SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE
MATHEMATICS DEPARTMENT 2023
SECONDARY 3 ELEMENTARY MATHEMATICS

SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

Name: Solutions ( ) Class: S__-__

E MATHS UNIT 3 - COORDINATE GEOMETRY

a. ENDURING UNDERSTANDING

At the end of the topic, students will understand that

- the length of a line segment can be **measured** using Pythagoras' Theorem
- the gradient of a line is **measured** using the ratios of sides of similar right-angled triangles
- the **diagram** of a linear graph is equivalent to the equation of a straight line, providing an algebraic structure to solve geometric problems

b. ESSENTIAL QUESTIONS

- How can we **measure** length in Cartesian space using Pythagoras' Theorem?
- How can we **measure** slope in Cartesian space using similarity?
- Why do parallel lines have the same gradient?
- How does the equation of a straight line relate the variables?
- How can we use the equation of a straight line to solve problems?

c. KNOWLEDGE & SKILLS

At the end of the topic, students will be able to

- calculate the length of a line segment given the coordinates of its end points.
- calculate the gradient of a straight line given the coordinates of two points on it.
- interpret and find the equation of a straight line graph in the form $y = mx + c$.
- find the gradient of parallel lines.
- solve geometric problems involving the use of coordinates.

d. RESOURCES

1. New Syllabus Mathematics Textbook (Shinglee Publishers) Chapter 4 (pg. 99 to 122)
2. BBC Bitesize
   http://www.bbc.co.uk/schools/gcsebitesize/maths/geometry/linesegmentsrev2.shtml

e. COMMON SYMBOLS/ LANGUAGE USED IN THIS CHAPTER

- Coordinates, Length, Gradient, Equation of a straight line, collinear, vertex, parallel lines

TEACHING TO THE BIG IDEA

Lesson sequence in the unit								
Student Learning Outcomes	Dimensions (Please tick the appropriate boxes)							
	FUNCTIONS F	INVARIANCE I	NOTATIONS N	DIAGRAMS D	MEASURES M	EQUIVALENCE E	PROPORTIONALITY P	MODELS M
Length of a line segment				√	√			
Gradient of straight line				√	√			
Equation of straight line				√				

UNIT CHECKLIST

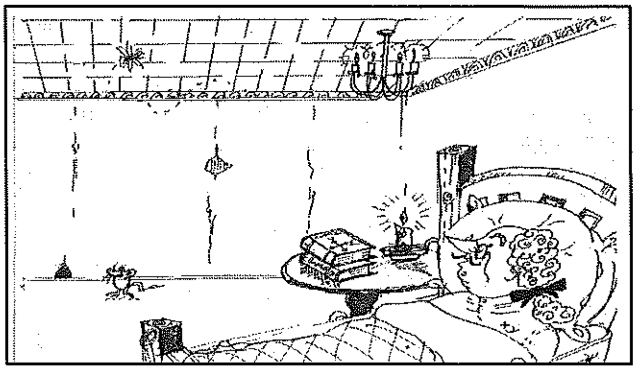
Section 1.1: Length of a line segment		
Cognitive Level	**Know, Understand, Demonstrate**	**Checklist**
Level 0: Memorisation	State the formula for the length of a line segment	
Level 1: Procedural tasks without connections	Calculate the length of a line segment given two coordinates	
	Find the coordinates of a point given another pair of coordinates and the length of the line segment	

Section 1.2: Gradient of a straight line		
Cognitive Level	**Know, Understand, Demonstrate**	**Checklist**
Level 0: Memorisation	State the formula for the gradient given two coordinates	
	State that the gradients of two parallel lines are equal	
	State that the gradients of lines between collinear points are equal to each other	
Level 1: Procedural tasks without connections	Calculate the gradient of a line given two coordinates	

Section 1.3: Equation of a straight line		
Cognitive Level	**Know, Understand, Demonstrate**	**Checklist**
Level 0: Memorisation	State that the equation of all linear graphs are in the form $y = mx + c$	
Level 1: Procedural tasks without connections	Manipulate a given equation of a line to become of the form $y = mx + c$	
Level 2: Procedural tasks with connections	Interpret and find the equation of a linear graph in the form $y = mx + c$	
Level 3: Problem Solving	Solve geometric problems involving the use of coordinates	

<u>The Story of René Descartes (1596 – 1650) and the Fly</u>

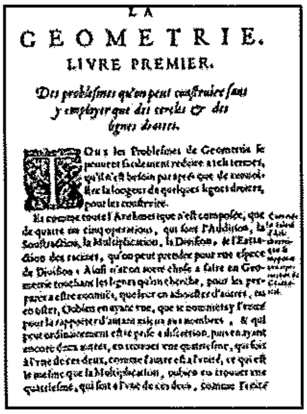
René Descartes, a famous French philosopher and mathematician, was born to middle-class parents on March 31, 1596 in La Haye, France. His mother died of tuberculosis when he was only one. He was a sickly child. His teachers would let him sleep until noon because of his poor health, but while in bed, he developed new ideas. He believed that by reasoning, he could overcome his troubles. "I think this inclination caused [my illness]," he later wrote, "gradually to disappear."



There is a long-standing myth that describes how Descartes discovered analytic geometry.

One night, Descartes was lying in bed and he looked up at the ceiling in his bedroom and noticed a fly was asleep on the ceiling. He began to think about how he might be able to describe the exact position of the fly. Descartes decided that if he drew two lines at right angles to each other, then he might be able to come up with a way of describing the exact position of the fly.

Grabbing a sheet of paper, he drew a graph of the ceiling. He drew a horizontal line and a vertical line and marked the point where the fly was located. With growing excitement he watched the fly land on point after point and realized that every point could be described with a pair of numbers: its distance from the horizontal line (x-axis) and its distance from the vertical line (y- axis).



LA

GEOMETRIE.

LIVRE PREMIER.

Although there is no evidence that this story is true, the idea of a horizontal and vertical axis and the origin used to describe a point in space was published in his book La Géométrie (Geometry) in 1637.

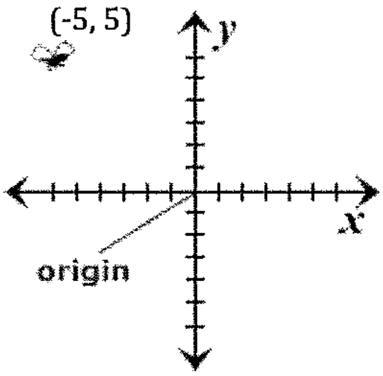
René Descartes who lived till 1650 has since been regarded as the father of Analytical Geometry, which is often called Cartesian Geometry or Coordinate Geometry. It is interesting to note that the terminology "Cartesian plane" and "Cartesian coordinate system" is derived from Decartes' Latin name *Renatius Cartesius*.

Sources:
<u>http://www.projectmaths.ie/documents/coordinate%20geometry_student_activities.pdf</u>
<u>http://www.mdhc.org/files/579_Paper_Junior_Pritt.pdf</u>

> "I am thinking, therefore I exist." or "I think, therefore I am."
> (Latin: Cogito ergo sum)
> *René Descartes (Discours de la Méthode, 1637)*

Cartesian Coordinate System

As mentioned on the earlier page, the idea for the Cartesian plane came to René Descartes as he watched a fly walk across his ceiling.

