① $f(t) = (t-1)^4$

$$f(t) \begin{cases} (t-1)^4 & t > 4 \\ 0 & t < 4 \end{cases}$$

$$\therefore \ L(f(t)] = e^{-4s} L[(t+3)^4]$$

$$L[(t+3)^4] = L[t^4 + 3t^3 + 3^2 t^2 + 3^3 t + 3^4]$$

$$= \left[\frac{\sqrt{5}}{t^5} + \frac{3\sqrt{4}}{t^4} + \frac{3^2\sqrt{3}}{t^3} + \frac{3^3\sqrt{2}}{t^2} + \frac{3^4}{t}\right]$$

Ans.- $e^{-4s}\left[\frac{\sqrt{5}}{t^5} + \frac{3\sqrt{4}}{t^4} + \frac{9\sqrt{3}}{t^3} + \frac{27\sqrt{2}}{t^2} + \frac{81}{t}\right]$

②

$$f(t) = \begin{cases} t & 0 < t < 1/2 \\ t-1 & 1/2 < t < 1 \\ 0 & t > 1 \end{cases}$$

$$= \int_0^{1/2} e^{-st}\, t \, dt$$

$$+ \int_{1/2}^{1} e^{-st} (t-1)\, dt + \int_1^{\infty} 0 \, dt$$

$$= t \int_0^{1/2} e^{-st} - \int_0^{1/2} \int e^{-st}$$

$$= \left[t \frac{e^{-st}}{-s} \right]_0^{1/2} - \left[\frac{e^{st}}{s^2} \right]_0^{1/2} = -\frac{e^{-s/2}}{2s} - \frac{e^{-st/2}}{s^2} + 1/s^2$$

$$\int_{1/2}^{1} e^{-s}(t-1) = \left[(t-1)\,e^{-st}\right]_{1/2}^{1} - \int_{1/2}^{1} \frac{e^{-st}}{-s}$$

$$= -\frac{1}{2}\frac{e^{-s/2}}{s} - \quad - \sim$$

$$= -\frac{1}{2}\frac{e^{-s/2}}{s} - \frac{e^{-s/2}}{2s}$$

$$- \frac{e^{-s}}{s^2} + \frac{1}{s^2}$$

$$= -\frac{e^{-s/2}}{s} - \frac{e^{-s}}{s^2} + 1/s^2$$

③ $f(t) = \sin^2(t-\pi) \quad t > \pi$

$$L(f(t)) = e^{-\pi s} L[\sin^2 t]$$

$$= e^{-\pi s} L\left[\frac{1-\cos 2\theta}{2}\right]$$

$$= e^{-\pi s}\left[\frac{1}{2s} - \frac{1}{2}\frac{s}{s^2+4}\right]$$

④ $\cos t \cos 2t \cos 3t$

$$\cos A \cos B = \frac{1}{2}(\cos(A+B) + \cos(A-B))$$

$$\cos t \cos 2t \cos 3t = \frac{1}{2}(\cos 3t + \cos t)\cos 3t$$

$$= \frac{1}{4}[\cos 6t + 1 + \cos 4t + \cos 2t]$$

$$L[\,] = \frac{1}{4}\left[\frac{6}{s^2+36} + \frac{1}{s} + \frac{4}{s^2+16} + \frac{2}{s^2+4}\right]$$

(5) $L\left[(\sqrt{t}-1)^2\right]$

$$= L\left[\, t - 2\sqrt{t} + 1 \,\right]$$

$$= \frac{1}{s^2} - 2\,\frac{\Gamma(3/2)}{s^{3/2}} + \frac{1}{s}$$

(6) $\dfrac{\cos\sqrt{t}}{\sqrt{t}}$

$$\cos x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$$\cos\sqrt{t} = 1 - \frac{t}{2!} + \frac{t^2}{4!} - \cdots$$

$$\frac{\cos\sqrt{t}}{\sqrt{t}} = t^{-1/2} - \frac{t^{1/2}}{2!} + \frac{t^{3/2}}{4!} - \cdots$$

$$L\left[\frac{\cos\sqrt{t}}{\sqrt{t}}\right] = \frac{\Gamma 1/2}{s^{1/2}} - \frac{\Gamma 3/2}{2!\, s^{3/2}} + \frac{\Gamma 5/2}{4!\, s^{5/2}} - \cdots$$

$$= \frac{1}{s^{1/2}}\left[\frac{\Gamma 1/2}{s^{0}} - \frac{\frac{1}{2}\,\Gamma 1/2}{2 \times s^{2}} + \frac{\frac{\cancel{3}}{2}\,\frac{1}{2}\,\Gamma 1/2}{4 \times \cancel{3} \times 2\, s^{3}}\right.$$

$$= \frac{\Gamma 1/2}{s^{1/2}}\left[1 - \frac{1}{(4s)^{2}} + \frac{1}{2!\,(4s)^{3}} + \cdots\right]$$

$$= \frac{\Gamma 1/2}{s^{1/2}}\, e^{-4s}$$

⑦ $\mathcal{L}[\sin\sqrt{t}] = \dfrac{\sqrt{\pi}}{2s\sqrt{s}}\, e^{-1/4s}$

$$\mathcal{L}[\sin 2\sqrt{t}] = \mathcal{L}[\sin\sqrt{4t}]$$

$$= \frac{1}{\cancel{4}} \frac{\sqrt{\pi}}{\cancel{2}\frac{s}{\cancel{4}}\frac{\sqrt{s}}{\cancel{2}}}\, e^{-\frac{1}{\cancel{4}\frac{s}{4}}}$$

$$= \frac{\sqrt{\pi}}{s\sqrt{s}}\, e^{-1/s}$$

⑧ $\sinh(t/2)\cdot\sin(\sqrt{3}\,t/2)$

$$\frac{e^{t/2}}{2}\sin\left(\frac{\sqrt{3}\,t}{2}\right) - \frac{e^{-t/2}}{2}\sin\left(\frac{\sqrt{3}}{2}t\right)$$

$$\mathcal{L}[\quad] = \frac{1}{2}\,\frac{\sqrt{3/4}}{(s-1/2)^2 + 3/4} - \frac{1}{2}\,\frac{\sqrt{3/4}}{(s+1/2)^2 + 3/4}$$

$$= \frac{1}{2}\frac{\sqrt{3}}{2}\left[\frac{1}{s^2-s+1} - \frac{1}{s^2+s+1}\right]$$

$$= \frac{\sqrt{3}}{4}\frac{s^2+s+1-s^2+s-1}{(s^2+s+1)(s^2-s+1)}$$

$$= \frac{\sqrt{3}}{4}\frac{\times 2s}{(s^2+1)^2-s^2}$$

$$= \frac{\sqrt{3}}{2}\frac{s}{(s^4+s^2+1)}$$

⑨ $e^{4t}\sin^3 t$

$$\sin t = \frac{e^{it}-e^{-it}}{2i} \quad \therefore \sin^3 t = \frac{(e^{it}-e^{-it})^3}{8i}$$

$$= \frac{e^{3it}-3e^{it}+3e^{-it}-e^{-3it}}{8i} = \frac{\sin 3t}{4} - 3\frac{\sin t}{4}$$

$$\mathcal{L}(\sin^3 t) = \frac{1}{4}\,\frac{s}{s^2+9} - \frac{3}{4}\,\frac{s}{s^2+1}$$

$$\mathcal{L}[\sin^3 t\, e^{4t}] = \frac{s-4}{4}\left[\frac{1}{(s-4)^2+9} - \frac{3}{(s-4)^2+1}\right]$$

