School of Science and Technology, Singapore
2023 Secondary 3 Additional Mathematics
Unit 04. Further Coordinate Geometry

Name: Solutions ( ) Class: S3-0_______

ENDURING UNDERSTANDING

Students understand that

- the ratios of sides of similar right angled triangles determine the midpoints and gradients of lines in Cartesian space.
- the equation of a straight line provides an algebraic structure to relate two variables in a connected equation to describe geometrical features of a straight line and solve geometric problems.
- the area of a rectilinear figure is determined by the determinant of a matrix using coordinates.

ESSENTIAL QUESTIONS

- How can we find midpoints in Cartesian space?
- How can we measure slope in Cartesian space using similarity?
- Why do parallel lines have the same gradients?
- What is the relationship between gradients of perpendicular lines?
- How does the equation of a straight line relate the variables?
- How can we use the equation of a straight line to solve problems?
- How can we find the area of a rectilinear figure using coordinates in a Cartesian space?

BIG IDEAS

- Diagrams: Diagrams are succinct, visual representations of real-world or mathematical objects that serve to communicate properties of the objects and facilitate problem solving. For example, graphs in coordinate geometry are used to represent the relationships between two sets of values.
- Measures: Numbers are used to measure or quantify a property of various real-world or mathematical objects, so that they can be analysed, compared and ordered.

KNOWLEDGE & SKILLS

At the end of the topic, students will be able to

- Relate gradient to the angle of inclination
- Find the equation of a line that is parallel to a given line
- Find how the gradients of two perpendicular lines are related
- Find the equation of a line that is perpendicular to a given line
- Solve problems involving the midpoint of a line segment
- Use the formula for area of a triangle and a quadrilateral

COMMON SYMBOLS/LANGUAGE USED

- Mid-point, collinear, vertex, bisector, perpendicular bisector

RESOURCES

- New Syllabus Mathematics 3 Textbook (Shinglee Publishers) Chapter 4 (pg 101 to 122)
- Thong, H.S., Khor, N. H., Yan, K. C. (2015). "Additional Mathematics 360". Singapore: Marshall Cavendish Education.
- Additional Mathematics 360 Textbook (Marshall Cavendish Education) 2nd Ed - Chapter 7 (pg 156 to 191)
- BBC Bitesize http://www.bbc.co.uk/schools/gcsebitesize/maths/geometry/linesegmentsrev2.shtml

TEACHING TO THE BIG IDEA

Lesson sequence in the unit								
Student Learning Outcomes	Dimensions (Please tick the appropriate boxes)							
	FUNCTIONS F	INVARIANCE I	NOTATIONS N	DIAGRAMS D	MEASURES M	EQUIVALENCE E	PROPORTIONALITY P	MODELS M
Length of a Line Segment								
Midpoint of a Line Segment								
Parallel Lines and Perpendicular Lines								
Bisectors and Perpendicular Bisectors								
Areas of Triangles and Quadrilaterals								

UNIT CHECKLIST

Section 4.1 Parallel Lines & Perpendicular Lines		
Cognitive Level	**Know, Understand, Demonstrate**	**Checklist**
Level 0: Memorisation	State the formula for the gradient given two coordinates	
	State that the gradients of two parallel lines are equal	
	State that the gradients of lines between collinear points are equal to each other	
	Find how the gradients of two perpendicular lines are related	
Level 1: Procedural tasks without connections	Calculate the gradient of a line given two coordinates	
	Deduce the relationship between the gradients of two parallel lines	
	Deduce the relationship between the gradients of two perpendicular lines	
	Find the equation of a line that is parallel to a given line	
	Find the equation of a line that is perpendicular to a given line	
Level 2: Procedural tasks with connections	Interpret and find the equation of a linear graph that is parallel to a given line	
	Interpret and find the equation of a linear graph that is perpendicular to a given line	
Level 3: Problem Solving	Solve geometric problems involving the use of coordinates	
	Relate gradient to the angle of inclination	

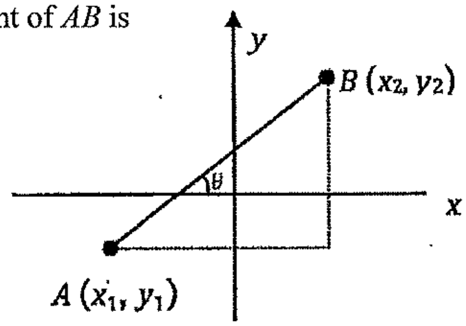
Section 4.2 Midpoint of a Line Segment		
Cognitive Level	**Know, Understand, Demonstrate**	**Checklist**
Level 0: Memorisation	State the formula for the midpoint of a line segment	
Level 2: Procedural tasks with connections	Use the midpoint of a line segment to find the equation of the perpendicular bisector of the line segment	
Level 3: Problem Solving	Solve problems involving midpoint of a line segment	

Section 4.3 Areas of Triangles and Quadrilaterals		
Cognitive Level	**Know, Understand, Demonstrate**	**Checklist**
Level 0: Memorisation	Use the formula for the area of a triangle	
	Use the formula for the area of a quadrilateral	

Section 4.1 – Parallel Lines and Perpendicular Lines

For any two given points $A(x_1, y_1)$ and $B(x_2, y_2)$, the gradient of AB is

$$m_{AB} = \boxed{\frac{y_2-y_1}{x_2-x_1} \text{ or } \frac{y_1-y_2}{x_1-x_2}}$$



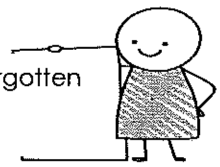
Gradient is a measure of the steepness of a line. Is angle of inclination a measure of steepness too? Why?

m_{AB} can also be written as ___tan θ___, where θ is the angle AB makes with the x-axis.