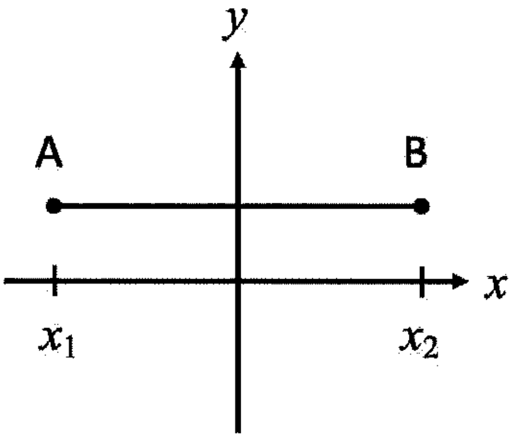


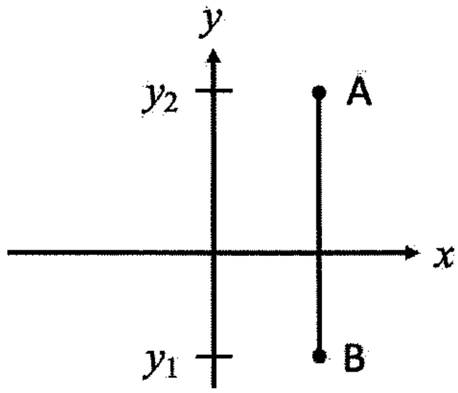
*Descartes represented the fly's location as an **ordered pair** of numbers (x, y). For any given point (x, y), x is known as the **x-coordinate** or abscissa of the point whereas y is known as the **y-coordinate** or ordinate of the point.*

*The plane containing these points is called the **Cartesian plane** (in honour of Descartes). Together, the x-axis, the y-axis, the Cartesian plane, and all the coordinates make up the **Cartesian Coordinate System**.*

Section 1.1 – Length of a Line Segment (Distance Between Two Given Points)

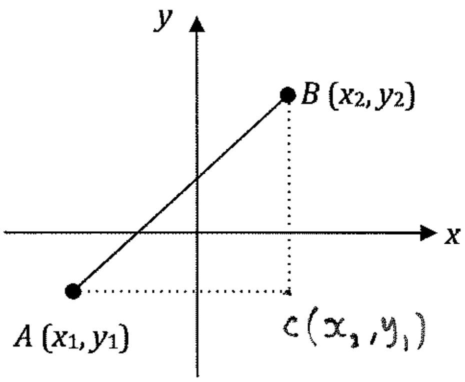
If ***A*** (x_1, y_1) and ***B*** (x_2, y_2) are two given points in the Cartesian plane that lie on the line segments as shown in the diagrams below. Find the distance between *A* and *B*.





Distance between *A* and *B* is $x_2 - x_1$ units

Distance between *A* and *B* is $y_2 - y_1$ units



Distance between *A* and *B* is

[redacted]

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example 1
Find the length of the line segment joining each of the following pairs of points.
(a) $A(4, 5)$, $B(4, -3)$ (b) $P(4, -3)$, $Q(-1, -7)$

$AB = \sqrt{(4-4)^2+(-3-5)^2}$
$= 8$ units.

$PQ = \sqrt{(-1-4)^2+(-7+3)^2}$
$= \sqrt{25+16}$
$= \sqrt{41}$ units
$= 6.40$ units (3 sf)

Example 2
The distance between the points $A(16, k)$ and $B(1, 1)$ is 17. Find the possible values of k.

$AB = 17$
$\sqrt{(16-1)^2+(k-1)^2} = 17$
$225 + (k-1)^2 = 289$
$(k-1)^2 = 64$
$k-1 = 8$ or $k-1 = -8$
$k = 9$ or $k = -7$ #

Example 3
The coordinates of the vertices of a triangle are (0, 4), (2, 0), (4, 2).
Prove that the triangle is **isosceles**.

Recall: An isosceles triangle has 2 equal sides.

Let the vertices be $A(0,4)$, $B(2,0)$, $C(4,2)$
$AB = \sqrt{(2-0)^2+(0-4)^2} = \sqrt{20}$ units
$AC = \sqrt{(4-0)^2+(2-4)^2} = \sqrt{20}$ units
$\therefore AB = AC$
$\triangle ABC$ is isosceles.

Example 4
The coordinates of two points are $A(-2, 6)$ and $B(9, 3)$.
Find the coordinates of the point C on the x-axis such that $AC = BC$.

Let $C = (x, 0)$
$AC = BC$
$\sqrt{(x+2)^2+(0-6)^2} = \sqrt{(x-9)^2+(0-3)^2}$
$x^2+4x+4+36 = x^2-18x+81+9$
$22x = 50$
$x = \frac{25}{11}$ #

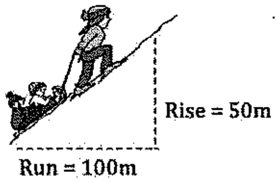
Each problem that I solved became a rule which served afterwards to solve other problems.
René Descartes (Discours de la Méthode, 1637)

Section 1.2 – Gradient of a Straight Line

Suppose you are trekking up a snowy hill to embark on a snowboarding adventure. As you trek up the hill, you will find yourself moving in two directions - upward in a vertical direction and horizontally, away from the bottom of the hill.

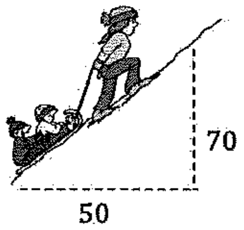
Let's describe your snowboarding adventure in numbers. Suppose each time you go 100m in a horizontal direction, you also go another 50m upward in a vertical direction. In geometry terms, you **rise** 50m every time you **run** 100m. We can symbolize your trek up the hill in the following way:

$$\frac{vertical\ disance\ travelled}{horizontal\ disance\ travelled} = \frac{rise}{run} = \frac{50m}{100m} = \frac{1}{2}$$



When we compare the rise and the run in this way, we call the result the **slope** or **gradient**. Gradient is the measure of incline. It is how steep something is.

Sign of gradient



$$Gradient = \frac{change\ in\ y}{change\ in\ x} = \frac{70}{50} = \frac{7}{5}$$

i.e. The line has a positive / negative gradient

$$Gradient = \frac{change\ in\ y}{change\ in\ x} = \frac{-10}{100} = -\frac{1}{10}$$

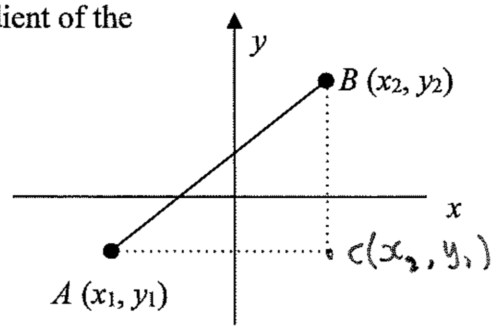
i.e. The line has a positive / negative gradient

