

Name: Solutions ()

Class: S3—

Enduring Understandings

At the end of the topic, students will understand that

- functions contain algebraic structures that describe the relationship between variables based on real-world situations
- functions represent the rules that exist within a system: in order for a function to exist, there needs to be a unique output for very specific input.
- the model of input-process-output is a good representation of a function.
- graphs are pictorial representations of the nature of the relationship connecting 2 or more variables.
- the gradient of a function might not be the same across the domain of the function.

Essential Questions

- How do functions describe the relationship between variables based on real-world situation?
- What conditions must be satisfied in order for the function to be considered well defined?
- How do graphs demonstrate the nature of the behaviour of a function?
- How does the gradient of a function change across the domain of the function?

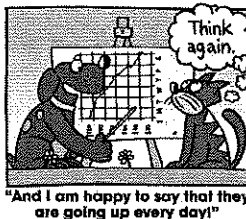
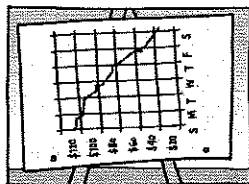
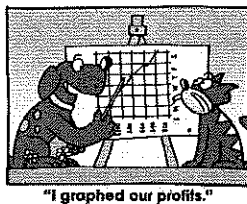
Big Ideas

- Functions are relationships between two sets of objects that expresses how an element from the 1st set (input) uniquely determines to an element from the second set (output) according to a rule.
- Diagrams are succinct, visual representations of real-world or mathematical objects that serve to communicate properties of the objects and facilitate problem solving.
- Models are abstractions of real-world situations or phenomena using mathematical objects and representations.

Learning Objectives

At the end of the unit, students will be able to

- differentiate between sketching and plotting.
- sketch and draw graphs of quadratic functions given in factorised form and in completed square forms.
- sketch and draw the graphs of $y = ax^n$ where $n = -2, -1, 0, 1, 2, 3$ and simple sums of not more than three of these.
- sketch and draw the graphs of exponential function $y = ka^n$ where a is a positive integer.
- estimate the gradient of a curve by drawing a tangent.
- solve equations in one variable by graphical method.



www.bigideasmath.com

Unit Checklist

Sketch and draw graphs			
Cognitive Level	Know, Understand, Demonstrate	Checklist	
		$y = ax^n$	$y = ka^x$
Level 0: Memorisation	Recognise/match the graph sketch to the correct equation		
Level 1: Procedural tasks without connections	Sketch the graph of a given equation		
	Plot and draw the graph of a given equation		

Graphical Solutions of Equations		
Cognitive Level	Know, Understand, Demonstrate	Checklist
Level 1: Procedural tasks without connections	Plot and draw the graph of a given equation	
Level 2: Procedural tasks with connections	Use the graph to find the intercepts and/or values of x for a given y value (or vice versa).	
	Draw a tangent line to find the gradient (rate of change) at a given x value	
Level 3: Problem Solving	Solve equations in one variable by graphical method (finding point(s) of intersection of two graphs)	

Teaching to the Big Idea

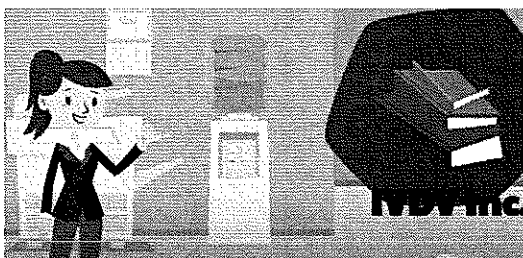
Lesson sequence in the unit								
Student Learning Outcomes	Dimensions (Please tick the appropriate boxes)							
	FUNCTIONS	INVARIANCE	NOTATIONS	DIAGRAMS	MEASURES	EQUIVALENCE	PROPORTIONALITY	MODELS
	F	I	N	D	M	E	P	M
Sketch and draw the graphs of $y = ax^n$	✓			✓			✓ (gradient)	✓
Sketch and draw the graphs of $y = ka^x$	✓			✓				✓
Solve equations in one variable by graphical method	✓			✓				✓
Estimate the gradient of a curve by drawing a tangent line	✓			✓			✓	✓

Sections

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IV. Graphical Solution of Equations.....	19
V. Gradients of Graphs	24
VI. Graph Drawing	26

Video introducing the use of functions in real life.

<https://www.youtube.com/watch?v=oeYUd5SFJys>

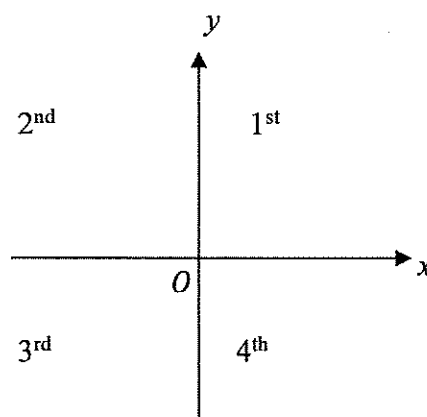


I. Asymptotes

Quadrants

The x -axis and y -axis divide the plane into four regions called quadrants.

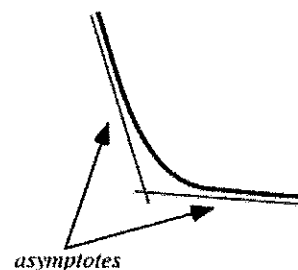
The quadrants are labelled in an anticlockwise direction as shown in the diagram.



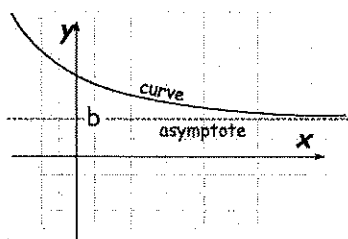
Asymptotes

An asymptote is a line or curve that approaches a given curve arbitrarily closely, as illustrated in the diagram.

[source: <http://mathworld.wolfram.com/Asymptote.html>]

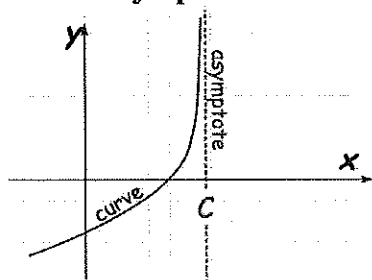


Horizontal Asymptotes



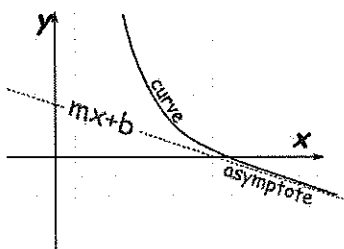
It is a Horizontal Asymptote when:
as x goes to infinity ($+\infty$) [or $-\infty$ ($-\infty$)], the curve approaches some constant value b

Vertical Asymptotes



It is a Vertical Asymptote when:
as x approaches some constant value c (from the left or right), the curve goes towards infinity (or $-\infty$).

Oblique Asymptotes



It is an Oblique Asymptote when:
as x goes to infinity (or $-\infty$) then the curve goes towards a line $y = mx+b$
(note: m is not zero as that is a Horizontal Asymptote).

Source: <http://www.mathsisfun.com/>

Example 1 [2017 SST S3 EOY EM P2]

The variables x and y are connected by the equation

$$y = \frac{5}{x^2} + x, \quad x \neq 0.$$

Some corresponding values of x and y , correct to 1 decimal place, are given in the following table.

x	-2	-1.5	-1	1	1.3	2	3	4
y	-0.8	0.7	4.0	6.0	4.3	3.3	3.6	4.3

The points given in the table is plotted in the axes below for $-2 \leq x \leq 4$.

(a) Will there be an asymptote for the graph of this function for $-2 \leq x \leq 4$?

Yes/ No (circle the answer)

(b) State the equation of the asymptote (if any): $y = x$

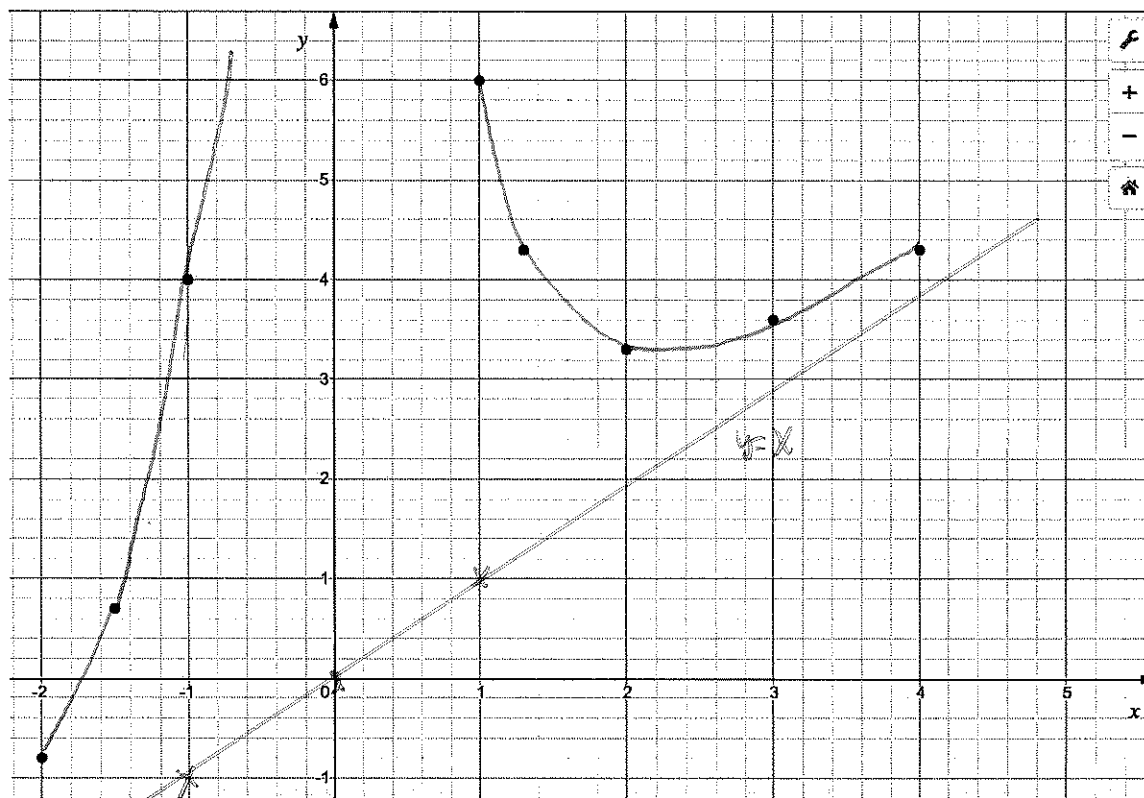
(c) Why is there an asymptote for the graph of this function?

