School of Science and Technology, Singapore
Mathematics Department

2023 Secondary 3 MATHEMATICS
Unit 01. Indices and Its Applications

SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

Name Solutions ( ) Class S3-0

Enduring Understanding

Students will understand the following:
- Expressing numbers in index **notation** facilitates the study of exponents and allows the laws of indices to be expressed compactly.
- Algebraic expressions remain **equivalent** when they are simplified using indices laws.
- Producing a series of **equivalent** equations by applying laws of indices provides an algorithm for solving exponential equations.
- Standard form **notation** represents very big or very small numbers in a compact way that facilitates calculations.

Essential Questions

- Why do we represent numbers and indices laws in index notation?
- What undergirds the simplification of algebraic expressions?
- What undergirds the solving of exponential equations?
- How is the standard form notation useful in real world applications?

Big Ideas

- Notations: Notations are symbols and conventions of writing used to represent mathematical objects, and their operations and relationships in a concise and precise manner. Examples in this unit include index notations that allow the laws of exponents to be compactly expressed, and scientific notation that represents very big or very small numbers.
- Equivalence: Equivalence is a relationship that expresses the 'equality' of two mathematical objects that may be represented in two different forms based on a criterion. Transformation or conversion of an expression or equation from one form to another equivalent form is the basis of many manipulations for analysing and comparing them and algorithms for finding solutions.

Learning Objectives

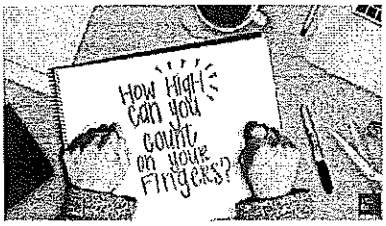
At the end of the unit, students should be able to
- State and apply the Laws of Indices
- State and use the definitions of zero, negative and rational indices
- Represent very large or very small numbers using standard form
- Solve simple exponential equations using equality of indices
- Appreciate why indices and standard form have useful applications in real life

Lesson sequence in the unit								
Student Learning Outcomes	Dimensions (Please tick the appropriate boxes)							
	FUNCTIONS **F**	INVARIANCE **I**	NOTATIONS **N**	DIAGRAMS **D**	MEASURES **M**	EQUIVA-LENCE **E**	PROPOR TIONALITY **P**	MODELS **M**
Laws of Indices			✓			✓		
Zero and Negative Indices			✓			✓		
Rational Indices			✓			✓		
Applications of Indices - Compound Interest - Standard Form			✓			✓		

Introductory Video

How high can you count on your fingers?
Source: https://youtu.be/UixU1oRW64Q

See, Think, Wonder… Why are indices useful?

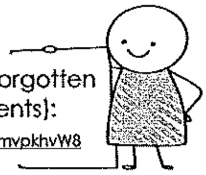


1.1 Laws of Indices

Index Notation

$$
\begin{aligned}
3\times 3 &= 3^2\\
3\times 3\times 3 &= 3^3\\
3\times 3\times 3\times 3 &= 3^4\\
&\vdots\\
\underbrace{3\times 3\times \ldots \times 3}_{7\text{ times}} &= 3^7
\end{aligned}
$$

Watch this video if you've forgotten what are indices (or exponents):
https://www.youtube.com/watch?v=zUmvpkhvW8



Written in this way, 3^7 is called the **index notation** (or index form) of $\underbrace{3\times 3\times \ldots \times 3}_{7\text{ times}}$, where 3 is called the **base** and 7 is called the **index** (or exponent).

3^7 is read as "3 to the power of 7".

Numbers with a base of '3', e.g. 3^1, 3^2 3^3, …, are called **powers** of 3.

$$\text{base} \longrightarrow a^{n} \longleftarrow \text{index}$$

BIG idea
Notations

The **index notation** is used to represent a number multiplied by itself n times in a *precise and concise manner*. The index or exponent indicates the number of times the base appears as a factor.

Note that $3^1 = 3$, 3 is the simplified form of 3^1.

Activity 1 **[Inquiry]**

Learning objective: To explore the laws of indices to facilitate the operations on expressions in index notations.

1. **(a)** Let's explore how to simplify $a^m \times a^n$ and $a^m \div a^n$.

Examples for exploration	**Examples for exploration**	**Generalising**
$5^3 \times 5 = (5\times5\times5)\times(5) = 5^{\boxed{4}}$	$a^3 \times a = a^4$	
$5^3 \times 5^2 = 5^5$	$a^3 \times a^2 = a^5$	$a^m \times a^n = a^{m+n}$
$5^3 \times 5^3 = 5^6$	$a^3 \times a^3 = a^6$	
$5^3 \times 5^4 = 5^7$	$a^3 \times a^4 = a^7$	
$5^6 \div 5^2 = \dfrac{5\times5\times5\times5\times5\times5}{5\times5} = 5^{\boxed{4}}$	$a^6 \div a^2 = a^4$	
$5^5 \div 5^2 = 5^3$	$a^5 \div a^2 = a^3$	$a^m \div a^n = a^{m-n}$
$5^4 \div 5^2 = 5^2$	$a^4 \div a^2 = a^2$	
$5^3 \div 5^2 = 5$	$a^3 \div a^2 = a^1$	

(b) From the above, we observe that for positive integers *m* and *n*,

$$a^m \times a^n = a^{m+n}$$

and for $m > n$,

$$a^m \div a^n = a^{m-n}$$

I wonder...