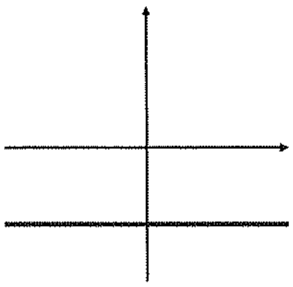
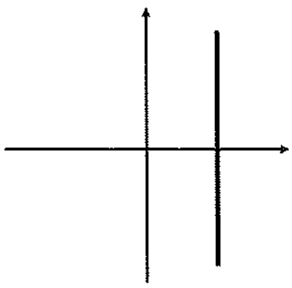
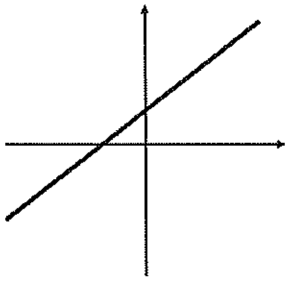
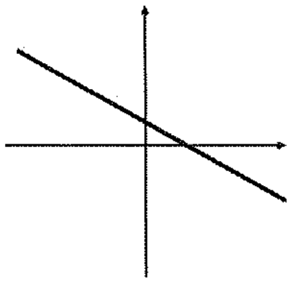
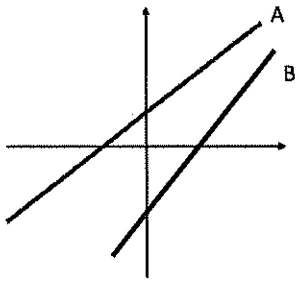
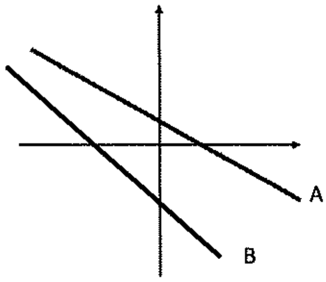
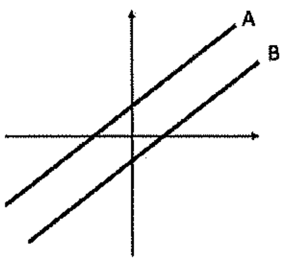
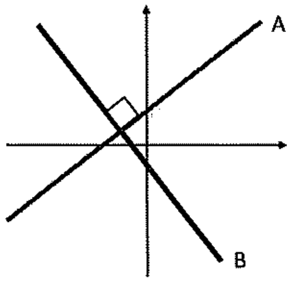
Gradient of a Line

For any two given points $A(x_1, y_1)$ and $B(x_2, y_2)$, the gradient of the line AB is

$m_{AB} =$ [redacted]

$$\frac{y_2 - y_1}{x_2 - x_1}$$



Horizontal line 	**Vertical line** 
$m =$ 0	$m =$ undefined.
Upward-sloping line 	**Downward-sloping line** 
$m =$ positive	$m =$ negative
Upward-sloping lines 	**Downward-sloping lines** 
m_A < m_B	m_A > m_B
Parallel lines 	**Perpendicular lines** 
m_A = m_B	**Coming soon in AM Coordinate Geometry!**

Example 5

Find the gradient of the straight line determined by each of the following pairs of points.

(a) (−2 , 4) and (−5, 8)

$$\text{gradient} = \frac{8-4}{-5+2}$$

$$= -\frac{4}{3}$$

(b) (1 , 7) and (−5, 6)

$$\text{gradient} = \frac{6-7}{-5-1}$$

$$= \frac{-1}{-6}$$

$$= \frac{1}{6}$$

Example 6

If the gradient of the line joining the points (−3, −7) and (4, b) is $\frac{3}{5}$, find b.

$$\frac{b+7}{4+3} = \frac{3}{5}$$

$$b+7 = \frac{3\times 7}{5}$$

$$b = \frac{21}{5} - 7$$

$$= -\frac{14}{5}$$

Example 7

$A(t , 3t)$, $B(t^2 , 2t)$, $C(t-2 , t)$ and $D(1 , 1)$ are four distinct points.
If AB is parallel to CD, find the possible values of t.

Since AB is parallel to CD

gradient of AB = gradient of CD

$$\frac{2t-3t}{t^2-t} = \frac{1-t}{1-(t-2)}$$

$$\frac{-\cancel{t}^{1}}{{}_{1}\cancel{t}(t-1)} = \frac{1-t}{3-t}$$

$$-3+t = (t-1)(1-t)$$

$$-3+t = -t^2+2t-1$$

$$t^2-t-2=0$$

$$(t-2)(t+1)=0$$

$$t=2 \text{ or } t=-1$$

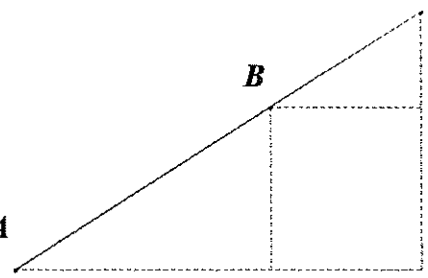
Collinear Points

If 3 or more points lie on the same straight line, we say that they are ***collinear***.

If points A, B, C are collinear, then

[redacted]

$m_{AB} = m_{BC} = m_{AC}$



Example 8

Given that the points $A(1, -1)$, $B(2, 2)$ and $C(4, t)$ are collinear, find the value of t.

Given A, B, C are collinear,

$m_{AB} = m_{BC}$

$\frac{2-(-1)}{2-1} = \frac{t-2}{4-2}$

$\frac{3}{1} = \frac{t-2}{2}$

$t-2=6$

$t=8$

Example 9

Show that the points $(-5, 1)$, $(5, 5)$ and $(10, 7)$ are collinear.

Let the points be $A(-5,1)$, $B(5,5)$ and $C(10,7)$

gradient of $AB = \frac{5-1}{5-(-5)} = \frac{2}{5}$

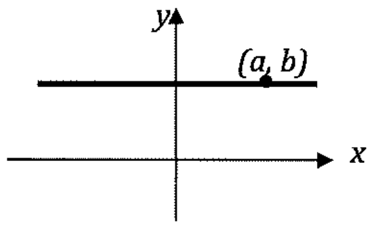
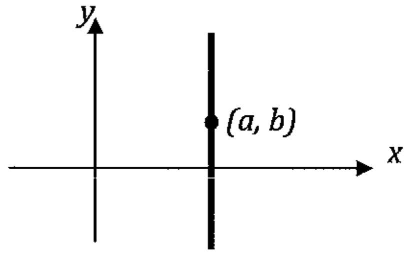
gradient of $BC = \frac{7-5}{10-5} = \frac{2}{5}$

since gradient of AB = gradient of BC

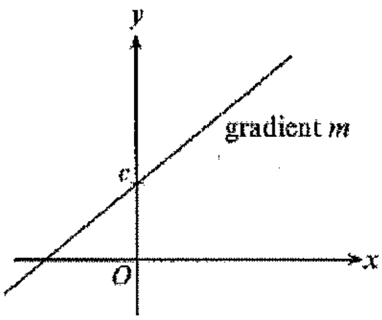
$\therefore$ The points are collinear.

1.3 Equation of a Straight Line

(I) Equations of Special Lines

Horizontal Line	Vertical Line
A horizontal line is parallel to the x-axis. Every point on the line has the same y-coordinate.  The gradient of a horizontal line is 0. The equation of a horizontal line is $y = b$.	A vertical line is parallel to the y-axis. Every point on the line has the same x-coordinate.  The gradient of a vertical line is undefined. The equation of a vertical line is $x = a$.

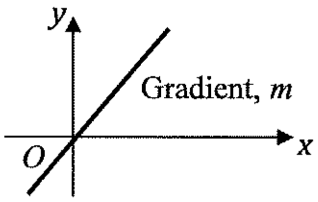
(II) Gradient-Intercept Form of Equation of a Straight Line $y = mx + c$



The equation of a straight line with gradient m and y-intercept $(0, c)$ is [redacted]

$y = mx + c$

This is known as the **gradient-intercept form** of the equation of a straight line.



The equation of a straight line with gradient m and passes through the origin is [redacted]

$y = mx$

Example 10

For each of the following equations, express in gradient-intercept form and write down the gradient and y-intercept of the line.

