School of Science and Technology, Singapore
Mathematics Department

SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

Secondary 2 – Trigonometric Ratios
Notes

Name Suggested Solutions ( ) Class ___________

Unit EU

Students will be able to understand that:

1. The ratio of the lengths of sides can be used as a form of **measure** for an acute angle in a right-angled triangle.
2. The acute in a right-angled triangle can be obtained using the inverse **functions** of the trigonometric ratios.

Unit EQ

1. How can the common ratios of corresponding sides in similar triangles be applied in real world situations?
2. How can we obtain the acute angle in a right-angled triangle from the ratios of corresponding sides?

Learning Objectives

At the end of the unit, students should be able to:

1. Explain what trigonometric ratios of acute angles are.
2. Find the unknown sides and angles in right-angled triangles.
3. Solve simple practical problems in two and three dimensions.

Reference Textbook

- Think! Mathematics New Syllabus Mathematics 8th Edition Textbook Secondary 2B. SL Education.

Lesson sequence in the unit								
Student Learning Outcomes	Dimensions (Please tick the appropriate boxes)							
	FUNCTIONS F	INVARIANCE I	NOTATIONS N	DIAGRAMS D	MEASURES M	EQUIVALENCE E	PROPORTIONALITY P	MODELS M
Pythagoras' Theorem			✓			✓	✓	
Trigonometric Ratios	✓				✓			

Cognitive Level	Know, Understand, Demonstrate	Checklist
Level 0: Memorisation	Identify the correct ratios of the lengths of a right angle triangle for sine, cosine and tangent.	
Level 1: Procedural tasks without connections	Apply the Trigonometric ratios to find the unknown lengths or angles of a right angle triangle.	
Level 2: Procedural tasks with connections	Apply the Trigonometric ratios to solve simple geometrical problems.	
Level 3: Problem Solving	Apply the Trigonometric ratios to solve simple real life problems involving angles of elevation and angles of depression.	

ACTIVITY 1 - The Trigonometric Ratios

In the space below, draw a right-angled triangle *ABC*
where angle *A* = _______________ (To be assigned to you by your teacher).

Measure and write down the lengths of *AB*, *BC* and *AC*.

Direct measurements	**Lengths**
AB	
BC	
AC	

Indirect measurements	**Ratios**
AB / *BC*	
AC / *BC*	
AC / *AB*	

Compare your answers with your classmates who were assigned the same angle *A*.

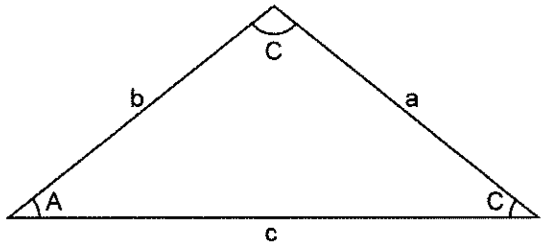
What do you notice?

Same Ratios

DISCUSSION 1 - Introduction to Trigonometry

Trigonometry refers to the mathematical discipline dealing with the relationships between the sides and angles of triangles. It comes from the Greek words trigonon meaning three angles and metron meaning ***MEASURE.***

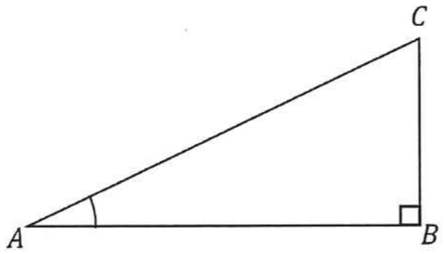
In an earlier topic on congruent triangles, we know that there are **6 measurements** (3 sides and 3 angles) in a triangle. Generally, we need at least 3 out of the 6 measurements to identify a particular triangle. Eg. SSS, SAS, AAS, RHS.



If we measure the angles and sides of this triangle with the aid of a protractor and ruler, we are doing **direct measurements**.

- Is it reasonable to measure everything in our environment using direct measure?
- Can we make indirect measurements of lengths and heights with the help of Trigonometry?

In ΔABC,



Hypotenuse is the longest side, AC, it is the side opposite to the right angle

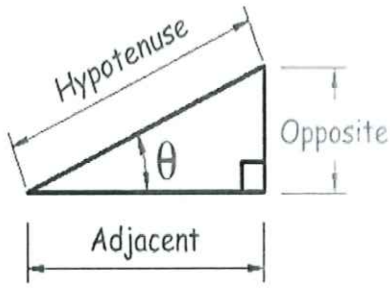
In Relation to $\angle A$

Adjacent side is the side AB.