What if $m = n$?
i.e. can an index be zero?

$$a^{m-n} = a^{m-m}$$

$$= \frac{a^m}{a^m}$$

$$= 1$$

$$\therefore a^0 = 1$$

2. **(a)** Let's explore how to simplify $a^m \div a^m$.

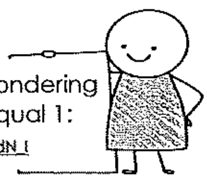
Examples for exploration	**Using results from 1(b)**	**Generalising**
$a^5 \div a^5 = \dfrac{a \times a \times a \times a \times a}{a \times a \times a \times a \times a} = 1$	$a^5 \div a^5 = a^{5-5} = a^0$	
$a^4 \div a^4 =$ 1	$a^4 \div a^4 = a^{4-4} = a^{\boxed{0}}$	$a^m \div a^m$ $= a^0$ $=$ 1
$a^3 \div a^3 =$ 1	$a^3 \div a^3 =$ $a^0 = 1$	

(b) From the above, we deduce that

$a^0 =$ 1

where a is a real number such that $a \neq 0$.

Watch this video if you're still wondering why numbers to power of 0 is equal 1:
https://www.youtube.com/watch?v=mYfmSx_dN_I



I wonder...

Can an index be negative?

3. **(a)** Let's explore what a^{-n} is.

Examples for exploration	Using results from 1(b)	Generalising
$a^4 \div a^5 = \dfrac{a \times a \times a \times a}{a \times a \times a \times a \times a} = \dfrac{1}{a}$	$a^4 \div a^5 = a^{4-5} = a^{-1}$	
$a^3 \div a^5 = \dfrac{a \times a \times a}{a \times a \times a \times a \times a} = \dfrac{1}{a^2}$	$a^3 \div a^5 = a^{3-5} = a^{\boxed{-}2}$	$a^{-n} = \dfrac{1}{a^n}$
$a^2 \div a^5 = \dfrac{1}{a^3}$	$a^2 \div a^5 = a^{-3}$	

(b) From the above, we deduce that

$$a^{-n} = \frac{1}{a^n}$$

where a is a real number such that $a \neq 0$.

I wonder...

Are the results from 1(b) still true for negative indices?

Conclusion

Law 1 of Indices:

$$a^m \times a^n = a^{m+n}$$

where base a is a real number such that $a \neq 0$.

Law 2 of Indices

$$a^m \div a^n = a^{m-n} \quad \text{or} \quad \frac{a^m}{a^n} = a^{m-n}$$

where base a is a real number such that $a \neq 0$.

Definition 1:

$$a^0 = 1$$

where base a is a real number such that $a \neq 0$.

Definition 2:

$$a^{-n} = \frac{1}{a^n}$$

where base a is a real number such that $a \neq 0$.

4. **(a)** Let's explore how to simplify $(a^m)^n$.

Examples for exploration	Examples for exploration	Generalising
$(5^5)^2 = 5^5 \times 5^5$ $= 5^{5+5}$ $= 5^{5\times\boxed{2}}$	$(a^5)^2 = a^{10}$	
$(5^4)^2 = 5^{4\times2} = 5^8$	$(a^4)^2 = a^8$	
$(5^3)^2 = 5^{3\times2} = 5^6$	$(a^3)^2 = a^6$	$(a^m)^n = a^{mn}$
$(5^3)^3 = 5^{3\times3} = 5^9$	$(a^3)^3 = a^9$	
$(5^3)^4 = 5^{3\times4} = 5^{12}$	$(a^3)^4 = a^{12}$	

(b) From the above, we observe that for positive integers m and n,

$$(a^m)^n = a^{mn}$$

Conclusion

Law 3 of Indices:

$$(a^m)^n = a^{m \times n}$$

where base a is a real number such that $a \neq 0$.

Compare the Laws 1 and 3 for $a^m \times a^n$ and $(a^m)^n$ respectively.

(i) How are the two laws different?

(ii) How do you know when to use which law?

Law 1 : $a^m \times a^n = a^{m+n}$

The indices are added up

It is used when 2 indices are multiplied

Law 3 : $(a^m)^n = a^{mn}$

The indices are multiplied

It is used when you take the power of one index.

5. **(a)** Let's explore how to simplify $a^m \times b^n$ and $a^n \div b^n$.

