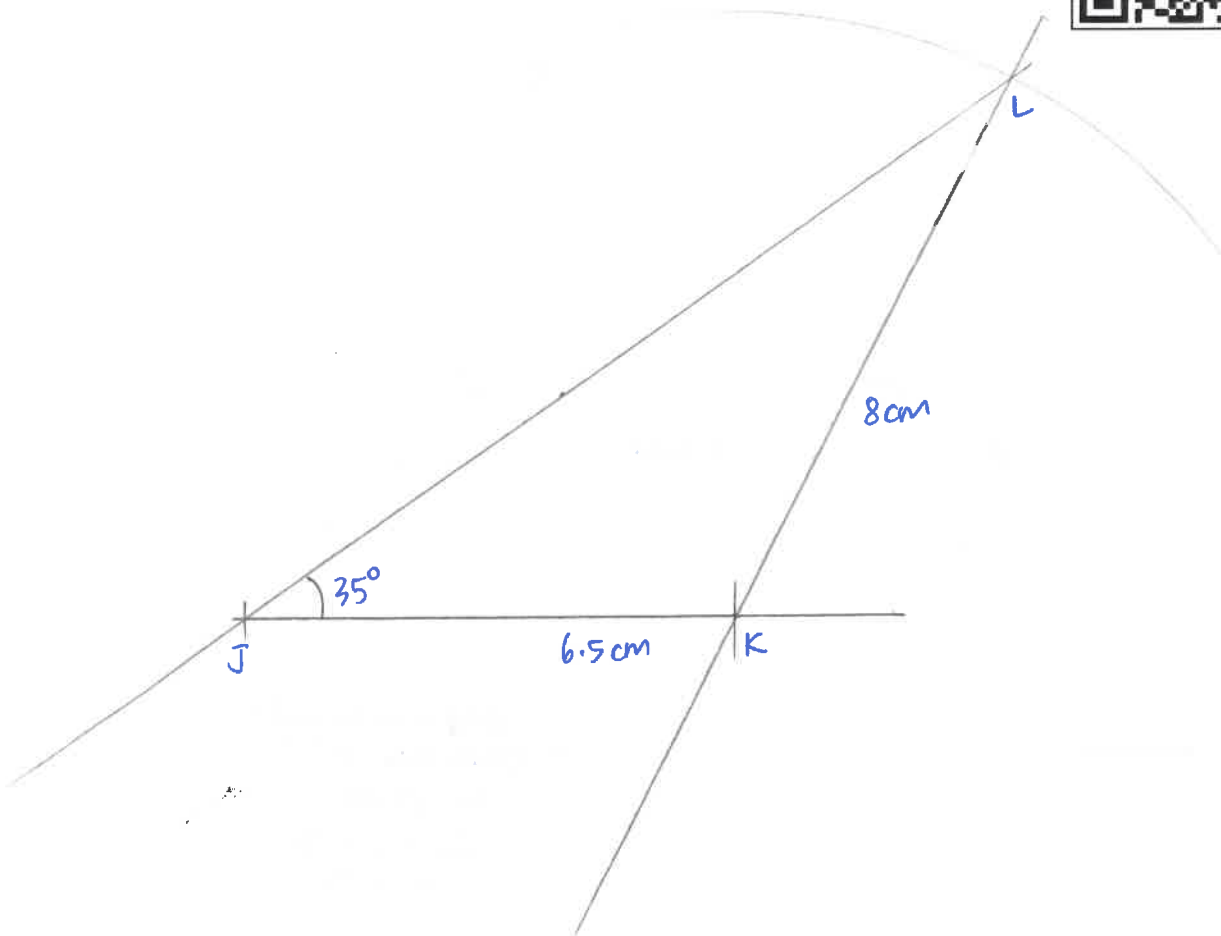
Estimated duration: 10 min

Construct $\triangle JKL$ where
 $JK = 6.5$ cm, $KL = 8$ cm and $\angle LJK = 35^\circ$.

Sketch – label the triangle with the given information.