Opposite side is the side BC.

Right Angled Triangle

The triangle of most interest is the right-angled triangle.



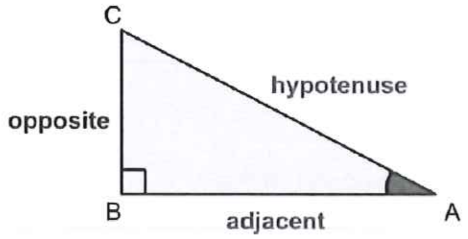
The right angle is shown by the little box in the corner.

We usually know another angle θ.

And we give names to each side:

- **Adjacent** is adjacent (next to) to the angle θ
- **Opposite** is opposite the angle θ
- the longest side is the **Hypotenuse**

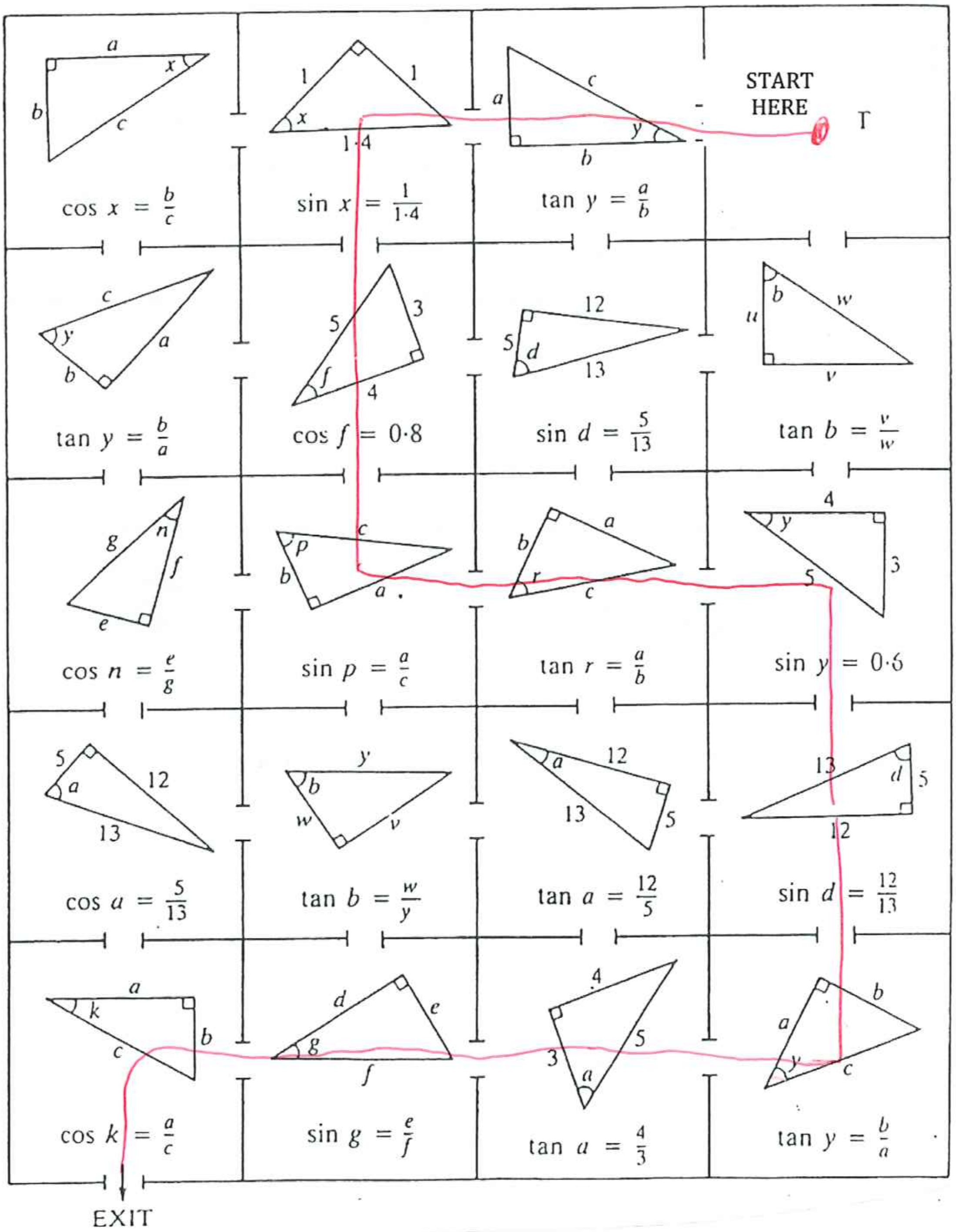
The 3 basic Trigonometric Ratios are namely, **Sine** Ratio, **Cosine** Ratio and **Tangent** Ratio.