A horizontal line is parallel to the ___x-axis___. Its gradient is ___0___.

A vertical line is parallel to the ___y-axis___. Its gradient is ___undefined___.

Watch this video if you've forgotten what is gradient:
https://youtu.be/zVSq5b3PPfY



Example 1

Find the acute angle that the line segment joining $P(-2, -3)$ and $Q(4, 6)$ makes with the positive direction of the x-axis.

$\tan\theta = \frac{6-(-3)}{4-(-2)} = \frac{3}{2}$

$\theta = 56.3^\circ$ (1 dp)

Example 2

$P(k^2, 3k)$, $Q(k, k-2)$, $R(k, k+2)$ and $S(1, 1)$ are four points. Find the possible values of k such that PQ is parallel to RS.

$\frac{3k-(k-2)}{k^2-k} = \frac{(k+2)-1}{k-1}$

$\frac{2k+2}{k^2-k} = \frac{k+1}{k-1}$

$\frac{2(k+1)}{k(k-1)} = \frac{k+1}{k-1}$

$2(k+1) = k(k+1)$

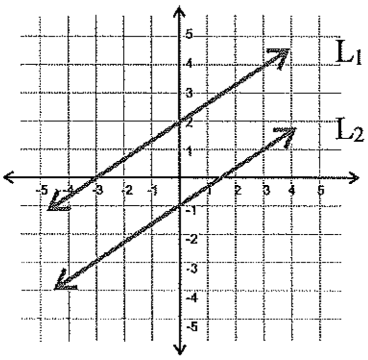
$k(k+1) - 2(k+1) = 0$

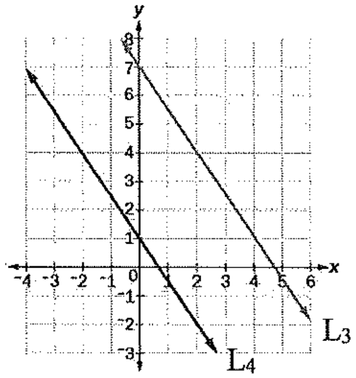
$(k-2)(k+1) = 0$

$k = 2$ or $k = -1$

Gradient of Parallel Lines

Compute the gradients of the lines L_1, L_2, L_3 and L_4 shown in the diagrams below.





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Gradient of L_1 = $\frac{2-0}{0-(-3)} = \frac{2}{3}$

Gradient of L_3 = $\frac{7-4}{0-2} = -\frac{3}{2}$

Gradient of L_2 = $\frac{0-(-1)}{1.5-0} = \frac{2}{3}$

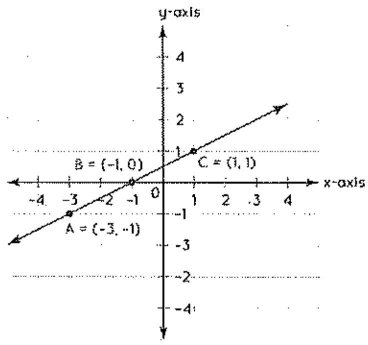
Gradient of L_4 = $\frac{4-(-2)}{-2-2} = -\frac{3}{2}$

What do you observe about the gradients of parallel lines?

Gradients of parallel lines are equal.
ie. $m_1 = m_2$

Collinear Points

A set of points are considered to be collinear, if they all lie in the same line. For example, if we plot the following three points $A(-3,-1)$, $B(-1,0)$, and $C(1,1)$ on a cartesian plane, we find that they lie on a straight line.



Therefore, we say that the points A, B and C are collinear.

To show that a set of points are collinear, choose the line segments in between the points (eg AB and BC) and establish that they have:

- a common direction (equal gradients)
- a common point (eg B)

Example 3

Show that the points $P(2\,,\, k+2)$, $Q(-2\,,\, k-2)$ and $R(3,\, k+3)$ are collinear.

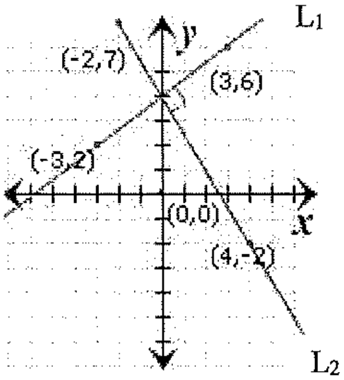
gradient of PQ $= \dfrac{(k+2)-(k-2)}{2-(-2)} = 1$

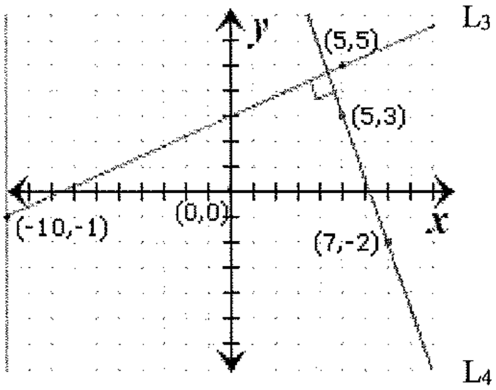
gradient of QR $= \dfrac{(k+3)-(k-2)}{3-(-2)} = 1$

$\therefore$ PQR is collinear.