<https://youtu.be/C7lwKsfG8sk>



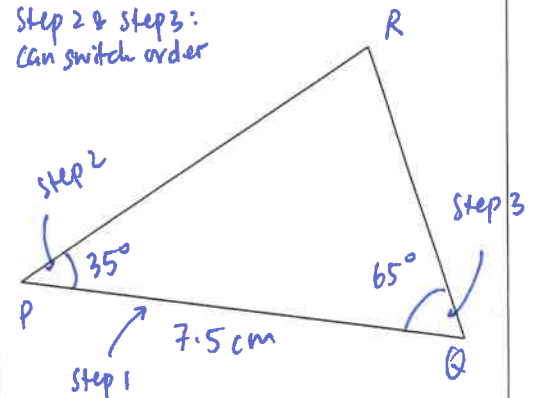
(d) Construction of Triangle given 1 side and 2 angles

Estimated duration: 10 min

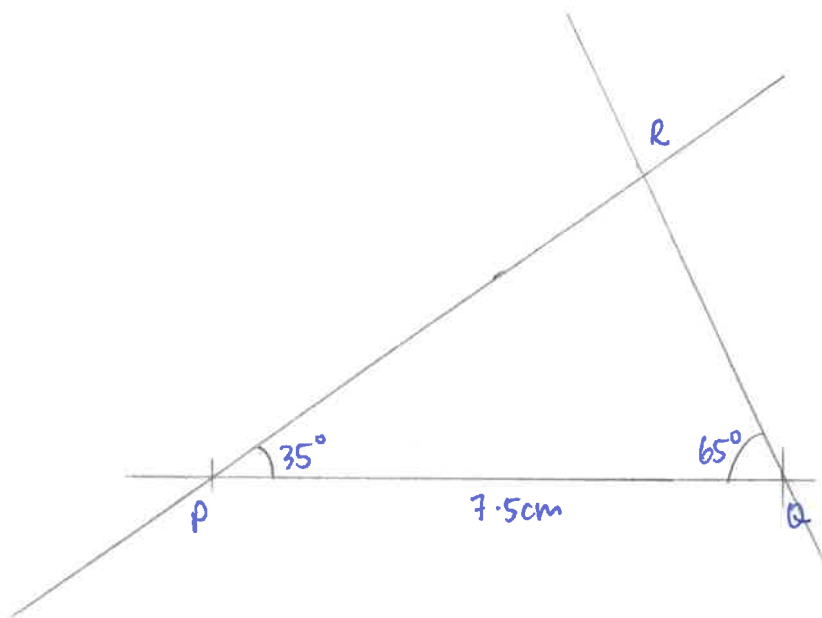
Construct $\triangle PQR$ where
 $PQ = 7.5$ cm, $\angle RPQ = 35^\circ$ and $\angle PQR = 65^\circ$.

Sketch – label the triangle with the given information.

Step 2 & step 3:
Can switch order



<https://youtu.be/mnzYT0jzlk4>

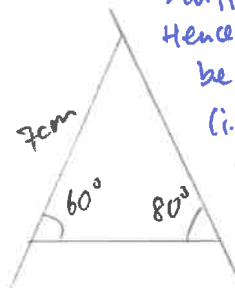
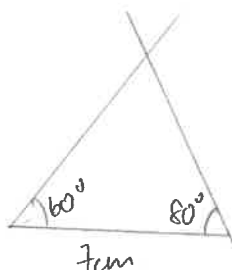


POINT(S) to PONDER...

You are asked to construct a triangle of a side 7 cm, and two angles 60° and 80° .

- How many different triangles can you construct?

3 different triangles
Hence info needs to
be specific
(i.e. the vertices
are given)



(e) Construction of Triangle given 3 angles

Estimated duration: 10 min

Construct $\triangle STU$ where $\angle STU = 55^\circ$, $\angle TUS = 25^\circ$ and $\angle UST = 100^\circ$.

Note 1: Do you notice any 'redundant' information given in the question? Why is it redundant?

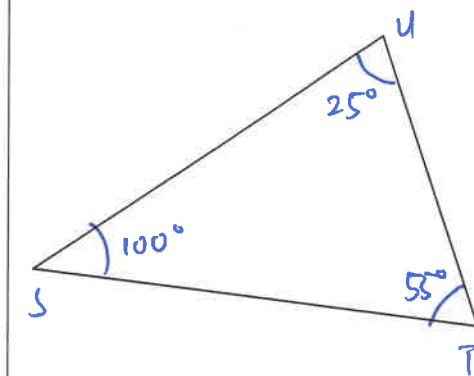
Since sum of angles of a triangle is 180° , only need to give 2 of the angles. The 3rd angle can be obtained through $180^\circ - (\text{sum of 2 given angles})$