(10) $\dfrac{\cos 2t \sin t}{e^{t}}$

$$\cos 2t \sin t = \frac{1}{2}\sin(3t) + \frac{1}{2}\sin(t)$$

$$L[\cos 2t \sin t) = \frac{1}{2} \frac{s}{s^2+9} + \frac{1}{2} \frac{s}{s^2+1}$$

$$L[e^{-t} \cos 2t \sin t] = \frac{1}{2} \frac{s+1}{(s+1)^2+9} + \frac{1}{2} \frac{s+1}{(s+1)^2+1}$$

(11) $e^{-4t} \sinh t . \sin t$

$$= \frac{e^{-4t}}{2} e^{t} \sin t + \frac{e^{-4t}}{2} e^{-t} \sin t$$

$$L[\sin t] = \phi(s) = \frac{s}{s^2+1}$$

$$L\left[e^{-5t} \sin t\right] = \frac{s+5}{(s+5)^2+1}$$

$$L\left[e^{-3t} \sin t\right] = \frac{s+3}{(s+3)^2+1}$$

$$\therefore \text{Ans} = L[\quad] = \frac{1}{2} \frac{s+5}{(s+5)^2+1} + \frac{1}{2} \frac{s+3}{(s+3)^2+1}$$

(12) $e^{2t}(1+t)^2$

$$= e^{2t}(1+2t+t^2)$$

$$L[e^{2t}(1+t)^2] = L[e^{2t}] + 2L[e^{2t}t] + L[e^{2t}t^2]$$

$$= \frac{1}{s-2} + \frac{2}{(s-2)^2} + \frac{2}{(s-2)^3}$$

(13) $L[f(t)] = \dfrac{s}{s^2+s+4}$

$$L[f(2t)] = \frac{1}{2}\,\frac{\frac{s}{2}}{\frac{s^2}{4}+\frac{s}{2}+4} = \frac{s}{s^2+2s+16}$$

$$L\left[e^{-3t} f(t)\right] = \phi(s+3)$$

$$= \frac{s+3}{s^2+6s+9+2s+6+16}$$

$$= \frac{s+3}{s^2+8s+31}$$

(14) $\left(1+te^{-t}\right)^3 = 1+3te^{-t} + 3t^2e^{-2t} + t^3e^{-3t}$

$$L\left[\left(1+te^{-t}\right)^3\right] = L[1] + 3L[te^{-t}]$$

$$+ 3L[t^2e^{-2t}] + L[t^3e^{-3t}]$$

$$= \frac{1}{s} + \frac{3}{(s+1)^2} + \frac{3\times 2}{(s+2)^3}$$

$$+ \frac{3\times 2}{(s+3)^3}$$

(15) $t \sin^3 t$

$$\sin^3 t = \left(\frac{e^{it} - e^{-it}}{2i}\right)^3$$

$$= \frac{e^{3it} - 3e^{it} + 3e^{-it} - e^{-3it}}{2i}$$

-4

$$= \frac{-1}{4}\left[\sin 3t - 3\sin t\right]$$

$$= \frac{3}{4}\sin t - \frac{\sin 3t}{4}$$

$$L[\sin^3 t] = \frac{3}{4}\frac{1}{s^2+1} - \frac{3}{4}\frac{3}{s^2+9}$$

$$L[t \sin^3 t] = -\frac{d}{ds}\phi(s) = +\frac{3}{4}\frac{2s}{(s^2+1)^2} - \frac{3}{4}\frac{2s}{(s^2+9)^2}$$

(16) $t^5 \cosh t = \frac{1}{2}e^{t} t^5 + \frac{1}{2}e^{-t} t^5$

$$L[t^5 \cosh t] = \frac{1}{2}\frac{\sqrt{6}}{(s-1)^6} + \frac{1}{2}\frac{\sqrt{6}}{(s+1)^6}$$

$$= \frac{120}{2}\left[\frac{1}{(s-1)^6} + \frac{1}{(s+1)^6}\right]$$

$$= 60\left[\frac{1}{(s-1)^6} + \frac{1}{(s+1)^6}\right]$$

(17) $t\sqrt{1+\sin t}$

$$\sqrt{1+\sin t} = \sqrt{(\cos t/2 + \sin t/2)^2}$$

$$= \cos t/2 + \sin t/2$$

$$L[t\sqrt{1+\sin t}] = \frac{d}{ds} L[\cos t/2] + \frac{d}{ds} L[\sin t/2]$$

$$= -\frac{d}{ds}\left(\frac{s}{s^2+1/4}\right) + \frac{d}{ds}\left(\frac{1/2}{s^2+1/4}\right)$$

$$= -\left(\frac{-2s^2}{(s^2+1/4)^2} + \frac{1}{s^2+1/4} + \frac{1}{2}\frac{2s}{(s^2+1/4)^2}\right)$$

$$= -\left(\frac{-2s^2 + s^2+1/4 + s}{(s^2+1/4)^2}\right)$$

$$= \frac{+s^2 - s - 1/4}{(s^2+1/4)^2}$$

(18) $t\left(\frac{\sin t}{e^t}\right)^2 = t\,e^{-2t}\sin^2 t$

$\sin^2 t = \frac{1}{2} - \frac{\cos 2\theta}{2}$

$L[\sin^2 t] = L[1/2] - \frac{1}{2}L[\cos 2\theta]$

$$= \frac{1}{2s} - \frac{1}{2}\,\frac{s}{s^2+4}$$

$$L[t\sin^2 t] = -\frac{d}{ds}\,L\{\sin^2 t\}$$

$$= -\left(\frac{-1}{2s^2} - \frac{1}{2}\left[\frac{1}{s^2+4}\right]\right.$$

$$\left. + \frac{1}{\cancel{2}}\,\frac{\cancel{2}s^2}{(s^2+4)^2}\right)$$

$$= -\left(-\frac{1}{2s^2} - \frac{1}{2(s^2+4)} + \frac{s^2}{(s^2+4)^2}\right)$$

$$L[e^{-2t}(t\sin^2 t)] = \phi(s+2)$$

$$= \frac{-1}{2(s+2)^2} - \frac{1}{2(s^2+4s+8)}$$

$$+ \frac{(s+2)^2}{(s^2+4s+8)^2}$$

$$= -\frac{1}{2}\left[-\frac{1}{(s+2)^2} - \frac{s^2+4s+8}{(s^2+4s+8)^2} \right.$$

$$\left. + \frac{2s^2+8s+8}{(s^2+4s+8)^2} \right.$$

$$= -\frac{1}{2}\left[-\frac{1}{(s+2)^2} + \frac{s^2+4s}{(s^2+4s+8)^2} \right]$$

(19) $$L[f(t)] = \frac{s+3}{s^2+s+1}$$

$$L[f(2t)] = \frac{1}{2}\,\frac{\frac{s}{2}+3}{\frac{s^2}{4}+\frac{s}{2}+1}$$

$$= \frac{s+6}{s^2+2s+4}$$

$$L[t f(2t)] = -\frac{d}{ds} L[f(2t)] = \frac{s^2+2s+4-(s+6)(2s+2)}{-(s^2+2s+4)^2}$$

$$= \frac{s^2+2s+4-2s^2-14s-12}{(s^2+2s+4)^2}$$

$$= -\frac{(-s^2-12s-8)}{(s^2+2s+4)^2}$$

$$= +\frac{(s^2+12s+8)}{(s^2+2s+4)^2}$$

(20) $t e^{-2t} \sinh 4t = t e^{-2t} \frac{e^{4t}-e^{-4t}}{2}$

$$= \frac{1}{2} t e^{2t} - \frac{t e^{-6t}}{2}$$

$$L[\quad] = \frac{1}{2} \frac{1}{(s-2)^2} - \frac{1}{2} \frac{1}{(s+6)^2}$$

(21) $t\cos(\omega t-\alpha) = t\,(\sin\omega t\sin\alpha)$
$+\,t\,(\cos\omega t\cos\alpha)$