OR Why are there no asymptotes for the graph of this function?

As $x \rightarrow \infty$, $\frac{5}{x^2} \rightarrow 0$. $\therefore y = \frac{5}{x^2} + x \rightarrow x$

(d) On the axes, join the points with a smooth curve.

(e) Verify your answer with a graphing software.



Example 2

The variables x and y are connected by the equation $y = \frac{1}{3}(x^3 - 6x^2 + 5x)$.

Some corresponding values of x and y , correct to 1 decimal place, are given in the following table.

x	-0.5	0	0.5	1.0	2.0	2.5	3.0	4.0	4.5	5.0	5.5
y	-1.4	0	0.4	0	-2	-3.1	-4	-4	-2.6	0	4.1

The points given in the table is plotted in the axes below for $-0.5 \leq x \leq 5.5$.

(a) Will there be an asymptote for the graph of this function for $-0.5 \leq x \leq 5.5$?

Yes / No (circle the answer)

(b) State the equation of the asymptote (if any): _____

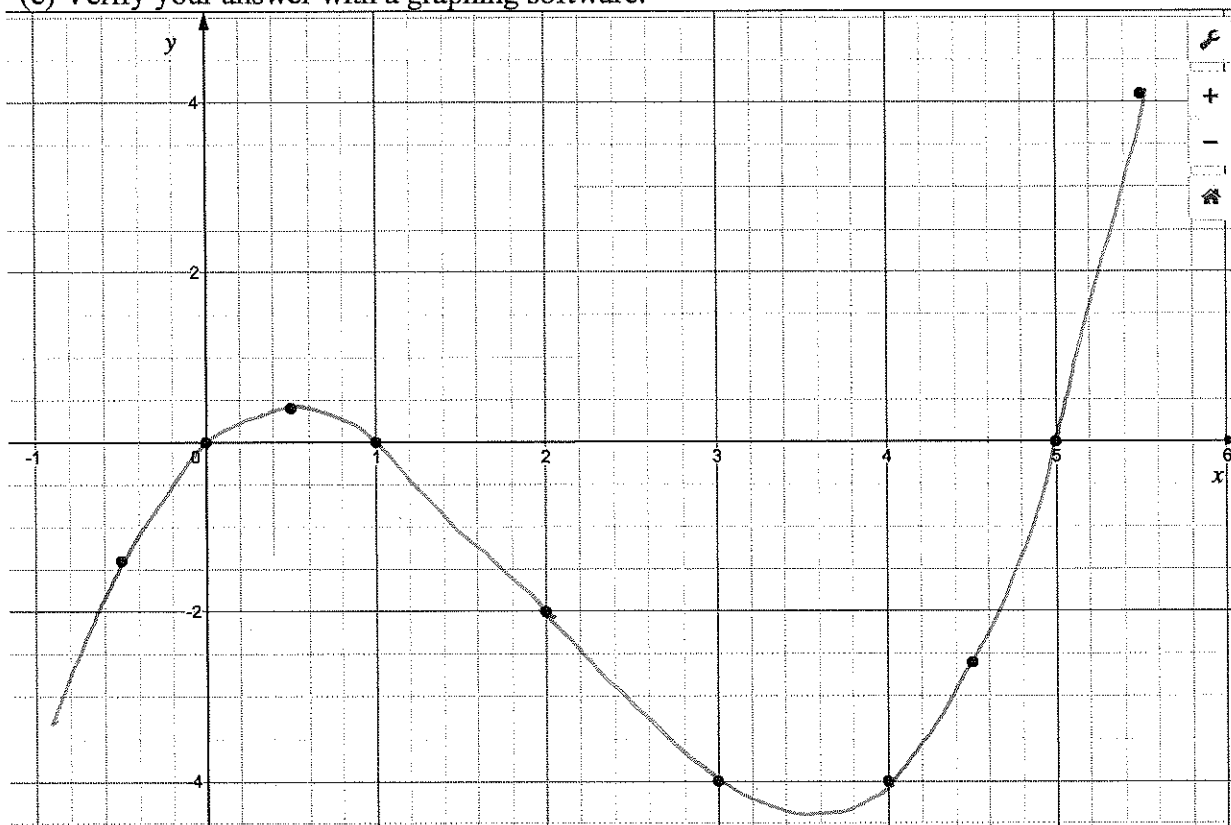
(c) Why is there an asymptote for the graph of this function?

OR Why are there no asymptotes for the graph of this function?

As $x \rightarrow \infty$, $y \rightarrow \infty$.

(d) On the axes, join the points with a smooth curve.

(e) Verify your answer with a graphing software.

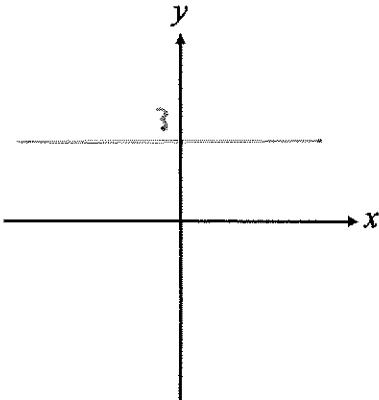
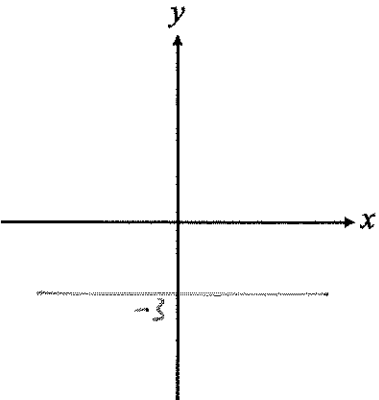
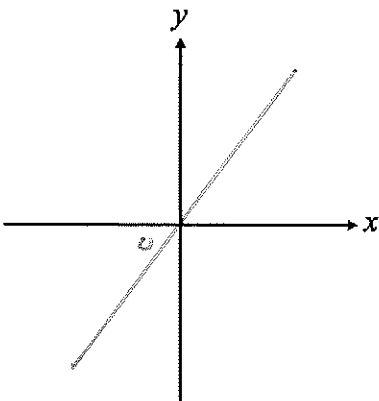
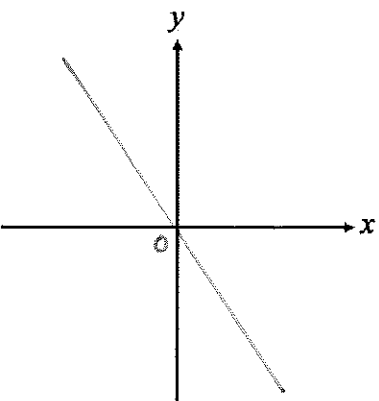
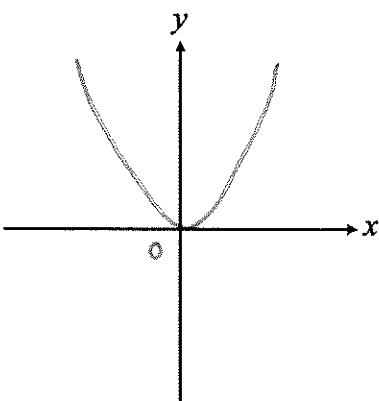
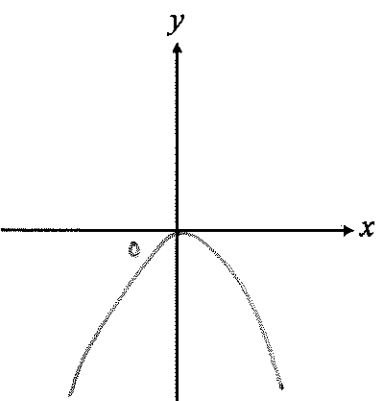


II. Graphs of Power and Exponential Functions

A power function has the form $y = ax^n$, $a \neq 0$.

In Secondary 1 and 2, we have learnt power functions where $n = 0, 1, 2$.

Can you sketch the graphs below?

Value of n	$a > 0$ e.g. $y = 3$	$a < 0$ e.g. $y = -3$
0		
1		
2		

In Secondary 3, we will learn how power functions look like, where $n = 3, -1, -2$.

