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MATHEMATICS
Secondary ONE
Year 2021

SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

Name: Suggested Solution ( ) Class:

Unit 9A Polygons (Chapter 11)

Topical Enduring Understanding

- Geometry is a branch of mathematics that studies the size, shape and position of 2 dimensional shapes and 3 dimensional figures where these shapes and figures are formed by a collection of points.
- In geometry, one explores spatial sense and geometric reasoning and develop problem solving skills.
- Geometry can be found in art, architecture, engineering, robotics, land surveys, astronomy, sculptures, space, nature, sports, machines, cars, etc. It is used daily by architects, engineers, architects, physicists and land surveyors.
- Angle properties of a polygon is **invariant** regardless how the shape or size is changed

Topical Enduring Questions

- How can angle and side measures enable us to classify triangles?
- How are the special quadrilaterals alike and different?
- How to classify special quadrilaterals?
- What is the relationship between the exterior and interior angles in a polygon?

Key Points

- Properties of triangles, special quadrilaterals and regular polygons (pentagon, hexagon, octagon and decagon), including symmetrical properties
- Angle sum of interior and exterior angles of any convex polygon
- Application of properties of polygons to solve geometrical problems
- Useful application of polygons in real life

Textbook: *Think! Mathematics New Syllabus Mathematics 1B (8th edition)* Chapter 11
Online resource: Student Learning Space (learning.moe.edu.sg)

Pre-requisites (Primary Mathematics Syllabus, implementation, starting with 2013 Primary 1 cohort)

- Properties of triangles (isosceles, equilateral, right-angled) and quadrilaterals (square, rectangle, parallelogram, rhombus, trapezium)
- Finding unknown angles in the above shapes as well as composite geometric figures involving them
- Properties of angles formed by two parallel lines and a transversal

What I already know — What I just discovered

<u>OBJECTIVE</u>: Apply our pre-requisite knowledge to discover interesting properties of hexagons and its relationship to Triangles.

Below are 4 regular hexagons.
We are going to add lines to form familiar shapes to discover new properties.
Follow the order.

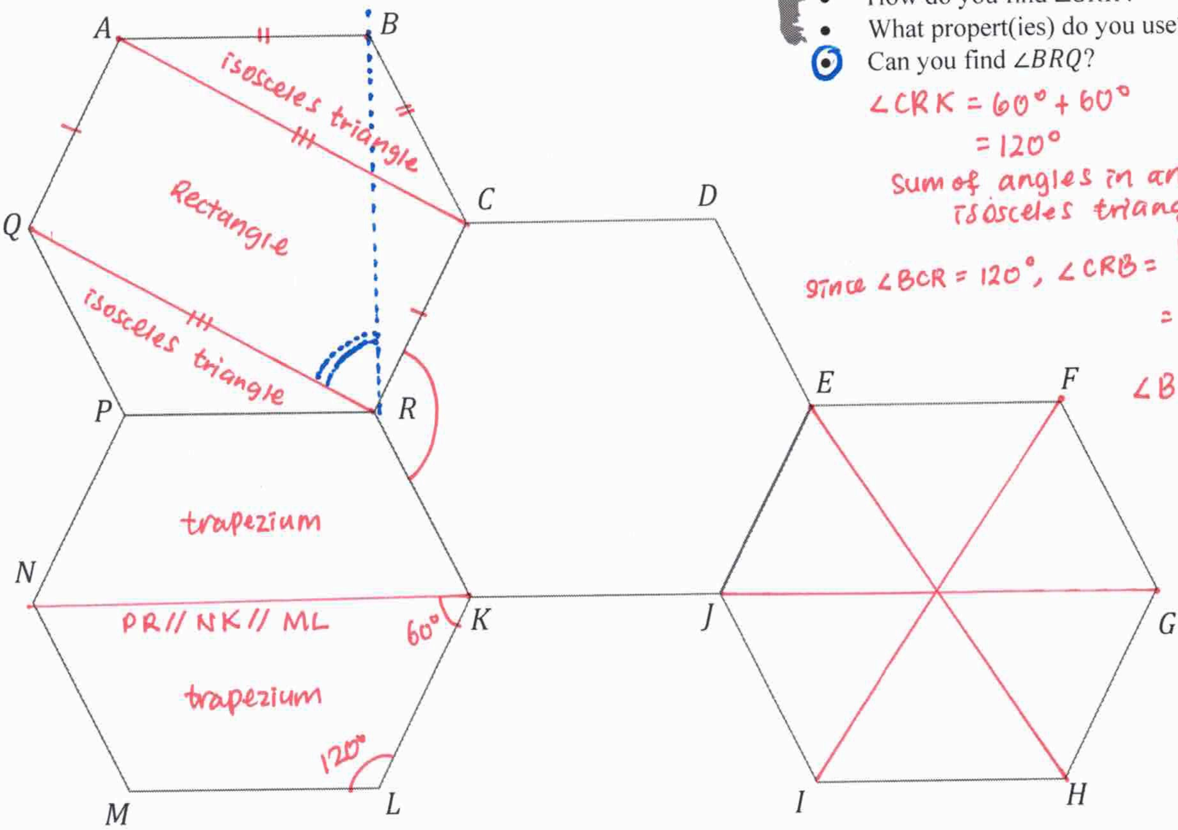
1 Join A to C. Join Q to R.
What shapes do you get? Be specific.
(make reference to the lengths and the angles)

