MATHEMATICS
Secondary ONE
Year 2020

Name: **(   )** **Class:**

Unit 11 Number Sequences

Topical Enduring Understanding

- Patterns in real life and numbers can be represented (including finding an algebraic expression for the nth term) mathematically.

Topical Enduring Questions

- How can patterns in real life be generalised?

Big Idea – Proportionality

- Proportionality is a relationship between two quantities that allows one quantity to be computed from the other based on multiplicative reasoning.

Online resource:
Student Learning Space (learning.moe.edu.sg)

11. 1 Number Sequences

Knowledge: At the end of the lesson, you should be able to

- recognise simple patterns from various number sequences.
- determine the next few terms and find an algebraic expression for the n^{th} term.

Introduction

Familiar to us, some examples of number patterns are:

	Sequence	**Rule**
Positive even numbers	2, 4, 6, 8, 10, ___, ___, …	Start with ___, then add ___ to each term to get the next term.
Multiples of 3	3, 6, 9, 12, 15, ___, ___, …	Start with ___, then add ___ to each term to get the next term.
Powers of 2	1, 2, 4, 8, 16, ___, ___, …	Start with ___, then multiply each term by ___ to get the next term.

Key IDEA: Every sequence has a rule to follow.

Take a Guess? **What is likely the first number sequence you have learnt?**

CLASS ACTIVITY

For each of the following sequence, write down the next four terms.

(a) 42, 39, 36, 33, 30, …

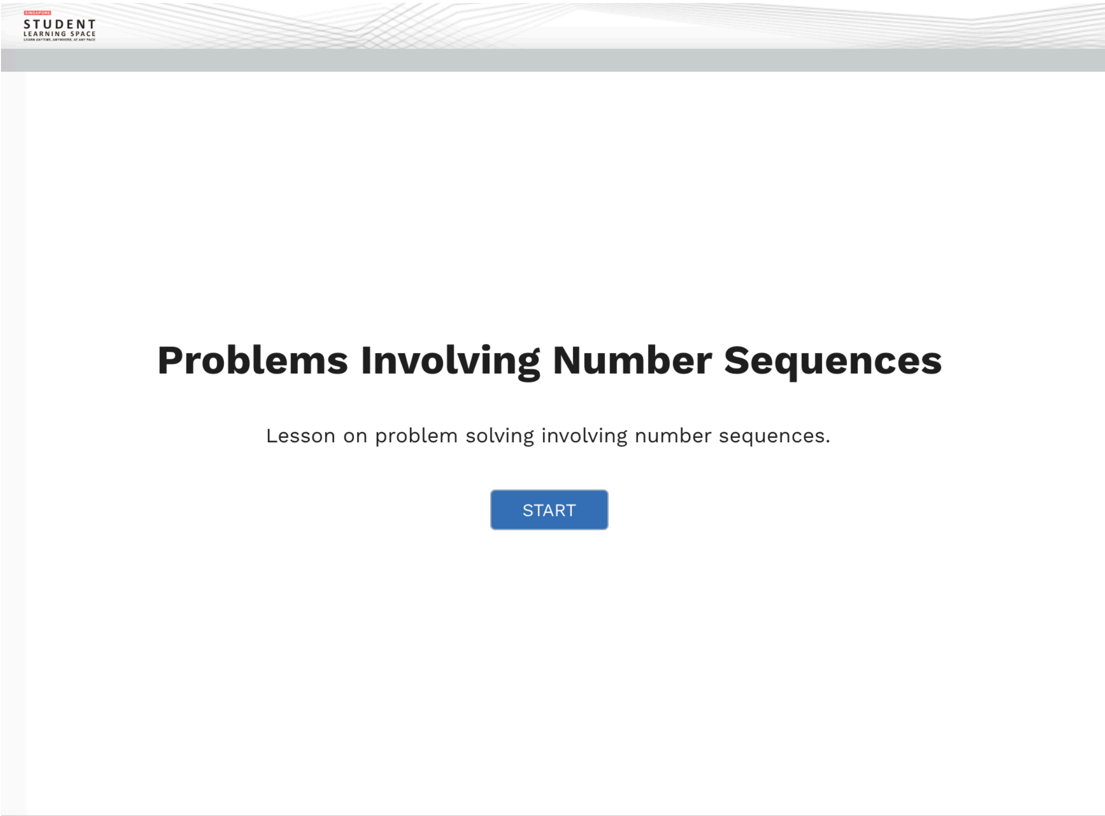
(b) –22, –18, –14, –10, –6, …

(c) –1, 1, –1, 1, –1, …

11.2 Student Learning Space Activity:

Login into SLS and complete the following activity assigned to you. (approx. 20-30 minutes activity)

The SLS Activity will also introduce you to how Number Sequences can be found in real life around us through the **Fibonacci Sequence.**



11.3 General Terms of Simple Sequences

An ordered list of numbers is called a ***sequence*** and it may be generated from shapes, patterns or rules.