$$\mathcal{L}[\qquad] = \mathcal{L}[t\sin\omega t]\sin\alpha$$
$$+\,\mathcal{L}[t\cos\omega t]\cos\alpha$$

$$= -\frac{d}{ds}\mathcal{L}\{\sin\omega t]\sin\alpha$$
$$-\frac{d}{ds}\mathcal{L}[\cos\omega t]\cos\alpha$$

$$= -\frac{d}{ds}\left(\frac{\omega}{s^2+\omega^2}\right)\sin\alpha$$
$$-\frac{d}{ds}\left(\frac{s}{s^2+\omega^2}\right)\cos\alpha$$

$$= \frac{-\omega\,2s\sin\alpha}{(s^2+\omega^2)^2} - \frac{(s^2+\omega^2-2s^2\cos\alpha)}{(s^2+\omega^2)^2}$$

$$= \frac{(s^2-\omega^2)\cos\alpha - 2\omega s \sin\alpha}{(s^2+\omega^2)^2}$$

(22) $$(t \sinh 2t)^2 = t^2\left(\frac{e^{2t}-e^{-2t}}{2}\right)^2$$

$$= \frac{t^2}{4}\left[e^{4t} - 2 + e^{-4t}\right]$$

$$= \frac{e^{4t}t^2}{4} - \frac{t^2}{2} + \frac{e^{-4t}t^2}{4}$$

$$L(\quad) = \frac{1}{4}L[e^{4t}t^2] - \frac{1}{2}L[t^2] + \frac{1}{4}[e^{-4t}t^2]$$

$$= \frac{1}{4}\frac{1}{(s-4)^3} - \frac{1}{2}\frac{1}{s^3} + \frac{1}{4}\frac{}{(s+4)^3}$$

(23) $(t+\sin 2t)^2$

$$= t^2 + 2t\sin 2t + \sin^2 2t$$

$$\sin^2 2t = \frac{1}{2} - \frac{\cos 4t}{2}$$

$$\therefore \mathcal{L}[\ ] = \mathcal{L}[t^2] + 2\mathcal{L}[t\sin 2t]$$

$$+ \mathcal{L}[1/2] - \frac{1}{2}\mathcal{L}[\cos 4t]$$

$$= \frac{2}{s^3} - 2\frac{d}{ds}\left(\frac{2}{s^2+4}\right)$$

$$+ \frac{1}{s} - \frac{1}{2}\frac{s}{s^2+16}$$

$$= \frac{2}{s^3} + \frac{8s}{(s^2+4)^2} + \frac{1}{s} - \frac{1s}{2(s^2+16)}$$

(24) $\frac{1}{t}(1-\cos t)$

$$L(1-\cos t) = L(1) - L(\cos t)$$

$$= \frac{1}{s} - \frac{s}{s^2+1}$$

$$L\left[\frac{1}{t}(1-\cos t)\right] = \int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+1}\right) ds$$

$$= \left[\ln s - \frac{1}{2}\ln(s^2+1)\right]_s^\infty$$

$$= \left[\ln\left(\frac{s}{\sqrt{s^2+1}}\right)\right]_s^\infty$$

$$= -\ln\frac{s}{\sqrt{s^2+1}} = \ln\frac{\sqrt{s^2+1}}{s}$$

(25) $\frac{1}{t} e^{-t} \sin t$

$$L[\sin t] = \frac{1}{s^2+1}$$

$$L[e^{-t} \sin t] = \frac{1}{(s+1)^2+1} = \frac{1}{s^2+2s+2}$$

$$L\left[\frac{1}{t} e^{-t} \sin t\right] = \int_s^{\infty} \frac{1}{s^2+2s+2} \, ds$$

$$= \int_s^{\infty} \frac{1}{(s+1)^2+1} = \left[\tan^{-1}(s+1)\right]_s^{\infty}$$

$$= \frac{\pi}{2} - \tan^{-1}(s+1)$$

26 $\dfrac{\sin^2 2t}{t}$

$$L[\sin^2 2t] = L\left[\tfrac{1}{2}\right] - L\left[\tfrac{1}{2}\cos 4t\right]$$

$$= \frac{1}{2s} - \frac{1}{2}\left(\frac{s}{s^2+16}\right)$$

$$L\left[\frac{\sin^2 2t}{t}\right] = \frac{1}{2}\int_s^\infty \frac{1}{s} - \frac{s}{s^2+16}\, ds$$

$$= \frac{1}{2}\left[\ln s - \frac{1}{2}\ln(s^2+16)\right]_s^\infty$$

$$= \frac{1}{2}\left[\ln \frac{s}{\sqrt{s^2+16}}\right]_s^\infty$$

$$= -\frac{1}{2}\ln\left[\frac{s}{\sqrt{s^2+16}}\right] = \frac{1}{2}\ln\frac{\sqrt{s^2+16}}{s}$$

22) $\dfrac{1-\cos t}{t^2}$

$$L\left[\frac{1-\cos t}{t}\right] = \frac{1}{2}\log\left(\frac{s^2+1}{s^2}\right)$$

$$L\left[\frac{1-\cos t}{t^2}\right] = \frac{1}{2}\int_s^\infty \log\left(\frac{s^2+1}{s^2}\right) ds$$

(28) $L\left[\frac{\sin at}{t}\right]$

$$= \int_s^\infty L(\sin at)\, ds$$

$$= \int_s^\infty \frac{a}{s^2+a^2}\, ds = \int_s^\infty \frac{1}{\left(\frac{s}{a}\right)^2+1}$$

$$= \left[\tan^{-1}\left(\frac{s}{a}\right)\right]_s^\infty = \left(\frac{\pi}{4} - \tan^{-1}\left(\frac{s}{a}\right)\right]$$

$$L\left(\frac{\cos at}{t}\right) = \int_s^\infty \frac{s}{s^2+a^2} = \left[\frac{\ln(s^2+a^2)}{2}\right]_s^\infty$$

does not exist as $\ln \infty$ does not exist

(2a) $\dfrac{\cosh 2t \sin 2t}{t}$

$$\frac{e^{2t}\sin 2t}{2t} + \frac{e^{-2t}\sin 2t}{2t}$$

$$L[\;] = \frac{1}{2}\int_s^{\infty} \frac{2}{(s-2)^2+2^2} + \frac{2}{(s+2)^2+2^2}\,ds$$

$$= \left[\frac{1}{2}\tan^{-1}\left(\frac{s-2}{2}\right) + \frac{1}{2}\tan^{-1}\left(\frac{s+2}{2}\right)\right]_s^{\infty}$$

$$= \frac{\pi}{2} - \frac{1}{2}\left(\tan^{-1}\left(\frac{s-2}{2}\right) + \tan^{-1}\left(\frac{s+2}{2}\right)\right)$$

(30) $\dfrac{e^{-at} - \cos at}{t}$

$$= \int_s^\infty L[e^{-at} - \cos at] \, ds$$

$$= \int_s^\infty \left(\frac{1}{s+a} - \frac{s}{s^2+a^2} \right) ds$$

$$= \left(\ln(s+a) - \frac{1}{2}\ln(s^2+a^2) \right)_s^\infty$$

$$= \ln\left[\frac{s+a}{\sqrt{s^2+a^2}} \right]_s^\infty$$

$$= -\ln\left(\frac{s+a}{\sqrt{s^2+a^2}} \right) = \ln\left(\frac{\sqrt{s^2+a^2}}{s+a} \right)$$