Equation	Gradient-intercept form	Gradient	y-intercept	Coordinates of y-intercept
(a) $2x + y - 3 = 0$	$y = -2x + 3$	-2	3	$(0, 3)$
(b) $x - 4y - 8 = 0$	$4y = x - 8$ $y = \frac{1}{4}x - 2$	$\frac{1}{4}$	-2	$(0, -2)$
(c) $\frac{x}{5} + \frac{y}{3} = 1$	$\frac{y}{3} = -\frac{x}{5} + 1$ $y = -\frac{3}{5}x + 3$	$-\frac{3}{5}$	3	$(0, 3)$

Example 11

Write down the equation of the line
(a) with gradient, $m = -2$ and y-intercept, $c = 7$
(b) with gradient, $m = 3$, and passes through $A(1, 4)$
(c) with gradient, $m = -\frac{4}{5}$, and passes through $B(-2, -7)$

(a) $y = -2x + 7$

(b) Let the equation be $y = 3x + c$, c is a constant

$4 = 3(1) + c$

$c = 1$

$\therefore y = 3x + 1$

(c) Let the equation be $y = -\frac{4}{5}x + c$, c is a constant

Subs $B(-2, -7)$ into the equation.

$-7 = -\frac{4}{5}(-2) + c$

$c = -\frac{43}{5}$

$\therefore y = -\frac{4}{5}x - \frac{43}{5}$

Example 12

For each of the following, write down the equation of the line that passes through the two given points *A* and *B*.

(a) $A(-3, 5)$, $B(0, 7)$

$m = \frac{7-5}{0-(-3)} = \frac{2}{3}$

$y = \frac{2}{3}x + c$, c is a constant

subs $(0, 7)$ into equation

$7 = \frac{2}{3}(0) + c$

$c = 7$

$\therefore y = \frac{2}{3}x + 7$

(b) $A(-3, -2)$, $B(2, -3)$

$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{y - (-2)}{x - (-3)} = \frac{-3 - (-2)}{2 - (-3)}$

$\frac{y+2}{x+3} = \frac{-1}{5}$

$y + 2 = -\frac{1}{5}x - \frac{3}{5}$

$y = -\frac{1}{5}x - \frac{13}{5}$

(c) $A(-5, -6)$, $B(8, -6)$

$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{y - (-6)}{x - (-5)} = \frac{-6 - (-6)}{8 - (-5)} = 0$

$\therefore y + 6 = 0$

$y = -6$

What information do we need to find the equation of a straight line?

① gradient and y-intercept

② gradient and one pair of coordinates.

③ 2 pairs of coordinates.

[redacted]

Line Graphs in Mathematical modelling

Discussion: Think of an example of how graphs of linear functions can be used to model real- world situations (such as parking rates, power bills, etc).

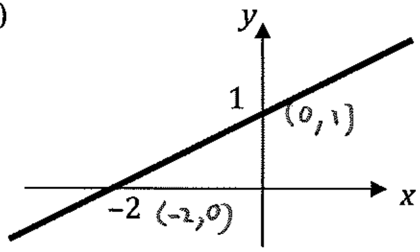
Key points to consider:

- highlight how the y-intercept, x-intercept, and gradient should be interpreted within the context, and
- models are approximations, idealisations and simplifications of what they represent, and that they come with assumptions and have limitations.

Example 13

Write down the equation of each of the following lines.

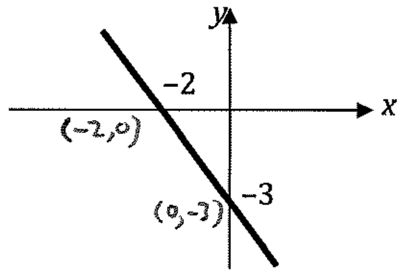
(a)



$m = \frac{1-0}{0-(-2)} = \frac{1}{2}$

$\therefore y = \frac{1}{2}x + 1$

(b)



$m = \frac{0-(-3)}{-2-0} = -\frac{3}{2}$

$y = -\frac{3}{2}x - 3$

Example 14

Find the gradient and y-intercept of the line $2x - 3y - 6 = 0$.

$2x - 3y - 6 = 0$

$3y = 2x - 6$

$y = \frac{2}{3}x - 2$

Gradient $= \frac{2}{3}$

y-intercept $= -2$

Example 15

Given that the point (2, −5) lies on the straight line $y = 2x + c$, find the value of c. Hence find the point where the given straight line meets the x-axis.

subs $(2,-5)$ into $y = 2x + c$

$-5 = 2(2) + c$

$c = -9$

$\therefore y = 2x - 9$

When $y = 0$

$2x - 9 = 0$

$x = \frac{9}{2}$

Point when straight line meets x-axis $= (\frac{9}{2}, 0)$

Example 16

Find the equation of the straight line with gradient $-\frac{2}{3}$ and passing through (−3, 5). If this line also passes through the point (a, 3), find a.

$y = -\frac{2}{3}x + c$, c is a constant.

$5 = -\frac{2}{3}(-3) + c$

$c = 3$

$\therefore y = -\frac{2}{3}x + 3$

$3 = -\frac{2}{3}(a) + 3$

$-\frac{2}{3}a = 0$

$a = 0$

Example 17

Find the equation of the straight line passing through (3, -2) and parallel to the line $2y = 5x + 7$.

$2y = 5x + 7$

$y = \frac{5}{2}x + \frac{7}{2}$ is parallel to the straight line.

Equation of line: $y = \frac{5}{2}x + c$, c is a constant

subs (3, −2) into equation

$-2 = \frac{5}{2}(-3) + c$

$c = -\frac{19}{2}$

$\therefore y = \frac{5}{2}x - \frac{19}{2}$

Recall: When two lines are parallel, they have the same gradient.

Example 18

Find the equation of the line which passes through the point (3, −2) and is parallel to the line $2x - 3y - 2 = 0$.

$2x - 3y - 2 = 0$

$3y = 2x - 2$

$y = \frac{2}{3}x - \frac{2}{3}$

$y = \frac{2}{3}x + c$, c is a constant

$-2 = \frac{2}{3}(3) + c$

$c = -4$

$\therefore y = \frac{2}{3}x - 4$

Example 19

The straight lines $kx = 4y + 5$ and $(2k + 2)x = 7 - 6y$ are parallel. Find k.

$kx = 4y + 5$

$4y = kx - 5$

$y = \frac{k}{4}x - \frac{5}{4}$ —(1)

$(2k+2)x = 7 - 6y$

$6y = -(2k+2)x + 7$

$y = -\left(\frac{2k+2}{6}\right)x + \frac{7}{6}$ —(2)

gradient in (1) = gradient in (2)

$\frac{k}{4} = -\left(\frac{2k+2}{6}\right)$

$\frac{k}{4} = -\frac{k}{3} - \frac{1}{3}$

$\frac{7k}{12} = -\frac{1}{3}$

$k = -\frac{12}{3 \times 7} = -\frac{4}{7}$

Example 20
(a) Given that the line $mx = ny + 2$ is parallel to the x-axis, find the value of m.
(b) State the condition for the line to be parallel to the y-axis instead.