The **Sine** Ratio is defined as: $\sin\ A = \frac{BC}{AC}$ In short, $\sin A = \frac{opp}{hyp}$	The **Cosine** Ratio is defined as: $\cos\ A = \frac{AB}{AC}$ In short, $\cos A = \frac{adj}{hyp}$
The **Tangent** Ratio is defined as: $\tan\ A = \frac{BC}{AB}$ In short, $\tan A = \frac{opp}{adj}$	

Trigonometric Ratios: Practice 1

Starting from the top right box, move into the box which displays a correct trigonometrical ratio. Carry on in the same manner until you exit.

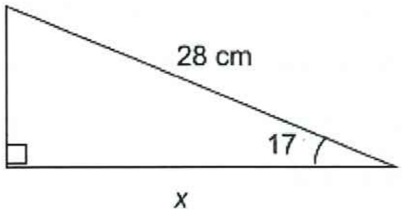


Use a calculator to obtain the values of the following:

i. $\sin 0.5°$ 0.00873 (3sf)	ii. $\cos 78.4°$ 0.201 (3sf)
iii. $4 \tan 35.3°$ 2.83 (3sf)	iv. $2 \tan 45° - 0.5 \cos 60°$ $\frac{7}{4}$
v. $\dfrac{\sin 40°}{8}$ 0.0803 (3sf)	vi. $\dfrac{2\tan 10°}{\sin 60°}$ 0.407 (3sf)
vii. $\dfrac{6(\sin 30°)(\cos 45°)^2}{2\tan 30°}$ 1.30 (3sf)	viii. $\dfrac{5\sin 0° + \tan 45°}{3\cos 0° - 2\sin 30°}$ $\frac{1}{2}$

2) Find the unknown length x in each of the following figures:

a)



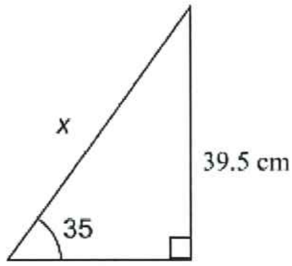
$\cos 17^\circ = \frac{x}{28}$

$x = 28 \cos 17^\circ$

$= 26.8$ cm

(3sf)

b)



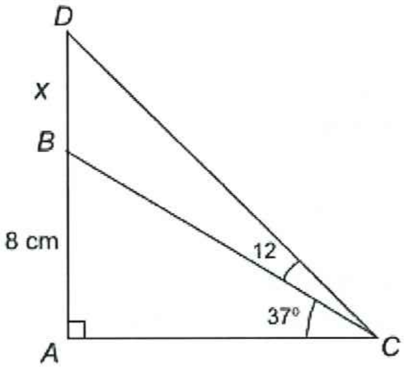
$\sin 35^\circ = \frac{39.5}{x}$

$x = \frac{39.5}{\sin 35^\circ}$

$= 68.9$ cm

(3sf)

c)



$\tan 37^\circ = \frac{8}{AC}$

$AC = \frac{8}{\tan 37^\circ}$

$\tan 49^\circ = \frac{x+8}{AC}$

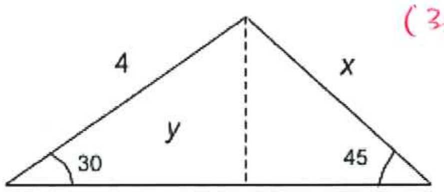
$x+8 = AC \cdot \tan 49^\circ$

$x = AC \cdot \tan 49^\circ - 8$

$= 4.21$ cm

(3sf)

d)



$\sin 30^\circ = \frac{y}{4}$

$y = 4 \sin 30^\circ$

$= 2$ units.

$\cancel{\tan}$ sin $45^\circ = \frac{y}{x}$

$x = \frac{y}{\sin 45^\circ}$

$= \frac{y}{\sin 45^\circ}$

$= 2.83$ units

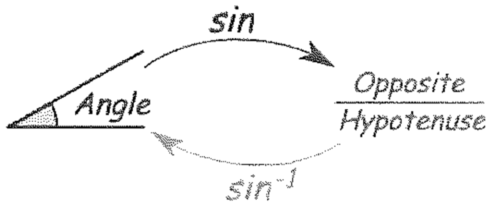
(3sf)

Assignment 1 -
Complete the questions from Exercise 10A and Exercise 10B selected by your teacher.