31

$$f(t) = \begin{cases} t+1 & 0 \le t \le 2 \\ 3 & t > 2 \end{cases}$$

Q32 $L\left[\frac{d}{dt}\left(\frac{\sin 3t}{t}\right)\right]$

$$= s\,L\left[\frac{\sin 3t}{t}\right] - f(0)$$

$$f(0) = \frac{\sin 3t}{t} = \frac{3\cos 3t}{1} = 3$$

L'Hopital

$$L\left[\frac{\sin 3t}{t}\right] = \int_s^\infty L(\sin 3t)\,ds$$

$$= \int_s^\infty \frac{3}{s^2+3^2}\,ds$$

$$= \left[\tan^{-1}\left(\frac{s}{3}\right)\right]_s^\infty$$

$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{3}\right)$$

$\therefore$ Answer $= s\left(\frac{\pi}{2} - \tan^{-1}(s/3)\right) - 3$

(33) $\operatorname{erf}\sqrt{t} = \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{t}} e^{-u^2}\, du$

$$L[\operatorname{erf}\sqrt{t}] = \frac{2}{\sqrt{\pi}} L\left[\int_0^{\sqrt{t}} e^{-u^2}\, du\right]$$

Let $u^2 = v$

$2u\,du = dv$

$du = \frac{dv}{2\sqrt{v}}$

u	0	$\sqrt{t}$
v	0	t

$$= \frac{2}{\sqrt{\pi}} L\left[\int_0^{t} \frac{e^{-v}\, dv}{2\sqrt{v}}\right]$$

$$= \frac{2}{\sqrt{\pi}} \frac{1}{s} L\left[\frac{e^{-v}}{2\sqrt{v}}\right] = \frac{1}{s\sqrt{\pi}} L\left[e^{-v} v^{-1/2}\right]$$

$$= \frac{1}{s\sqrt{\pi}} \frac{\Gamma 1/2}{(s+1)^{1/2}}$$

(34) $\operatorname{erf} 2\sqrt{t} = \operatorname{erf}\sqrt{4t}$

$$L\{\operatorname{erf}\sqrt{t}\} = \frac{1}{s\sqrt{s+1}}$$

$$L\{\operatorname{erf}\sqrt{4t}\} = \frac{1}{4}\,\frac{1}{\frac{s}{4}\frac{\sqrt{s+4}}{2}}$$

$$= \frac{2}{s\sqrt{s+4}}$$

(35) $e^{3t}\, t \operatorname{erf}\sqrt{t}$

$$L\{\operatorname{erf}\sqrt{t}\} = \frac{1}{s\sqrt{s+1}}$$

$$\therefore L\{\operatorname{erf}\sqrt{t}\; t\} = -\frac{d}{ds}\,\frac{1}{s\sqrt{s+1}}$$

$$= \frac{\sqrt{s+1} + \frac{s}{2\sqrt{s+1}}}{+\left(s(\sqrt{s+1})\right)^2}$$

$$= \frac{3s+2}{2s^2(s+1)^{3/2}}$$

$$L\left[e^{+3t}\,\mathrm{erf}\sqrt{t}\right] = \frac{3(s-3)+2}{2(s-3)(s-2)^{3/2}}$$

(36) $L\left[\int_0^t\int_0^t\int_0^t t\sin t\,(dt)^3\right]$

$$L[\sin t] = \frac{1}{s^2+1}$$

$$L\{t\sin t\} = \frac{2s}{(s^2+1)^2}$$

$$L\left[\iiint t\sin t\right] = \frac{2s}{s^3(s^2+1)^2} = \frac{2}{s^2(s^2+1)^2}$$

(37) $\int_0^t u e^{-3u}\cos^2 2u\,du$

$$L[\cos^2 2t] = L\left[\tfrac{1}{2}\right] + L\left[\tfrac{1}{2}\cos 4t\right]$$

$$= \frac{1}{2s} + \frac{s}{2(s^2+16)}$$

$$L\left[u\cos^2 2u\right] = \frac{1}{2s^2} - \frac{\left(s^2+16-2s^2\right)}{2\left(s^2+16\right)^2}$$

$$= \frac{1}{2s^2} + \frac{s^2-16}{2\left(s^2+16\right)^2}$$

$$L\left[e^{-3u}u\cos^2 2u\right] = \frac{1}{2(s+3)^2} + \frac{(s+3)^2-16}{2\left((s+3)^2+16\right)^2}$$

$$L\left(\int e^{-3u}u\cos^2 2u\right) = \frac{1}{2s(s+3)^2} + \frac{s^2+6s-7}{2s\left(s^2+6s+25\right)^2}$$

(38) $\int_0^t \frac{1-e^{-au}}{a}\,du$

$$L[1-e^{-au}] = \frac{1}{s} - \frac{1}{s+a}$$

$$L\left[\frac{1-e^{-au}}{a}\right] = \int_s^\infty \frac{1}{s} - \frac{1}{s+a}$$

$$= \left[\ln\left(\frac{s}{s+a}\right)\right]_s^\infty$$

$$= \ln\left(\frac{s+a}{s}\right)$$

$$\therefore L[\int \quad] = \frac{1}{s}\ln\left(\frac{s+a}{s}\right)$$

(39) $\frac{1}{t}\int_0^t e^{-u}\sin u\,du$

$$\mathcal{L}\{e^{-u}\sin u\} = \frac{1}{(s+1)^2+1}$$

$$\mathcal{L}\left\{\int_0^t e^{-u}\sin u\right\} = \frac{1}{s((s+1)^2+1)}$$

$$\mathcal{L}\left\{\frac{1}{t}\int_0^t e^{-u}\sin u\right\} = \int_s^\infty \frac{1}{s}\,\frac{1}{((s+1)^2+1)}\,ds$$

(40) $e^{-4t}\int_0^t u\sin 3u$

$$L(\sin 3u) = \frac{3}{s^2+9}$$

$$L[u\sin 3u] = +\frac{6s}{(s^2+9)^2}$$

$$L\left[\int_0^t u\sin 3u\right] = \frac{6}{(s^2+9)^2}$$

$$L\left[e^{-4t}\int_0^t u\sin 3u\,du\right] = \frac{6}{((s+4)^2+9)^2}$$

$$= \frac{6}{(s^2+8s+25)^2}$$

(41) $\cosh t \int_0^t e^u \cosh u \, du$

$$L\{e^u \cosh u\} = \frac{1}{2} L\{e^u e^u + e^{-u} e^u\}$$

$$= \frac{1}{2} L\{e^{2u}\} + \frac{1}{2} L\{1\}$$

$$= \frac{1}{2(s-2)} + \frac{1}{2s}$$

$$L\left(\int_0^t e^u \cosh u\right) = \frac{1}{2s(s-2)} + \frac{1}{2s^2}$$

$$L\left\{\cosh t \int_0^t e^u \cosh u \, du\right\} = \frac{1}{4}\left[\frac{1}{(s+1)(s-1)} + \frac{1}{(s+1)^2}\right.$$

$$\left. + \frac{1}{(s-1)(s-3)} + \frac{1}{(s-1)^2}\right]$$

(42) $\int_0^t u e^{-3u} \sin^2 u$

$$\mathcal{L}[\sin^2 u] = \mathcal{L}[1/2] + \frac{1}{2}\mathcal{L}[\cos 2u]$$

$$= \frac{1}{2s} + \frac{1}{2} \frac{s}{s^2+4}$$

$$\mathcal{L}[u \sin^2 u] = \frac{1}{2s^2} - \frac{1}{2} \frac{s^2+4-2s^2}{(s^2+4)^2}$$

$$\mathcal{L}[e^{-3u} u \sin^2 u] = \frac{1}{2(s+3)^2} + \frac{(s+3)^2-4}{2(s^2+6s+13)^2}$$

$$\mathcal{L}\left[\int_0^t e^{-3u} u \sin^2 u\right] = \frac{1}{s}\left[\frac{1}{2(s+3)^2} + \frac{1}{2}\frac{s^2+6s+5}{(s^2+6s+13)^2}\right]$$