4
- How do you find $\angle CRK$?
- What propert(ies) do you use?
- Can you find $\angle BRQ$?

$\angle CRK = 60° + 60°$
$= 120°$
Sum of angles in an isosceles triangles.

Since $\angle BCR = 120°$, $\angle CRB = \frac{180° - 120°}{2}$
$= 30°$ (Base $\angle$s of isos $\triangle$)

$\angle BRQ = 90° - 30°$
$= 60°$



3 Join N and K.
- What shape do you get?
- What propert(ies) do you use to identify this shape?
- Can you find all the angles in this shape?

Yes. Apply what we found / the concept in #2

2 Join F and I. Join G and J. Join H and E.
- Describe the shapes you get (as a result)?
- What do you discover about the angles of these shapes?

6 Isosceles triangles.
However, because angles in the centre of hexagon are 60°, then these triangles are equilateral triangles

BIG IDEAs

Diagrams help us visualise the given information so that we can think of a solution.

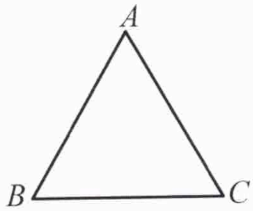
Notations help to convey ideas in a *concise* and *precise* manner. e.g. Pentagon *ABCDE* means that its vertices must be in this order: *A*, *B*, *C*, *D* and *E*. However, it does not matter whether the order is in the clockwise or clockwise direction.

Invariance refers to a property of mathematical object which remains unchanged when the object undergoes some form of transformation. e.g. The sum of angles of a triangle is always 180°, regardless what type of triangle or the size of the triangle.

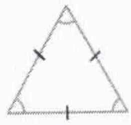
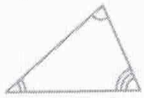
Lesson 1 Triangles

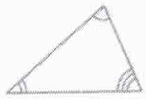
The figure on the right shows a triangle which can be denoted as ΔABC, formed by three sides AB, BC and CA.

- The points A, B and C are called the **vertices** (singular: vertex) of the triangle.
- Angle ABC, angle BCA and angle BCA are called the interior angles of ΔABC or simply the **angles** of ΔABC.



Source: Textbook 1B Chap 11 (p98-p99)

Name	Definition	Figure	Properties
Equilateral triangle	A triangle with ____ equal sides		All the angles in an equilateral triangle are equal, i.e. 60°. (abbreviation: $\angle$s of equilateral $\triangle$)
Isosceles triangle	A triangle with at least ____ equal sides		The base angles of an isosceles triangle are equal. (abbreviation: base $\angle$s of isos. $\triangle$)
Scalene triangle	A triangle with no equal sides		All the angles in a scalene triangle are different.

Name	Definition	Figure
Acute-angled triangle	A triangle with 3 acute angles	
Right-angled triangle	A triangle with a right angle	
Obtuse-angled triangle	A triangle with an obtuse angle	

Food for Thought...
About classifying different types of triangles

Information

Euclid defined an isosceles triangle to have exactly 2 equal sides (this is an *exclusive definition* as it excludes the equilateral triangle).
Nowadays, many people use the *inclusive definition* of an isosceles triangle, i.e. an isosceles triangle has at least 2 equal sides, to include the equilateral triangle.
Therefore, based on the inclusive definition, an equilateral triangle is a special type of isosceles triangle.

SLS Lesson 1 Triangles

Attempt the SLS lesson to learn more about the properties of triangles. You will come across similar properties as we learn quadrilaterals and polygons.

You may attempt this SLS lesson before or after *Triangles*; however, you are strongly encouraged to complete this before we start *Polygons*.