ACTIVITY and DISCUSSION 2 - Inverse of Trigonometric Functions

Sine, Cosine and Tangent can be understood as ***FUNCTIONS****. As such, we are also able to obtain the acute angle in a triangle using the inverse of the functions given the ratio.*

Well, the Sine function ***"sin"*** takes an angle and gives us the **ratio** "opposite/hypotenuse",



But ***sin^{-1}*** (called "inverse sine") goes the other way ...
... it takes the **ratio** "opposite/hypotenuse" and gives us an angle.

Note: $\sin^{-1}(\ldots)$ is a function that takes a **ratio** and gives us an angle. It is NOT to be mistaken as the reciprocal of $\sin(\ldots)$!
That is to say,

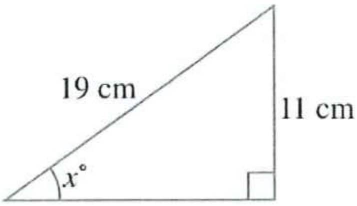
$$\sin^{-1}(x) \neq \frac{1}{\sin(x)}$$

Remember, $\sin^{-1}(\ldots)$ takes a **ratio** whereas $\sin(\ldots)$ takes an **angle.** So x can be the input of both $\sin^{-1}(\ldots)$ and $\sin(\ldots)$ at the same time.

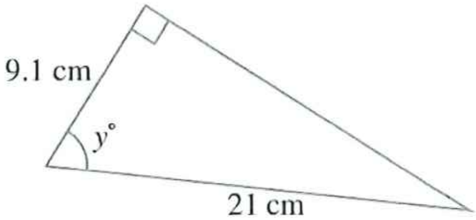
PRACTICE 2 - Inverse of Trigonometric Functions

1) Find the value of the unknown in each of the following right-angled triangles.

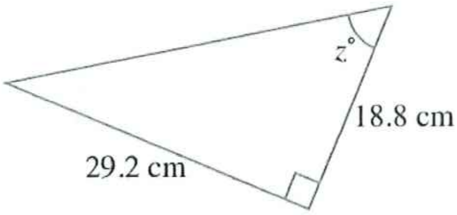
(a)



(b)



(c)



(a) $\sin x^\circ = \frac{11}{19}$

$x^\circ = 35.4^\circ$ (1dp)

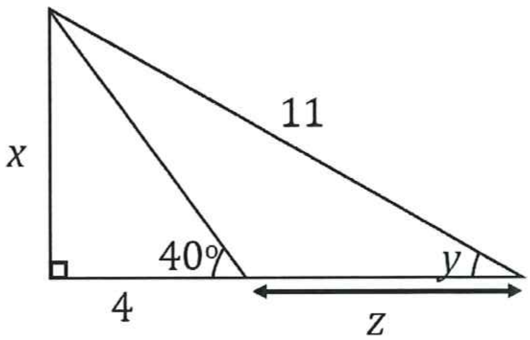
(b) $\cos y^\circ = \frac{9.1}{21}$

$y^\circ = 64.3^\circ$ (1dp)

(c) $\tan z = \frac{29.2}{18.8}$

$z = 57.2^\circ$ (1dp)

2) Find the value of x, y and z in the following diagram.



$\tan 40^\circ = \frac{x}{4}$

$x = 4 \tan 40^\circ$

$= 3.36$ (3sf)

$\sin y^\circ = \frac{x}{11}$

$y = 17.8^\circ$ (1dp)

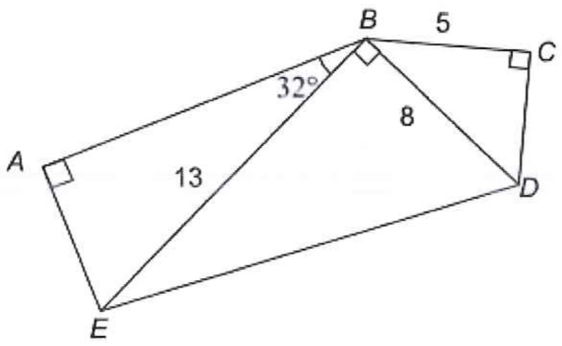
$\cos y^\circ = \frac{z+4}{11}$

$z + 4 = 11 \cos y^\circ$

$z = 11 \cos y^\circ - 4$

$= 6.48$ (3sf)

3) In the following diagram, $\angle EAB$, $\angle EBD$and $\angle BCD$ are each 90° and $\angle ABE$is 32°.
BE = 13 cm, BD = 8 cm and BC = 5 cm.