Gradient of Perpendicular Lines

Compute the gradients of the lines L_1, L_2, L_3 and L_4 shown in the diagrams below.





Gradient of L_1 = $\dfrac{6-2}{3-(-3)}$

$= \dfrac{2}{3}$

Gradient of L_2 = $\dfrac{7-(-2)}{-2-4}$

$= -\dfrac{3}{2}$

Gradient of L_3 = $\dfrac{5-(-1)}{5-(-10)}$

$= \dfrac{2}{5}$

Gradient of L_4 = $\dfrac{3-(-2)}{5-7}$

$= -\dfrac{5}{2}$

What do you observe about the gradients of perpendicular lines?

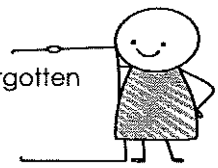
The product of gradients of perpendicular lines = −1

ie $m_1 m_2 = -1$

Is the converse also true?

Yes. If the product of gradients of 2 lines is −1, then the lines are perpendicular.

Watch this video if you've forgotten what is perpendicular lines:
https://youtu.be/tXxlwT6ZSwQ



Example 4

The line joining $A(a, 3)$ and $B(2, -3)$ is perpendicular to the line joining $C(10, 1)$ and B. Find the value of a.

$$m_{AB} \times m_{BC} = -1$$

$$\frac{3-(-3)}{a-2} \times \frac{-3-1}{2-10} = -1$$

$$\frac{6}{a-2} = -2$$

$$6 = -2a + 4$$

$$2a = -2$$

$$a = -1$$

Example 5

The equation of a line L_1 is given by $2y = 3x + 2$. The line L_2 passes through the points (1, 6) and (7, 2). Show that L_1 is perpendicular to L_2.

$$2y = 3x + 2$$

$$y = \frac{3}{2}x + 1$$

Gradient of $L_1 = \frac{3}{2}$

Gradient of $L_2 = \frac{6-2}{1-7}$

$$= \frac{4}{-6}$$

$$= -\frac{2}{3}$$

$$m_{L_1} \times m_{L_2} = \frac{3}{2} \times \left(-\frac{2}{3}\right)$$

$$= -1$$

$\therefore L_1$ is perpendicular to L_2.

Example 6

Prove that the points $A(4, 2)$, $B(2, 8)$ and $C(14, 12)$ are the vertices of a right-angled triangle.

$$m_{AB} = \frac{8-2}{2-4} = -3$$

$$m_{BC} = \frac{12-8}{14-2} = \frac{1}{3}$$

Since $m_{AB} \times m_{BC} = -3\left(\frac{1}{3}\right)$

$$= -1$$

$\therefore$ ABC are vertices of a right-angled triangle. with AB perpendicular to BC.

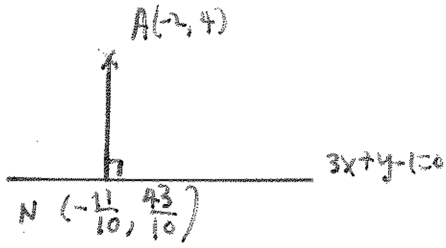
Example 7

(i) Find the equation of the line through $A(-2, 4)$ and perpendicular to the line $3x + y - 1 = 0$.
(ii) Given that the two lines intersect at the point N, find the coordinates of N.
(iii) Hence, find the shortest distance from A to the line $3x + y - 1 = 0$.

(i) $3x + y - 1 = 0$
$y = -3x + 1$ —(1)
Gradient of the line through $A = -\frac{1}{-3} = \frac{1}{3}$
$y = \frac{1}{3}x + c$, c is a constant
At $(-2, 4)$
$4 = \frac{1}{3}(-2) + c$
$c = \frac{14}{3}$
$\therefore y = \frac{1}{3}x + \frac{14}{3}$ —(2)

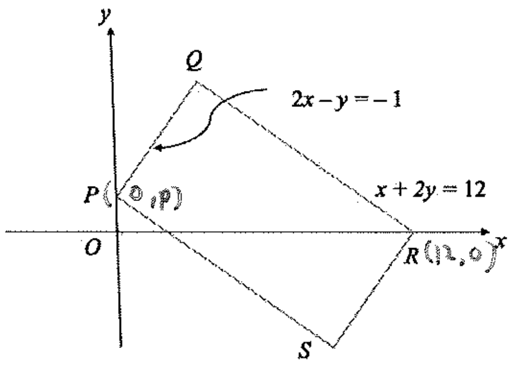
(ii) (1) = (2):
$-3x + 1 = \frac{1}{3}x + \frac{14}{3}$
$\frac{10}{3}x = -\frac{11}{3}$
$x = -\frac{11}{10}$
when $x = -\frac{11}{10}$,
$y = \frac{43}{10}$
$N = \left(-\frac{11}{10}, \frac{43}{10}\right)$