(a) $mx = ny + 2$

$y = \left(\frac{m}{n}\right)x - \frac{2}{n}$

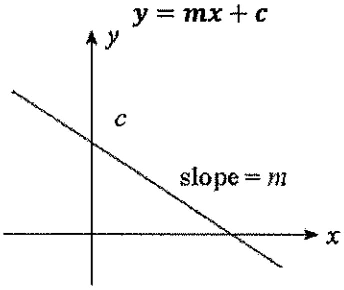
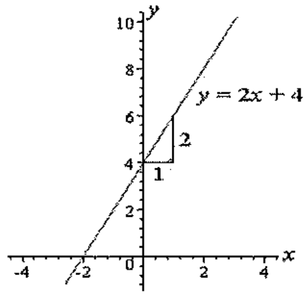
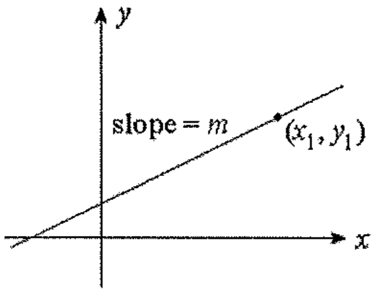
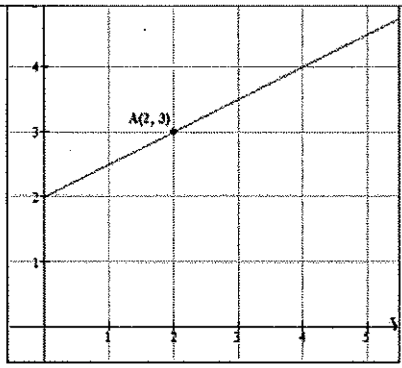
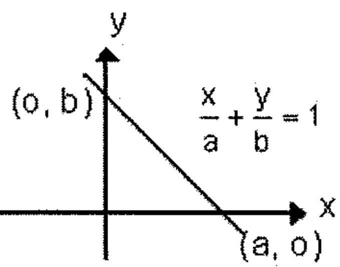
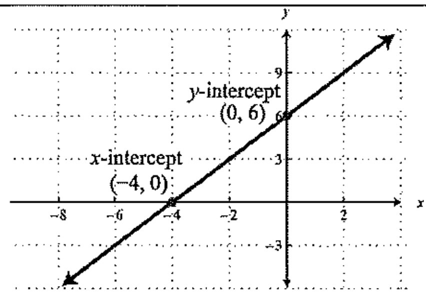
When line is parallel to x-axis, $\frac{m}{n} = 0$.

$\therefore m = 0$

(b) When line is parallel to y-axis, $n = 0$

Different Forms of Equation of a Straight Line

The graph of a linear function is represented by a straight line and the general form of a straight line is $ax + by = c$.

Form	Information given	Example
Gradient-intercept form $y = mx + c$  	Gradient = m y-intercept = c	
Gradient-point form $y - y_1 = m(x - x_1)$  	Gradient = m Point on the line (x_1, y_1)	 $y - 3 = \frac{1}{2}(x - 2)$
Intercept-intercept form $\frac{x}{a} + \frac{y}{b} = 1$  	x-intercept = a y-intercept = b	 $\frac{x}{-4} + \frac{y}{6} = 1$ $\frac{y}{6} - \frac{x}{4} = 1$

To solve coordinate geometry problems, it is useful to **identify and interpret the keywords** in the question. Identifying and interpreting the keywords help us to **understand the problem** and devise a plan to solve the problem (Stage 1 of Polya's Problem Solving Techniques).

Example 21

The line l has equation $x + 4y - 36 = 0$.

(a) Find the gradient of the line l.

(b) Given that the point C $(p, 2p)$ lies on the straight line l, find the value of p.

(c) The line $2y - x - 30 = 0$ intersects the line l at the point D. Find coordinates of D.

(d) Find the length of CD.

[Answer: (a) $-\frac{1}{4}$ *(b)* $p = 4$ *(c)* $D(-8, 11)$ *(d) 12.4 units]*

(a) $x + 4y - 36 = 0$

$4y = -x + 36$

$y = -\frac{1}{4}x + 9$ —(1)

gradient $= -\frac{1}{4}$

(b) subs $(p, 2p)$

$4(2p) = -p + 36$

$9p = 36$

$p = 4$

(c) $2y - x - 30 = 0$

$2y = x + 30$

$y = \frac{1}{2}x + 15$ —(2)

(1) = (2):

$-\frac{1}{4}x + 9 = \frac{1}{2}x + 15$

$\frac{3}{4}x = -6$

$x = -\frac{4 \times 6}{3} = -8$

when $x = -8$, $y = 11$

coordinates of D

$= (-8, 11)$

(d) coordinates of $C = (4, 8)$

Length of CD

$= \sqrt{(-8-4)^2 + (11-8)^2}$

$= \sqrt{153}$

$= 12.4$ units (3 sf)

Example 22

O A B C

The coordinates of the points O, A, B and C are (0, 0), (1, 5), (3, 4) and (2, −3) respectively. Find

(a) AB^2;

(b) the gradient of BC;

(c) the equation of the line passing through O and parallel to AC.

(a) $AB^2 = (3-1)^2 + (4-5)^2$

$= 5$

(b) gradient of BC $= \frac{-3-4}{2-3}$

$= 7$

(c) gradient of AC $= \frac{-3-5}{2-1} = -8$

Equation of line: $y = -8x$

Example 23

A straight line passes through the points $A(0, 3)$ and $B(8, 9)$.

(a) Find the equation of the line AB.

(b) Calculate the length of line segment AB.

(c) Another line, parallel to the y-axis and passing through the point (5, 1) meets AB at point C.

Calculate the coordinates of the point C.

ass through poin 5, 1)	Point (5, 1) is on the vertical Equation of this line is

[Answer: (a) $y = 0.75x + 3$ *(b) 10 units (c)* $C(5, 6.75)$*]*

(a) $\frac{y-3}{x-0} = \frac{9-3}{8-0} = \frac{3}{4}$

$y - 3 = \frac{3}{4}(x-0)$

$y = \frac{3}{4}x + 3$

(b) Length of AB

$= \sqrt{(8-0)^2 + (9-3)^2}$

$= 10$ units

(c) Equation of line : $x = 5$

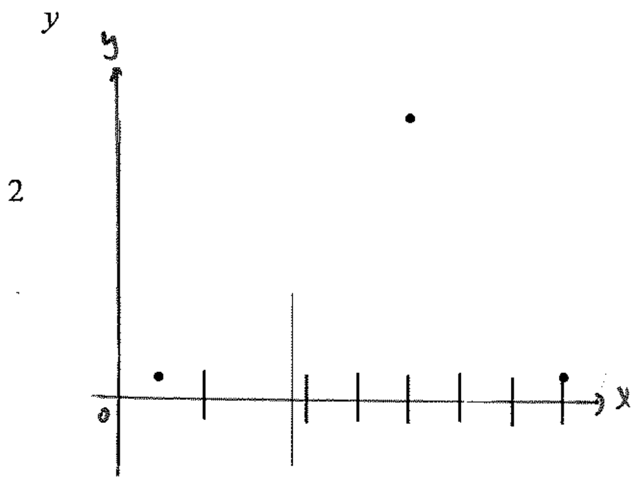
when $x = 5$, $y = \frac{3}{4}(5) + 3$

$= \frac{27}{4}$

$\therefore$ coordinates of C = $(5, \frac{27}{4})$

Example 24

.5) and $C(7, 3)$.



Keywords	Interpretation

(a) Write down the equation of line AB.