Calculate
(a) AE,
(b) $\angle CBD$.

(a) $\sin 32^\circ = \frac{AE}{13}$

$AE = 13 \sin 32^\circ$

$= 6.89$ cm (3sf)

(b) $\cos \angle CBD = \frac{5}{8}$

$\angle CBD = 51.3^\circ$ (1dp)

Assignment 2 - Complete the questions from Exercise 10C selected by your teacher.

ACTIVITY and DISCUSSION 3 - Applications of Trigonometric Ratios

WHAT'S MY HEIGHT?

Let us explore how trigonometry can be applied to real-life problems.

A. Problem

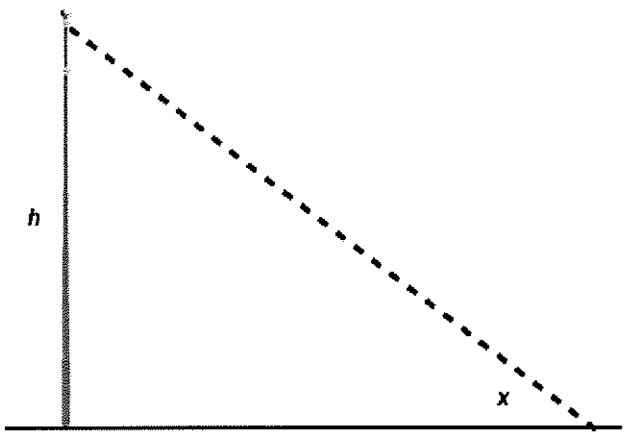
While training the morning assembly flag raising personnel, a question was raised on whether the flagpoles in SST (ISH and Atrium) were of the same height. In groups of two or three, you are tasked to investigate the height of flagpoles in these two locations.

B. Background

Direct measurement of the height of the atrium is neither feasible nor practical. How can we use our understanding of trigonometry to calculate the height of the flagpole? There are also many situations where direct measurement is not possible or could even be dangerous; hence trigonometry can be applied.

C. Trigonometry

In the following diagram, let the height of the flagpole be h. Using a clinometer to measure angle x, discuss a strategy within your groups to do an indirect measurement of the height of the flagpoles.



PRE-ACTIVITY KNOWLEDGE

1. Assumptions

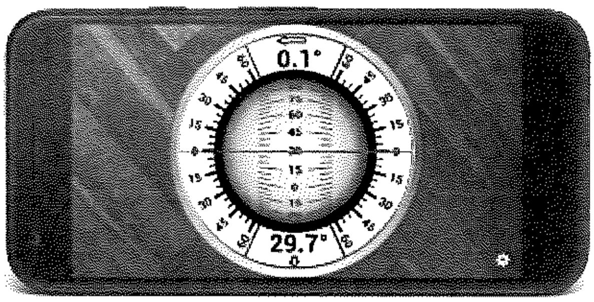
   In engineering and design, making assumption is an important aspect and it is taking into consideration as they will help in producing products as accurately as possible.

   What is/are your assumption(s) when measuring angle of elevation?

2. Use of Clinometer

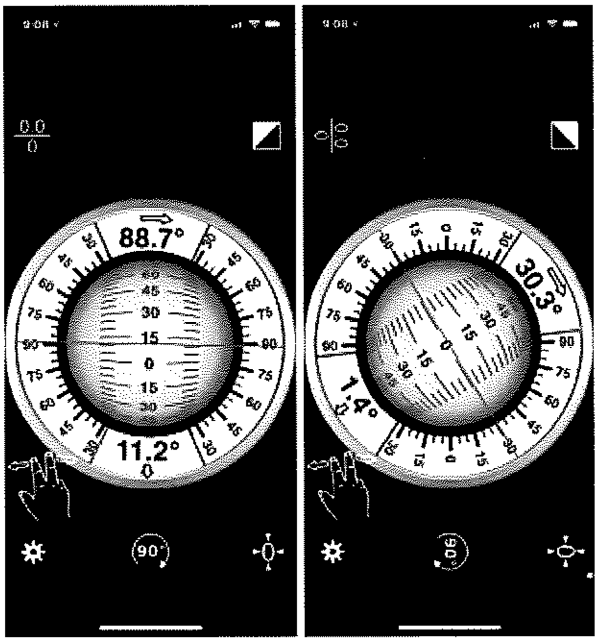
   A clinometer is an instrument used to measure the angle of elevation.
   In our activity, we will be using a clinometer app.