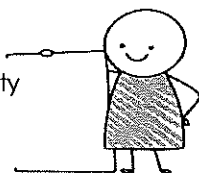


Imagine, think of how we may describe a graph's characteristics?

It is important to use a consistent set of methods to describe a graph, and we may use the following points to describe a graph (**STAIRS** – full details will be on page 14)

1. Shape
2. Turning points and/or points of inflexion
3. Asymptotes: vertical, horizontal or oblique asymptotes
4. Intercepts: x-intercept, y-intercept
5. Region: domain, range, which quadrants the graphs lie on
6. Symmetry: line of symmetry, order of symmetry

Self-directed learning Activity

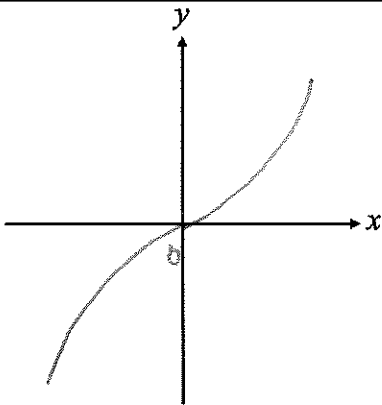
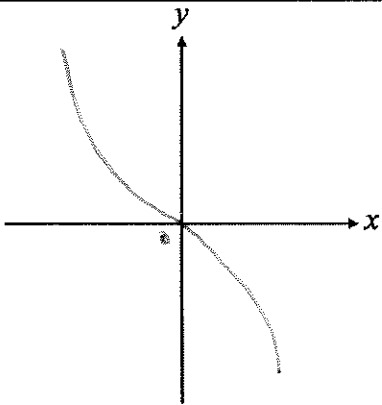
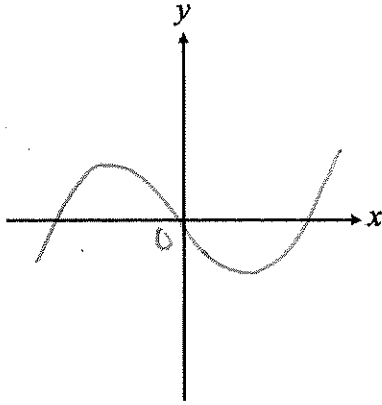
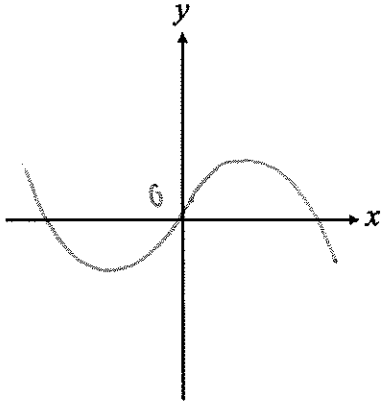


Go to SLS Activity: “Sec 3 Mathematics Graphs of Power Functions”

As you go through the SLS, complete pages 10 to 12.

Cubic Graph ($n = 3$)

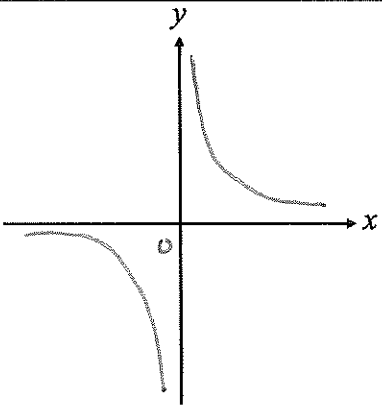
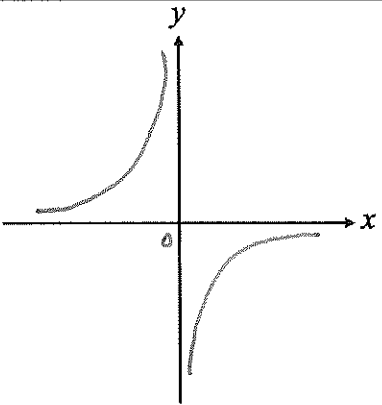
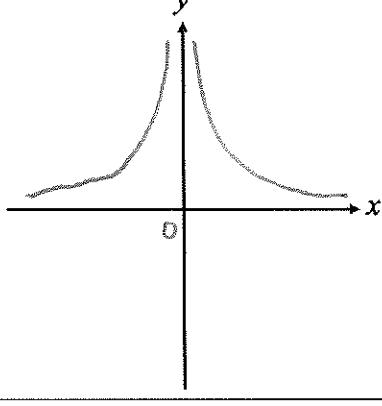
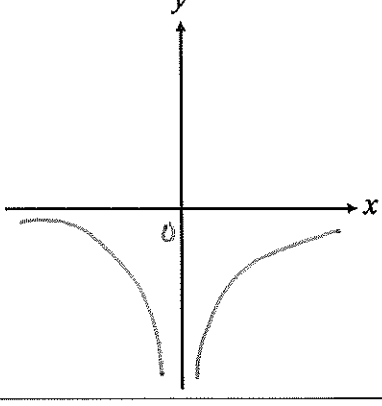
Sketch the following graphs upon attempting SLS.

	$a > 0$	$a < 0$
$y = ax^3$		
$y = ax^3 + bx^2 + cx + d$		

	$y = ax^3$
Turning point	point of inflexion $(0,0)$
Asymptotes	nil
Intercepts	$(0,0)$
Region	Domain: $\begin{cases} \text{As } x \rightarrow \infty, y \rightarrow \infty \\ \text{As } x \rightarrow -\infty, y \rightarrow -\infty \end{cases}$ Domain: $(-\infty, \infty)$ Range: $\begin{cases} \text{As } x \rightarrow \infty, y \rightarrow \infty \\ \text{As } x \rightarrow -\infty, y \rightarrow -\infty \end{cases}$ Range: $(-\infty, \infty)$
Symmetry	Rotational symmetry: Order 3 about $(0,0)$

Reciprocal Graphs ($n = -1, -2$)

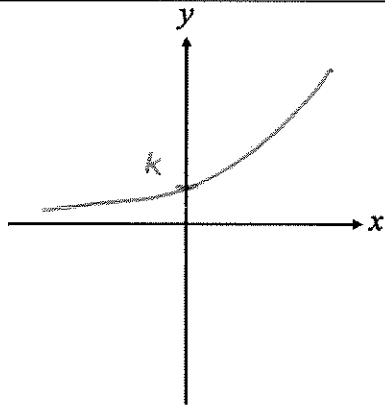
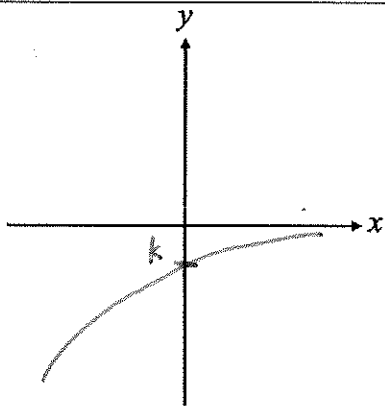
Sketch the following graphs upon attempting SLS.

	$a > 0$	$a < 0$
$y = \frac{a}{x}$		
$y = \frac{a}{x^2}$		

	$y = \frac{a}{x}$	$y = \frac{a}{x^2}$
Turning point	nil	nil
Asymptotes	x -axis, $y = 0$ y -axis, $x = 0$	x -axis, $y = 0$ y -axis, $x = 0$
Intercepts	nil	nil
Region	For $a > 0$, the graph lies in <u>1st</u> and <u>3rd</u> quadrants. For $a < 0$, the graph lies in <u>2nd</u> and <u>4th</u> quadrants.	For $a > 0$, the graph lies in <u>1st</u> and <u>2nd</u> quadrants. For $a < 0$, the graph lies in <u>3rd</u> and <u>4th</u> quadrants.
Symmetry	Rotational symmetry: order 2 about $(0, 0)$	Line of symmetry: $x = 0$

Exponential Graphs ($y = ka^x$)

Sketch the following graphs upon attempting SLS.

	$k > 0$	$k < 0$
$y = ka^x$		

	$y = ka^x$
Turning point	nil
Asymptotes	x -axis: $y = 0$
Intercepts	$(0, k)$
Region	For $k > 0$, the graph lies entirely <u>above</u> the x -axis. For $k < 0$, the graph lies entirely <u>below</u> the x -axis.
Symmetry	nil

Example 3

Match the functions to their respective graphs.