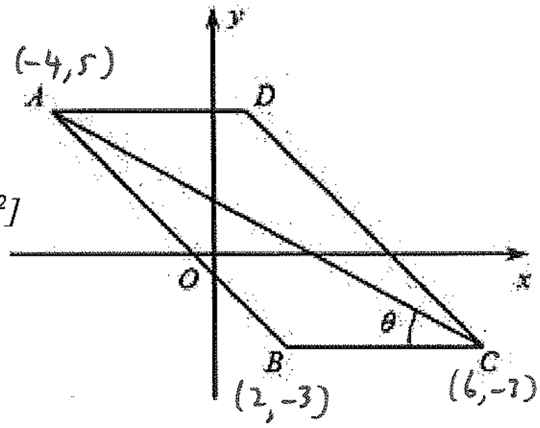
(b) Find the coordinates of point D such that $ABCD$ forms a parallelogram.

[Answer: (a) $y = 0.5$ *(b)* $(-1, 3)$ *]*

NA.

Example 25

The diagram shows a parallelogram $ABCD$ with vertices $A(-4,5)$ and $B(2,-3)$.
AC is parallel to the line $5y = -4x$ and BC is parallel to the x-axis.



(a) Find the equation of AC.
(b) Find the length of BC.
(c) Find the angle θ, in degrees.
(d) Find the area of triangle ABC.

[Answer: (a) $y = -\frac{4}{5}x + \frac{9}{5}$ (b) 4 unit (c) 38.7° (d) 16 units²]

(a) $5y = -4x$
$y = -\frac{4}{5}x$
Equation of AC:
$y = -\frac{4}{5}x + p$, p is a constant
When $x = -4$, $y = 5$
$\therefore 5 = -\frac{4}{5}(-4) + p$
$p = \frac{9}{5}$
$\therefore y = -\frac{4}{5}x + \frac{9}{5}$

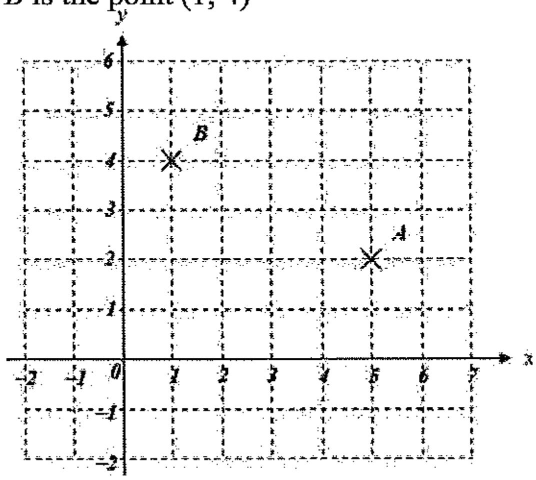
(b) When $y = -3$, $-3 = -\frac{4}{5}x + \frac{9}{5}$
$\frac{4}{5}x = \frac{24}{5}$
$x = 6$
coordinates of $C = (6,-3)$
$BC = 6 - 2 = 4$ units

(c) $\tan\theta$
$= \frac{5-(-3)}{6+4}$
$= \frac{4}{5}$
$\theta = 38.7°$ (1dp)

(d) Area of triangle ABC
$= \frac{1}{2}(6-2)(5-(-3))$
$= 16$ units^2

Example 26

The diagram shows one side of ΔABC. A is the point (5, 2) and B is the point (1, 4) marked on the graph above.



(a) The triangle is symmetrical about the line $y = 2$.
  (i) Write down the coordinates of point C.
  (ii) Calculate the area of ΔABC.
(b) Calculate the gradient of the line AB.
(c) Find the equation of line AB.

[Answer: (a)(i) (1, 0) (ii) 8 (b) -0.5 (c) $y = -0.5x + 4.5$]

(a)(i) coordinates of $C = (1,0)$
(ii) Area of $\Delta ABC = \frac{1}{2} \times 4 \times 4$
$= 8$ units^2

(b) gradient of $AB = \frac{4-2}{1-5} = -\frac{1}{2}$

(c) $y = -\frac{1}{2}x + p$, p is a constant
subs (1,4)
$4 = -\frac{1}{2}(1) + p$
$p = \frac{9}{2}$
$y = -\frac{1}{2}x + \frac{9}{2}$

Example 27

The points A, B and C have coordinates $A(1, 1)$, $B(p, q)$ and $C(4, 7)$. Line BC is parallel to the line $-4y + 6 = -2x$. A point P has coordinates $(r, 4)$.

(a) Find the equation of line BC, expressing y in terms of x.
(b) Given that B is a point on the line $x = 2$, find the coordinates of B.
(c) Given that B, C and P are collinear, find the value of r.
(d) The line $y = 1$ is the line of symmetry of ΔACT. State the coordinates of T.
(e) Find the area of ΔACT.

[Answer: (a) y=0.5x +5 (b) B (2,6) (c) r = –2 (d) T (4, –5) (e) 18 units²]

(a) $-4y+6=-2x$

$4y=2x+6$

$y=\frac{1}{2}x+\frac{3}{2}$

Let equation of BC be $y=\frac{1}{2}x+k$, k is constant.

At $(4,7)$

$7=\frac{1}{2}(4)+k$

$k=5$

$\therefore y=\frac{1}{2}x+5$

(b) Let coordinates of $B=(2,y)$

$y=\frac{1}{2}(2)+5$

$=6$

coordinates of B $=(2,6)$

(c) B, C, P are collinear.

$m_{BC}=m_{CP}$

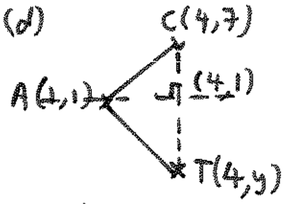
$\frac{4-7}{r-4}=\frac{7-6}{4-2}$

$\frac{-3}{r-4}=\frac{1}{2}$

$r-4=-6$

$r=-2$

(d)



Let the coordinates of T be 4, y)

$7-1=1-y$

$y=-5$

coordinates of $T=(4,-5)$

(e) Area of ΔACT

$=\frac{1}{2}(4-1)(7+5)$

$=18$ units^2

Example 28

P is the point (2, 3) and Q is the point (9, 5).

(a) Find the equation of the line joining PQ.
(b) Find the coordinates of the point where the line PQ intersects the x-axis.
(c) The line $y = 5$ is the line of symmetry of ΔPQR. Find the coordinates of R.
(d) Calculate the area of ΔPQR.
(e) Calculate the length of PQ.
(f) Hence calculate the perpendicular distance from R to the line PQ.

(a) $\frac{y-5}{x-9}=\frac{3-5}{2-9}$

$-7(y-5)=-2(x-9)$

$-7y+35=-2x+18$

$7y=2x+17$

$y=\frac{2}{7}x+\frac{17}{7}$

(b) When $y=0$,

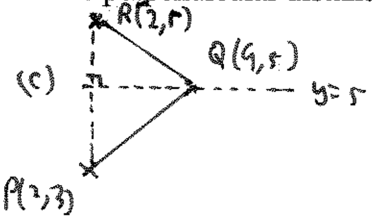
$\frac{2}{7}x+\frac{17}{7}=0$

$2x+17=0$

$x=-\frac{17}{2}$

coordinates $=(-\frac{17}{2},0)$

(c)



Let coordinates of R $=(2,r)$

$r-5=5-3$

$r=7$

coordinates of r $=(2,7)$

(d) Area $=\frac{1}{2}(7-3)(9-2)$

$=14$ units^2

(e) Length $=\sqrt{(9-2)^2+(5-3)^2}$

$=\sqrt{53}$ units

$=7.28$ units (3 sf)

(f) Perpendicular distance

$=\frac{2\times 14}{7.2801}$

$=3.85$ units (3 sf)

Summary: Coordinate Geometry (E Math)

1. Given two points (x_1, y_1) and (x_2, y_2),
   - Distance between 2 points/ length of line segment $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
   - Gradient of straight line $=\dfrac{y_2-y_1}{x_2-x_1}$
   - Equation of a straight line: $y=mx+c$, where m is the gradient and c is the y-intercept.