BIG IDEA: Invariance

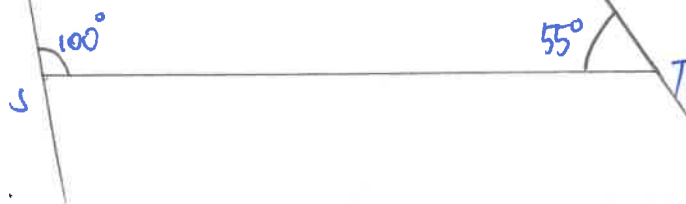
Note 2: Compare the triangle that you constructed with your classmates' – are they exactly the same? What can you say about these triangles that all of you have constructed?

All triangles come in different sizes as we would have used different lengths to begin with. However, all the angles will be the same size regardless of the size of triangle. All these triangles are SIMILAR TRIANGLES

Sketch – label the triangle with the given information.



<https://youtu.be/8R5soEyRUY0>



Lesson 3 Quadrilaterals

At the end of the lesson, you should be able to construct the following quadrilaterals

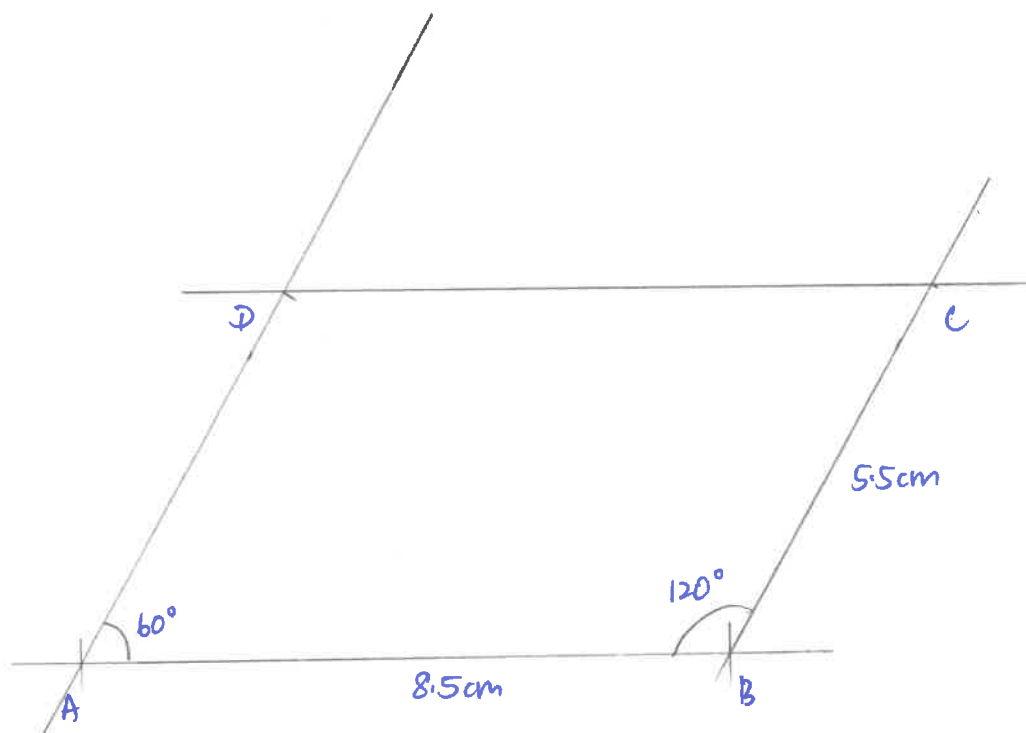
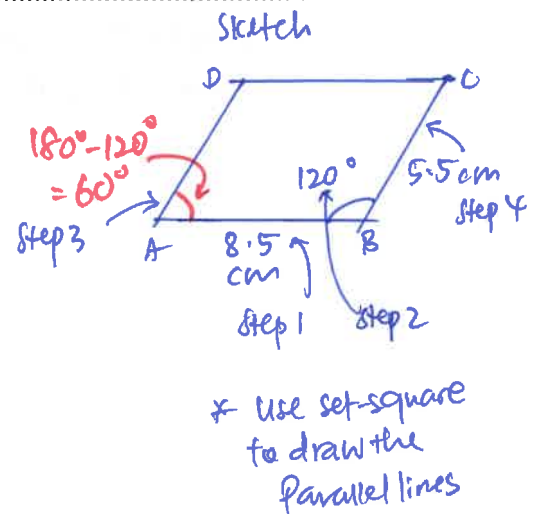
(a) Parallelogram (b) Rhombus (c) Quadrilateral

(a) Construction of a Parallelogram

Estimated duration: 15 min

Construct a parallelogram $ABCD$ such that $AB = 8.5$ cm, $BC = 5.5$ cm and $\angle ABC = 120^\circ$.
Measure and write down the length of the diagonals.