   Download Rotating Sphere Clinometer, which is available on Google Play Store and the App Store.



   (a) Initialise the app and the following image will appear.

Align your device horizontally to the ground and next to your head (at eye-level) to read the measurement of angle x.



(b) Place the device directly in front of your eyes. Ensure that the clinometer reads the value 0 at the beginning. Steady your device as you move your head upwards to look at the end of the flagpole.

Your partner should read the value on your phone for you.

GROUP TASK

In the following table, you are required to:

1. Indicate the name of your group members
2. Use the mobile application to measure the angles

Measurement: Atrium

Name	First Reading		Second Reading	
	Angle of Elevation	Distance from Flagpole	Angle of Elevation	Distance from Flagpole

Measurement: Indoor Sports Hall

Name	First Reading		Second Reading	
	Angle of Elevation	Distance from Flagpole	Angle of Elevation	Distance from Flagpole

Compare and contrast the readings above and deduce the height of the flagpoles at the respective venues.

Name	Atrium		Indoor Sports Hall	
	Calculated Height from first readings	Calculated Height from second readings	Calculated Height from first readings	Calculated Height from second readings
Mean Height				

By using scientific hypothesis and reasonings, describe how you conduct the experiment to ensure that it is accurate. You are required to provide as much details as possible to ensure clarity of your work.

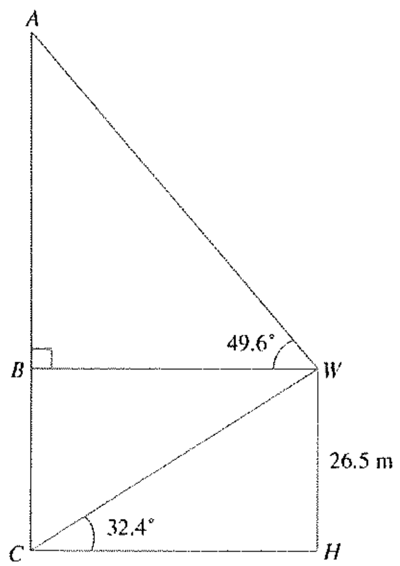
Share one possible strategy to improve the accuracy of the readings on the clinometer.

Prepare a short class presentation on this task for the next lesson.

PRACTICE 3 - Application of Trigonometric Ratios

(a) The height of a tower TF is 40 m. Given that A is a point on level ground such that AT makes an angle of 37° with the horizontal ground, find the distance FA.

(b) The height of a warehouse WH is 26.5 m. ABC is a vertical mast in front of the warehouse. BW is a horizontal cable attached to the mast from the top of the warehouse. Given that angle AWB = 49.6° and angle WCH = 32.4°, find the height of the mast.

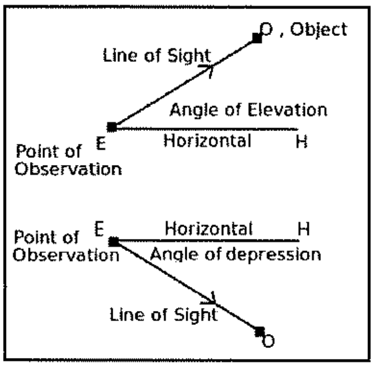


Assignment 3 - Complete the questions from Exercise 10D selected by your teacher.

ACTIVITY and DISCUSSION 4 - Line of Sight

The line of sight is an imaginary line from the eye to a perceived object

- Suppose you are looking at an object in the distance.
- If the object is above you, then the angle of elevation is the angle your eyes look up.
- If the object is below you, the angle of depression is the angle your eyes look down.
- Angles of elevation and depression are measured from the horizontal.
- It is a common mistake not to measure the angle of depression from the horizontal.

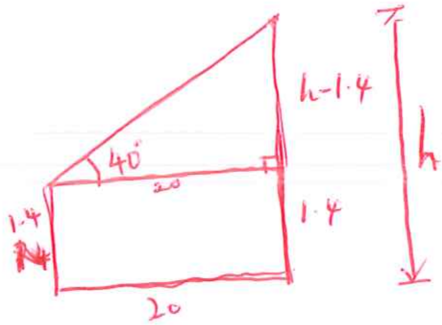


We have investigated the angle of elevation in our previous activity of finding the height of the flag poles. Let us now look at an example of the angle of depression.