2. Parallel lines have equal gradients.

3. Collinear points: 3 or more points lie on the same straight line.
   If points A, B and C are collinear, then gradient of AB = gradient of BC = gradient of AC

4. For a point on the x-axis, the y-coordinate is 0. i.e. point is $(x, 0)$
   For a point on the y-axis, the x-coordinate is 0. i.e. point is $(0, y)$

5. To find equation of a straight line, we need
   (a) gradient of the straight line (b) a point on the straight line.

6. To find gradient of a straight line from an equation,
   (1) Make y the subject
   (2) Gradient of straight line = coefficient of x.

E Math Assignment 01 Coordinate Geometry

E Math Textbook: Shinglee think! Mathematics Book 3A (8^{th} Edition)

Tier A

- Textbook Exercise 4A (pages 97): Questions 1(a), 1(d), 3
- Textbook Exercise 4B (pages 103): Questions 1(b), 1(e), 3, 4
- Textbook Exercise 4C (pages 109 to 110): Questions 2, 3(b), 3(h), 4, 5

Tier B

- Textbook Exercise 4A (pages 97): Questions 4, 5, 8
- Textbook Exercise 4B (pages 103): Questions 5, 7, 8
- Textbook Exercise 4C (pages 109 to 110): Questions 8, 9, 11, 13

Tier C

- Textbook Exercise 4A (pages 97): Questions 11
- Textbook Exercise 4B (pages 103): Questions 9
- Textbook Exercise 4C (pages 109 to 110): Questions 16, 17

Computational Thinking in Coordinate Geometry

1 Given two points, find the equation of the line that passes through the points.
Function: EqnofLine
Input: Coordinate of two points
Output: Equation of the line in the form $y = mx + c$ or $x = a$.

2 Given the co-ordinates of 3 non-collinear points, determine the type of triangle formed by these 3 points. State whether it is a scalene, isosceles or equilateral, and whether it is an acute-angled, right-angled or obtuse-angled triangle.
Function: TypesofTriangle
Input: Coordinates of 3 non-collinear points
Output: Scalene/isosceles/equilateral, acute-angled/right-angled/ obtuse-angled

3 Given the co-ordinates of the vertices of a quadrilateral (in order), determine the type of quadrilateral. State whether it is a square, rectangle, parallelogram, trapezium, rhombus, kite or none of these.
Function: TypesofQuad
Input: Co-ordinates A, B, C and D (in order)
Output: Square/rectangle/parallelogram/trapezium/rhombus/kite or quadrilateral (if none of the above)

4 Given 3 points, determine whether the points form a triangle or are collinear.
Function: IsTriangle
Input: Co-ordinates of P, Q and R
Output: True if it is a triangle, False otherwise

5 Given the coordinates of two triangles, determine whether they are similar, congruent or not at all.
Function: IsSimilar
Input: Coordinates of two triangles, ABC and PQR.
Output: Similar, Congruent or Otherwise

6 Given a line and a point, find the equation of the line that passes through the point and is perpendicular to the line
Function: PerpendicularLine
Input: Equation of line in the form $ax + by = c$ and coordinates of point
Output: Equation of the perpendicular line passing through the point

7 Given a line L and a point P, find the distance of P from the line L.
Function: DistancetoLine
Input: Equation of line in the form $ax + by = c$ and coordinates of point
Output: Distance of the point from the line

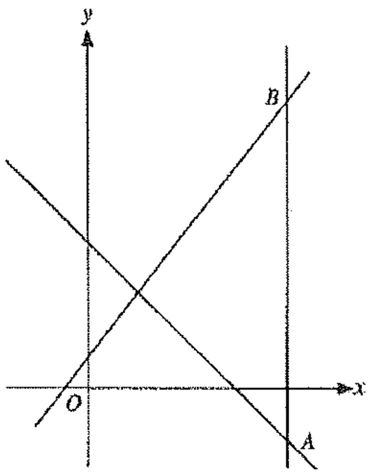
8 Given two points A and B, find the equation of the perpendicular bisector of AB.
Function: EqnofPerpendicularBisector
Input: Coordinates of the two points A and B
Output: Equation of the perpendicular bisector of AB

9 Given 3 non-collinear points (in ascending order of the x-coordinates) on the coordinate plane in the first quadrant, find the area of the triangle formed by the 3 points.
Function: AreaofTriangle
Input: 3 non-collinear points in the first quadrant (given in ascending order of the x-coordinates)
Output: Area of triangle formed by the 3 points

Past Years GCE 'O' Level Questions

1. [N11/I/19]

The diagram, which is not drawn accurately, shows the three lines $x = 8$, $y = 6 - x$ and $2y = 3x + 2$.



(a) Find the coordinates of A and B. [2]

(b) Find the gradient of the line $y = 6 - x$. [1]

(c) The point $(0, k)$ is the same distance from A as it is from B.
Find the value of k. [1]

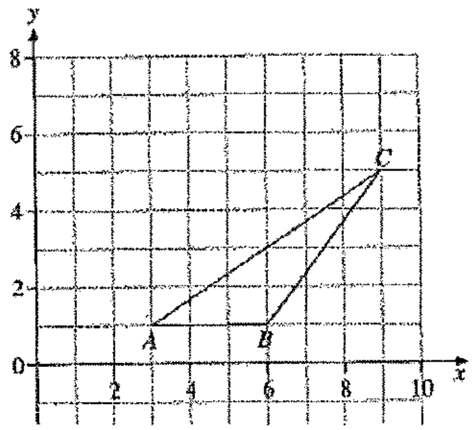
2. [N10/I/20]

A, B and C are the points $(3\,,1)$, $(6\,,1)$ and $(9\,,5)$.

(a) Find the area of triangle ABC. [1]

(b) $ABCD$ is a trapezium with AB parallel to DC.
The area of the trapezium is 14 units^2.
Find the coordinates of the point D. [2]

(c) E is the point $(2, k)$ and the area of triangle ABE is 9 units^2. Find the two possible values of k. [2]



3. [N09/I/25]

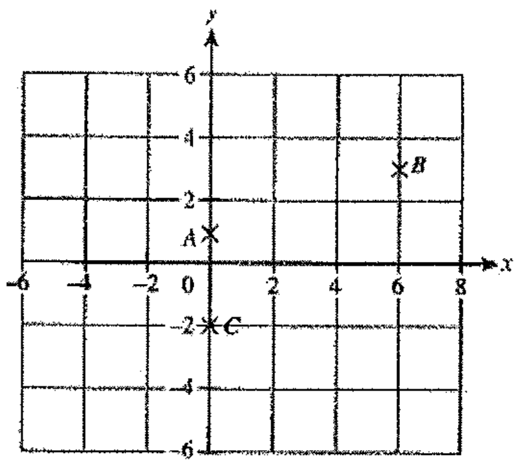
On the axes shown, A is $(0\,,1)$, B is $(6\,,3)$ and C is $(0\,,-2)$. Find

(a) the gradient of AB, [1]

(b) the equation of the line AB, [1]

(c) the area of triangle ABC, [1]

(d) the coordinates of **two** possible points D, such that the four points A, B, C and D are the four vertices of a parallelogram. [2]



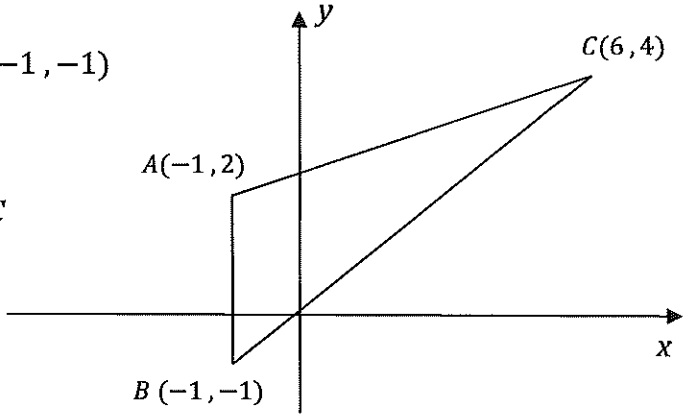
Additional Questions on Coordinate Geometry

Question 1

It is given that $A(3,-8)$ and $B(-2,2)$.
(a) Calculate the length of AB.
(b) Find the equation of the line that passes through A and B.
(c) $C(1, k)$ is on the line AB. Find the value of k.