Each number in a sequence is called a ***term*** and it is identified by its position in the ordered list. The terms are usually denoted by T_n, where n is the sequence place.

Given the following sequence

$$2, 4, 6, 8, 10,$$

the terms of the sequence is denoted by

$$T_1 = 2,$$
$$T_2 = 4,$$
$$T_3 = 6,$$
$$T_4 = 8,$$
$$T_5 = 10.$$

Every set of sequences follows a rule that enables us to derive a general term of the sequence.

Based on the above sequence, we can deduce that $T_n = 2n$.

CLASS ACTIVITY

1. For each of the following sequences, use the table provided to find a formula for the general term, and hence, state the 100th term (T_{100}).

   (a) ***Multiples of 3*** : 3, 6, 9, 12, 15, …
   Hence, T_n = _____.
   And, T_{100} = _____.

   (b) ***Perfect squares*** : 1, 4, 9, 16, 25, …
   Hence, T_n = _____.
   And, T_{100} = _____.

2. Given the n^{th} term, T_n, of a sequence is $T_n = 5n - 3$, find
   (a) the 3rd term,
   (b) the difference between the 3rd term and 5th term,
   (c) the product of the 6th term and 10th term,
   of the sequence.

General Terms of Complicated Sequences

Consider the sequence 2, 5, 8, 11, 14, …
How do we find a formula for the general term?

LOOKING FOR A PATTERN

Notice that the difference between consecutive terms are all equal to a constant. Thus, we say that the ***common difference*** of this sequence is equal to 3.

$$
\begin{array}{lccccccccc}
\text{Position } n & 1 & & 2 & & 3 & & 4 & & 5 \\
\text{Term } T_n & 2 & \xrightarrow[+3]{} & 5 & \xrightarrow[+3]{} & 8 & \xrightarrow[+3]{} & 11 & \xrightarrow[+3]{} & 14
\end{array}
$$

Therefore, we can express each term as follows:

$$
\begin{array}{lclclcl}
T_1 & = & 2 & = & 2 & = & 2 + 0 \text{ x } 3 \\
T_2 & = & 5 & = & 2 + 3 & = & 2 + 1 \text{ x } 3 \\
T_3 & = & 8 & = & 2 + 3 + 3 & = & 2 + 2 \text{ x } 3 \\
 & & & & \vdots & & \\
T_n & & & = & 2 + \underbrace{3 + 3 + \ldots + 3}_{(n-1) \text{ terms}} & = & 2 + (n-1) \text{ x } 3
\end{array}
$$

By looking at the above pattern, we can infer that

$$
\begin{aligned}
T_n &= 2 + (n-1) \text{ x } 3 \\
&= 2 + 3n - 3 \\
&= 3n - 1
\end{aligned}
$$

TRANSFORMING TO ANOTHER SEQUENCE

Since the common difference is equal to 3, we can transfer this sequence to another sequence which consists of terms that are multiples of 3, such that the first term must be 3, so that we know that the general term is given by $3n$ immediately.

$$
\begin{array}{lccccc}
\text{Position } n & 1 & 2 & 3 & 4 & 5 \\
\textbf{Term } \boldsymbol{T_n} & \mathbf{2} & \mathbf{5} & \mathbf{8} & \mathbf{11} & \mathbf{14} \\
 & \downarrow{\scriptstyle +1} & \downarrow{\scriptstyle +1} & \downarrow{\scriptstyle +1} & \downarrow{\scriptstyle +1} & \downarrow{\scriptstyle +1} \\
\text{Term } T_n + 1 & 3 & 6 & 9 & 12 & 15
\end{array}
$$

We are concerned with the term T_n. By adding 1 to each term of the first sequence to derive the terms in the second sequence, we have to subtract 1 from $3n$.