PRACTICE 4 - Application of Angle of Elevation and Angle of Depression

1. A boy whose eye level is 1.4 m above the ground finds that the angle of elevation of a bird on a tree is 40º. If the boy is 20 m from the tree,

   (a) How high is the bird above the ground?



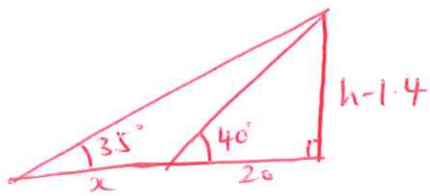
$\tan 40^\circ = \dfrac{h-1.4}{20}$

$h - 1.4 = 20 \tan 40^\circ$

$h = 20 \tan 40^\circ + 1.4$

$= 18.2 \text{ m} \quad (3sf)$

   (b) If he moves a distance x m away from the tree, the angle of elevation becomes 35°. How far did he move from the original position?



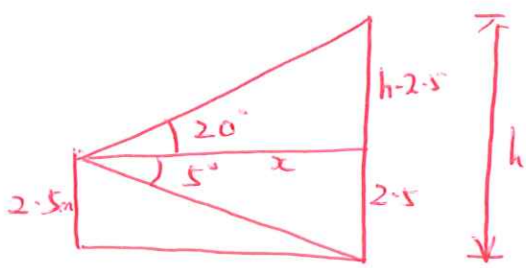
$\tan 35^\circ = \dfrac{h-1.4}{x+20}$

$x + 20 = \dfrac{h-1.4}{\tan 35^\circ}$

$x = \dfrac{h-1.4}{\tan 35^\circ} - 20 = 3.97 \text{ m}$ (3sf)

2. An observer whose eye is 2.5 m above the ground observes the angle of depression of the foot of the building to be 5º, and the angle of elevation of the top to be 20º. Calculate the horizontal distance of the observer from the building, and the height of the building, each to the nearest metre.

Let horizontal distance be x m.



$\tan 5^\circ = \dfrac{2.5}{x}$

$x = \dfrac{2.5}{\tan 5} = 28.575\ldots$

$= 29 \text{ m}$ (nearest m)

$\tan 20 = \dfrac{h-2.5}{x}$

$h - 2.5 = x \tan 20^\circ$

$h = x \tan 20^\circ + 2.5$

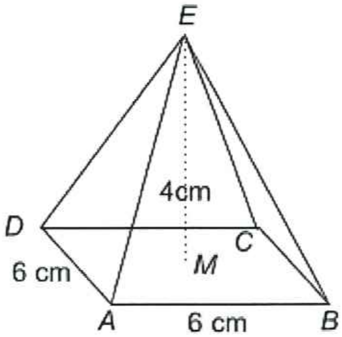
$= 12.900497\ldots$

$= 13 \text{ m}$ (nearest m)

PRACTICE 5 - Application of Trigonometric Ratios in 3-Dimension

Example 1

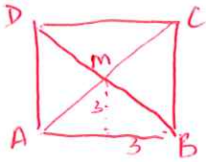
The diagram shows a square pyramid *ABCDE* with *E* 4 cm vertically above *M*, the mid-point of the base, and *ABCD* is of side 6 cm.



(a) State the vertical height of the pyramid.
(b) Find the length of *EB*.
(c) Find the angle *AEM*.
(d) Is $\angle AEM$ the same as $\angle BEM$? Justify your answer.
(e) Find the angle between *AE* and *AC*.

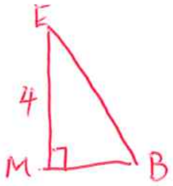
(a) Vertical height = 4 cm.

(b)

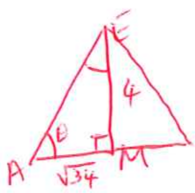


$MB = \sqrt{3^2 + 3^2}$
$= \sqrt{18}$

$EB = \sqrt{4^2 + 18}$
$= \sqrt{34}$
$= 5.83$ cm (3sf)



(c)



$\tan \angle AEM = \frac{\sqrt{34}}{4}$

$\angle AEM = 55.6°$ (1dp)

(d) Yes. Because $\Delta AEM \equiv \Delta BEM$.