Examples for exploration	Examples for exploration	Generalising
$5^3 \times 2^3 = (5\times5\times5)\times(2\times2\times2)$ $= (5\times2)\times(5\times2)\times(5\times2)$ $= (5\times2)^{\boxed{3}}$ $5^3 \times 3^3 = (5\times3)^3$ $= 15^3$ $5^4 \times 3^4 = (5\times3)^4$ $= 15^4$	$a^3 \times b^3 = (ab)^3$ $a^3 \times b^3 = (ab)^3$ $a^4 \times b^4 = (ab)^4$	$a^n \times b^n = (ab)^n$
$5^3 \div 2^3 = \frac{5\times5\times5}{2\times2\times2}$ $= \frac{5}{2}\times\frac{5}{2}\times\frac{5}{2}$ $= \left(\frac{5}{2}\right)^{\boxed{3}}$ $5^3 \div 3^3 = \left(\frac{5}{3}\right)^3$ $5^4 \div 3^4 = \left(\frac{5}{3}\right)^4$	$a^3 \div b^3 = \left(\frac{a}{b}\right)^3$ $a^3 \div b^3 = \left(\frac{a}{b}\right)^3$ $a^4 \div b^4 = \left(\frac{a}{b}\right)^4$	$a^n \div b^n =$ or $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$

(b) From the above, we observe that for positive integers m and n,

$a^n \times b^n =$

and $a^n \div b^n = \left(\frac{a}{b}\right)^n$ or $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$

Conclusion

Law 4 of Indices:

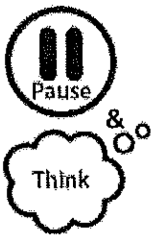
$$a^n \times b^n = (a \times b)^n$$

where bases a and b are non-zero real numbers.

Law 5 of Indices

$$a^n \div b^n = (a \div b)^n \quad \text{or} \quad \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

where bases a and b are non-zero real numbers.



Compare the Laws 1 and 4 for $a^m \times a^n$ and $a^n \times b^n$ respectively.

(i) How are the two laws different?

(ii) How do you know when to use which law?

Similarly, compare the Laws 2 and 5 for $a^m \div a^n$ and $a^n \div b^n$ respectively.

(i) How are the two laws different?

(ii) How do you know when to use which law?

Law 1: $a^m \times a^n = a^{m+n}$

(i) same base, diffeant power

Law 4: $a^n \times b^n = (ab)^n$

diffeant base, same power

Law 2 $a^m \div a^n = a^{m-n}$

same base, different power

Law 5: $a^n \div b^n = \left(\frac{a}{b}\right)^n$

different base, same power

Worked Example 1
Applying Law 1 of Indices

Simplify each of the following, leaving your answer in index notation where appropriate.

(a) $6^4 \times 6^7$

(b) $x^5 \times x^6$

(c) $5pq^2 \times 7p^3q^4$

(d) $4a^{-1}b^8 \times a^3b^{-2}$

(a) $6^4 \times 6^7 = 6^{4+7} = 6^{11}$

(b) $x^5 \times x^6 = x^{5+6} = x^{11}$

(c) $5pq^2 \times 7p^3q^4 = 35p^4q^6$

(d) $4a^{-1}b^8 \times a^3b^{-2} = 4a^{3-1}b^{8-2}$
$= 4a^2b^6$

Worked Example 2
Applying Law 2 of Indices

Simplify each of the following, leaving your answer in index notation where appropriate.

(a) $6^7 \div 6^4$

(b) $x^7 \div x^6$

(c) $35p^3q^4 \div 7pq^4$

(d) $4a^5b^8 \div a^3b^{-2}$

(a) $6^7 \div 6^4 = 6^{7-4} = 6^3$

(b) $x^7 \div x^6 = x^{7-6} = x$

(c) $35p^3q^4 \div 7pq^4$
$= 5p^2$

(d) $4a^5b^8 \div a^3b^{-2} = 4a^2b^{10}$

Problem Solving STrategy
When there are coefficients involved, we may multiply or divide the coefficients directly.

Worked Example 3
Applying Laws 1 to 3 of Indices

Simplify each of the following, leaving your answer in index notation where appropriate.

(a) $(6^7)^4$

(b) $(7^p)^5 \times (7^2)^p \div 7$

(c) $(a^5)^2 \div (a^3)^{-2} \times (a^{-1})^{-3}$

(a) $(6^7)^4 = 6^{28}$

(b) $(7^p)^5 \times (7^2)^p \div 7 = 7^{5p+2p-1}$
$= 7^{7p-1}$

(c) $(a^5)^2 \div (a^3)^{-2} \times (a^{-1})^{-3}$
$= a^{10+6+3}$
$= a^{19}$

When applying the power law, every term in the bracket will be raised to the power.

Worked Example 4
Applying Laws 1 to 4 of Indices

Simplify each of the following, leaving your answer in positive index form where appropriate.

(a) $(3h^{-2})^4$

(b) $(-ab^3)^3$

(c) $(3c^3d)^0$

(d) $\dfrac{(-xy^2)^2 \times (2x^3y^{-1})^4}{4(x^{-2}y)^{-1}}$

(a) $(3h^{-2})^4 = 3^4(h^{-2})^4$
$= \frac{81}{h^8}$

(b) $(-ab^3)^3 = -a^3b^9$

(c) $(3c^3d)^0 = 1$

(d) $\dfrac{(-xy^2)^2 \times (2x^3y^{-1})^4}{4(x^{-2}y)^{-1}}$
$= \dfrac{\cancel{x^2}y^4 \times 16x^{12}\cancel{y^4}}{4\cancel{x^2}y^{-1}}$
$= 4x^{12}y$

We can change a negative index to a positive index by "taking reciprocal of the base".
E.g. $\dfrac{a^{-2}}{b^3} = \dfrac{1}{a^2b^3}$; $\dfrac{2}{x^{-3}} = 2x^3$

Worked Example 5
Applying Laws 1 to 5 of Indices

Simplify each of the following, leaving your answer in positive index form where appropriate.