Length of diagonals:



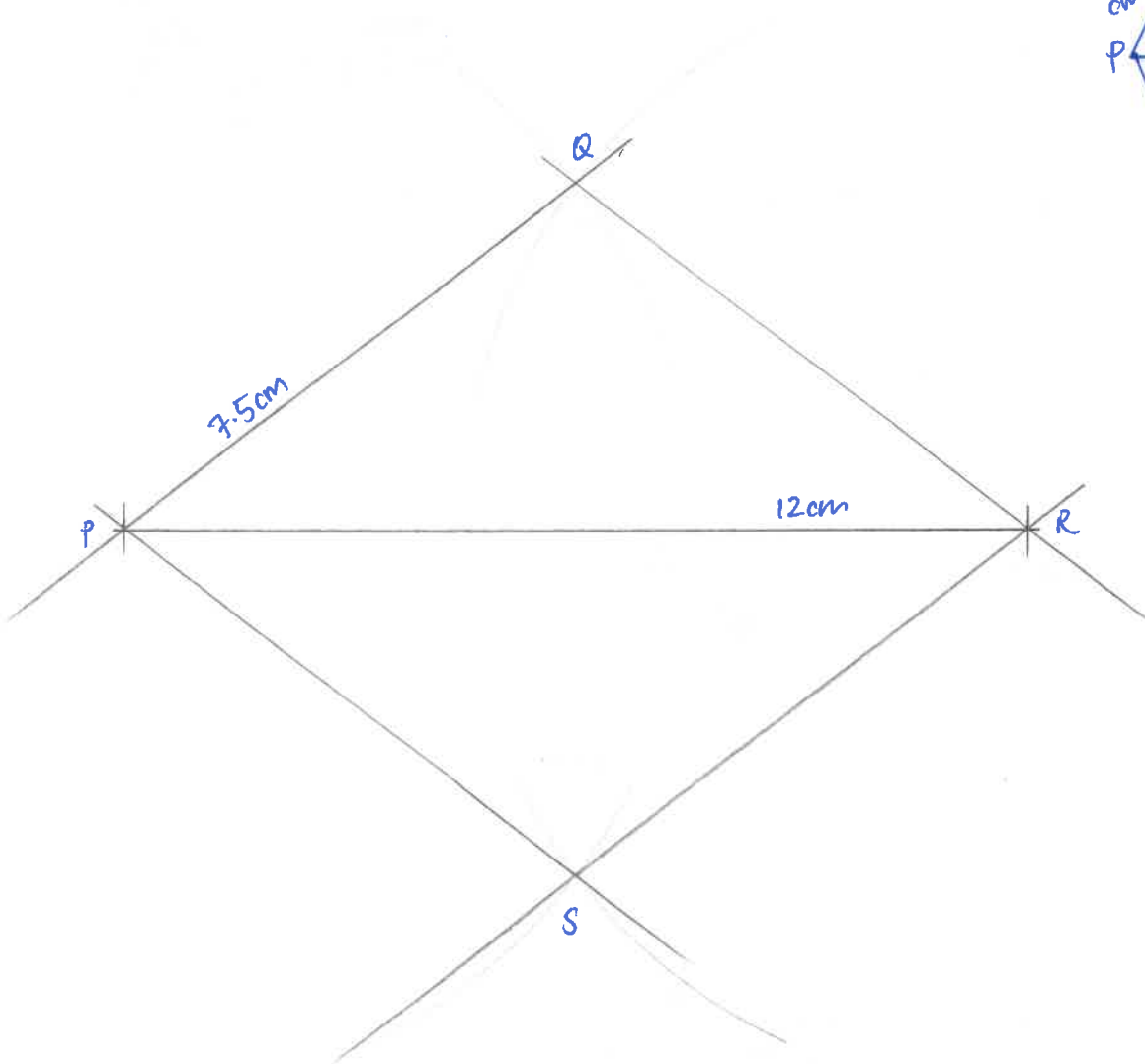
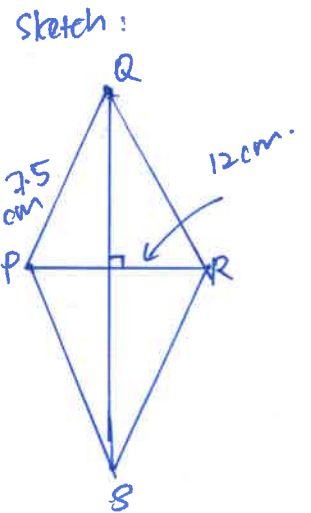
(b) Construction of a Rhombus