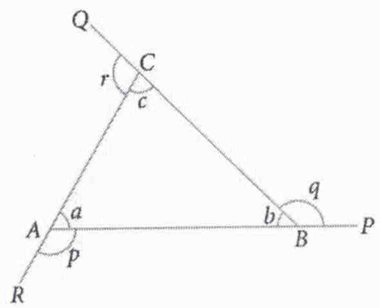
Angle Properties of Triangles

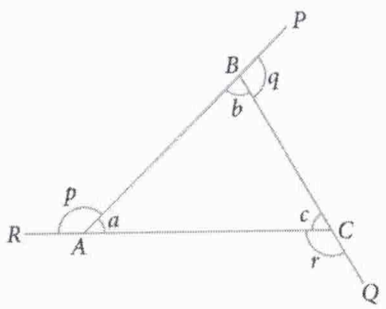
(1) Basic Properties

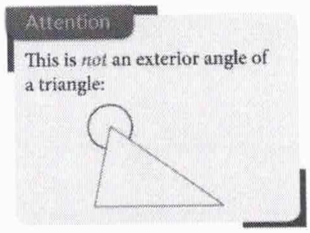
- The largest angle of a triangle is ......opposite........... the longest side; and the smallest angle is .....opposite......... the shortest side.
- The sum of the lengths of any two (shorter) sides of a triangle must be ....longer......... than the length of the third side.
- The sum of interior angles of a triangle is 180° (∠ sum of Δ)

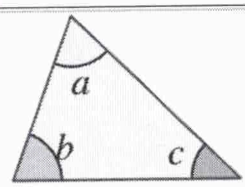
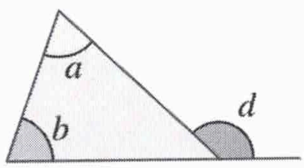
(2) Properties involving interior and exterior angles

Recognising ***interior*** *and* ***exterior angles*** of a triangle.







The sum of interior angles of a triangle = 180° (∠ sum of triangle) $a + b + c = 180°$	
An exterior angle of a triangle is equal to the sum of the interior opposite angles. (Ext. ∠= sum of int. opp. ∠s) $d = a + b$	

Class Work 1

Source: Textbook Exercise 11A

Show your working clearly. Remember to indicate the properties.

Question 6 AB // CD

In the figure, the lines AB and CD are cut by the transversal XY. MOP is a straight line. If $OM = ON$, $\angle AMN = \angle CNY = 90°$ and $\angle OPN = 31°$, find $\angle MON$.

$\angle BMP = 31°$ (alternate angles, AB // CD)

$\angle BMY = 90°$ (adj angles on a line)

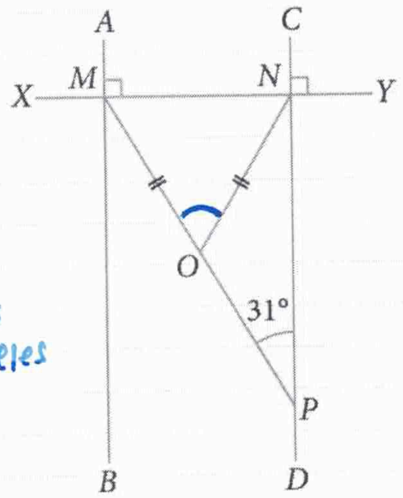
$\angle PMN = 90° - 31°$

$= 59°$

Since $OM = ON$, $\angle ONM = 59°$

$\therefore \angle MON = 180° - 2(59°)$ (Base angles of an isosceles triangle)

$= 62°$ (Ans)



Question 12

The figure shows ΔABC inscribed in a circle with centre O. If $\angle CBO$ is twice of $\angle CAO$ and $\angle BAO$ is one and a half times of $\angle CBO$, calculate $\angle CAO$.

NOTE: Let $\angle CAO$ be x. Form expressions for the angles and hence the equation to solve for x.

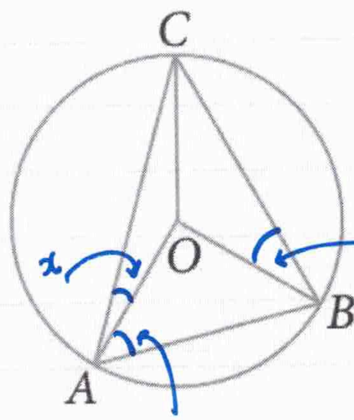
Sum of angles of $\Delta ABC = 180°$

$2(x) + 2(2x) + 2(3x) = 180°$

$2x + 4x + 6x = 180°$

$12x = 180°$

$x = 15°$ (Ans)



$2 \times \angle CAO$

$= 2x$

x

$1\frac{1}{2} \times \angle CBO$

$= \frac{3}{2} \times (2x)$

$= 3x$

Note: Since O is the centre of the circle, OA = OB = OC = radius of circle.