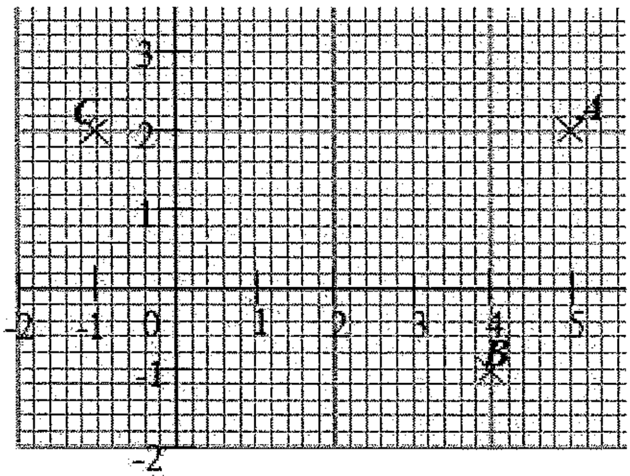
Question 2

The vertices of a triangle are $A(-1,2)$, $B(-1,-1)$ and $C(6,4)$.
(a) Find the gradient of BC.
(b) Find the length of AB.
(c) Find the equation of the line through C which is parallel to AB.
(d) Find the area of triangle ABC.
(e) Find coordinates of D if $ABCD$ is a parallelogram.



Question 3

In the diagram the points A, B and C are marked.



(a) Write the coordinates of the point B.
(b) Calculate the area of triangle ABC.
(c) The point D is such that $ABCD$ is a kite. Write down the coordinate of D.
(d) Find the equation of BC.

Question 4

The equation of a straight line L is $\frac{x}{4} - \frac{y}{3} = 1$. Find
(a) the gradient of the line L,
(b) the equation of the line K, which is parallel to L, and passes through the point $(1,-4)$,
(c) if $(2a, a+1)$ is a point on the line L, find the value of a.

Question 5

The line $2x + 3y + 11 = 0$ meets another line $2y + 5 = x$ at the point P.
(a) Find the coordinates of P.
(b) Find the equation of the line parallel to the line $4x + 2y = 1$ and passing through P.

Question 6

The coordinates of the points A and B are $(-2,15)$ and $(3,-6)$ respectively.
(a) Find the length of AB.
(b) Find the equation of the line that passes through AB.
(c) Hence, find the coordinates of the point where the line crosses the x-axis.
(d) Determine whether the point $(2,-9)$ lies on the line.

Question 7

The equation of the line l is $3y - x = 10$. Find the equation of the line which is parallel to l and which passes through the point (6, 10).

Question 8

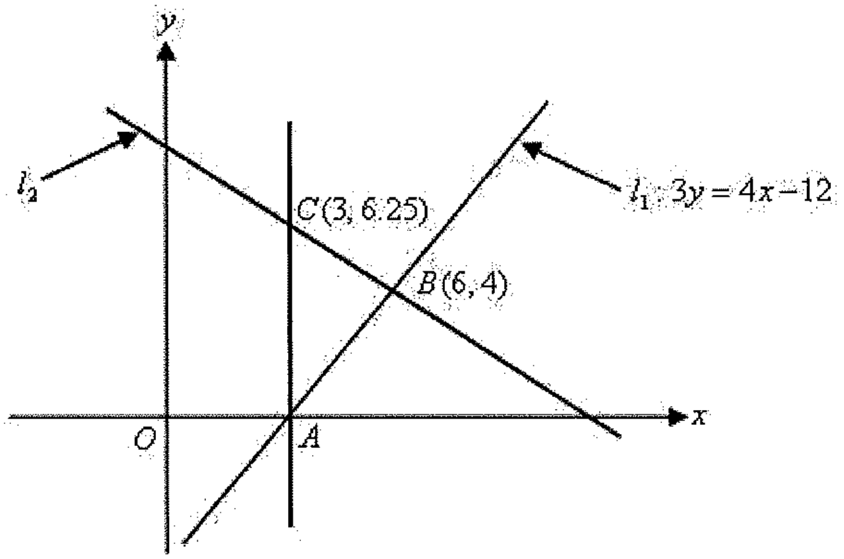
The line passing through points $A(-2, 3)$ and $P(p, 5)$ is parallel to the line $4x + 3y - 5 = 0$. Find the

(a) value of p.

(b) distance AP.

Question 9

$A(-2, 3)$



In the graph above, the ling l_1 has the equation $3y = 4x - 12$. A second line l_2 intersects l_1 at $B(6, 4)$. The vertical Line AC intercepts the x axis at A and l_2 at $C(3, 6.25)$.

Find

(a) the gradient of the line l_1.

(b) the coordinates of A.

(c) the equation of the line l_2 that passes through point B and C.

(d) the area enclosed by ΔABC.

Question 10

Find the equation of the line

(a) given that its gradient is $\frac{1}{3}$ and its y-intercept is -2.

(b) that passes through the points $A(0, 7)$ and $B(-1, -5)$.

(c) that passes through (3, 14) and (7, 4).

Question 11

The gradient of the line $\frac{1}{2}x + ky + 3 = 0$ is -3. Find

(a) the value of k,

(b) the y-intercept of the line.

[Answer Key]

1. (a) 11.2 units (b) $y=-2x-2$ (c) -4

2. (a) $\frac{5}{7}$ (b) 3 (c) $x=6$ (d) 10.5 (e) $D(6, 7)$

3. (a) $(4,-1)$ (b) 9units^2 (c) $(4,5)$ (d) $y=-\frac{3}{5}x+1\frac{2}{5}$

4. (a) $\frac{3}{4}$ (b) $y=\frac{3}{4}x-\frac{19}{4}$ (c) $a=8$

5. (a) P(-1, -3) (b) $y=-2x-5$

6. (a) Length = $21.6units(3sf)$ (b) $y=-\frac{21}{5}x+\frac{33}{5}$ (c) $\left(\frac{11}{7},0\right)$

7. $y=\frac{1}{3}x+8$

8. (a) $-\frac{7}{2}$ (b) 2.5 unit

9. (a) $\frac{4}{3}$ (b) $A(3, 0)$ (c) $4y=-3x+34$ (d) 9.375 cm^2

10. (a) $y=\frac{1}{3}x-2$ (b) $y = 12x + 7$ (c) $y=-\frac{5}{2}x+\frac{43}{2}$

11. (a) $\frac{1}{6}$ (b) -18