(a) $\left(\dfrac{3}{h^2}\right)^4$

(b) $\left(-\dfrac{b^3}{a}\right)^3 \times \dfrac{(ab)^2}{b^3}$

(c) $\left(\dfrac{4x^2}{y^3}\right)^{-1} \div \dfrac{6x^7}{y^{-2}}$

(a) $\left(\dfrac{3}{h^2}\right)^4 = \dfrac{81}{h^8}$

(b) $\left(-\dfrac{b^3}{a}\right)^3 \times \dfrac{(ab)^2}{b^3}$

$= \left(-\dfrac{b^9}{a^3}\right) \times \dfrac{a^2b^2}{b^3}$

$= -\dfrac{b^8}{a}$

(c) $\left(\dfrac{4x^2}{y^3}\right)^{-1} \div \dfrac{6x^7}{y^{-2}}$

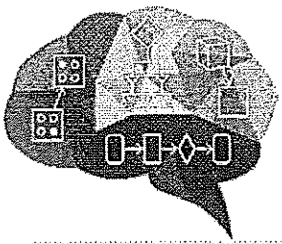
$= \dfrac{y^3}{4x^2} \times \dfrac{y^{-2}}{6x^7}$

$= \dfrac{y}{24x^9}$

Concept Quiz 1 | Assignment 1

Textbook 3A Exercise 3A

Extension
Computational Thinking



The binary number system is a positional numeral system using 2 as the base and so requiring only two different symbols for its digits, 0 and 1, instead of the usual 10 different symbols needed in the decimal system. The importance of the binary system to information theory and computer technology derives mainly from the compact and reliable manner in which 0s and 1s can be represented in electromechanical devices with two states – "on-off".

(Source: https://www.britannica.com/science/binary-number-system)

How to convert a decimal number into a binary number?

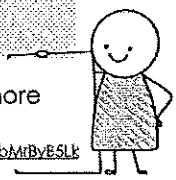
Your task…

Think of an algorithm for the above conversion.

You may present your algorithm in one of the following:

- A flowchart
- A pseudo-code
- A program (in any language)

Watch this video to learn more about the binary numbers:
https://www.youtube.com/watch?v=7CbMrByE5Lk



1.2 Rational Indices

We have learnt in Secondary 1 that if a is a non-negative number such that $a^n = b$ for some non-negative number b, then b is the **positive n^{th} root** of a, and we write $b = \sqrt[n]{a}$.
For example, since $3 \times 3 \times 3 \times 3 = 81$, $3 = \sqrt[4]{81}$.

BIG idea
Notations

An expression that involves the **radical sign** $\sqrt[n]{\ }$ is called a radical expression.

Since $3^2 = 9$ and $(-3)^2 = 9$, there are *two square roots* of 9, namely 3 and -3.
However, we use the notation $\sqrt{\ }$ to represent the **positive** square root so that there is no ambiguity, i.e. $\sqrt{9} = 3$ and $-\sqrt{9} = -3$.

We can now link the n^{th} root to indices that are non-integer rational numbers.

Activity 2a **[Inquiry]**
Learning objective: To make sense of rational indices

1. **(a)** What is $5^{\frac{1}{2}}$ equal to?

Let $p = 5^{\frac{1}{3}}$.

Then $p \times p \times p = 5^{\frac{1}{3}} \times 5^{\frac{1}{3}} \times 5^{\frac{1}{3}}$

$= (5^{\frac{1}{3}})^3$ *(Definition of indices)*

$= 5^1$ *(Law 3 of indices)*

$= 5$

$\therefore$ $p = \sqrt[3]{5}$ *(Definition of n^{th} root)*

i.e. $5^{\frac{1}{3}} = \sqrt[3]{5}$

(b) What is $5^{\frac{1}{2}}$ equal to?

Let $p = 5^{\frac{1}{2}}$.

Then $p \times p = 5^{\frac{1}{2}} \times 5^{\frac{1}{2}}$

$= (5^{\frac{1}{2}})^2$ *(Definition of indices)*

$= 5^1$ *(Law 3 of indices)*

$= 5$

$\therefore$ $p = \pm\sqrt{5}$ *(Definition of n^{th} root)*

Since we want $y = a^{\frac{1}{x}}$ to be a function, i.e. for every value of x, there is exactly one value of y.
Hence $p = \sqrt{5}$

i.e. $5^{\frac{1}{2}} = \sqrt{5}$

(c) From the above, we observe that for positive integers a and n,

$a^{\frac{1}{n}} = \sqrt[n]{a}$

Conclusion

> **Definition 3a:**
>
> $$a^{\frac{1}{n}} = \sqrt[n]{a}$$
>
> where base a is a real number such that $a \geq 0$,
> and n is a positive integer.

I wonder...

> If $a^{\frac{1}{n}} = \sqrt[n]{a}$,
>
> (i) what happens for $a < 0$?
>
> (ii) what happens for $a = 0$

(i) No solution.

