Mr Jo's copy (w answer)

School of Science and Technology, Singapore
Mathematics Department
Secondary 2

SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

Name: SOLUTION ( ) Class: S2-0 __

Big Ideas: Ratio and proportion

https://www.youtube.com/watch?v=_VktSY6SIH0
https://archi-monarch.com/architectural-scale/

Unit Essential Understanding

1. Geometry reflects natural phenomena that could be modeled mathematically.
2. A ratio is the relationship of two or more quantities or measurements.

Unit Essential Questions

1. How are ratio and scale related mathematically?
2. How do you determine the real distance of 2 towns from the Google map?

Teaching To The Big Idea …

Lesson sequence in the unit								
Student Learning Outcomes	Dimensions (Please tick the appropriate boxes)							
	FUNCTIONS F	INVARIANCE I	NOTATIONS N	DIAGRAMS D	MEASURES M	EQUIVALENCE E	PROPORTIONALITY P	MODELS M
Similarity, Enlargement and Scale Drawings		✓	✓	✓	✓		✓	✓

Scale Drawing

A map that is the same size as the area it represents is not very useful. So, we **scale down** measurements to make the map small enough to be conveniently carried around by users such as motorists, cyclists and bushwalkers.

A ratio is used in the scale drawing of map/drawing. That is:
Scale = Map distance: Actual distance
= Drawing length: Actual length

note:
1 cm = 10 mm
1 m = 100 cm
1 km = 1000 m = 100000cm

A scale is usually expressed in one of two ways:

- Using units as in 1 cm to 1 km
- Without explicitly mentioning units as in 1: 100 000.

Sometimes, the ratios are represented as fractions:

	Scale Ratio	Representative Fraction (R.F.)
Map Scale	$1: r$	$\frac{1}{r}$

If $1: r$ and $\frac{1}{r}$ do not have units, or '1' and 'r' are based on the same units, we say that the scale is a **representative fraction**.

Example 1

Given that the scale of a map 1 cm: 25 km, express it in the following forms:

(a) $1: r$

(b) $\frac{1}{r}$

(a) 1 cm : 25 km

1 cm : 25000 m

1 cm : 2500000 cm

1 : 2500000

(b) $\frac{1}{2500000}$

step 1. map distance : actual distance

step 2. convert to same units (either cm or km)

step 3. simplify (there should not be any units in this case.

model /actual

Example 2

Given that the scale of a map is $\frac{1}{15\,000}$, find the distance represented on the map for the following:

a) 750 m

b) 3.9 km

note the units

reverse process to example 1

actual represents model

step 1. actual distance : map distance

step 2. convert to same units (either cm or km)

step 3. find required distance

(a) 15000 cm represents 1 cm

150 m is represented by 1 cm

750 m is represented by $\frac{750}{150} \times 1 = 5$ cm

(b) 0.15 km is represented by 1 cm

3.9 km is represented by $\frac{3.9}{0.15} = 26$ cm

Area of Maps

If a map scale is $1: r$, the area scale is $1: r^2$.

note: similar figures

Example 3

Given that the scale of a map is 1: 35 000, find

(a) the actual area, in km^2, of an area which is 12 cm^2 on the map,

(b) the map area in cm^2, of an area which is 28 km^2.

1 cm represents 35 000 cm

(a) 1 cm represents 0.35 km

$1\ cm^2$ represents $0.35^2 = 0.1225\ km^2$ squared because of similar figures

$12\ cm^2$ represents $0.1225 \times 12 = 1.47\ km^2$ approx to 3 sf as it is not exact

(b) Map area $= \frac{28}{0.1225} = 228.6 = 229\ cm^2$ (3sf).

Example 4

Given that 2 cm represented 5 km on a certain map, find the

(a) scale of the map in the form $1 : r$; ratio form

(b) scale of the map in the form $\frac{1}{r}$; fraction form

(c) actual distance between two police stations that are 6.6 cm apart on the map;

(d) distance between two libraries as represented on the map if they are actually 9 km apart on the ground.

map. : actual

(a) 2 cm : 5 km

prefers 'represents' than ':'

2 cm : 500 000 cm

1 cm : 250 000 cm

scale is 1 : 250 000 map. : actual

(b) $\frac{1}{250000}$

(c) Actual distance $= \frac{6.6}{2} \times 5 = 16.5$ km

(d) Map distance $= \frac{9}{5} \times 2 = 3.6$ cm

Example 5

The scale of a map is 1: 40000.

(a) If a road is represented in 2.5 cm on the map, find the actual length of the road, in km.

(b) If the actual area of a plot of land is 0.5 km^2, find its area on the map, in cm^2.

map. : actual

(a) 1 : 40 000

1 cm represents 40 000 cm

2.5 cm represents 100 000 cm

2.5 cm represents 1 km

(b) 1^2 km^2 is represented by $2.5^2 = 6.25$ cm^2

0.5 km^2 is represented by $\frac{0.5}{1} \times 6.25 = 3.13$ cm^2

Example 6

The actual area of a pond which is16 km^2 is represented on map A by an area of 400 cm^2.

(a) If map A is drawn to a scale of 1: k, find the value of k.

(b) The distance of two locations on map A is 8.6 cm. Find the distance of the same two locations on map B which has a scale of 1 cm: 0.4 km.

(c) State which map has a longer map distance of the two locations.

actual : map

(a) 16 km^2 is represented by 400 cm^2

400 cm^2 : 16 km^2 — ratio of AREA of similar ponds

20 cm : 4 km — ratio of corresponding lengths

20 cm : 400000 cm — change to same units

1 cm : 20000 cm

1 : 20000 — simplification to make it 1: k

$\therefore k = 20000$

(b) Actual distance $= \frac{8.6 \text{ cm}}{20 \text{ cm}} \times 4 \text{ km} = 1.72 \text{ km}$

Distance on map B $= \frac{1.72}{0.4} = 4.3 \text{ cm}$

(c) MAP A : 1: 20000

MAP B: 1: 40000 — data given / found

Map B has a longer map distance — conclusion by comparing the data presented / found

Summary

Map Scale

A map scale is the ratio of a single unit of distance on the map to the actual distance on the ground.

Map scale can be represented as a:

1. Ratio of the form 1: r
2. Representative Fraction, or R.F. of the form $\frac{1}{r}$

Areas on a Map

To find the relationship between the area used to represent a plot of land on a map and the actual area on the ground, we simply square the map scale.

If the map scale is 1: r, the area scale is 1: r^2.