Hence ΔAOB, ΔBOC, ΔAOC are isosceles triangles

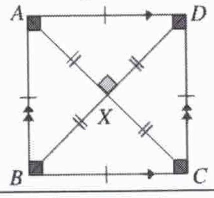
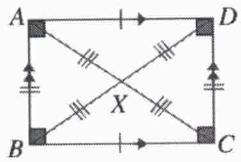
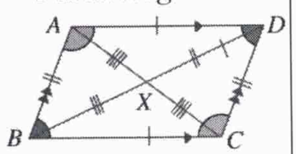
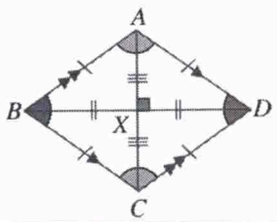
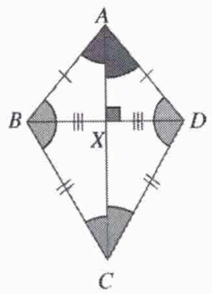
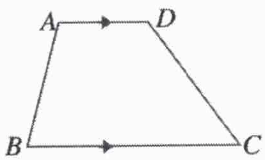
Assignment

- *Remember to state the properties clearly.*

Source: Textbook 1B Review Exercise 11 (p138-p139) Q2, Q5c

Lesson 2 Quadrilaterals

4-sided polygon with 4 interior angles. When one pair of opposite vertices are joined with a straight line (***diagonal***), two triangles are formed.

Name	Parallel sides	Equal sides	Interior angles	Diagonals
Square 	There are two pairs of parallel opposite sides	All sides are equal	All four interior angles are right angles	• Two diagonals are equal in length • Diagonals **bisect** each other at **right angles** • Diagonals **bisect** the **interior angles**
Rectangle 	There are two pairs of parallel opposite sides	The opposite sides are equal in length	All four angles are right angles	• Two diagonals equal in length • Diagonals **bisect** each other
Parallelogram 	There are two pairs of parallel opposite sides	The opposite sides are equal in length	Opposite angles are equal	• Diagonals **bisect** each other
Rhombus 	There are two pairs of parallel opposite sides	All sides are equal	Opposite angles are equal	• Diagonals **bisect** each other at **right angles** • Diagonals **bisect** the **interior angles**
Kite 	There are no parallel sides	There are two pairs of equal adjacent sides		• The longer diagonal **bisects** the interior angles, ∠*ABC* and ∠ ADC • The longer diagonal **bisects** the shorter diagonal at right angle
Trapezium 	There is one pair of parallel opposite sides			

SLS Lesson 2 Quadrilaterals

In this SLS lesson, you will

- *learn to classify quadrilaterals based with reference to the unique properties that each of them has (e.g. number of parallel sides).*
- *examine and learn how different/ similar the diagonals behave in each of these quadrilaterals.*

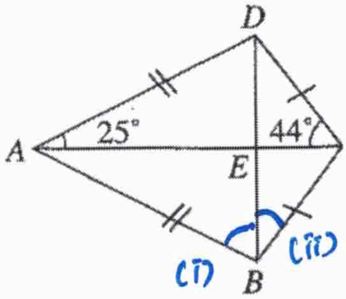
Class Work 2

Source: Textbook Exercise 11B (p112-p113)

Show your working clearly. Remember to indicate the properties.

Question 7

The diagam shows a kite $ABCD$ where $AB = AD$, $BC = CD$ and the diagonals AC and BD intersect at E.
Find (i) $\angle ABD$, (ii) $\angle CBD$.



(i) $\angle BAE = 25°$

$\therefore \angle ABD = 180° - 90° - 25°$ (angle sum of triangle)

$= 65°$ (Ans)

AC and BD intersect at 90° since ABCD is a kite

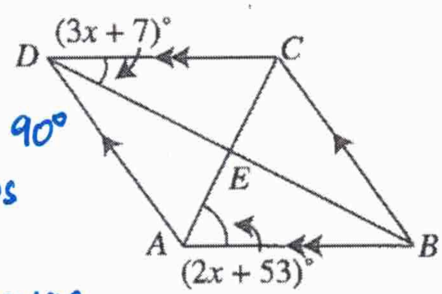
(ii) similarly, $\angle BCE = 44°$ (since $\triangle BCD$ is an isosceles triangle)

$\angle CBD = 180° - 90° - 44°$ (angle sum of triangle)

$= 46°$ (Ans)

