

## Secondary 2 Notes

### Unit: Direct and Inverse Proportions

Name: \_\_\_\_\_ (      ) Class: \_\_\_\_\_

#### Unit EUs

Students will be able to understand that:

- 1) Many variables in the real world have a directly proportional or inversely proportional relationships. Proportional relationships can be represented using tables, graphs or algebraic functions.
- 2) When modeled as algebraic functions, many problems encountered in everyday life can be solved mathematically.
- 3) The function  $y = kx$  describes a directly proportional relationship in which  $y$  varies directly to  $x$ .
- 4) The function  $y = \frac{k}{x}$  describes an inversely proportional relationship in which  $y$  varies inversely as  $x$ .

#### Unit EQs

- 1) How can proportional relationships be represented using tables, graphs or algebraic functions?
- 2) How can algebraic functions be used to model real-world situations and be used to solve everyday problems?
- 3) How can we apply  $y = kx$  to represent directly proportional relationships in various context?
- 4) How can we apply  $y = \frac{k}{x}$  to represent inversely proportional relationships in various context?

#### Unit Objectives

- Identify the type of relationship between two given variables.
- Create a suitable algebraic equation to relate the two variables.
- Apply the equation to solve future problems.
- Understand the graphical representation of the given relationship.

## Teaching To The Big Idea ...

Student Learning Outcomes	Dimensions (Please tick the appropriate boxes)							
	FUNCTIONS F	INVARIANCE I	NOTATIONS N	DIAGRAMS D	MEASURES M	EQUIVALENCE E	PROPORTIONALITY P	MODELS M
Direction Proportion		✓					✓	✓
Algebraic and Graphical Representation of Direct Proportion		✓		✓			✓	✓
Other Forms of Direct Proportion		✓					✓	✓
Inverse Proportion		✓					✓	✓
Algebraic and Graphical Representation of Inverse Proportion		✓		✓			✓	✓

### 9.1 DIRECT PROPORTION

If it costs \$2000 to build a treehouse, how much does it cost to build 5 treehouses?

Complete the following table.

No. of treehouse ( $x$ )	1	2	3	4	5
Cost (\$ $y$ )					
Rate, $\frac{y}{x}$					

- a. From the above table, what do you notice about  $y$  when  $x$  increases?

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- b. What do you notice about the rate  $\frac{y}{x}$ ?

The rate is a c \_\_\_\_\_.

- c. Formulate an equation connecting  $x$  and  $y$ .

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**When one quantity increases or decreases at the same *rate* as another, the two are said to be in \_\_\_\_\_ proportion.**

Conclusion:

If  $y$  is directly proportional to  $x$ , then  $\frac{y}{x} = k$  or  $y = kx$ , where  $k$  is a constant.

**Example 1:**

If  $y$  is directly proportional to  $x$  and if  $y = 6$  when  $x = 2$ , find

- (a) an equation connecting  $x$  and  $y$ ,
- (b) the value of  $y$  when  $x = 10$ ,
- (c) the value of  $x$  when  $y = 9$ .

**Example 2:**

The cost of hiring a bike for a certain number of days is shown in the table.

No. of days, $x$	1	2	3	4
Cost, $y$ (\$)	5	10	15	$f$

- a. Find the value of  $f$ .
- b. Find the equation connecting  $x$  and  $y$ .
- c. Hence, find the number of days of rental if the cost is \$125.

**Extension:**

- a. Can you represent the information graphically?
- b. What would the axes be?
- c. Which point will the line always pass through?



**Example 3:**

It takes 9 workmen 4 days to build a fence. How many fences will 12 workmen build in 4 days? Use algebra in your working. What are the assumptions made in this example?

**Classwork (Tier A)**

1. Given that  $y$  is proportional to  $x$  and that  $y = 3$  when  $x = 2$ , find
  - (a)  $y$  when  $x = 5$ ,
  - (b)  $x$  when  $y = 11$

2. The length stretched by a spring is directly proportional to the amount of force applied. A force of 20 kg stretches a certain spring 15 cm. How long will a force of 25 kg stretch it?