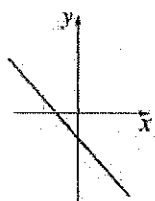


Figure 1

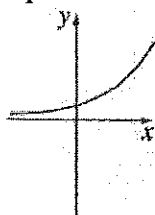


Figure 2

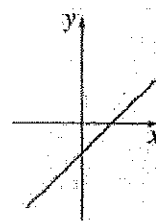


Figure 3

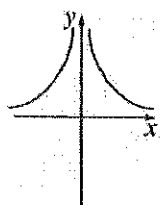


Figure 4

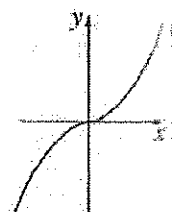


Figure 5

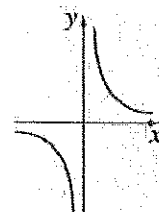


Figure 6

1. $y = 2^x$

Figure 2

2. $y = x^3$

Figure 5

3. $y = x - 1$

Figure 3

4. $y = \frac{3}{x^2}$

Figure 4

5. $y = -x - 2$

Figure 1

6. $y = \frac{5}{x}$

Figure 6

Example 4 [2020 SST S3 EM EOY P1]

The sketch shows the graph of $y = ka^{-x} + 1$. The points $A(-2, -79)$ and $B(0, -4)$ lie on the graph. Find the values of k and a .

$$\begin{aligned} y &= ka^{-x} + 1 \\ -79 &= ka^{-(-2)} + 1 & -4 &= ka^{-0} + 1 \\ ka^2 &= -80 - (1) & k &= -5 - (1) \end{aligned}$$

Subs (2) into (1):

$$-5a^2 = -80$$

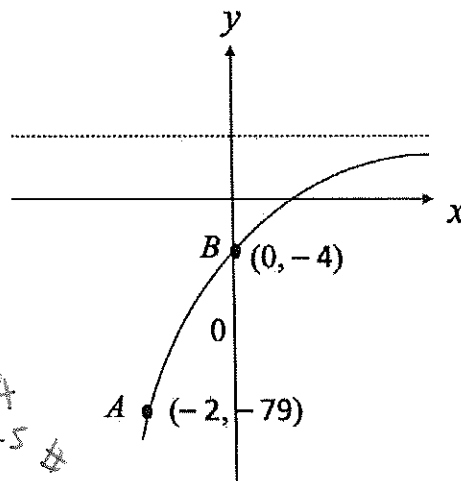
$$a^2 = 16$$

$$a = 4 \text{ or } -4 \text{ (NA, } a > 0)$$



Now is the time to complete Assignment 1!

$$\therefore a = 4 \\ k = -5$$



III. Graph Sketching

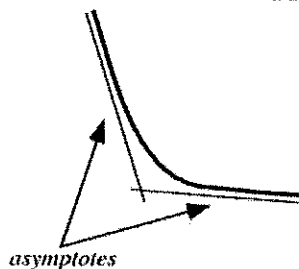
The key features of a graph are:

1. **Shape**
2. **Turning Points [Stationary Points (point where gradient = 0)]**
 - Maximum point
 - Minimum point
 - Point of inflexion

Note: Maximum point and minimum point are also known as turning points.

3. **Asymptote**

A line or curve that approaches a given curve arbitrarily closely:



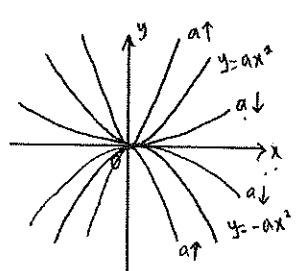
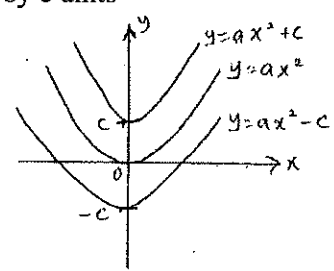
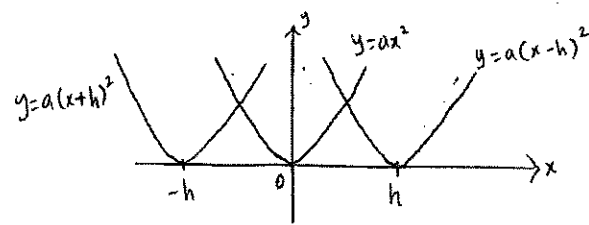
(Refer to notes page 4 for more information)

4. **Intercepts (Axial Intercepts)**
 - To find x-intercepts, let $y = 0$
 - To find y-intercept, let $x = 0$
5. **Region**
 - **Domain:** range of values of x
 - **Range:** range of values of y
6. **Symmetry**
 - Line of symmetry
 - Rotational symmetry about a point
 - Note: rotational symmetry of order 2 about a point means the shape maps onto itself twice by rotation in 360°

STAIRS is a useful acronym in remembering the key features of a graph.

Graphical Transformation

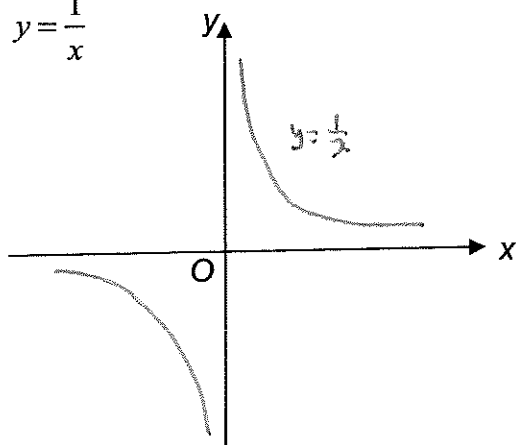
Use a graphing software, draw the graph of a power function or exponential function.
Vary the coefficients in the function and observe the following graphical transformations.

	Effect of changing the coefficient
$y = a \times f(x)$	<p>When the absolute value of a increases, the graph becomes steeper. When the absolute value of a decreases, the graph becomes less steep.</p> <p>Example: $y = ax^2$</p> 
$y = a \times f(x)$ VS $y = -a \times f(x)$	Reflection about the x-axis, $y = 0$
$y = af(x)$ VS $y = af(-x)$	Reflection about the y-axis, $x = 0$
$y = af(x) \pm c$	<p>$+c$: the graph translates vertically upwards by c units $-c$: the graph translates vertically downwards by c units</p> <p>Example: $y = ax^2 \pm c$</p> 
$y = af(x \pm h)$	<p>$+h$: the graph translate horizontally to the left by h units $-h$: the graph translate horizontally to the right by h units</p> <p>Example: $y = a(x \pm h)^2$</p> 

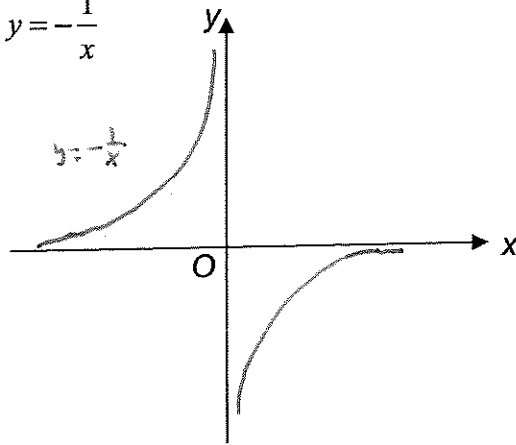
Example 5

Sketch and label the graphs of the following functions on the given axes.

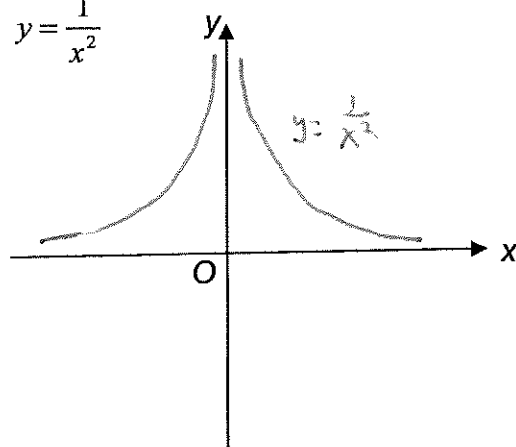
(a) $y = \frac{1}{x}$



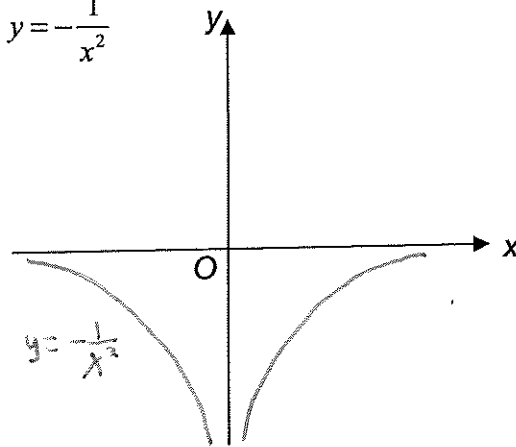
(b) $y = -\frac{1}{x}$



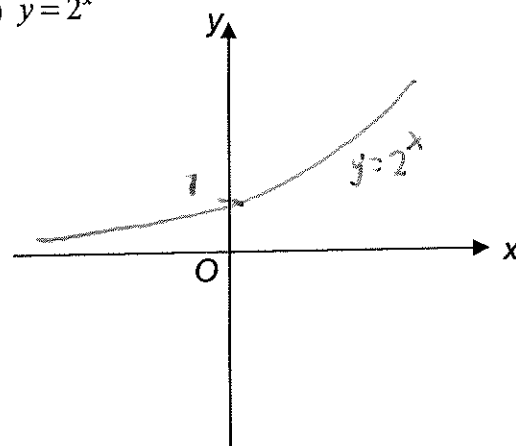
(c) $y = \frac{1}{x^2}$



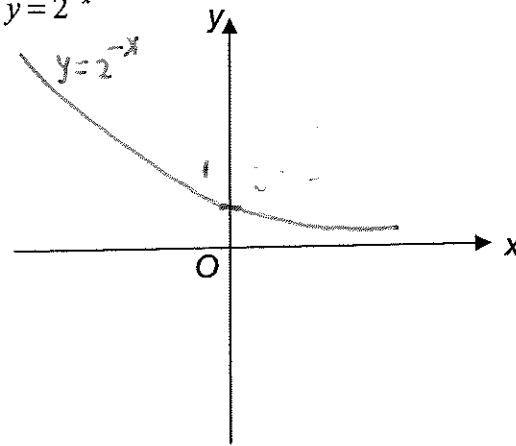
(d) $y = -\frac{1}{x^2}$



(e) $y = 2^x$



(f) $y = 2^{-x}$



Example 6 [N2002/I/17]