Question 14

The diagram shows a rhombus $ABCD$ where the diagonals AC and BD intersect at E. Find the value of x.



since it is a rhombus, AC and BD meet at 90°

$\angle DCA = (2x + 53)°$ (corresponding angles DC // AB)

$3x + 7 + 2x + 53 + 90 = 180°$ (angle sum of triangle)

$5x + 150 = 180$

$5x = 30$

$x = 6$ (Ans)

Assignment

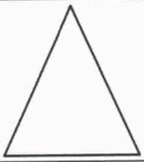
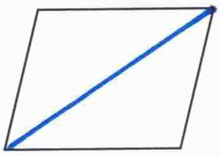
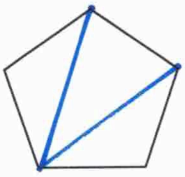
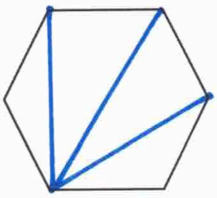
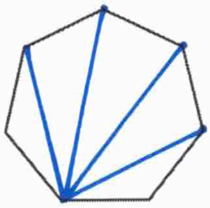
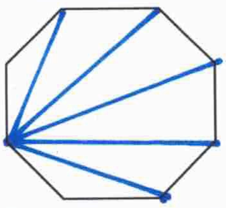
- *Remember to state the properties clearly.*

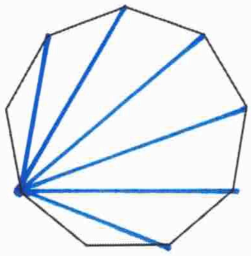
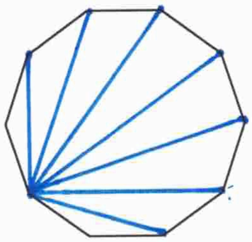
Source: Textbook 1B Review Exercise 11 (p138-p139) Q4, Q5a

Lesson 3 Polygons

A polygon is a closed plane figure with three or more straight-line segments as its sides.
A **regular** polygon is one in which all its sides are **equal in length** and all its **angles** (all interior and exterior angles) **are equal**.
E.g. A square is a regular polygon; however, a rectangle is **not a regular** polygon.

Activity 1 *Fill in the blanks*

Name	Shape	Number of sides	Number of triangles formed	Sum of interior angles
Triangle		3-sided	1	$1 \times 180^\circ$ $= 180^\circ$
Quadrilateral		4-sided	2	$2 \times 180^\circ$ $= 360^\circ$
Pentagon		5-sided	3	$3 \times 180^\circ$ $= 540^\circ$
Hexagon		6-sided	4	$4 \times 180^\circ$ $= 720^\circ$
Heptagon		7-sided	5	$5 \times 180^\circ$ $= 900^\circ$
Octagon		8-sided	6	$6 \times 180^\circ$ $= 1080^\circ$

Name	Shape	Number of sides	Number of triangles formed	Sum of interior angles
Nanogon		9-sided	7	$7 \times 180°$ $= 1260°$
Decagon		10-sided	8	$8 \times 180°$ $= 1260°$

You may wish to write down any question/ observations/ notes as you attempt the above activity.

■ SLS Lesson 3 Polygons

There are 3 mini-lessons in SLS that will help us to deepen our understanding on the Properties of Polygons.

Lesson 3A Introduction to Polygons
Key learning point:
3 ***Properties*** *of a Polygon:*

1. Formed by 3 or more straight lines / edges
2. It is a CLOSED figure
3. It is a plane figure where all the points lie on the same plane

Lesson 3B Properties of Polygons
Write down the key points discussed, about the ***interior and exterior angles*** *of polygon (i.e. formulae)*

Sum of interior angles of a n-gon $= (n-2) \times 180°$

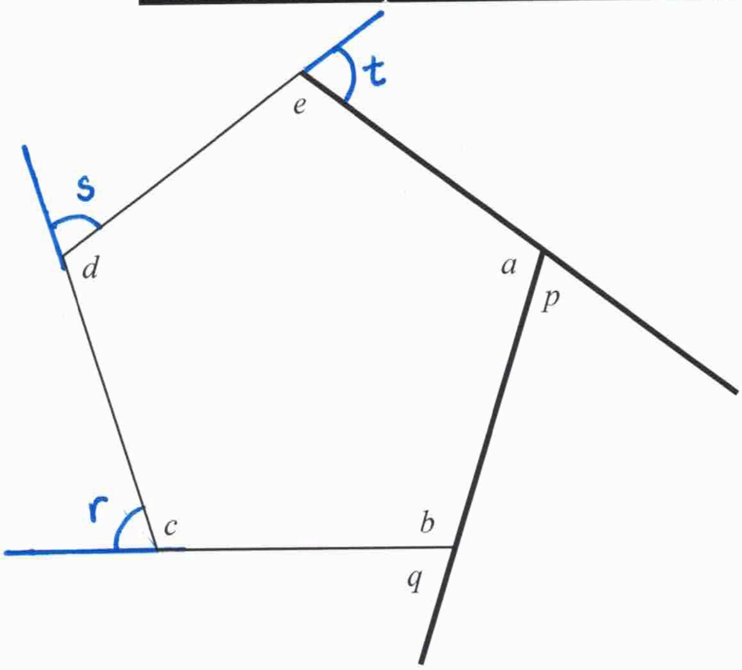
Sum of exterior angles of a polygon $= 360°$