(43) $\frac{1}{t}(\cos at - \cos bt)$

$$= \int_s^\infty \frac{s}{s^2+a^2} - \frac{s}{s^2+b^2} \cdot ds$$

$$= \frac{1}{2}\left[\ln\left(\frac{s^2+a^2}{s^2+b^2}\right)\right]_0^\infty$$

$$= \frac{1}{2} \ln\left(\frac{s^2+b^2}{s^2+a^2}\right)$$

(44) $L(\operatorname{erf}\sqrt{t}) = \frac{1}{s\sqrt{s+1}}$

$$L(\operatorname{erf}\sqrt{at} = \frac{1}{a}\frac{1}{\frac{s}{a}\sqrt{\frac{s+9}{3}}} = \frac{3}{s\sqrt{s+9}}$$

$$L\left[\cosh t\ \text{erf}\ 3\sqrt{t}\right] = \frac{3}{2(s+2)\sqrt{s+11}} + \frac{3}{2(s-2)\sqrt{s+7}}$$

(43) $$L\left(2\sqrt{\frac{t}{\pi}}\right) = \frac{1}{s^{3/2}}$$

Scaling

$$L\left[\sqrt{\frac{t}{\pi}}\right] = \frac{4}{4^{3/2}\, s^{3/2}} = \frac{4}{2\cdot 4\, s^{3/2}}$$

Division by t

$$L\left[\frac{1}{\sqrt{\pi t}}\right] = \frac{1}{2}\int_s^\infty \frac{1}{s^{3/2}}\,ds$$

$$= -\frac{1}{2}\left[\frac{2}{\sqrt{s}}\right]_s^\infty$$

$$= \frac{\quad}{\sqrt{s}}$$

(46) $f(t) + 2\int_0^t f(t)\,dt = \cosh 2t$

$$L[f(t)] + 2\,\frac{L[f(t)]}{s} = L[\cosh 2t]$$

$$\therefore \quad A + 2\frac{A}{s} = \frac{1}{2(s+2)} + \frac{1}{2(s-2)}$$

$$\therefore \quad A\left(1 + \frac{2}{s}\right) = \frac{1}{2}\left(\frac{1}{(s+2)} + \frac{1}{(s-2)}\right)$$

$$\therefore A = \frac{1}{2}\left[\frac{1}{(s+2)} + \frac{1}{(s-2)}\right]\frac{s}{s+2}$$

$$= \frac{s^2}{(s+2)(s^2-4)}$$

(1) $\int_0^\infty e^{-2t} \sin^3 t \, dt$

$$\sin^3 t = \left(\frac{e^{it} - e^{-it}}{2i}\right)^3$$

-4

$$= -\frac{1}{4} \frac{e^{3it} - 3e^{it} + 3e^{-it} - e^{-3it}}{2i}$$

$$= -\frac{1}{4}\left[\sin 3t - 3\sin t\right]$$

$$L\left[\sin^3 t\right] = -\frac{1}{4}\left[\frac{3}{s^2+9} - \frac{3}{s^2+1}\right]$$

$$-\frac{1}{4}\left[\frac{3}{13} - \frac{3}{5}\right] = \frac{(15 - 39)}{65 \times 4}$$

$$= \frac{24}{4 \times 65} = \frac{6}{65}$$

② $$\int_0^\infty e^{-2t} \sin(t+\alpha)\cos(t-\alpha)\,dt = 3/8$$

$$= \frac{1}{2}\int_0^\infty e^{-2t}\left(\sin(t+\alpha+t-\alpha) + \sin(t+\alpha-t+\alpha)\right)$$

$$= \frac{1}{2}\int_0^\infty e^{-2t}\left[\sin(2t) + \sin(2\alpha)\right]$$

$$\frac{1}{2}\mathcal{L}\left[\sin 2t + \sin 2\alpha\right] = \frac{1}{2}\left(\frac{2}{s^2+4} + \frac{\sin 2\alpha}{s}\right)$$

at $s = 2$

$$\frac{3}{\cancel{8}\,4} = \frac{1}{\cancel{2}}\left(\frac{2}{8} + \frac{\sin 2\alpha}{2}\right)$$

$$\alpha = \pi/4$$

③ $\int_0^\infty e^{-3t}\, t \sin t$

$$L[t \sin t] = -\frac{d}{ds}\,\frac{1}{s^2+1}$$

$$= \frac{2s}{(s^2+1)^2}$$

at $s = 3$

$$= \frac{2\times 3}{100} = 3/50$$

④ $\int_0^\infty e^{-3t}\, t\, J_0 4t$

$$L[J_0] = \frac{1}{\sqrt{s^2+1}}$$

$$L[J_0(4t)] = \frac{1}{4}\,\frac{1}{\sqrt{\frac{s^2+16}{4}}} = \frac{1}{\sqrt{s^2+16}}$$

$$L\{t\, J_0(4t)\} = \frac{\cancel{\pi}\, s}{\cancel{\pi}\,(s^2+16)^{3/2}}$$

$$= \frac{s}{(s^2+16)^{3/2}}$$

at $s = 3$

$$= \frac{3}{(25)^{3/2}} = 3/125$$

Hence proved.

⑤ $\displaystyle\int_0^\infty e^{-2t}\, t^2 \sin 3t$

$$L\{\sin 3t\} = \frac{3}{s^2+9}$$

$$L\{t^2 \sin 3t\} = \frac{d^2}{ds^2}\frac{3}{s^2+9} = 3\frac{d}{ds}\frac{-2s}{(s^2+9)^2}$$

$$= -6\left[\frac{1}{(s^2+9)^2} - \frac{4s^2}{(s^2+9)^3}\right] = \frac{18}{2197}$$

⑥ $\displaystyle\int_0^\infty \frac{\cos at - \cos bt}{t}\, dt$

$$L\left[\frac{\cos at - \cos bt}{t}\right] = \int_s^\infty \frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}$$

$$= \frac{1}{2}\log \frac{s^2+b^2}{s^2+a^2}$$

at $s=0$

$$= \frac{1}{2}\log \frac{b^2}{a^2} = \log b/a$$

⑦ $\displaystyle\int_0^\infty e^{-st}\, \frac{\sin^2(at/2)}{t}$

$$L\left(\sin^2\left(\tfrac{a}{2}t\right)\right] = L\left[\frac{1}{2}\right] - L\left[\frac{1}{2}\cos at\right]$$

$$= \frac{1}{2s} - \frac{1}{2}\,\frac{s}{s^2+a^2}$$

$$L\left[\frac{\sin^2 at/2}{t}\right] = \int \frac{1}{2s} - \frac{1}{2}\,\frac{s}{s^2+a^2}$$

$$= \left(\frac{1}{2}\ln s - \frac{1}{4}\ln(s^2+a^2)\right]_s^\infty$$

$$= \left[\frac{1}{2}\ln\frac{s}{\sqrt{s^2+a^2}}\right]_s^\infty$$

$$= \frac{1}{2}\log\left(\frac{\sqrt{s^2+a^2}}{s}\right)$$

(8) $\displaystyle\int_0^\infty e^{-st}\,\frac{\sin t\,\sinh t}{t}$

$$L\{\sin t\} = \frac{1}{s^2+1}$$

$$L\{\sinh t\,\sin t\} = \frac{1}{2(s-1)^2+2} - \frac{1}{2(s+1)^2+2}$$

$$L\left[\frac{\sinh t\,\sin t}{t}\right] = \frac{1}{2}\left[\tan^{-1}(s-1) - \tan^{-1}(s+1)\right]_s^\infty$$