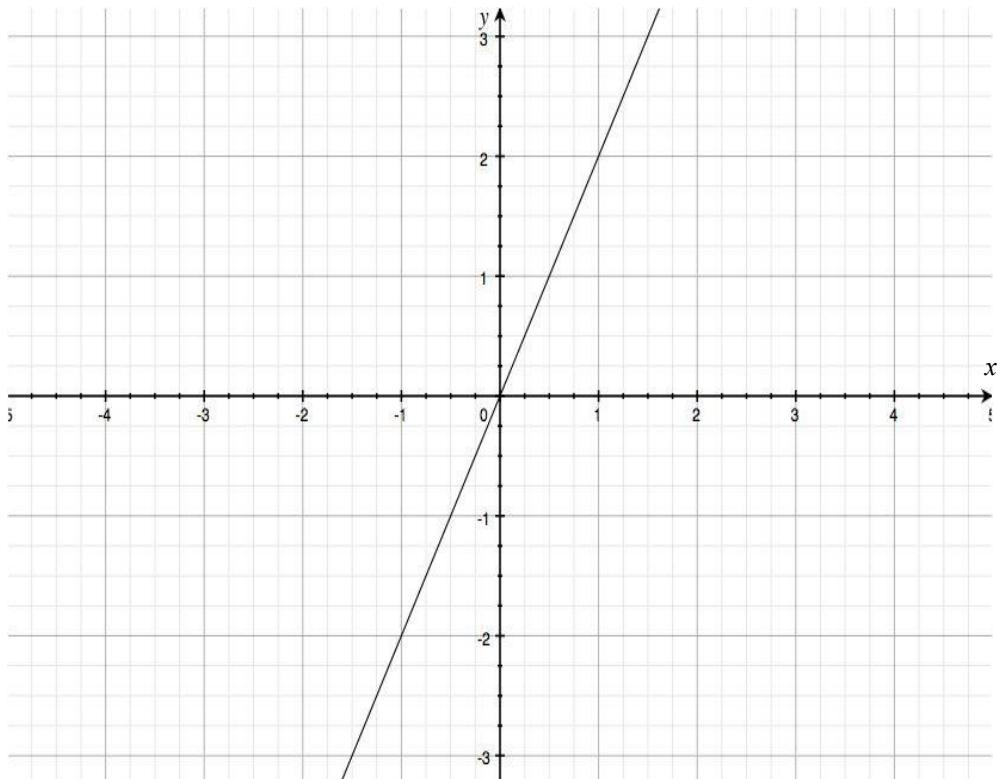
**Class Work (Tier B)**

3. Given that  $p$  is directly proportional to  $q^3$ , it is known that  $p=24$  for a particular value of  $q$ . Find the value of  $p$  when this value of  $q$  is halved. (N04I/15)

**Class Work (Tier C)**

4. A open cylinder has a circular base of variable area. The radius of the circular base is directly proportional to the height of the cylinder. When the volume of the cylinder is  $375\pi \text{ cm}^3$ , the radius is found to be 5 cm.
- (a) Write down the relationship between radius and height as a formula.  
(b) Write down the volume of the cylinder in terms of  $r$ .  
(c) Find the total surface area in terms of  $r$ . (Assumption: negligible thickness)

5. The graph shows a linear relationship.



Find the equation of the line in terms of  $x$  and  $y$ .

## 9.2 INVERSE PROPORTION

If 5 men take 6 hours to complete a task,

- (a) How many hours would 3 men take to complete the same task?
- (b) How many men would be needed to complete the task in 15 hours?
- (c) Complete the following table.

No. of men, $m$	1	2	3	4	5
No. of hours, $h$					
No. of man-hours, $mh$					

- a. From the above table, what do you notice about  $h$  when  $m$  increases?

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- b. What do you notice about the product  $mh$ ?

The rate is a c \_\_\_\_\_.

- c. Formulate an equation connecting  $m$  and  $h$ .

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**When one quantity increases while the second \_\_\_\_\_ at the same *rate* as another, the two are said to be in \_\_\_\_\_ proportion.**

Conclusion:

If  $y$  is inversely proportional to  $x$ , then  $yx = k$  or  $y = \frac{k}{x}$ , where  $k$  is a constant and  $k \neq 0$

### **Example 1:**

Two variables  $a$  and  $b$  are inversely proportional to each other. If  $a = 3$  when  $b = 27$ , find the value of  $a$  when  $b = 54$ .

**Extension:**

How do you think the graph will look like? Draw a preliminary sketch.  
Confirm your prediction using a graphing software

Your prediction:



The actual:



**Example 2:**

Given  $y$  is inversely proportional to  $x^2$ , it is known that  $y=4$  when  $x=3$ . Find the value of  $y$  when  $x=10$ .  
(N04I/15)

**Example 3:**

The number of days  $d$  required to renovate a house is inversely proportional to the number of men available,  $n$ . When 6 men are doing the job, the renovation takes 8 days. If it takes 12 days to complete the job, how many men are doing the job?

**Class Work (Tier A)**

1. The frequency,  $f$  of a radio wave is inversely proportional to its wavelength,  $w$ . Given that  $w = 3000$  m and  $f = 100$  kHz, find the frequency when the wavelength is 500 m. Find the wavelength when the frequency is 800 kHz.

**Class Work (Tier B)**

2. The force of attraction,  $F$  newtons, between two magnets is inversely proportional to the square of the distance,  $x$  centimeters, between them.
    - (a) When the magnets are 4 cm apart, the force is 3 newtons. Find the formula connecting  $F$  and  $x$ .
    - (b) Find the force when the magnets are 2 cm apart.
    - (c) When the magnets are a certain distance apart, the force is 1.25 newtons.
- Write down the force when the distance is halved. (N07I/19)

**Class Work (Tier C)**

3. The mass of an object is inversely proportional to the square of the distance from the object to the centre of the earth. A certain astronaut weighs 80 kg at sea level (6500 km from the centre of the earth).
- (i) How much does the astronaut weigh when orbiting  $2.5 \times 10^4$  km above the sea level?
  - (ii) How far above the earth, to the nearest km, will an astronaut weigh half of sea level mass?

**Secondary 2 Notes**

**Unit: Direct and Inverse Proportions**

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**Assignment 9.1**

(7.2 Algebraic and graphical representations of direct proportion)

(7.3 Other forms of direct proportion)

(7.5 Algebraic and graphical representations of inverse proportion)

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Exercise 7A Q4, 12

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Exercise 7B Q2, 6

**Assignment 9.2**

(7.5 Algebraic and graphical representations of inverse proportion)

(7.6 Other forms of inverse proportion)

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Exercise 7C Q3, 7

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Exercise 7D Q2, 8