(ii) $a^{\frac{1}{n}} = 0$

Activity 2b **[Inquiry]**
Learning objective: To make sense of rational indices

2. **(a)** What is $5^{\frac{2}{3}}$ equal to?

(i)

$$\begin{aligned} 5^{\frac{2}{3}} &= 5^{2 \times \frac{1}{3}} \\ &= \left(5^2\right)^{\frac{1}{3}} && \textit{(Law 3 of indices)} \\ &= \sqrt[3]{5^2} && \textit{(Definition of } a^{\frac{1}{n}}\textit{)} \end{aligned}$$

(ii)

$$\begin{aligned} 5^{\frac{2}{3}} &= 5^{\frac{1}{3} \times 2} \\ &= \left(5^{\frac{1}{3}}\right)^2 && \textit{(Law 3 of indices)} \\ &= 5^{\frac{2}{3}} && \textit{(Definition of } a^{\frac{1}{n}}\textit{)} \end{aligned}$$

(b) From the above, we observe that for positive integers a, m and n,

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

Conclusion

> **Definition 3b:**
>
> $$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{or} \quad \left(\sqrt[n]{a}\right)^m$$
>
> where base a is a real number such that $a \geq 0$,
> and m and n are positive integers.

With the definition of rational indices, the five Laws of Indices can be extended to include all rational indices:

Law 1 of Indices:	$a^m \times a^n = a^{m+n}$	if $a > 0$
Law 2 of Indices:	$a^m \div a^n = a^{m-n}$	if $a > 0$
Law 3 of Indices:	$(a^m)^n = a^{m \times n}$	if $a > 0$
Law 4 of Indices:	$a^n \times b^n = (a \times b)^n$	if $a, b > 0$
Law 5 of Indices:	$a^n \div b^n = (a \div b)^n$	if $a, b > 0$

Notice that some conditions for the bases a and b are now different.

I wonder...

(i) Why is it necessary for $a < 0$ in Laws 1 to 3?

(ii) Why is it necessary for $a, b > 0$ in Laws 4 to 5?

What happens if we are not careful about the conditions of the base?
The following shows a ridiculous proof that concludes that $1 = -1$.
Explain what is wrong with the proof.

$$\begin{aligned} 1 &= \sqrt{1} \\ &= \sqrt{(-1)\times(-1)} \\ &= \sqrt{-1}\times\sqrt{-1} \\ &= \left(\sqrt{-1}\right)^2 \\ &= (-1)^{\frac{1}{2}\times 2} \\ &= (-1)^1 \\ &= -1 \end{aligned}$$

Worked Example 6

Applying Laws of Indices involving rational indices

Simplify each of the following, leaving your answer in positive index form.

(a) $\sqrt[3]{a}\times\sqrt[4]{a^2}\div\sqrt{a^5}$

(b) $(pq)^{\frac{2}{3}}\div\left(p^{\frac{3}{4}}q^{\frac{1}{3}}\right)^2\times\left(p^{-\frac{1}{2}}\right)^{-3}$

(c) $\left(\dfrac{32x^{10}}{y^{-15}}\right)^{-\frac{1}{5}}\times\dfrac{\left(3xy^{-1}\right)^0}{\left(\sqrt{x}y\right)^6}$

(a) $\sqrt[3]{a}\times\sqrt[4]{a^2}\div\sqrt{a^5}$

$= a^{\frac{1}{3}}\times a^{\frac{1}{2}}\times a^{-\frac{1}{5}}$

$= a^{\frac{1}{3}+\frac{1}{2}-\frac{1}{5}}$

$= a^{\frac{25}{30}-\frac{6}{30}}$

$= a^{\frac{19}{30}}$

(b) $(pq)^{\frac{2}{3}}\div\left(p^{\frac{3}{4}}q^{\frac{1}{3}}\right)^2\times\left(p^{-\frac{1}{2}}\right)^{-3}$

$= p^{\frac{2}{3}}q^{\frac{2}{3}}p^{-\frac{3}{2}}q^{-\frac{2}{3}}p^{\frac{3}{2}}$

$= p^{\frac{2}{3}}$

(c) $\left(\dfrac{32x^{10}}{y^{-15}}\right)^{-\frac{1}{5}}\times\dfrac{(3xy^{-1})^0}{(\sqrt{x}\,y)^6}$

$= 2x^{-2}y^{-3}\times\dfrac{1}{x^3y^6}$

$= \dfrac{2}{x^5y^9}$ #

In a fractional index $a^{\frac{m}{n}}$, it is useful to think of
- numerator m as 'power' (because power 'comes from above')
- denominator n as 'root' (because roots 'grow below')

$\therefore\quad a^{\frac{m}{n}}=\sqrt[n]{a^m}$

Psst!

When a term is given in radical form, e.g. $\sqrt{y}$ or $\sqrt[3]{x^2}$, convert them into index form, e.g. $y^{\frac{1}{2}}$ or $x^{\frac{2}{3}}$, so that we can apply the laws of indices easily.

Psst!

It is necessary to know how to compute indices without using calculators so that we are able to apply the laws well when solving equations, or simplifying expressions involving indices and unknowns.