(e) Let the angle between AE and AC be θ.

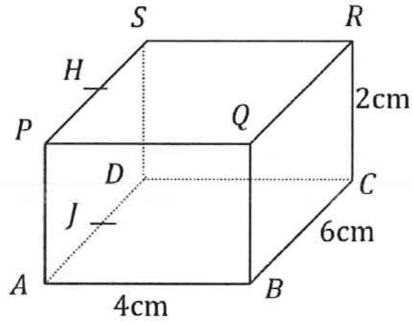
$\tan\theta = \frac{4}{\sqrt{34}}$

$\theta = 34.4°$ (1dp)

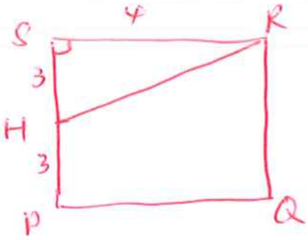
Example 2

H and *J* are respectively the midpoints of the sides *PS* and *AD* of the cuboid with dimensions 4 cm, 6 cm and 2 cm.

(a) Find the length of *HR*.
(b) Find the length of *JR*.
(c) Calculate angle *JRH*.



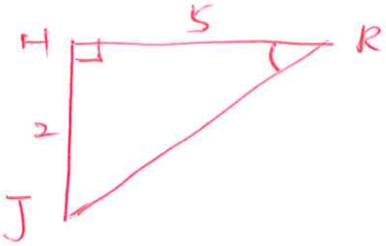
(a)



(pythagoras' Thm)

$$HR = \sqrt{3^2 + 4^2}$$
$$= 5 \text{ cm}$$

(b)



(pythagoras' Thm)

$$JR = \sqrt{2^2 + 5^2}$$
$$= \sqrt{29}$$
$$= 5.39 \text{ cm} \quad (3sf)$$

(c)
$$\tan \angle JRH = \frac{2}{5}$$
$$\angle JRH = 21.8^\circ \quad (1dp)$$

SUGGESTED PRACTICE QUESTIONS

Exercise 10A (page 65)

- **Basic - 2**
- **Intermediate - 5**

Exercise 10B (page 69)

- **Basic - 4**
- **Intermediate - 5, 6**
- **Advanced - 7, 8**

Exercise 10C (page 73)

- **Basic - 1, 2**
- **Intermediate - 3, 4, 5**
- **Advanced - 6, 7, 8**

Exercise 10D (page 80)

- **Basic - 1, 2, 3, 4**
- **Intermediate - 7, 8, 9**
- **Advanced - 13**

EXTENSION - Special Angles

Ratio of Sine, Cosine and Tangent of Special Angles

For some angles, it is easy to obtain the values of their trigonometric ratios without the use of a calculator.

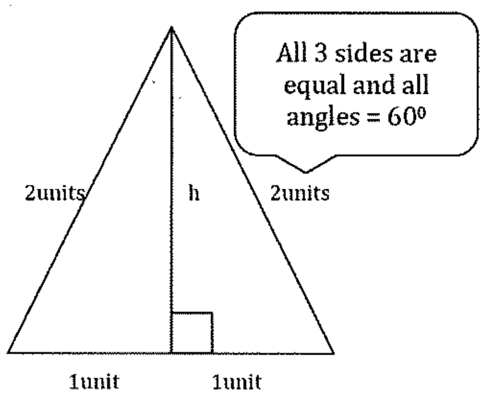
Consider an Equilateral Triangle.

Write down the value of h in surd form.

$h =$ _______________

Therefore,

sin 30° =	sin 60° =
cos 30° =	cos 60° =
tan 30° =	tan 60° =



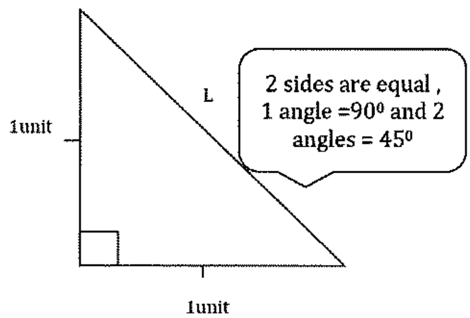
Consider an Isosceles Triangle

Write down the value of L in surd form.

$L =$ _______________

Therefore,

sin 45° =
cos 45° =
tan 45° =



What do you think the values of the following will be without using a calculator?

sin 0° =	sin 90° =
cos 0° =	cos 90° =
tan 0° =	tan 90° =