Estimated duration: 15 min

Construct a rhombus $PQRS$ such that $PQ = 7.5$ cm and the diagonal $PR = 12$ cm.

Measure and write down the size of $\angle QRS$.

$\angle QRS = \dots\dots\dots 75^\circ \dots\dots\dots$



(c) Construction of a Quadrilateral

Estimated duration: 15 min

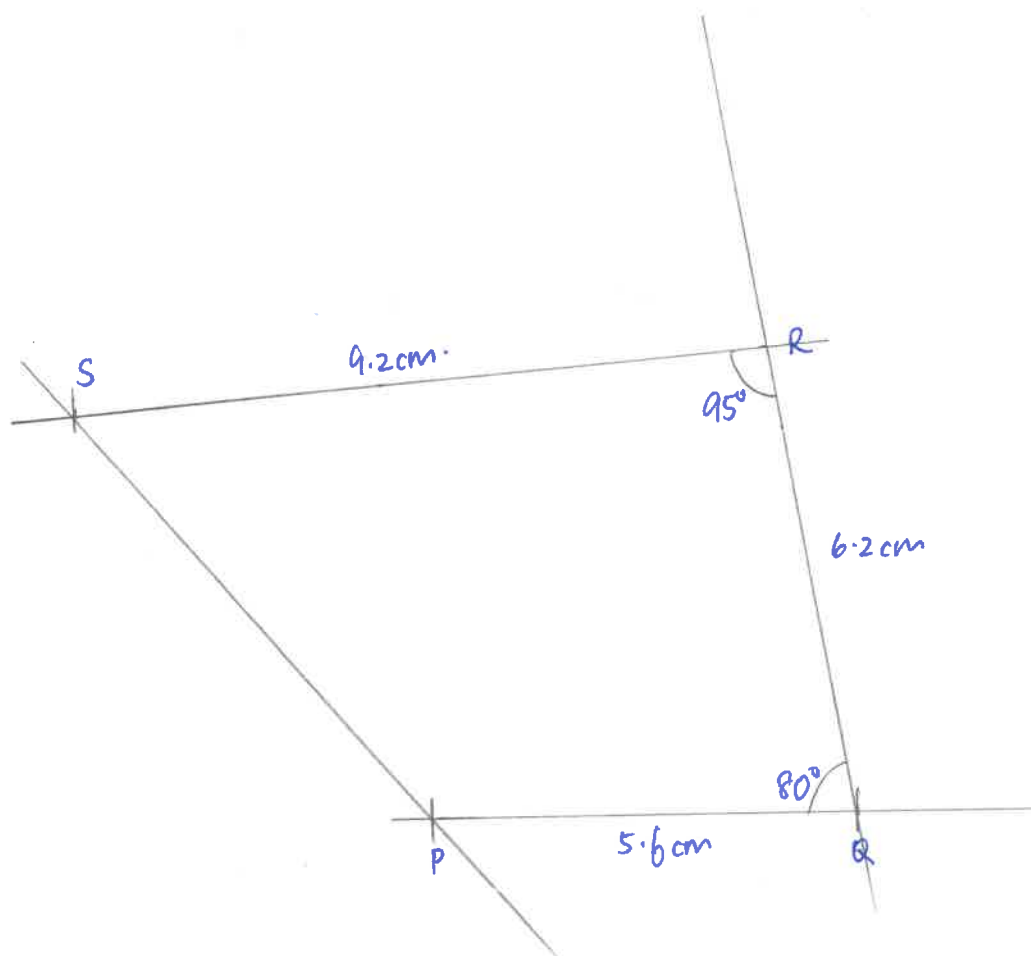
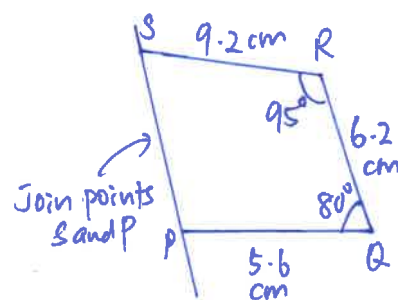
Construct a quadrilateral $PQRS$ such that $PQ = 5.6$ cm, $QR = 6.2$ cm, $RS = 9.2$ cm, $\angle PQR = 80^\circ$ and $\angle QRS = 95^\circ$.