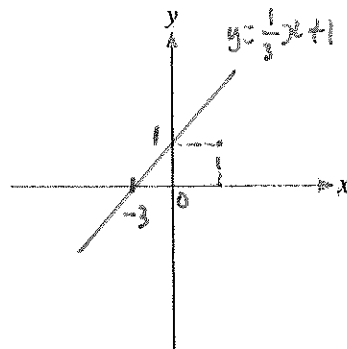
(b) The point (1, 1) is marked on each diagram in the answer space. On these diagrams, sketch the graphs of

(i) $y = \frac{1}{3}x + 1$,

(ii) $y = \frac{1}{x^2}$,

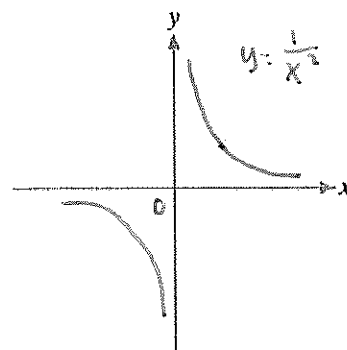
(iii) $y = 2^x$.

Answer (b)(i)



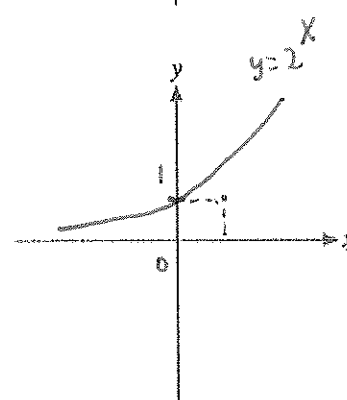
[1]

(ii)



[1]

(iii)

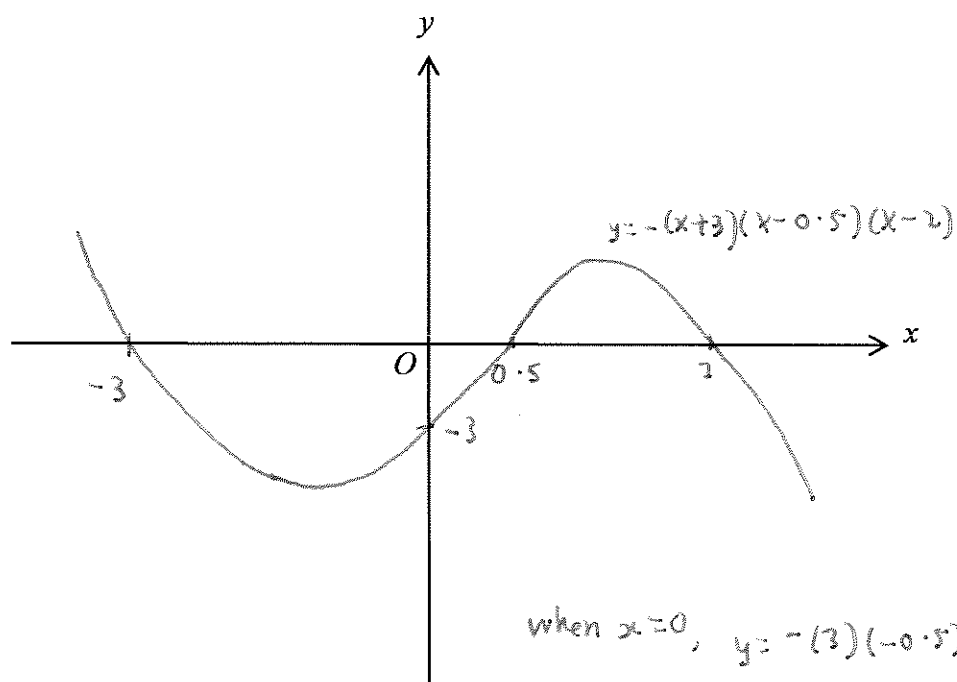


[1]

NP1/2002/17

Example 7 [2020 SST S3 EM CT2]

On the axes below, sketch the graph of $y = -(x + 3)(x - 0.5)(x - 2)$.



$$\begin{aligned}\text{when } x=0, \quad y &= -(3)(-0.5)(-2) \\ &= -3\end{aligned}$$

$$\begin{aligned}\text{when } y=0, \quad -(x+3)(x-0.5)(x-2) &= 0 \\ x &= -3, \quad 0.5 \text{ or } 2.\end{aligned}$$

IV. Graphical Solution of Equations

Activity

(1) Explain how to solve the following simultaneous equations using a graphical method.

$$x + y = 12,$$

$$2x - y = 3.$$

Draw the graphs $x+y=12$ and $2x-y=3$.

The point of intersection $(5,7)$, is the solution to the equations

(2) Four equations are presented below.

Equation 1: $\frac{x^2}{2} - \frac{21}{2} = 2x$

Equation 2: $x^2 - 21 = 4x$

Equation 3: $x^2 - 4x - 21 = 0$

Equation 4: $(x + 3)(x - 7) = 0$

(a) Why are these equations **equivalent**?

They have the same solution, $x = -3$ and $x = 7$

(b) Explain how we get the solution of the equation $\frac{x^2}{2} - \frac{21}{2} = 2x$ using a graphical method.

Plot $y = \frac{x^2}{2} - \frac{21}{2}$ and $y = 2x$. Find their intersection.

(b) What is/are other possible ways to find the solutions using a graphical method?

1. Plot $y = x^2 - 21$ and $y = 4x$. Find their intersection.

2. Plot $y = x^2 - 4x - 21$ and $y = 0$. Find their intersection.

3. Plot $y = (x+3)(x-7)$ and $y = 0$. Find their intersection.

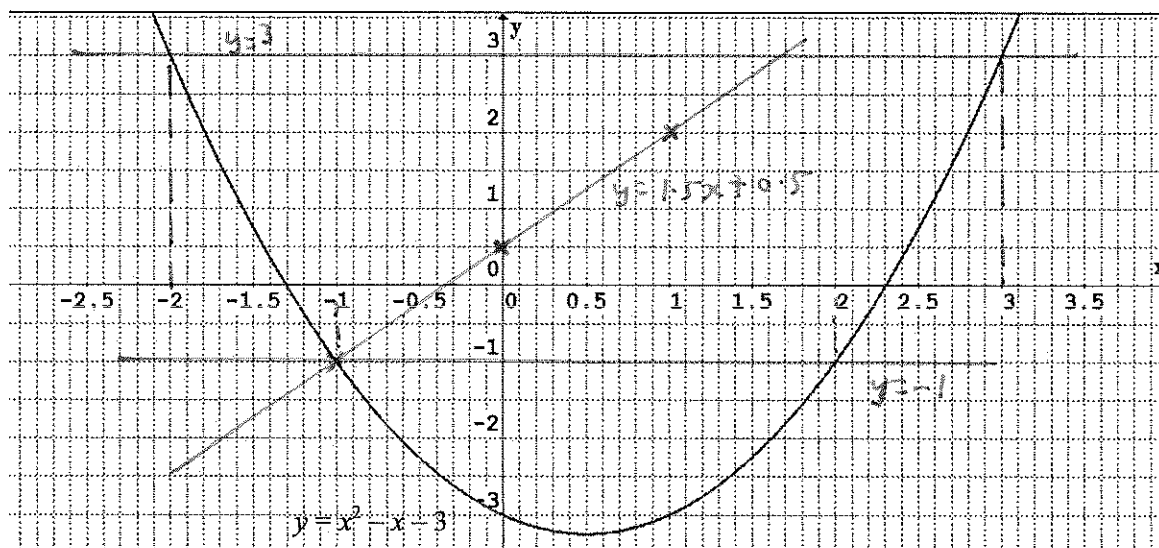
Conclusion

The points of intersection between graphs of 2 functions represents the solutions of a polynomial equation.

The polynomial equation is formed by substituting the equation of one function into the equation of the other function.

Example 8

The graph of $y = x^2 - x - 3$ is shown below, for $-2 \leq x \leq 3$.



By drawing suitable lines on the same axes, find the solution(s) to the following equations for $-2 \leq x \leq 3$.

(i) $x^2 - x - 3 = 3$,

Draw $y = 3$

From the graph, $x = -2$ or $x = 3$

(ii) $x^2 - x - 3 = 1.5x + 0.5$,

Draw $y = 1.5x + 0.5$

From the graph,

$x = -1$

x	$y = 1.5x + 0.5$
-1	-1
0	0.5
1	2

(iii) $x^2 - x - 2 = 0$.