(iii)



shortest distance
$= AN$
$= \sqrt{\left(-2 - \left(-\frac{11}{10}\right)\right)^2 + \left(4 - \frac{43}{10}\right)^2}$
$= \sqrt{\frac{9}{10}}$
$= 0.949$ units

Example 8



The diagram shows a parallelogram $PQRS$.
(i) Given that $P(0, p)$ is a point on the line $2x - y = -1$, find the value of p.
(ii) R is the point where the line $x + 2y = 12$ meets the x-axis. Find the coordinates of R.
(iii) Using the properties of a parallelogram, find the equation of PS and of RS.
(iv) Find the coordinates of S.

(i) $2x - y = -1$
$\therefore 2(0) - p = -1$
$p = 1$

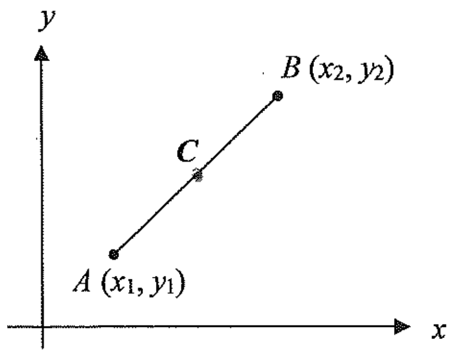
(ii) When $y = 0$,
$x + 2(0) = 12$
$x = 12$
coordinates of R: $(12, 0)$

(iii) $x + 2y = 12$
$2y = -x + 12$
$y = -\frac{1}{2}x + 6$
Gradient of $PS = -\frac{1}{2}$
Equation of PS:
$y = -\frac{1}{2}x + 1$ —(1)
$2x - y = -1$
$y = 2x + 1$
Gradient of $RS = 2$
$y = 2x + c$, c is a constant
$0 = 2(12) + c$
$c = -24$
$\therefore$ Equation of RS: $y = 2x - 24$ —(2)

(iv) (1) = (2): $-\frac{1}{2}x + 1 = 2x - 24$
$\frac{5}{2}x = 25$
$x = 10$
when $x = 10$, $y = -4$
coordinates of $S = (10, -4)$

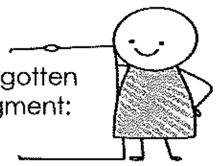
Section 4.2 – Midpoint of a Line Segment

A (x_1, y_1) and **B** (x_2, y_2) are two given points in the plane that lie on the line segments as shown in the diagram below. Find the coordinates of point **C**, the midpoint of the line joining A and B.



Coordinates of midpoint C

$= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$



Watch this video if you've forgotten what is midpoint of a line segment:
https://youtu.be/MpJUxVl_Egw

Example 9
Find the coordinates of the midpoint of the points (−4, −3) and (5, 7).

midpoint $= \left(\frac{-4+5}{2}, \frac{-3+7}{2}\right)$

$= \left(\frac{1}{2}, 2\right)$ #

Example 10
Given that $(p, 7)$ is the midpoint of the line segment joining the points $A(-3, 1)$ and $B(11, q)$. Find the values of p and q.

$(p, 7) = \left(\frac{-3+11}{2}, \frac{1+q}{2}\right)$

$\therefore p = \frac{-3+11}{2} = 4$ #

$\frac{1+q}{2} = 7$

$q = 13$ #

Example 11
Three of the vertices of a parallelogram $ABCD$ are $A(3, 0)$, $B(7, 3)$ and $C(1, 7)$.
(a) Find the coordinates of the midpoint of AC.
(b) Hence, find the coordinates of the fourth vertex, D.