Worked Example 7
Applying Laws of Indices to evaluate

Evaluate each of the following without using the use of calculators.

(a) 1.5^0

(b) $\dfrac{1}{2^{-3}}$

(c) $\left(\sqrt{16}\right)^{-1}$

(d) $\left(\sqrt[5]{32}\right)^3$

(e) $(5^6)^{\frac{2}{3}}$

(f) $\left(\sqrt[3]{3}\right)^9$

(g) $\left(\dfrac{81}{16}\right)^{-\frac{3}{2}}$

(a) $1.5^0 = 1$

(b) $\frac{1}{2^{-3}} = 2^3 = 8$

(c) $(\sqrt{16})^{-1} = \frac{1}{4}$

(d) $(\sqrt[5]{32})^3 = 2^3 = 8$

(e) $(5^6)^{\frac{2}{3}} = 5^4 = 625$

(f) $(\sqrt[3]{3})^9 = 3^3 = 27$

(g) $\left(\frac{81}{16}\right)^{-\frac{3}{2}} = \left(\frac{4}{9}\right)^3$

$= \frac{64}{729}$ #

What is your Problem Solving Strategy for such problem? Write it here...

Psst!

Consider the following two equations:

$$x^2 = 64$$

$$2^x = 64$$

What is the difference between the two equations?

Worked Example 8

Solving equations involving indices

Solve each the following equations.

(a) $2^x = 128$

(b) $3^x = \dfrac{1}{9}$

(c) $5^{2x-1} = 0.2$

(d) $\dfrac{1}{9^x} = 27^{\frac{x}{3}}$

(e) $2^{-x} \times 8 = 16^x$

(f) $5^{\frac{1}{x}} \div 625^x = 1$

(a) $2^x = 128$

$2^x = 2^7$

$x = 7$ #

(b) $3^x = \frac{1}{9}$

$3^x = 3^{-2}$

$x = -2$ #

(c) $5^{2x-1} = \frac{1}{5}$

$5^{2x-1} = 5^{-1}$

$2x - 1 = -1$

$2x = 0$

$x = 0$ #

(d) $\frac{1}{9^x} = 27^{\frac{x}{3}}$

$(3^{-2})^x = (3^3)^{\frac{x}{3}}$

$-2x = x$

$3x = 0$

$x = 0$ #

(e) $2^{-x} \times 8 = 16^x$

$2^{-x} \times 2^3 = 2^{4x}$

$-x + 3 = 4x$

$5x = 3$

$x = \frac{3}{5}$ #

(f) $5^{\frac{1}{x}} \div 625^x = 1$

$5^{\frac{1}{x}} \div 5^{4x} = 5^0$

$5^{\frac{1}{x} - 4x} = 5^0$

$\frac{1}{x} - 4x = 0$

$4x = \frac{1}{x}$

$4x^2 = 1$

$x^2 = \frac{1}{4}$

$x = \frac{1}{2}$ or $x = -\frac{1}{2}$ #

> To solve the equations in Worked Example 8, we convert both sides to the same base and observe that if $a^x = a^n$, then $x = n$, provided $a \neq -1$, 0 or 1.

Psst!

Worked Example 9

Solving equations involving indices

Solve each the following equations.

(a) $7^x \times 2^x = 14^3$

(b) $3^{x+4} \times 2^{x+4} = \frac{1}{6}$

(c) $\frac{12^{1-x}}{2^{1-x}} = 36^{2x+1}$

(d) $6^{2x-1} \times 2^{1-2x} = 9$

Before we solve these equations…

How are the equations in Worked Example 9 different from those in Worked Example 8?

(a) $7^x \times 2^x = 14^3$

$14^x = 14^3$

$x = 3$

(b) $3^{x+4} \times 2^{x+4} = \frac{1}{6}$

$6^{x+4} = 6^{-1}$

$x + 4 = -1$

$x = -5$

(c) $\frac{12^{1-x}}{2^{1-x}} = 36^{2x+1}$

$6^{1-x} = 6^{2(2x+1)}$

$1 - x = 4x + 2$

$5x = -1$

$x = -\frac{1}{5}$

(d) $6^{2x-1} \times 2^{1-2x} = 9$

$6^{2x-1} \div 2^{2x-1} = 9$

$3^{2x-1} = 3^2$

$2x - 1 = 2$

$2x = 3$

$x = \frac{3}{2}$

What is your Problem Solving Strategy for problems in Worked Example 9? Write it here…

Concept Quiz 2 | Assignment 2

Textbook 3A Exercise 3B
Questions 1 – 11, 13 – 17, 22

1.3 Standard Form

Activity 3 **[Inquiry]**
Learning objective: To explore standard form.