(i) Measure and write down the length of PS .

Length of $PS = 7.1$ cm

(ii) Measure and write down the size of $\angle PSR$.

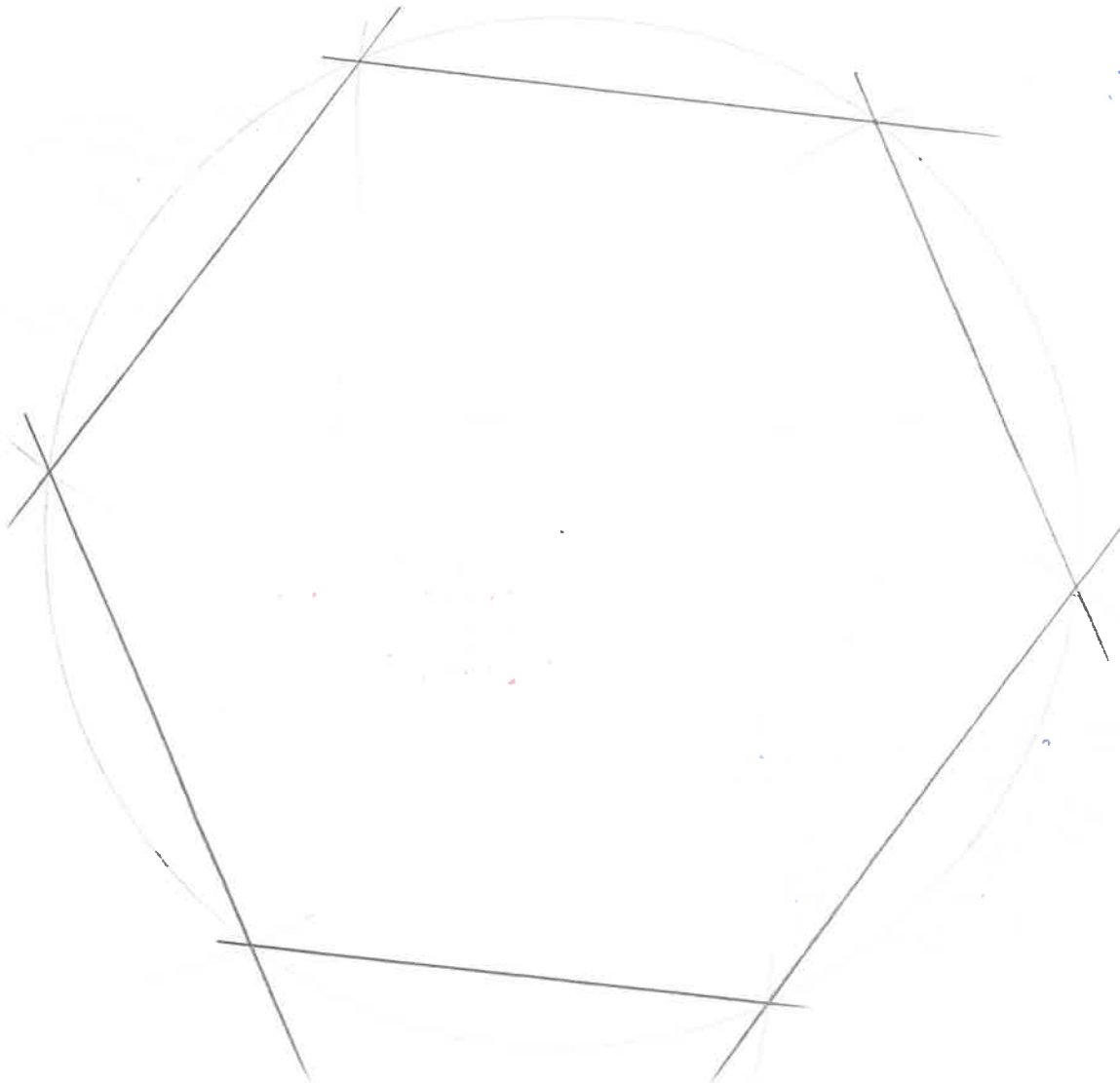
$\angle PSR = 126^\circ$



ACTIVITY (Tier C) Fun with Construction 🧐🧐

In the space below, construct a **regular hexagon** (each side measures 7 cm) using a **ruler and a pair of compasses only**. Label the diagram clearly.