Activity 2

In the pentagon below, there are 5 interior angles: a, b, c, d and e, and 5 exterior angles: p, q, r, s and t respectively.

To get the exterior angles, each side is extended. In the diagram, two sides have been extended, showing the pairs of interior and exterior angles (a and p; b and q).

- Complete the diagram by extending the sides and labelling the exterior angles.
- *From the diagram, can we find the **sum of all the exterior angles** in the pentagon?*
  Watch the video clip in SLS Lesson 3C Sum of the Exterior Angles of a Polygon



sum of interior angles of a pentagon
$= (5-2) \times 180°$
$= 540°$

$\underset{\sim}{a+p} + \underset{\sim}{b+q} + \underset{\sim}{c+r} + \underset{\sim}{d+s} + \underset{\sim}{e+t} = 5 \times 180°$
$= 900°$

Reorganising

$\underbrace{a+b+c+d+e}_{540°} + \underbrace{p+q+r+s+t}_{\text{sum of exterior angles}} = 900°$

sum of exterior angles $= 900° - 540°$
$= 360°$

In summary:

1. An n-gon will have ___n___ sides,

   ___n___ interior angles *and* ___n___ exterior angles

2. For each pair of interior and exterior angles,
   Interior angle + Exterior angle = ___$180°$___

3. The sum of interior angles of an n-sided polygon = ___$(n-2) \times 180°$___

   Size of each interior angle of an n-sided polygon = ___$\frac{(n-2) \times 180°}{n}$___

4. Sum of exterior angles of an n-sided polygon = ___$360°$___

   Size of each interior angle of an n-sided polygon = ___$\frac{360°}{n}$___

Class Work 3

Source: Textbook Exercise 11D (p134-p136)

Show your working clearly. Remember to indicate the properties.

Question 7

Three of the interior angles of an n-sided polygon are 76°, 169° and 105°, and the remaining interior angles are 146° each. Find the value of n.

sum of interior angles = $(n-2) \times 180°$

$76° + 169° + 105° + 146(n-3)° = (n-2) \times 180°$

$350 + 146n - 438 = 180n - 360$

$350 - 438 + 360 = 180n - 146n$

$272 = 34n$

$n = \frac{272}{34}$

$= 8$ (Ans)

Question 8

Three of the exterior angles of an n-sided polygon are 50° each, two of its interior angles are 127° and 135°, and the remaining interior angles are 173° each. Find the value of n.

exterior angle $= 53°$ exterior angle $= 45°$ exterior angle $= 7°$

sum of exterior angles $= 360°$

$3(50°) + 53° + 45° + (n-5) \times 7° = 360°$

$150° + 53° + 45° + 7n° - 35° = 360°$

$7n + 213 = 360$

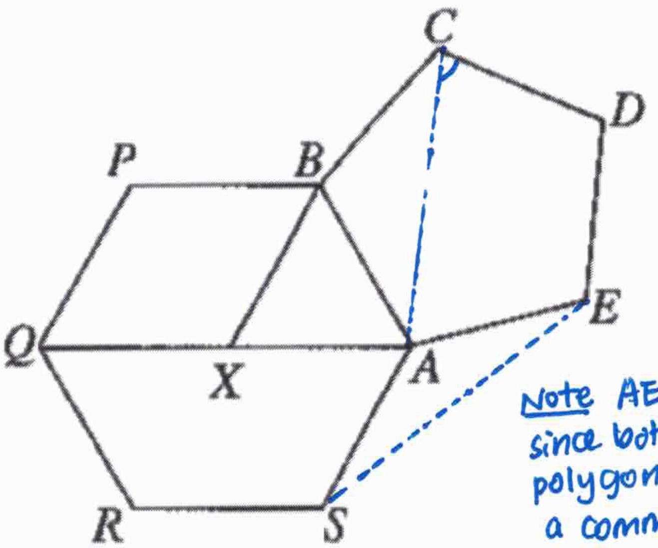
$7n = 147$

$\therefore n = 21$ (Ans)

Question 11

In the figure, $ABCDE$ is a regular pentagon and $ABPQRS$ is regular hexagon. X is the centre of the hexagon.