1. The following table shows some examples of measurements which involve very large or very small numbers.

		Ordinary notation	Standard form
(i)	Singapore's population in June 2022	5 637 000	5.637×10^{6}
(ii)	Distance between Earth and Sun	147 100 000 000 m	1.496×10^{11} m
(iii)	Diameter of a hair	0.000 075 m	7.5×10^{-5} m
(iv)	Mass of an oxygen atom	0.000 000 000 000 000 000 000 026 567 kg	2.6567×10^{-26} kg
(v)	Mass of Earth	5 972 200 000 000 000 000 000 000 kg	5.9722×10^{24} kg
(vi)	Boiling point of tungsten	5828 K	5.828×10^{3} K
(vii)	Wavelength of violet light	0.000 000 38 m	3.8×10^{-7} m
(viii)	Mass of a neutron	0.000 000 000 000 000 000 000 000 001 674 kg	1.674×10^{-27} kg

(a) The examples in **(i)** – **(ii)** inolve very large numbers.
What do you observe about the powers of 10 in each standard form? positive.

(b) The examples in **(iii)** – **(iv)** inolve very small numbers.
What do you observe about the powers of 10 in each standard form? negative

(c) Complete the last column for **(v)** – **(viii)**.

2. The following table shows numbers expressed in standard form and numbers not expressed in standard form.

	Standard form	Not standard form
(i)	5.637×10^{6}	56.37×10^{5}
(ii)	1.496×10^{11}	0.1496×10^{12}
(iii)	7.5×10^{-5}	75×10^{-6}
(iv)	3.8×10^{-7}	0.038×10^{-9}
(v)	1.0×10^{12}	10×10^{11}

For a number in the form $A\times10^{n}$ to be considered a standard form, what can you say about A and n?

$1 \leq A < 10$, $n \in \mathbb{Z}$

Conclusion

> **Definition:**
>
> A number is said to be expressed in **standard form** (or also referred to as scientific notation) when it is written as
>
> $$A \times 10^n,$$
>
> where $1 \leq A < 10$ and n is an integer.

Standard form is a system of writing numbers which can be particularly useful for working with very large or very small numbers. It is based on using powers of 10 to express how big or small a number is.

BIG idea

Notations

Scientists use standard form when working with the speed of light and distances between galaxies, which can be enormous. The size of bacteria or atoms may also be referred to in standard form as they are so tiny. Very large and very small numbers will not fit onto a standard calculator and they also take a while to write out if used in calculations. Standard form makes this easier. Standard form allows scientists to compare the order of magnitude of different quantities in a way that helps them to explain phenomena and make useful predictions.

Worked Example 10
Expressing numbers in standard form

Express each of the following numbers in standard form.

(a) 34 500 000
(b) 34 567
(c) 345.6
(d) 3 000
(e) 0.000 034
(f) 0.000 000 3

(a) 3.45×10^7
(b) 3.4567×10^4
(c) 3.456×10^2
(d) 3×10^3
(e) 3.4×10^{-5}
(f) 3.0×10^{-7}

> *What is your Problem Solving Strategy for problems in Worked Example 10? Write it here...*

Psst!

Worked Example 11
Using calculator to apply orders of operation in standard form

Evaluate each of the following, giving your answer in standard form, correct to 3 significant figures.

(a) $(1.35\times10^{3}+5.4\times10^{-1})\times(2.07\times10^{4})$

(b) $\dfrac{3.58\times10^{-3}-4.6\times10^{-4}}{\sqrt{5.17\times10^{5}}}$

(a) $(1.35\times10^{3}+5.4\times10^{-1})\times(2.07\times10^{4})$

$= (1.35\times10^{3}+0.00054\times10^{3})\times2.07\times10^{4}$

$= 1.35054\times10^{3}\times2.07\times10^{4}$

$= 2.80\times10^{7}$ #

(b) $\dfrac{3.58\times10^{-3}-4.6\times10^{-4}}{\sqrt{5.17\times10^{5}}} = \dfrac{3.58\times10^{-3}-0.46\times10^{-3}}{\sqrt{517000}}$

$= 4.33\times10^{-6}$ #

Worked Example 12
Using standard form in real-world context

(i) In this part, use the fact that 1 light year $= 9.46\times10^{15}$ metres.
The distance of the star Sirius from the Sun is 8.6 light years.
Calculate the distance, in kilometres, of Sirius from the Sun.
Give your answer in standard form.

(ii) The distance of the star Proxima Centauri from the Sun is 3.97×10^{13} km.
A space probe travels at 60 000 km/h.
Calculate the time taken for the probe to travel from the Sun to Proxima Centauri.
Give your answer in years, correct to three significant figures.

[GCEO2012/P2/Q4c]