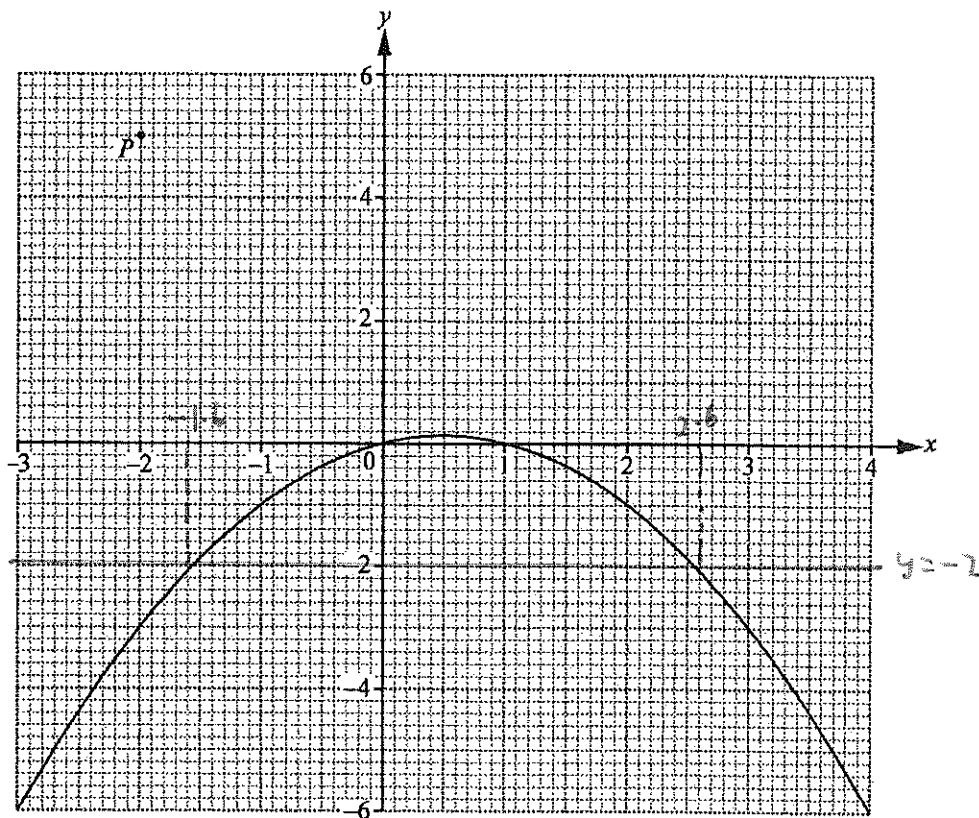
$$x^2 - x - 3 = -1$$

Draw $y = -1$

From the graph, $x = -1$ or $x = 2$

Example 9 [N18/I/23b]

The graph of $y = \frac{1}{2}(x - x^2)$ is drawn on the grid. (Point P is not needed for this question)



Use the graph to solve the equation $x - x^2 = -4$.

$$x - x^2 = -4$$

$$\frac{1}{2}(x - x^2) = -2$$

Plot $y = -2$

From the graph, $x = -1.6$ or $x = 2.6$

(Actual answer: $x = -1.562$ or $x = 2.562$)

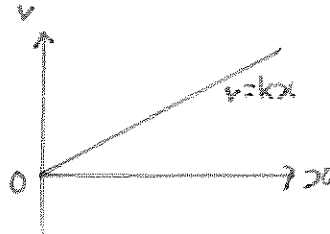


Extension STEM in Functions

1.



The velocity, v , of a fluid flow is given by $v=kx$. Sketch, on the same axes, the graphs of v against x for x between 0 and 10.



2.



$$y = \frac{9}{5}x + 32 \text{ (Type)}$$

Let $y=95x+32$. This function converts temperature from $^{\circ}\text{C}$ to $^{\circ}\text{F}$. Evaluate y when $x = 100$.

$$y = \frac{9}{5}x + 32$$

$$\text{When } x=100, y=212$$

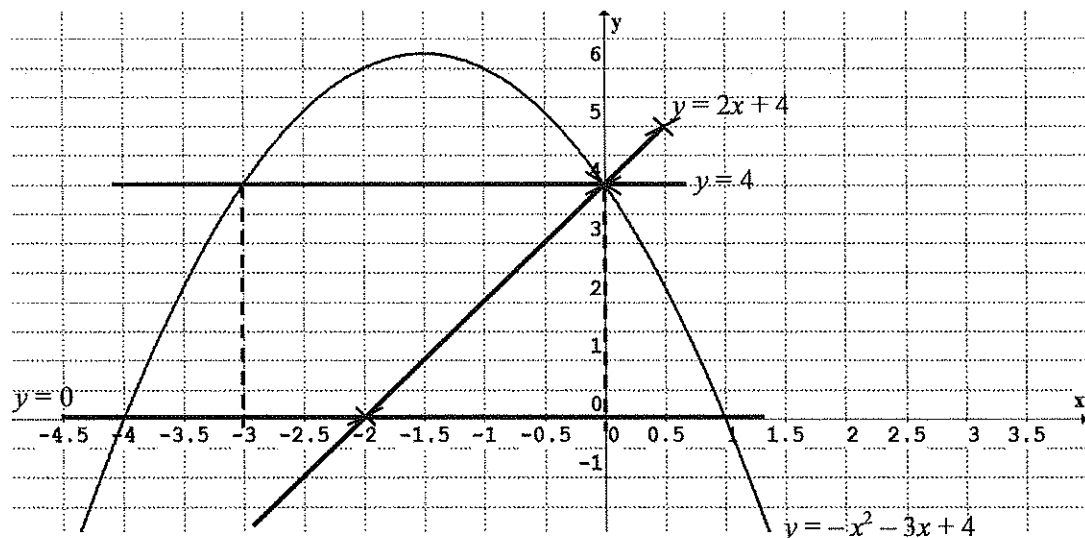
$$\therefore 100^{\circ}\text{C} = 212^{\circ}\text{F} \quad \text{A}$$

Summary: Graphical Solution of Equations

To use graphs to solve equations,

1. Make y the subject of the formula. In other words, the left hand side of the equation should be the equation of the quadratic graph drawn.
2. Draw a suitable line based on the right hand side of the equation, using a solid line. Label equation of line.
3. Find the solution(s), value(s) of x , from the graph. **Draw dotted lines for reading lines.**

Example: The graph of $y = -x^2 - 3x + 4$ is shown below, for $-4.5 \leq x \leq 1.5$.



By drawing suitable lines on the same axes, find the solution(s) to the following equations for $-4.5 \leq x \leq 1.5$.

(i) $-x^2 - 3x + 4 = 4$,

$$-x^2 - 3x + 4 = 4$$

$$y = 4$$

From graph, $x = -3$ or $x = 0$

(ii) $-x^2 - 3x = -4$,

$$-x^2 - 3x = -4$$

$$-x^2 - 3x + 4 = -4 + 4$$

$$-x^2 - 3x + 4 = 0$$

$$y = 0$$

From graph, $x = -4$ or $x = 1$

(iii) $-x^2 - 3x + 4 = 2x + 4$.

$$-x^2 - 3x + 4 = 2x + 4$$

$$y = 2x + 4$$

x	-2	0	0.5
y	0	4	5

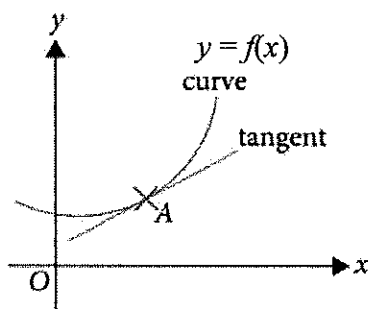
From graph, $x = 0$

V. Gradients of Graphs

You have learnt that the GRADIENT (aka the steepness of the SLOPE) of the graph of a linear function is CONSTANT across the whole domain of the function.

In the diagram below, we have drawn the graph of the function $y = f(x)$. We have also drawn a line that TOUCHES (it does NOT cut the graph of the function) at the point $y = f(x)$.

This line is known as the TANGENT to the function at point A .



(Adapted from: think! Mathematics Secondary Textbook 3A (8th Edition) (SL Education))

It does not take a lot of imagination to note that if we choose another point, say point B , and drew another tangent to the function $y = f(x)$, the GRADIENT of this tangent will be different than the gradient of the tangent drawn in the diagram above.

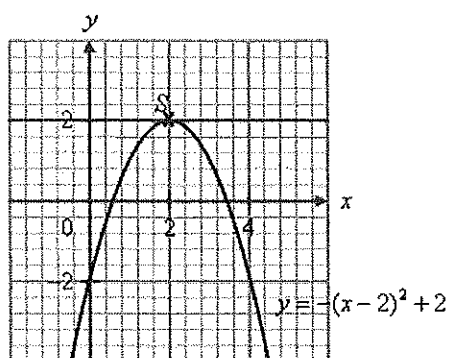
Note: Until next year, all our discussions regarding the TANGENT and its GRADIENT are simply ESTIMATIONS. The computation will be taught in A Math Calculus.

Recall: Given a line joined by a pair of points with coordinates (x_1, y_1) and (x_2, y_2) ,
gradient of a straight line = $\frac{y_2 - y_1}{x_2 - x_1}$.

Example 10

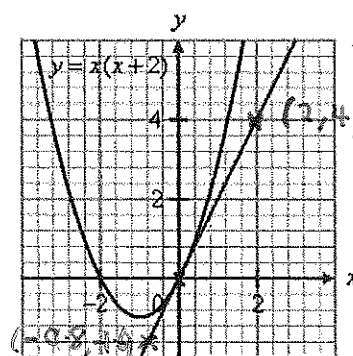
Calculate the gradient of the curve at the marked points.