Find
(i) $\angle ABP$,
(ii) $\angle PQX$,
(iii) $\angle AXB$,
(iv) $\angle ABC$,
(v) $\angle ACD$,
(vi) $\angle ASE$.



Note $AE = AS$ since both regular polygons share a common side AB

Size of 1 interior angle in pentagon

$= \dfrac{(5-2) \times 180°}{5} = 108°$

Size of 1 interior angle in hexagon

$= \dfrac{(6-2) \times 180°}{6} = 120°$

(i) $\angle ABP = 120°$

(ii) $\angle PXQ = \dfrac{360°}{6} = 60°$

Since $\triangle PXQ$ is an isosceles triangle,

$\angle PQX = 60°$

(iii) $\angle AXB = 60°$

(iv) $\angle ABC = 108°$

(v) $\angle BCA = \dfrac{180° - 108°}{2}$ Base angle of isosceles triangle

$= 36°$

$\therefore \angle ACD = 108° - 36°$

$= 72°$

(vi) $\angle SAE = 360° - 108° - 120°$

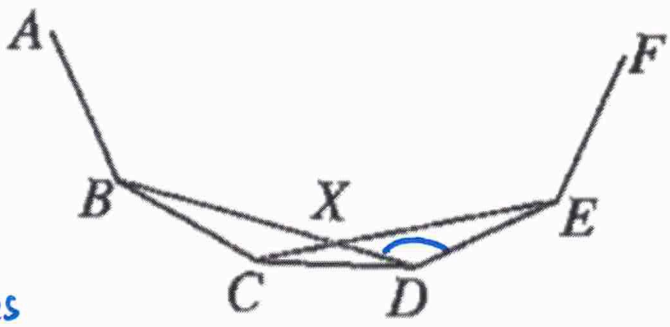
$= 132°$ (angles at a point)

$\angle ASE = \dfrac{180° - 132°}{2} = 24°$

(Base angle of isosceles triangle)

Question 12
In the figure, $ABCDEF$ is part of an n-sided regular polygon. Each exterior angle of this polygon is 36°.

Find
(i) the value of n,
(ii) $\angle BDE$,
(iii) $\angle CXD$.



(i) Since sum of exterior angles
$= 360°$

$36° \times n = 360°$
$\therefore n = 10$ (Ans)

(ii) Size of each interior angle $= 180° - 36°$
$= 144°$

since of $\angle CBD = \angle CDB$ as $BC = DC$
$\angle CDB = \frac{180° - 144°}{2}$ (Base angles of an isosceles triangle)
$= 18°$

$\therefore \angle BDE = 144° - 18°$
$= 126°$ (Ans)

(iii) $XC = XD$, i.e. $\triangle CXD$ is an isosceles triangle
$\angle CXD = 180° - 18° - 18°$
$= 144°$ (Ans)

Assignment
- *Remember to state the properties clearly.*

Source: Textbook 1B Review Exercise 11 (p138-p139) Q10, Q11, Q12, Q14

Lesson 4 Symmetry

(1) Line Symmetry

A **line of symmetry** divides a plane figure into two identical halves such that if you fold the figure along it, each half of the figure will overlap with the other exactly.

Activity 3

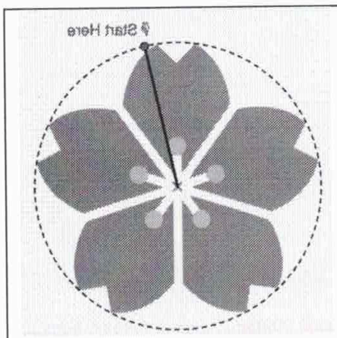
Draw the line of symmetry of the following letters

(2) Rotational Symmetry

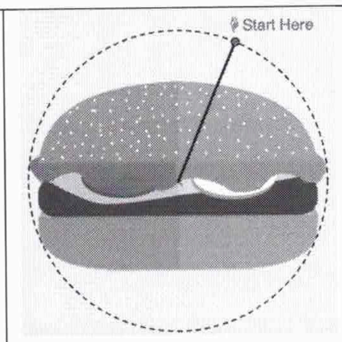
The **order of rotation** is the number of distinctive ways a plane figure maps onto itself in a rotation of 360°.