$$= \frac{1}{2}\left[\tan^{-1}(s+1) - \tan^{-1}(s-1)\right]$$

9. $$\int_0^\infty \frac{e^{-t} - \cos t}{t e^{4t}}$$

$$= \int_0^\infty e^{-4t} \frac{(e^{-t} - \cos t)}{t}$$

$$L\left[e^{-t} - \cos t\right] = \frac{1}{s+1} - \frac{s}{s^2+1}$$

$$L\left[\frac{e^{t} - \cos t}{t}\right] = \int_s^\infty \frac{1}{s+1} - \frac{s}{s^2+1}$$

$$\Rightarrow \left[\ln(s+1) - \frac{\ln(s^2+1)}{2}\right]_s^\infty$$

$$= -\ln\left[\frac{s+1}{\sqrt{s^2+1}}\right]$$

$$= \ln\left[\frac{\sqrt{s^2+1}}{s+1}\right]$$

at $s = 4$,

$$= \ln\left(\frac{\sqrt{17}}{5}\right)$$

(10) $$\int_0^\infty \frac{(\sin 2t + \sin 3t)e^{-t}}{t}$$

$$L(\sin 2t + \sin 3t) = \frac{2}{s^2+4} + \frac{3}{s^2+9}$$

$$L[t^{-1} \cdot \text{—}) = \cot^{-1}(s/2) + \cot^{-1}(s/3)$$

at $s = 1$

$$= 3\pi/4$$

(11) $\int_0^\infty e^{-2t} \sinh t \frac{\sin t}{t} dt$

$$L[\sinh t \sin t] = \frac{1}{2}\left[\frac{1}{(s-1)^2+1} - \frac{1}{(s+1)^2+1}\right]$$

from Eq

$$L\left[\frac{\sinh t \sin t}{t}\right] = = \frac{1}{2}\left[\tan^{-1}(s+1) - \tan^{-1}(s-1)\right]$$

at $s = 2$ $= 0.2318$

(12) $\int_0^\infty e^{-t} \int_0^t u^2 \sinh u \cosh u \, du$

$$\sinh u \cosh u = \frac{(e^u - e^{-u})}{2}\frac{(e^u + e^{-u})}{2}$$

$$= \frac{e^{2u} + 1 - 1 - e^{-2u}}{4}$$

$$= \frac{e^{2u} - e^{-2u}}{4}$$

$$L\{\sinh u \cosh u\} = \frac{1}{4}\left[\frac{1}{s-2} - \frac{1}{s+2}\right]$$

$$L\{u^2 \sinh u \cosh u\} = \frac{1}{4}\frac{d^2}{ds^2}\left[\frac{1}{s-2} - \frac{1}{s+2}\right]$$

$$= \frac{1}{4} - \frac{d}{ds}\left[\frac{1}{(s-2)^2} - \frac{1}{(s+2)^2}\right]$$

$$= \frac{1}{2}\left[\frac{1}{(s-2)^3} - \frac{1}{(s+2)^3}\right]$$

$$L\left[\int_0^t u^2 \sinh u \cosh u\right] = \frac{1}{2s}\left\{\frac{1}{(s-2)^3} - \frac{1}{(s+2)^3}\right\}$$

at $s = 1$

$$= \frac{1}{2}\left\{\frac{-1}{} - \frac{1}{27}\right\}$$

(13) $\int_0^\infty e^{-4t}\left[\cosh t \int_0^t e^u \cosh u \, du\right.$

$e^u \cosh u = \frac{e^{2u}}{2} + \frac{1}{2}$

$L\left[e^u \cosh u\right] = \frac{1}{2}\left[\frac{1}{(s-2)} + \frac{1}{s}\right]$

$L\left[\int_0^t e^u \cosh u\right] = \frac{1}{2s}\left[\frac{1}{(s-2)} + \frac{1}{s}\right]$

$L\left[\cosh t \int_0^t e^u \cosh u\right] = \frac{1}{4(s+1)}\left[\frac{1}{(s-1)} + \frac{1}{(s+1)}\right]$

$+ \frac{1}{4(s-1)}\left[\frac{1}{(s-3)} + \frac{1}{(s-1)}\right]$

at $s = 4$

$= \frac{1}{4 \cdot 5}\left[\frac{1}{3} + \frac{1}{5}\right] + \frac{1}{4 \cdot 3}\left[\frac{1}{1} + \frac{1}{3}\right].$

$$= \frac{1}{\cancel{k}}\left[\frac{\cancel{8}^{2}}{15\times 3} + \frac{\cancel{4}^{1}}{9} \right]$$

$$= \frac{18 + 15\times 3}{15\times 3\times 9}$$

$$= \frac{\cancel{93}^{31}}{15\times 3\times \cancel{9}\,3} = \frac{31}{225}$$

(14) $\int_0^\infty e^{-st} \frac{\sin bt + \sin at}{t}$

$$L[\sin bt + \sin at] = \frac{a}{s^2+a^2} + \frac{b}{s^2+b^2}$$

$$L\left[\frac{1}{t}(\sin bt + \sin at)\right] = \int_s^\infty \frac{a}{s^2+a^2} + \frac{b}{s^2+b^2} = \left[\tan^{-1}(s/a) + \tan^{-1}(s/b)\right]_0^\infty$$

$$= \pi - \tan^{-1}\left(\frac{s}{a}\right) - \tan^{-1}(s/b)$$

$$\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

$$\therefore \tan^{-1}\left(\frac{s}{a}\right) + \tan^{-1}(s/b) = \tan^{-1}\left(\frac{s/a + s/b}{1 - \frac{s^2}{ab}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{s(b+a)}{\cancel{ab}}}{\frac{ab-s^2}{\cancel{ab}}}\right)$$

$$= \tan^{-1}\left(\frac{s(b+a)}{ab-s^2}\right)$$

Hence proved

(15) $\int_0^\infty e^{-t} \sin^5 t$

$$\sin^5 t = \left[\frac{e^{it} - e^{-it}}{2i}\right]^5$$

$$= \frac{1}{2^4 i^4}\left[\frac{e^{5it} - 5e^{3it} + 10e^{it} - 10e^{-it} + 5e^{-3it} - e^{-5it}}{2i}\right]$$

$$= \frac{1}{2^4}\Big(\sin 5t - 5\sin 3t$$

$$+ 10\sin t\Big)$$

$$L[\sin^5 t] = \frac{1}{2^4}\left(\frac{5}{s^2+25} - \frac{15}{s^2+9} + \frac{10}{s^2+1}\right)$$

at $t = 1$ $= \frac{1}{2^4}\left(\frac{5}{26} - \frac{15}{10} + \frac{10}{2}\right)$

$$= \frac{1}{2^4}\left(\frac{5}{26} - \frac{3}{2} + 5\right)$$

Ans Not Match

(16) $\int_0^\infty \frac{\cos 4t - \cos 3t}{t}$

$$L(\cos 4t - \cos 3t) = \frac{s}{s^2+4^2} - \frac{s}{s^2+9}$$

$$L\left(\frac{\cos 4t - \cos 3t}{t}\right) = \int_s^\infty \frac{s}{s^2+4^2} - \frac{s}{s^2+9}$$

$$= \left(\frac{1}{2} \ln\left(\frac{s^2+16}{s^2+9}\right)\right)_s^\infty$$

$$= \frac{1}{2} \ln\left(\frac{s^2+9}{s^2+16}\right)$$

at $s = 0$

$$= \frac{1}{2} \ln(9/16) = \ln(3/4)$$

(17) $\int_0^\infty e^{-t} t^3 \sin t$

$L(\sin t) = \frac{1}{s^2+1}$

$L\{t^3 \sin t) = -\frac{d}{ds^3} \frac{1}{s^2+1}$

$= \frac{d}{ds^2} \frac{2s}{(s^2+1)^2}$

$= \frac{d}{ds}\left(\frac{2}{(s^2+1)^2} + \frac{-8s^2}{(s^2+1)^3}\right)$

$= \frac{-8s}{(s^2+1)^3} \frac{-16s}{(s^2+1)^3} \frac{+48s^3}{(s^2+1)^4}$

at $s=1$ $= -\frac{8}{2^3} - \frac{16}{2^3} + \frac{48}{2^3} = 0$

(18) $\int_{t=0}^{\infty}\int_{u=0}^{t} e^{-t} \frac{\sin u}{u} = \int_{t=0}^{\infty} e^{-t} \int_{u=0}^{t} \frac{\sin u}{u} du\, dt$