(a) Midpoint of $AC = \left(\frac{3+1}{2}, \frac{7+0}{2}\right)$

$= \left(2, \frac{7}{2}\right)$ #

(b) Let coordinates of $D = (x, y)$

$\left(\frac{x+7}{2}, \frac{y+3}{2}\right) = \left(2, \frac{7}{2}\right)$

$\therefore \frac{x+7}{2} = 2$ $\qquad \frac{y+3}{2} = \frac{7}{2}$

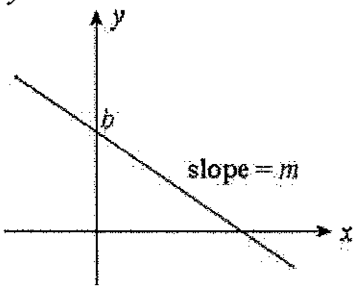
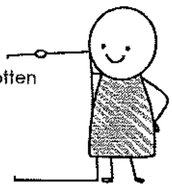
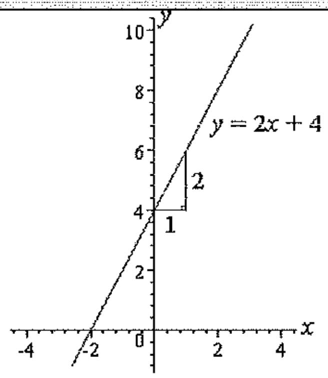
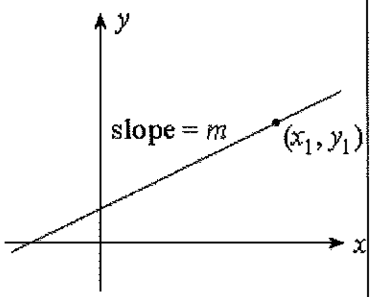
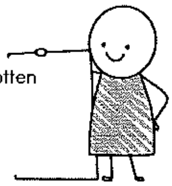
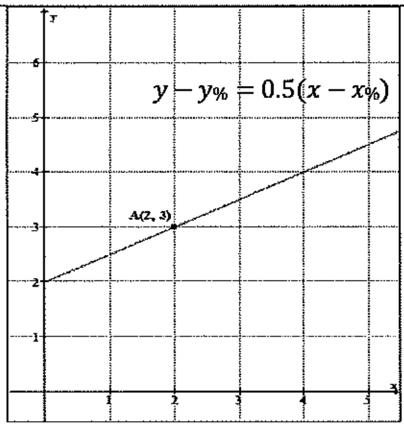
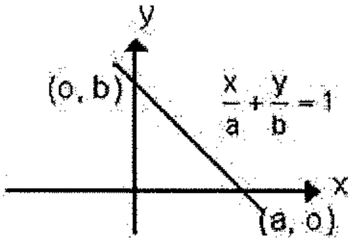
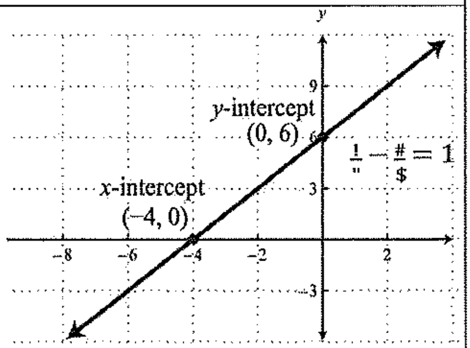
$x = -3$ $\qquad y = 4$

Coordinates of $D = (-3, 4)$ #

Section 4.3 – Bisectors and Perpendicular Bisectors

Equations of Straight Lines (Revision)

The graph of a linear function is represented by a straight line with general form $ax + by = c$ or $y = mx + c$. However, there are also different equations that can represent a straight line.

Form	Information about line	Example
Slope-intercept form $y = mx + b$ 	Gradient / slope = m y-intercept = b Watch this video if you've forgotten what is slope-intercept form: https://youtu.be/ys0Dxj-jKTk 	
Slope-point form $y - y_1 = m(x - x_1)$ 	Gradient / slope = m One point on the line is given. Watch this video if you've forgotten what is slope-point form: https://youtu.be/SemcMTLjSiw 	
Intercept-intercept form $\frac{x}{a} + \frac{y}{b} = 1$ 	x-intercept = a y-intercept = b	

When a line AB divides a line segment CD into 2 equal lengths, AB is known as a bisector. The intersection point between CD and AB is also known as the midpoint of CD.
Note: the bisector of CD does not pass through point C and point D.

What are the similarities and differences of a bisector and a perpendicular bisector?

Similarity: Both pass through / intersect the line segment at its midpoint

Difference : Perpendicular bisector is perpendicular to the line segment, but a bisector may not be.

Example 12

Find the equation of the straight line that is parallel to $2y - x = 7$ and bisects the line segment joining the points (3, 1) and (1, −5).

Midpoint $= \left(\frac{3+1}{2}, \frac{1-5}{2}\right)$

$= (2, -2)$

$2y - x = 7$

$2y = x + 7$

$y = \frac{1}{2}x + \frac{7}{2}$

$m = \frac{1}{2}$

$y = \frac{1}{2}x + c$

subs $(2, -2)$ into $y = \frac{1}{2}x + c$

$-2 = \frac{1}{2}(2) + c$

$c = -3$

$\therefore\ y = \frac{1}{2}x - 3$

Example 13

Find the equation of the perpendicular bisector of AB given the points $A(1, 2)$ and $B(3, 4)$.

Midpoint of AB $= \left(\frac{1+3}{2}, \frac{2+4}{2}\right)$

$= (2, 3)$

$m_{AB} = \frac{4-2}{3-1} = 1$

Let l be the perpendicular bisector of AB

$m_{AB}\, m_l = -1$

$\therefore m_l = -1$

l: $y = -x + c$, c is a constant

subs $(2, 3)$:

$3 = -2 + c$

$c = 5$

$\therefore$ Equation of perpendicular bisector:

$y = -x + 5$

Given a line and a point, find the equation of the line that passes through the point and is perpendicular to the line

Function: Perpendicular Line

Input: Equation of line in the form $ax + by = c$ and coordinates of point

Output: Equation of the perpendicular line passing through the point

Given two points A and B, find the equation of the perpendicular bisector of AB.

Function: Equation of Perpendicular Bisector

Input: Coordinates of the two points A and B

Output: Equation of the perpendicular bisector of AB

A Math Assignment 4A

A Math textbook: Marshall Cavendish Additional Mathematics 360 Textbook A 2nd Ed.