Requirement: Create the hexagon with a circle.



POINT(S) to PONDER...

1. We used a circle to construct the regular hexagon. Do you think this method can be used to construct a regular hexagon of any size? *YES. The construction rides on the angle properties unique to the hexagon (angles at a point)*
2. Why, by constructing a circle, will enable us to construct a regular hexagon? What are the key properties of a regular hexagon and circle used here?
Since the hexagon has 6 equal sides, the diagonals that divide it into 6 identical triangles will result in $6 \times 60^\circ$ at the centre of the circle

In addition, since the sides extending from the centre of the circle is the radius of the circle, these 6 triangles are isosceles triangles. With one of the angles being 60° , calculated base angles are each 60° , which implies all 6 triangles are equilateral triangles.

ASSIGNMENT

Textbook 1B

Exercise 11C

(p121)

Q9

Triangle

Q10

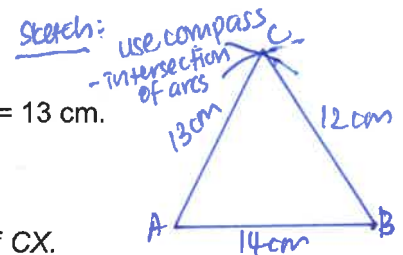
Rhombus (amend: $AC = 90$ mm)

9. Construct $\triangle ABC$ such that $AB = 8.4$ cm, $AC = 6.5$ cm and $\angle C = 50^\circ$.
Measure and write down the possible lengths of BC .

10. Construct a rhombus $ABCD$ such that $AB = 60$ mm and the diagonal $AC = 90$ mm.
(i) State a property of rhombus used in order to complete this construction.
(ii) Measure and write down the size of $\angle BAD$.

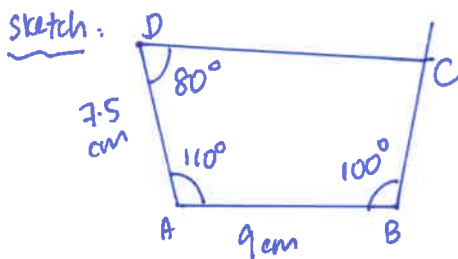
More Practice

1. Construct $\triangle ABC$ such that $AB = 14$ cm, $BC = 12$ cm and $AC = 13$ cm.
(i) Construct the perpendicular bisector of AB .
Construct the perpendicular bisector of BC .
The two bisectors meet at X .
(ii) Measure and write down the length of AX , of BX and of CX .
(iii) Construct a circle that will pass through all the three points A , B and C .



What can you conclude from the above observation?

2. (a) Construct a quadrilateral $ABCD$ such that $AB = 9$ cm, $\angle A = 110^\circ$, $\angle B = 100^\circ$, $AD = 7.5$ cm and $\angle D = 80^\circ$.
(b) A point X is equidistant from line BC and CD and $\angle CXB = 90^\circ$. Mark and label clearly this point X on your diagram. Bisector of $\angle BCD$
(c) A point is equidistant from points C and D , and $YB = 8$ cm. Mark and label clearly this point Y on your diagram. Perp bisector



Useful TIP: Apply POLYA's Problem Solving Model for Construction

Step 1: Understand the Problem

- Be clear what is the main shape to construct
- Circle all the relevant info (e.g. length, angle) required for the construction.

Step 2: Think of a Plan

- Sketch the shape
- Write down all the info given in the sketch
- Plan what mathematical instrument are needed
- Plan the sequence

Step 3: Carry out the Plan

- Ensure pencils is sharp, and all the necessary mathematical instrument are available
- Label the diagram: vertices, angles, lengths
- Answer any question

Step 4: Look Back

- Check whether all the parts of the questions are answered.
- Ensure the arcs and drawing are clear.
- Erase all the incorrect / irrelevant lines.