Activity 4

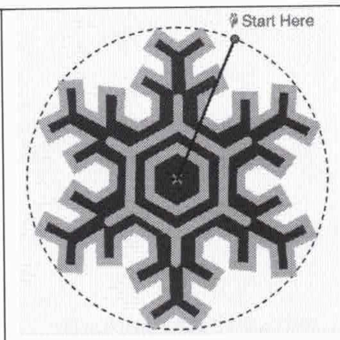
Access the following website ***https://www.geogebra.org/m/nxbm22u8***
Try the activities to find out the **order of rotation** of each figure, do **NOT** take into account the designs on the shapes/ cutouts.



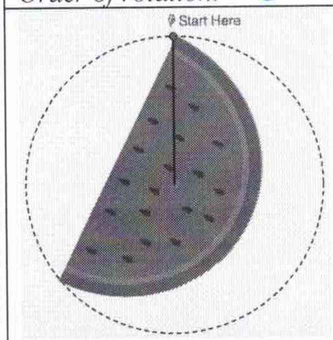
Order of rotation: 5



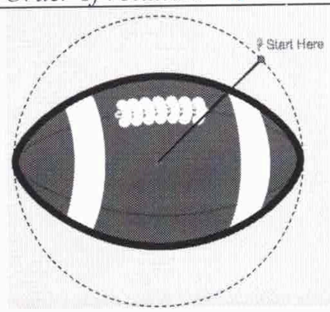
Order of rotation: 1



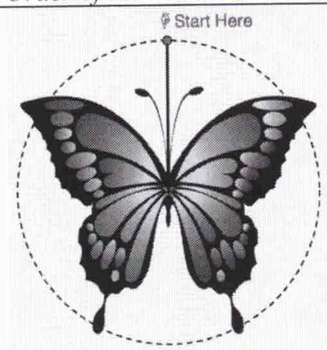
Order of rotation: 6



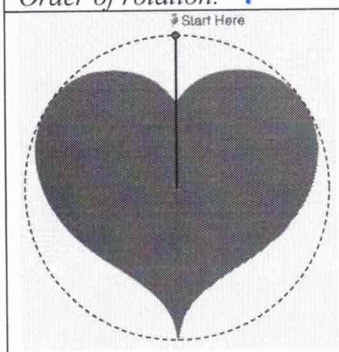
Order of rotation: 1



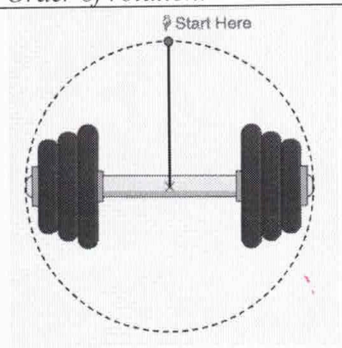
Order of rotation: 2



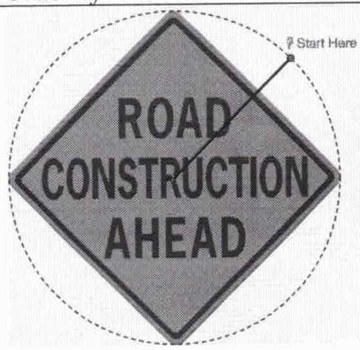
Order of rotation: 1



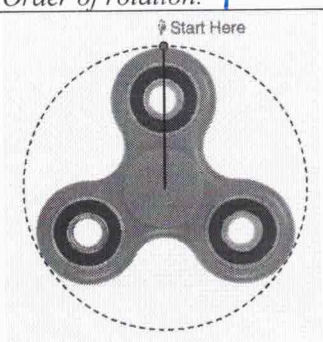
Order of rotation: 1



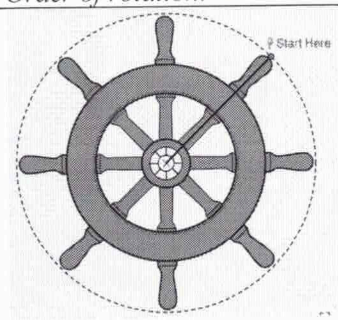
Order of rotation: 2



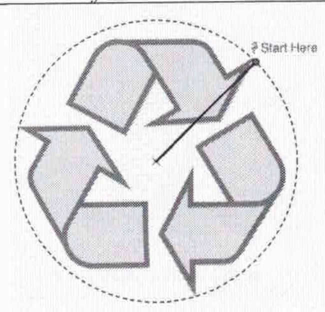
Order of rotation: 4



Order of rotation: 3



Order of rotation: 8



Order of rotation: 3