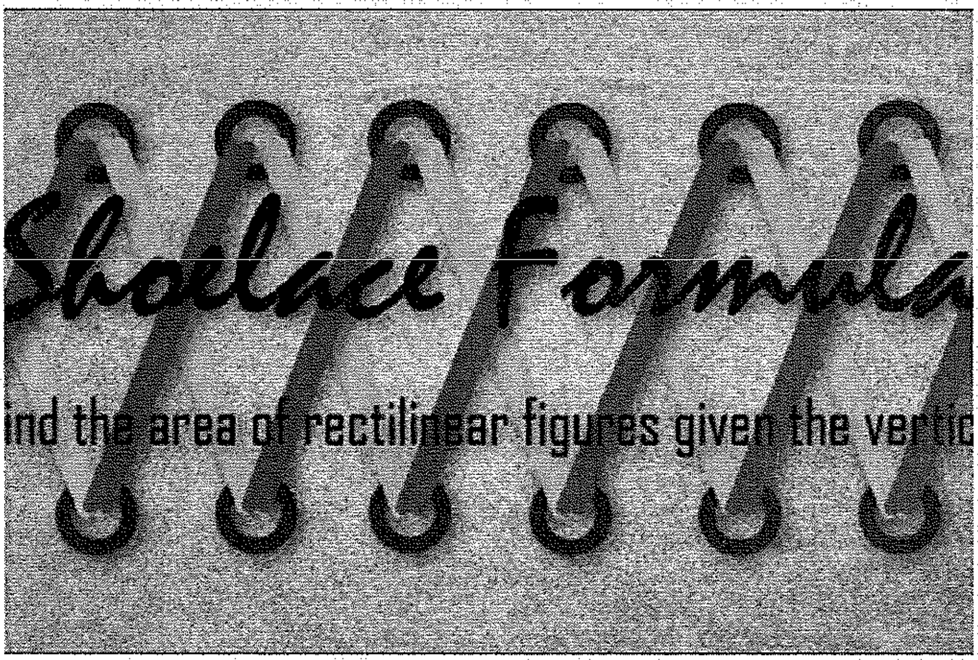
Tier A: Exercise 7.1 (pg 161) Q1, 3, 5, 9, 11
Exercise 7.2 (pg 169) Q1, 4, 5, 7, 8
Exercise 7.3 (pg 177) Q7, 8, 11, 12

Tier B: Exercise 7.1 (pg 161) Q14
Exercise 7.2 (pg 169) Q10, 13, 14
Exercise 7.3 (pg 177) Q17, 18, 19

Tier C: Exercise 7.1 (pg 161) Q17
Exercise 7.3 (pg 177) Q25, 26

Section 7.4 - Areas of Triangles and Quadrilaterals

Pre-lesson assignment: Visit SLS and attempt the lesson titled 'Shoe-lace' formula.

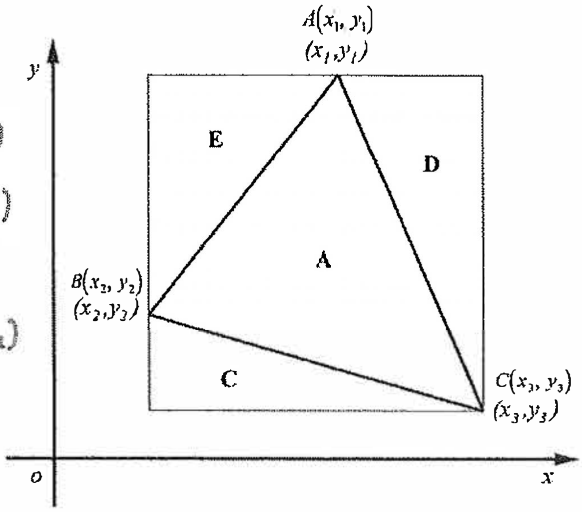


In this lesson, we will learn about how to find area of rectilinear figures given the coordinates of their vertices.

***the lesson will be assigned to you by your Mathematics teacher.**

The diagram shows a triangle ABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$.

Find the area of the triangle in terms of x and y.



Area of ΔABC = Area of $\square$ − Area of E − Area of D − Area of C

Area of $\square = (x_3-x_2)(y_1-y_3) = (x_3y_1+x_2y_3)-(x_3y_3-x_2y_1)$

Area of E $= \frac{1}{2}(x_1-x_2)(y_1-y_2) = \frac{1}{2}(x_1y_1+x_2y_2)-\frac{1}{2}(x_1y_2+x_2y_1)$

Area of D $= \frac{1}{2}(x_3-x_1)(y_1-y_3) = \frac{1}{2}(x_3y_1+x_1y_3)-\frac{1}{2}(x_1y_1+x_3y_3)$

Area of C $= \frac{1}{2}(x_3-x_2)(y_2-y_3) = \frac{1}{2}(x_3y_2+x_2y_3)-\frac{1}{2}(x_3y_3+x_2y_2)$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}\left|(x_2y_3-x_3y_2)-(x_1y_3-x_3y_1)+(x_1y_2-x_2y_1)\right|$

$= \frac{1}{2}\left|x_1y_2+x_2y_3+x_3y_1-x_2y_1-x_3y_2-x_1y_3\right|$

Hence, rearranging the terms, the area of a triangle, with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) can be written as:

Area of Triangle $ABC = \frac{1}{2}\left|\begin{array}{cccc} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{array}\right|$

$= \frac{1}{2}\left|(x_1y_2+x_2y_3+x_3y_1)-(x_2y_1+x_3y_2+x_1y_3)\right|$

This formula is known as the **shoelace formula** or shoelace algorithm.