(a)



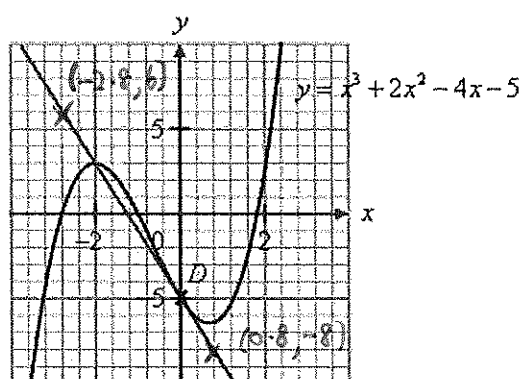
Gradient = 0

(b)



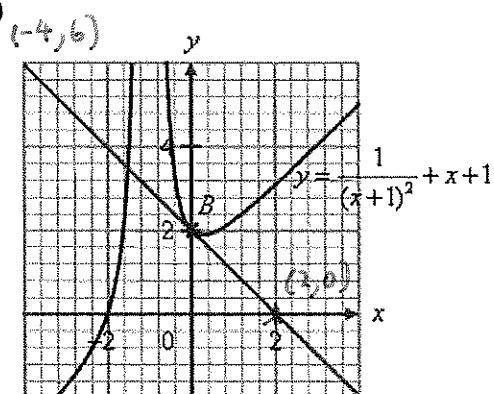
$$\text{Gradient} = \frac{4 - (-1.6)}{2 - (-0.8)} = \frac{5.6}{2.8} = 2$$

(c)



$$\text{Gradient} = \frac{6 - (-8)}{-2.8 - 0.8} = -3.89$$

(d)



$$\text{Gradient} = \frac{0 - 6}{2 - (-4)} = -1$$

VI. Graph Drawing

Example 11

Note: Parts (a) and (b) are done for you on the given graph paper. Use the given graph on the next page to complete parts (c), (d) and (e).

The variables x and y are connected by the equation $y = x^3 - 9x$.

Some corresponding values of x and y is given in the following table.

x	-3	-2.5	-2	-1	-0.5	0	0.5	1	2	2.5	3
y	a	6.9	10	8	4.4	0	-4.4	-8	-10	-6.9	0

- (a) Calculate the value of a . [1]

$$a = (-3)^3 - 9(-3)$$

$$a = 0$$

- (b) Taking 2 cm to represent 1 unit on the x -axis and 1 cm to represent 1 unit on the y -axis, draw the graph of $y = x^3 - 9x$ for the values of x in the range $-3 \leq x \leq 3$. [3]

(Graph on next page)

- (c) From your graph, estimate
(i) the solutions of the equation $x^3 - 9x = 6$. [1]

$$\text{Plot } y=6$$

$$x = -0.7 \text{ or } -2.6$$

- (ii) the value(s) of x where the gradient of the curve is zero. [2]

$$x = -1.7 \text{ or } x = 1.7$$

- (iii) the gradient of the graph when $x = -1$. [2]

$$\text{gradient} = \frac{11-2}{-1.5-0}$$

$$= -6$$

- (d) On the same axes, draw the graph of $y = 5 - 2x$ for $-3 \leq x \leq 3$. [2]

x	$y = 5 - 2x$
-3	11
0	5
3	-1

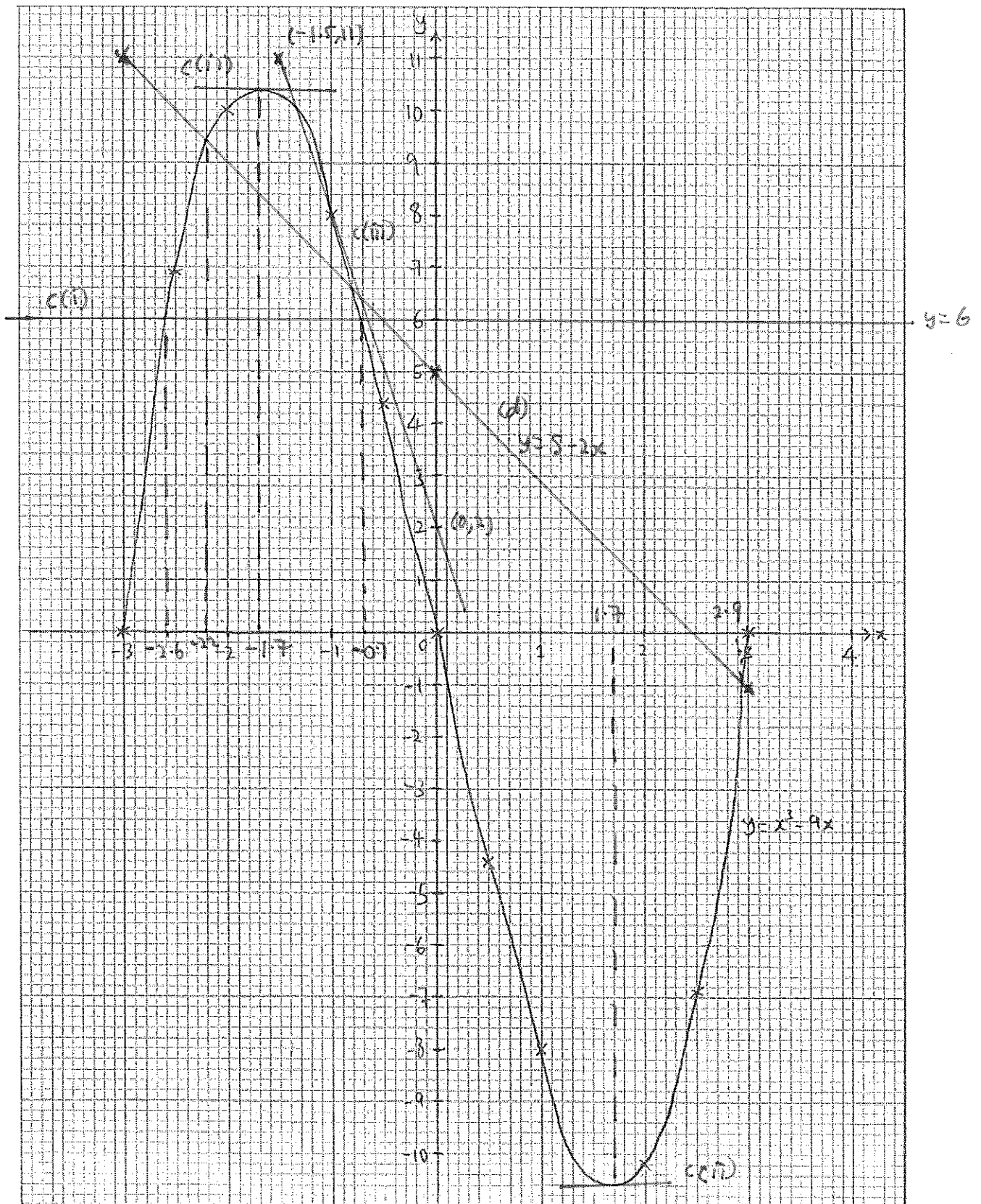
- (e) Hence solve the equation $x^3 - 7x - 5 = 0$. [2]

$$x^3 - 7x - 5 = 0$$

$$x^3 - 9x = -2x + 5$$

$$\text{From the graph, } x = -2.2 \text{ or } x = 2.9$$

Example 11(b) Graph



Example 12

Note: The function in this question is the same as Example 8 but some question parts are different. Parts (a) and (b) are done for you on the given graph paper. Use the given graph paper to complete parts (c), (d) and (e).

The variables x and y are connected by the equation $y = x^3 - 9x$.

Some corresponding values of x and y is given in the following table.

x	-3	-2.5	-2	-1	-0.5	0	0.5	1	2	2.5	3
y	a	6.9	10	8	4.4	0	-4.4	-8	-10	-6.9	0

- (a) Calculate the value of a . [1]

$$a = (-3)^3 - 9(-3)$$

$$a = 0$$

- (b) Taking 2 cm to represent 1 unit on the x -axis and 1 cm to represent 1 unit on the y -axis, draw the graph of $y = x^3 - 9x$ for the values of x in the range $-3 \leq x \leq 3$. [3]

(Graph on next page)

- (c) From your graph, find the coordinates of the point where the gradient is -6 . [2]

$$x = -1 \text{ or } x = 1$$

- (d) On the same axes, draw the graph of $y = 5 - 2x$ for $-3 \leq x \leq 3$. [2]

x	$y = 5 - 2x$
-3	11
0	5
3	-1

- (e) [1]

- (i) Write down the x -coordinates of the points where the two graphs intersect.

$$x = -2.2 \text{ or } x = 2.95$$

[3]

- (ii) These values of x are solutions of the equation $x^3 + Ax^2 + Bx + C = 0$. Find the value of A , of B and of C .