$L\{\sin u] = \frac{1}{u^2+1}$

$L[\frac{\sin u}{u}) = \int_{s}^{\infty} \frac{1}{u^2+1} = (\tan^{-1}(u))]_{s}^{\infty}$

$= \frac{\pi}{2} - \tan^{-1}(u)$

$L\{\int_{0}^{t} \frac{\sin u}{u}] = \frac{\pi}{2s} - \frac{\tan^{-1}(s)}{s})$

at $s=1$

$= \frac{\pi}{2} - \tan^{-1}(1) = \frac{\pi}{4}$

Q27 $\dfrac{1-\cos t}{t^2}$

Q31

$$f(t) = \begin{cases} t+1 & 0 \le t \le 2 \\ 3 & t > 2 \end{cases}$$

$L[f(t)]$

$L[f'(t)]$

$L[f''(t)]$

39 $\dfrac{1}{t}\displaystyle\int_0^t e^{-u} \sin u \, du$

12 $\displaystyle\int_0^\infty e^{-t} \int_0^t u^2 \sinh u \cosh u \, du \, dt$

(65) $\dfrac{4s+12}{s^2+8s+12}$

$$= \frac{4s+12}{s^2+8s+16-4} = \frac{4(s+4)-4}{(s+4)^2-2^2}$$

$$L^{-1}[\quad] = e^{-4t} L^{-1}\left[\frac{4s-4}{s^2-2^2}\right]$$

$$= e^{-4t}(4\cosh 2t - \sinh 2t)$$

(66) $\dfrac{s}{s^2+2s+2} = \dfrac{(s+1)-1}{(s+1)^2+1}$

$$L^{-1}[\quad] = e^{-t} L^{-1}\left[\frac{s-1}{s^2+1}\right]$$

$$= e^{-t}(\cos t - \sin t)$$

(67) $$\frac{s}{(2s+1)^2} = \frac{1}{4}\frac{s}{(s+1/2)^2}$$

$$\mathcal{L}^{-1}\left[\frac{s}{(2s+1)^2}\right] = \frac{1}{4} e^{-1/2 t}\mathcal{L}^{-1}\left[\frac{s-1/2}{s^2}\right]$$

$$= \frac{1}{4} e^{-1/2 t}\left[\mathcal{L}^{-1}[1/s] - \frac{1}{2}\mathcal{L}^{-1}[1/s^2]\right]$$

$$= \frac{1}{4} e^{-1/2 t}\left[1 - \frac{t}{2}\right]$$

$$\mathcal{L}^{-1}\left[\frac{s+2}{s^2+4s+5}\right] = \mathcal{L}^{-1}\left[\frac{s+2}{(s+2)^2+1}\right] = e^{-2t}\cos t$$

(68) $$\frac{s+1}{s^2-4} = \frac{s}{s^2-4} + \frac{1}{s^2-4}$$

$$L^{-1}\left[\frac{s+1}{s^2-4}\right] = \cosh 2t + \frac{1}{4}\sinh 2t$$

$$L^{-1}\left[\frac{s+4}{s^2-8s}\right] = L^{-1}\left[\frac{s+4}{(s-4)^2-4^2}\right] = e^{4t}L^{-1}\left[\frac{s+8}{s^2-4^2}\right]$$

$$= e^{4t}L^{-1}\left[\frac{s}{s^2-4^2} + \frac{8}{s^2-4^2}\right]$$

$$= e^{4t}\left[\cosh 4t + \frac{1}{2}\sinh 4t\right]$$

(69) $$\frac{s^2+2s-4}{(s^2+2s+5)(s^2+2s+2)}$$

$$= \frac{s^2+2s+2-6}{(s^2+2s+5)(s^2+2s+2)}$$

$$= \frac{-6}{(s^2+2s+5)(s^2+2s+2)} + \frac{1}{(s^2+2s+5)}$$

$$= 2\left[\frac{1}{s^2+2s+5} - \frac{1}{s^2+2s+2}\right] + \frac{1}{s^2+2s+5}$$

$$= 2\left[e^{-t}\frac{\sin 2t}{2} - e^{-t}\sin t\right] + \frac{\sin 2t}{2}e^{-t}$$

$$= \frac{3}{2}e^{-t}\sin 2t - 2e^{-t}\sin t$$

(20) $$\frac{s^2}{(s^2+a^2)(s^2+b^2)}$$

$$= \frac{1}{s^2+b^2} - a^2\left[\frac{1}{(s^2+a^2)(s^2+b^2)}\right]$$

$$= \frac{1}{s^2+b^2} - \frac{a^2}{a^2-b^2}\left[-\frac{1}{s^2+a^2} + \frac{1}{s^2+b^2}\right]$$

$\mathcal{L}^{-1}[\;]$

$$= \frac{1}{b}\sin bt - \frac{a^2}{a^2-b^2}\left[\frac{1}{b}\sin bt - \frac{1}{a}\sin at\right]$$

71

$$\frac{s}{(s^2+a^2)(s^2+b^2)}$$

$$= \frac{1}{a^2-b^2}\left[-\frac{s}{s^2+a^2} + \frac{s}{s^2+b^2}\right]$$

$$= \frac{1}{a^2-b^2}\left(-\cos at + \cos bt\right)$$

$$= \frac{1}{b^2-a^2}\left(\cos at - \cos bt\right)$$

(72) $\frac{5s^2+8s-1}{(s+3)(s^2+1)}$

$$= \frac{A}{s+3} + \frac{Bs+C}{s^2+1}$$

$$\therefore \; A(s^2+1) + (Bs+C)(s+3) = 5s^2+8s+1$$

$$\left.\begin{array}{l} A + B = 5 \\ C + 3B = 8 \\ A + 3C = 1 \end{array}\right\}$$

on solving

$A = 2.2$
$B = 2.8$
$C = -0.4$

$$= \frac{2.2}{s+3} + \frac{2.8s - 0.4}{s^2+1}$$

$$L^{-1}[\;] = 2.2\,e^{-3t} + 2.8\cos t - 0.4\sin t$$

(73) $\dfrac{2s}{s^4+4} = \dfrac{2s}{s^4+4s^2+4-4s^2}$

$$= \frac{2s}{(s^2+2)^2-(2s)^2}$$

$$= \frac{2s}{(s^2+2+2s)(s^2+2-2s)}$$

$$= \frac{1}{4}\left(\frac{-1}{(s^2+2+2s)} + \frac{1}{(s^2+2-2s)}\right)$$

$$L^{-1}\left[\frac{2s}{s^4+4}\right] = \frac{1}{2}\left[-L^{-1}\left[\frac{1}{(s+1)^2+1}\right] + \frac{1}{(s-1)^2+1}\right]$$