What are some conditions you need to take note of when applying the shoelace formula?

① Fill in the coordinates in anti-clockwise direction.

② start and end with the same pair of coordinates.

The same formula can be applied to any convex polygon. In general, if the vertices of an n-sided polygon taken in the **anticlockwise** direction are (x_1, y_1), (x_2, y_2), … ,(x_n, y_n), then

Area of polygon

$= \frac{1}{2}\left|\begin{array}{cccccc} x_1 & x_2 & x_3 & \cdots & x_n & x_1 \\ y_1 & y_2 & y_3 & \cdots & y_n & y_1 \end{array}\right|$

$= \frac{1}{2}\left|(x_1y_2+x_2y_3+\ldots+x_{n-1}y_n+x_ny_1)-(x_2y_1+x_3y_2+\ldots+x_ny_{n-1}+x_1y_n)\right|$

A simple sketch often helps in understanding the problem visually. The diagram helps us to see the order of vertices in the anticlockwise direction.

BIG idea

Diagrams

Example 14

Find the area of triangle ABC with vertices $A(-3, 5)$, $B(1, -4)$ and $C(4, 2)$.

Area of triangle ABC $= \frac{1}{2}\begin{vmatrix} -3 & 1 & 4 & -3 \\ 5 & -4 & 2 & 5 \end{vmatrix}$

$= \frac{1}{2}\left| 12 + 2 + 20 - 5 + 16 + 6 \right|$

$= \frac{1}{2}\left|51\right|$

$= \frac{51}{2}$ units^2

Example 15

P is the point (2, 3) and Q is the point (9, 5).
(a) Find the equation of the line joining PQ.
(b) Find the coordinates of the point where the line PQ intersects the x-axis.
(c) The line $y = 5$ is the line of symmetry of PQR. Find the coordinates of R.
(d) Calculate the area of PQR.
(e) Calculate the length of PQ and hence calculate the perpendicular distance from R to the line PQ.

(a) $m_{PQ} = \frac{5-3}{9-2} = \frac{2}{7}$

$y = \frac{2}{7}x + c$, c is a constant.

subs $(2,3)$:

$3 = \frac{2}{7}(2) + c$

$c = \frac{17}{7}$

$\therefore y = \frac{2}{7}x + \frac{17}{7}$

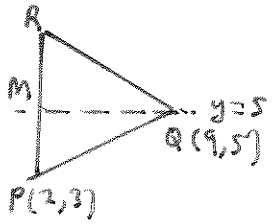
(b) When $y = 0$,

$0 = \frac{2}{7}x + \frac{17}{7}$

$x = -\frac{17}{2}$

The intersection is at $\left(-\frac{17}{2}, 0\right)$

(c)



$R = (2, 5-3+5)$

$= (2, 7)$

or M is the midpoint of PR.

$(2,5) = \left(\frac{2+x}{2}, \frac{3+y}{2}\right)$

$x = 2$, $y = 7$

$\therefore R = (2,7)$

(d) Area of $\triangle PQR = \frac{1}{2}(7-3)(9-2)$

$= 14$ units^2

OR

Area of $\triangle PQR$

$= \frac{1}{2}\begin{vmatrix} 2 & 9 & 2 & 2 \\ 3 & 5 & 7 & 3 \end{vmatrix}$

$= \frac{1}{2}\left|10 + 63 + 6 - 27 - 10 - 14\right|$

$= 14$ units^2

(e) $PQ = \sqrt{(9-2)^2 + (5-3)^2}$

$= \sqrt{53}$ units.

Let perpendicular distance be h.

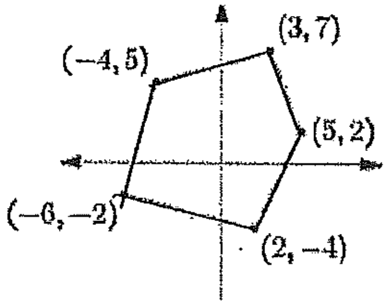
$\frac{1}{2} \times h \times \sqrt{53} = 14$

$h = \frac{2(14)}{\sqrt{53}}$

$= 3.85$ units^2 (3 sf)

Example 16

Find the area of the figure shown below.



$$\text{Area} = \frac{1}{2}\left|\begin{matrix} 3 & -4 & -6 & 2 & 5 & 3 \\ 7 & 5 & -2 & -4 & 2 & 7 \end{matrix}\right|$$

$$= \frac{1}{2}\left|(15+8+24+4+35)-(-28-30-4-20+6)\right|$$

$$= \frac{1}{2}|162|$$

$$= 81 \text{ units}^2$$

Example 17

The coordinates of the points O, A, B and C are (0, 0), (1, 5), (3, 4) and (2, −3) respectively. Find

(a) AB^2,

(b) the gradient of BC,

(c) the equation of the line passing through O and parallel to AC.