Thus, $T_n = 3n - 1$.

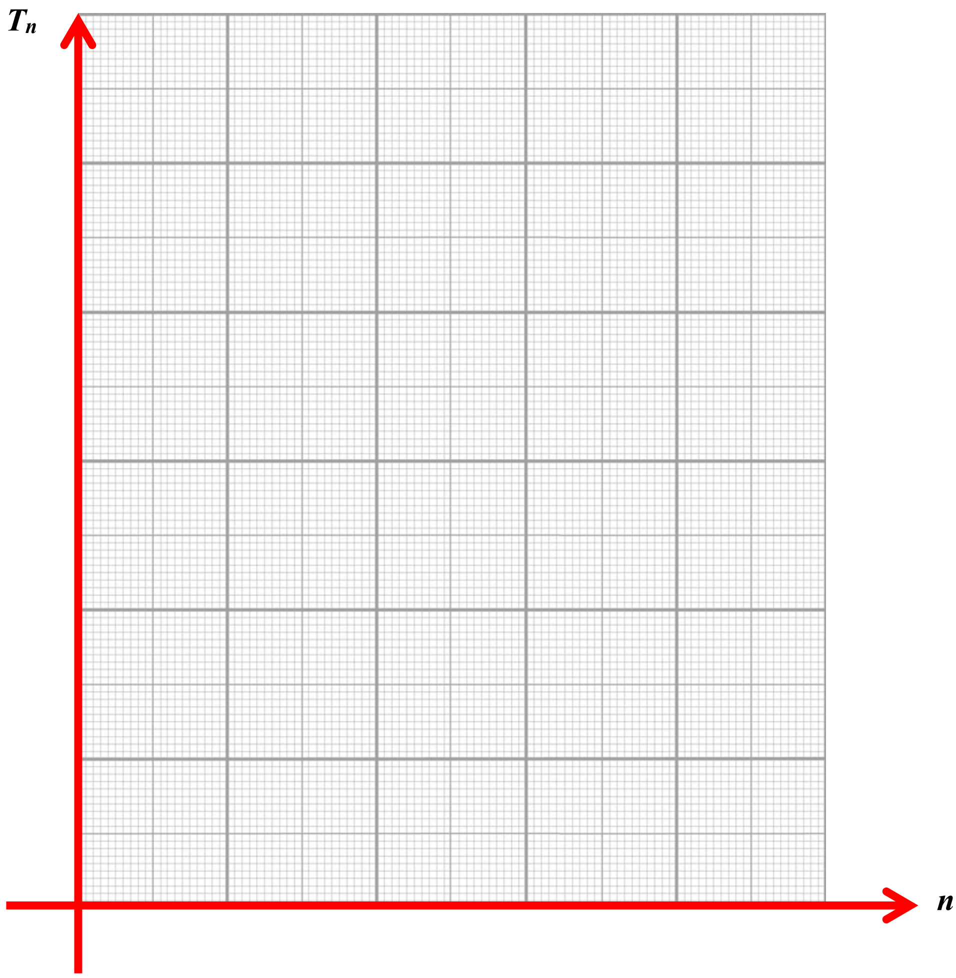
PRACTICE QUESTIONS

1. Find a general formula for the general term of sequence 8, 12, 16, 20, 24, …

2. Consider the sequence 3, 7, 11, 15, 19, …
   (a) Write down the next two terms of the sequence.
   (b) Find, in terms of n, a formula for the n^{th} term of the sequence.
   (c) Hence, find the 50^{th} term.

CONNECTING ON A GRAPH

Consider the same sequence, 2, 5, 8, 11, 14, …
Plot the number sequence on the graph.



What did you observe?

11.4 Problem solving with Number Patterns

At the end of the lesson, you should be able to

- solve **problems** involving number sequences and number patterns.

Number Patterns

Number patterns are derived from number sequences but in visual representation.