$$y = x^3 - 9x \quad \text{--- (1)}$$

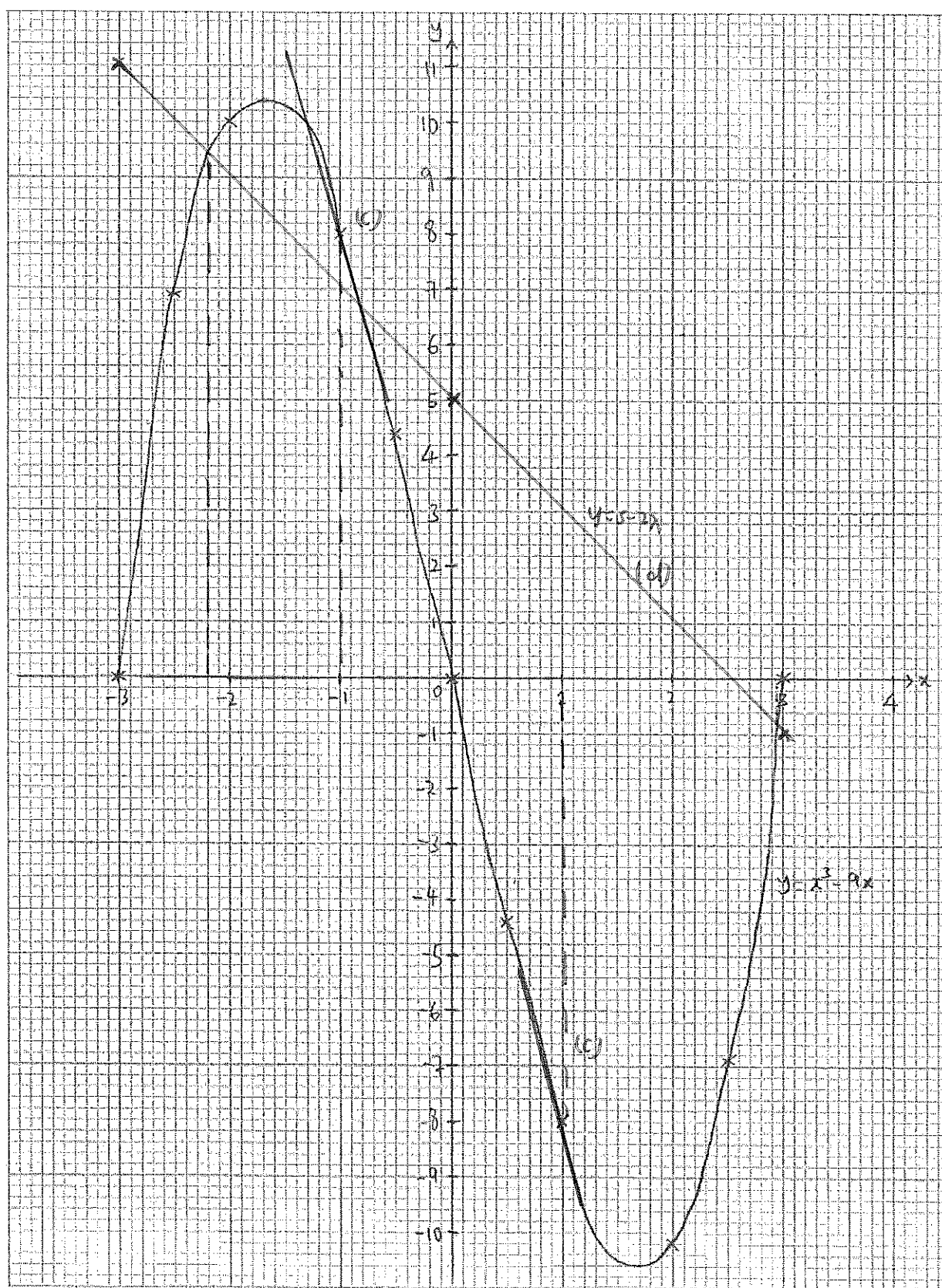
$$y = 5 - 2x \quad \text{--- (2)}$$

$$(1) = (2): \quad x^3 - 9x = 5 - 2x$$

$$x^3 - 7x - 5 = 0$$

$$A = 0, B = -7, C = -5$$

Example 12(b) Graph



Example 13 [2020 S3 EM EOY P2]

Aloysius deposits \$100 into a savings account.

After x years, Aloysius will have \$ y in his bank account.

The variables x and y are connected by the equation $y = 100 \times 1.04^x$.

The table below gives some of the values of x and the corresponding values of y , corrected to the nearest dollar.

x years	0	10	20	30	40
\$ y	100	p	219	q	480

- (a) Calculate the value of p and of q .

Answer $p = \dots\dots\dots$

$q = \dots\dots\dots$ [2]

- (b) On the grid opposite, using a scale of 2 cm to represent 5 years on the x -axis and 2 cm to represent \$50 on the y -axis, draw the horizontal x -axis for $0 \leq x \leq 40$ and the vertical y -axis for $0 \leq y \leq 500$.

On your axes, plot the points in the table and join them with a smooth curve. [3]

- (c) Use your graph to estimate

- (i) how much Aloysius will have in his bank account after 25 years,

Answer \$..... [1]

- (ii) how many years, to the nearest year, it takes for Aloysius to have \$200 in his bank account.

Answer years [1]

- (d) Belinda deposits \$100 into a savings account at 7% per year simple interest.

- (i) Show, by calculation, that after 20 years, Belinda will have \$240 in her bank account.

[1]

- (ii) How much will Belinda have after 40 years?

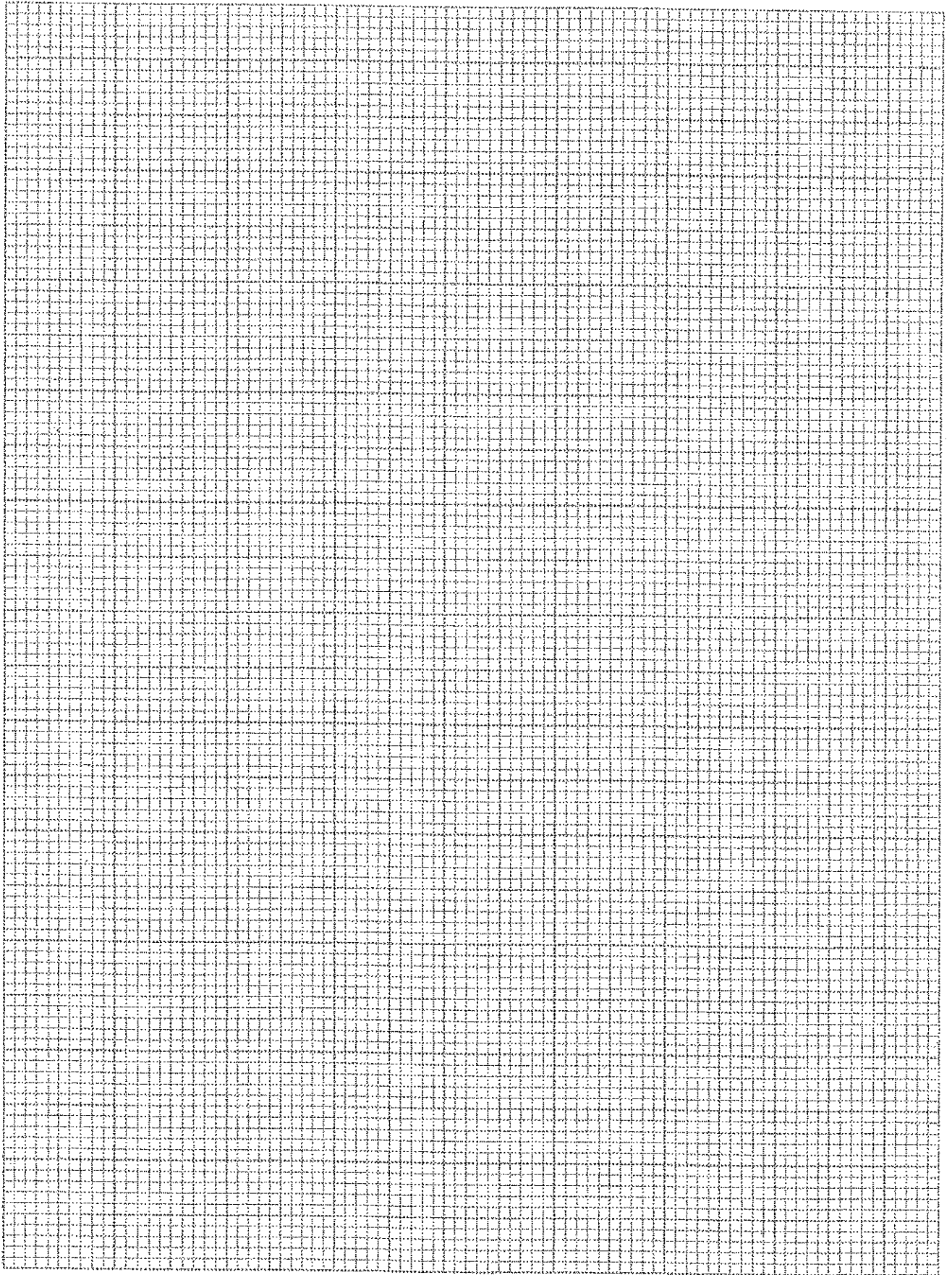
Answer \$..... [1]

- (iii) On the same axes, draw a graph to represent the increase of Belinda's savings during the 40 years.

[1]

- (e) Use your graphs to find how long it takes for Aloysius to save the same amount as Belinda.

Answer years [1]





Extension

Computational Thinking

1.



Write a computer function that converts temperature from Celsius to Fahrenheit, and another function that converts temperature from Fahrenheit to Celsius.

2.



Write a computer function that converts denary numbers to binary, and another function that converts binary numbers to denary.



If not, now is the time to complete Assignment 2 and of course, the End of Unit Summary Assignment!