(a) $AB^2 = (3-1)^2 + (4-5)^2$

$\therefore AB^2 = 5$

(b) $m_{BC} = \dfrac{-3-4}{2-3}$

$= 7$

(c) $m_{AC} = \dfrac{-3-5}{2-1}$

$= -8$

Subs $(0,0)$ into $y = -8x + c$

$\therefore c = 0$

$\therefore$ Equation : $y = -8x$

Example 18

The diagram shows a triangle with vertices at $A(6, 9)$, $B(-2, 3)$ and $C(4, -5)$. M and P are the midpoints of AB and AC respectively. The line through M and P meets the x-axis at the point Q.
Find (i) the coordinates of M and P,
(ii) the ratio $MP : PQ$.

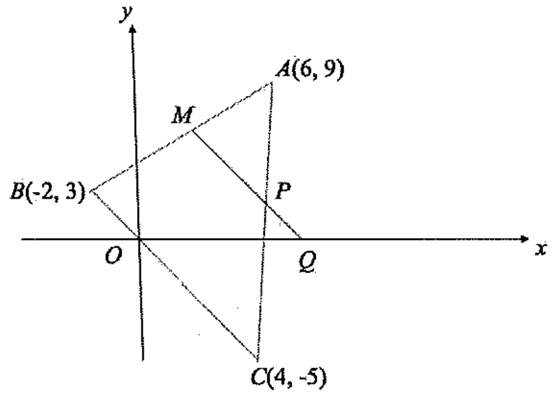
(i) $M = \left(\frac{-2+6}{2}, \frac{9+3}{2}\right) = (2, 6)$

$P = \left(\frac{6+4}{2}, \frac{9-5}{2}\right) = (5, 2)$

(ii) $MP : PQ = (6-2) : (2-0)$

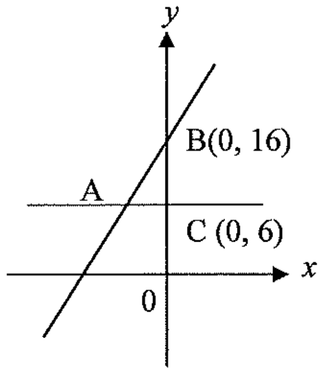
$= 4 : 2$

$= 2 : 1$



Example 19

In the diagram, B is the point (0, 16) and C is the point (0, 6).
The sloping line through B and the horizontal line that passes through C meet at the point A.
(a) Write down the equation of the line AC.
(b) Given that the gradient of the line AB is 2, find the equation of AB.
(c) Find the coordinates of A.
(d) Calculate the area of ABC.



(a) Equation of AC: $y = 6$

(b) $y = 2x + k$, k is a constant.

subs (0, 16),

$$\left.\begin{aligned} 16 &= 2(0) + k \\ k &= 16 \\ \therefore y &= 2x + 16 \end{aligned}\right\} \text{Is this necessary?}$$

(c) When $y = 6$

$6 = 2x + 16$

$x = -5$

$\therefore$ coordinates of $A = (-5, 6)$

(d) Area of $ABC = \frac{1}{2}(16-6)(0-(-5))$

$= 25$ units^2

OR Area of $ABC = \frac{1}{2}\begin{vmatrix} 0 & 0 & -5 & 0 \\ 6 & 16 & 6 & 6 \end{vmatrix}$

$= \frac{1}{2}\left|(0+0-30) - (0-80+0)\right|$

$= \frac{1}{2}|50|$

$= 25$ units^2

Example 20
The coordinates of W, X, Y and Z are (4, 10), $(a + 2, a)$, (11, 2) and (2, 2) respectively. Given that the area of $WXYZ$ is 21.5 units^2, find the value of a.

$$\text{Area of } WXYZ = \frac{1}{2}\begin{vmatrix} 4 & a+2 & 11 & 2 & 4 \\ 10 & a & 2 & 2 & 10 \end{vmatrix}$$

$$= \frac{1}{2}\left|(4a + 2a+4 + 22+20) - (10a+20+11a+4+8)\right|$$

$$= \frac{1}{2}\left|6a + 46 - 21a - 32\right|$$

$$= \frac{1}{2}\left|-15a+14\right|$$

$$\therefore \frac{1}{2}\left|-15a+14\right| = 21.5$$

$$\left|-15a+14\right| = 43$$

$$-15a + 14 = 43 \quad \text{or} \quad -15a+14 = -43$$

$$a = -\frac{29}{15} \quad \text{or} \quad a = \frac{57}{15} = \frac{19}{5}$$

COMPUTING Given 3 non-collinear points (in ascending order of the x-coordinates) on the coordinate plane in the first quadrant, find the area of the triangle formed by the 3 points.
Function: Area of Triangle
Input: 3 non-collinear points in the first quadrant (given in ascending order of the x-coordinates)
Output: Area of triangle formed by the 3 points

A Math Assignment 4B

A Math textbook: Marshall Cavendish Additional Mathematics 360 Textbook.
Tier A: Exercise 7.4 (pg 186) Q1, 2, 8, 9

Tier B: Exercise 7.4 (pg 186) Q11, 13, 14

Tier C: Exercise 7.4 (pg 186) Q16