(i) Distance $= 8.6\times9.46\times10^{15}$

$= 8.14\times10^{16}$ m #

(ii) Time $= \dfrac{3.97\times10^{13}}{60000} = 6.62\times10^{8}$ years #

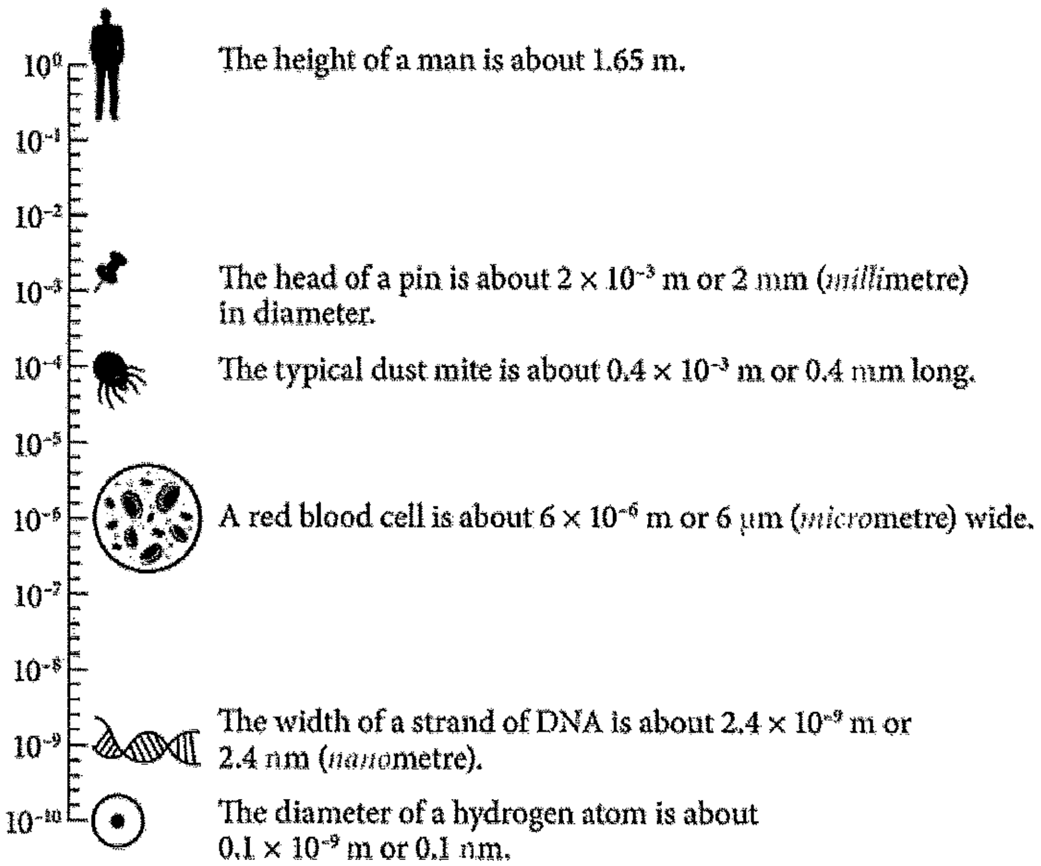
SI prefixes

SI unit (SI stands for International System of Unit) system is a metric system, i.e. multiples and submultiples of the units can be easily expressed as indices (or powers) of 10. In engineering, some indices of 10 are more popular than others, particularly the multiples of 3: 10^3, 10^6, 10^{-3}, etc. **SI prefixes** are a series of prefixes to SI units.

The table below lists some of the common prefixes and their symbols used for very large and very small numbers.

Power of 10	English word	SI prefix	Symbol
10^{12}	trillion	tera-	T
10^{9}	billion	giga-	G
10^{6}	million	mega-	M
10^{3}	thousand	kilo-	k
10^{-3}	thousandth	milli-	m
10^{-6}	millionth	micro-	μ
10^{-9}	billionth	nano-	n
10^{-12}	trillionth	pico-	p

The figure below shows a range of SI prefixes used in our daily lives.



Worked Example 13
Expressing SI prefixes in daily lives in standard form

For each of the following, give your answer in standard form.

(a) The speed of light is approximately 3×10^8 m/s. This means that the light from an object 1 m away from us takes 3.33 nanoseconds (ns) to reach our eyes.
Express this time in seconds (s).

(b) A steam power plant in Singapore has a capacity of 250 megawatts (MW).
Express this capacity in watts (W).

(c) In Singapore, most power socket installations have a current rating of 13 amperes (A). It is important to note that currents as low as 10 milliamperes (mA) can cause muscle contractions.
Express 13A in mA.

(Reference: https://www.consumerproductsafety.gov.sg/types-of-mains-plugs-suitable-for-use-in-singapore)

(a) Time $= 3.33 \times 10^{-9}$ s

(b) Capacity $= 250 \times 10^6$
$= 2.50 \times 10^8$ W

(c) 13 A $= 13 \times 10^3$ mA
$= 1.3 \times 10^4$ mA

*What is your **Problem Solving Strategy** for problems in Worked Example 10?*
Write it here...

Psst!

Concept Quiz 3 | Assignment 3

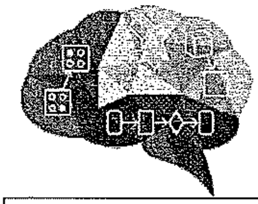
Textbook 3A Exercise 3C

Look it up!

Does a thumbdrive with a capacity of 1 gigabyte (GB) have exactly 1 billion bytes of storage space?

Because the SI prefixes strictly represent powers of 10, they should not be used to represent powers of 2. Thus, one kilobit, or 1 kbit, is 1000 bit and not 210 bit = 1024 bit. To alleviate this ambiguity, prefixes for binary multiples have been adopted by the International Electrotechnical Commission (IEC) for use in information technology.

Extension
Computational Thinking



How to convert a number given in ordinary notation to standard form?

Your task...

Think of an algorithm for the above conversion.
You may present your algorithm in one of the following:

- A flowchart
- A pseudo-code
- A program (in any language)

REVIEW & REFLECT