PRACTICE QUESTIONS

1. The first four figures of a sequence are as shown.

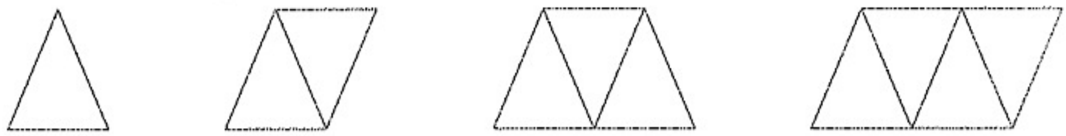


Figure 1 Figure 2 Figure 3 Figure 4

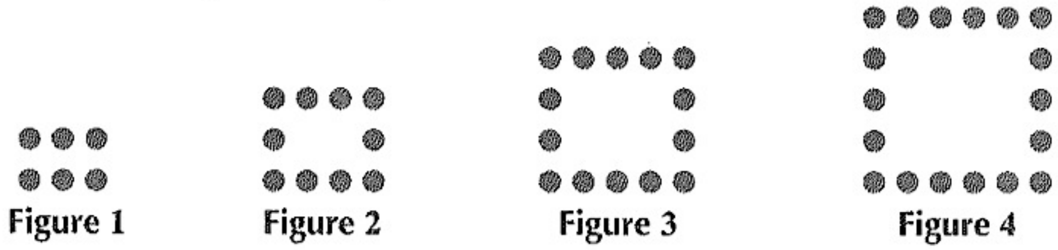
(a) Draw the next two figures of the sequence.

(b) Complete the table.

Figure Number	Number of Triangles	Number of Lines
1	1	1 + 1 x 2 = 3
2	2	1 + 2 x 2 = 5
3	3	1 + 3 x 2 = 7
4	4	1 + 4 x 2 = 9
5		
6		
⋮	⋮	⋮
n		

(c) Write down a formula connecting the number of triangles, T, and the number of lines, L, in the sequence shown above.

2. The first four figures of a sequence are as shown.



(a) Draw the next two figures of the sequence.

(b) Complete the table.

Figure Number	Number of Dots
1	$2 + 1 \times 4 = 6$
2	$2 + 2 \times 4 = 10$
3	$2 + 3 \times 4 = 14$
4	$2 + 4 \times 4 = 18$
5	
6	
⋮	⋮
n	

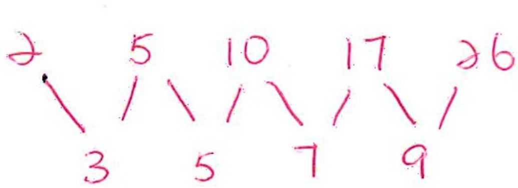
(c) Find the number of dots in 2013^{th} figure.

11.5 Extension (Optional) : Method of Differences

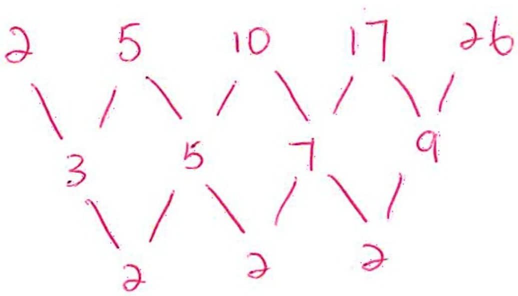
When faced with a sequence for which you need to find missing values or the next few values, you need to first look at it and see if you can get a "feel" of what is going on.

Given the following question:
Find the next number in the following sequence 2, 5, 10, 17, 26, … and provide a formula for the n^{th} term.

To find the pattern, first list the numbers, and find the difference for each pair of numbers.



Since these first differences are not the same, we continue subtracting.



Now, we notice that these values are all the same. This is when we stop. What is important is that the **second differences are the same**. And this shows that the polynomial of this sequence of values is a **quadratic**.

A quadratic form is given by $an^2 + bn + c$, for $a, b, c \in \mathbb{Z}$.

For instance, I know the first term (where $n = 1$) is 2, so I will put in 1 for n and 2 for the value:

$$a(1)^2 + b(1) + c = 2$$

And continuing with the second term,

$$a(2)^2 + b(2) + c = 5$$

With the third term,

$$a(3)^2 + b(3) + c = 10$$

This gives a system of three equations with three unknowns, which you can solve using your calculator.

From this, we are able to derive

$$a = 1, b = 0, c = 1$$

Putting this back into the equation $T_n = an^2 + bn + c$, we get

$$T_n = n^2 + 1$$

that is the general formula for the sequence.

CLASS ACTIVITY

1. Try to find the next number in the following sequence

$$0, 12, 10, 0, -12, -20, \ldots$$

and its general formula using $an^3 + bn^2 + cn + d$.