$$= \frac{1}{2}\left[-e^{-t}\sin t + \sin t\, e^{+t}\right] = \sinh t \sin t$$

24 $L^{-1}\left[\frac{1}{s^3+1}\right] = e^{-t} L^{-1}\left[\frac{1}{(s-1)^3+1}\right]$

$$= e^{-t}\left[\frac{1}{s^3-2s^2+2s+2}\right]$$

75

$$\frac{1}{s^3(s-1)} = \frac{-s+1+s}{s^3(s-1)}$$

$$= \frac{-1}{s^3} + \frac{1}{s^2(s-1)}$$

$$= \frac{-1}{s^3} + \frac{-1}{s^2} + \frac{1}{s(s-1)}$$

$$= \frac{-1}{s^3} + \frac{-1}{s^2} + \frac{-1}{s} + \frac{1}{s-1}$$

$$= -\left(\frac{t^2}{2} + t + 1\right) - e^{-t}$$

(76) $\dfrac{s}{(s+1)^2(s^2+1)}$

$$= \frac{1}{2}\left(\frac{-1}{s^2+2s+1} + \frac{1}{s^2+1}\right)$$

$$\mathcal{L}^{-1}[\quad] = \left(-\frac{1}{2} e^{-t} t + \frac{1}{2}\sin t\right)$$

(77) $\dfrac{5s^2-15s-11}{(s+1)(s-2)^2} = \dfrac{5s^2-15s-11}{(s+1)(s^2-4s+4)}$

$$= 5\left[\frac{1}{s+1} + \frac{1}{s^2-4s+4}\right] + 14\left[\frac{1}{(s+1)(s-2)^2}\right]$$

$$\frac{1}{(s+1)(s-2)^2} = \frac{s+1-s}{(s+1)(s-2)^2} = \frac{1}{(s-2)^2} - \frac{s}{(s-2)^2}$$

$$L^{-1}\left[\frac{s}{(s-2)^2}\right] = e^{2t} L^{-1}\left[\frac{s+2}{s^2}\right] = e^{2t}[1+2t]$$

$$\therefore L^{-1}[\,] = 5\left(e^{-t} + e^{2t} t\right) + 14 e^{2t} t - e^{2t} - 2te^{2t}$$

$$= 5e^{-t} + 17te^{2t} - e^{2t}$$

(19) $$L^{-1}\left[\frac{s^2}{(s+1)^3}\right] = e^{-t} L^{-1}\left[\frac{(s-1)^2}{s^3}\right]$$

$$= e^{-t} L^{-1}\left[\frac{s^2 - 2s + 1}{s^3}\right]$$

$$= e^{-t} L^{-1}\left[1/s - \frac{2}{s^2} + 1/s^3\right]$$

$$= e^{-t}\left(1 - 2t + \frac{1}{2}t^2\right]$$

(78) $$\frac{1}{(s^2+1)(s^2+4)(s^2+9)}$$

$$= \frac{A}{s^2+1} + \frac{B}{s^2+4} + \frac{C}{s^2+9}$$

$$= A(s^2+4)(s^2+9) + B(s^2+1)(s+9)$$
$$+ C(s^2+1)(s^2+4) = 1$$

$$\left.\begin{aligned} A+B+C &= 0 \\ 36A+9B+4C &= 1 \\ 13A+10B+5C &= 0 \end{aligned}\right\} \quad \begin{aligned} A &= 0.0417 \\ B &= -0.066 \\ C &= 0.025 \end{aligned}$$

$$L^{-1}\left(\frac{s}{(s^2+1)(s^2+4)\;(s^2+9)}\right)$$

$$= (0.0417)\cos t + (-0.66)\cos 2t$$

$$+ (0.025)\cos 3t$$

(80) $$\frac{3s-2}{s^{5/2}} + \frac{7}{3s+2}$$

$$= \frac{3}{s^{3/2}} - \frac{2}{s^{5/2}} + \frac{7}{3}\frac{1}{(s+2/3)}$$

$$L^{-1}\left(\frac{1}{s^n}\right) = \frac{t^{n-1}}{\Gamma n}$$

$$\therefore L^{-1}[\;\;] = \frac{3}{\Gamma 3/2}t^{1/2} - \frac{2t^{3/2}}{\Gamma 5/2} + \frac{7}{3}e^{-2/3t}$$

(81) $\mathcal{L}^{-1}\left[\dfrac{3}{s^2+6s+18}\right] = e^{-3t}\sin 3t$

$\mathcal{L}^{-1}\left[\dfrac{8}{4s^2+4s+1}\right] = \mathcal{L}^{-1}\left[\dfrac{8}{(2s+1)^2}\right]$

$= \mathcal{L}^{-1}\left[\dfrac{2}{(s+1/2)^2}\right]$

$= 2te^{-1/2 t}$

(82) $\dfrac{s}{(s^2+2-2s)(s^2+2+2s)} = \dfrac{1}{4}\left(\dfrac{-1}{s^2+2+2s}\ \dfrac{1}{(s^2+2-2s)}\right)$

$\mathcal{L}^{-1}[\ ] = \dfrac{1}{4}\left(-e^{-t}\sin t + e^{t}\sin t\right)$

$= \dfrac{1}{2}\sin t \sinh t$

(83) $\dfrac{s}{(s^2+1-s)(s^2+1+s)} = \dfrac{1}{2}\left(\dfrac{-1}{s^2+1-s} + \dfrac{1}{s^2+1+s}\right)$

$L^{-1}[\ ] = 2\left[-e^{+1/2}\sin 1/4 t + e^{1/2}\sin 1/4 t\right]$

$= 2 \sin 1/4 t \sinh 1/2 t$

(84) $\dfrac{1}{(s^2+16)(s^2+25)} = \dfrac{1}{9}\left[\dfrac{1}{s^2+25} - \dfrac{1}{s^2+16}\right]$

$L^{-1}[\ ] = \dfrac{1}{9}(5\sin 5t - 4\sin 4t)$

(85) $\dfrac{s}{(s^2+16)(s^2+25)} = \dfrac{s}{9}\left[\dfrac{1}{s^2+25} - \dfrac{1}{s^2+16}\right]$

$L^{-1}[\ ] = \dfrac{1}{9}(\cos 5t - \cos 4t)$

86 $\dfrac{s}{(s^2-16)(s^2-25)} = \dfrac{s}{-9}\left[\dfrac{1}{s^2-16} - \dfrac{1}{s^2-25}\right]$

$= \dfrac{-1}{9}(\cosh 4t - \cosh 5t)$

87 $\log\left(\dfrac{s+a}{s+b}\right) = \log(s+a) - \log(s+b)$

Let $\phi(s) = \log\left(\dfrac{s+a}{s+b}\right)$

$\phi'(s) = \dfrac{1}{s+a} - \dfrac{1}{s+b}$

$L^{-1}(\phi'(s)) = e^{-as} - e^{-bs}$

$-t\, L^{-1}(\phi(s)) = L^{-1}(\phi'(s))$

$\therefore L[\,] = \dfrac{-1}{t}(e^{-as} - e^{-bs})$

(88) $2\tanh^{-1}(s) = \log\left(\frac{1+s}{1-s}\right)$

Let $\phi(s) = \log\left(\frac{1+s}{1-s}\right)$

$$\phi'(s) = \frac{1}{1+s} - \frac{1}{1-s}$$

$$L^{-1}(\phi'(s)) = e^{-t} - e^{t}$$

$$-t L^{-1}(\phi(s)) = L^{-1}(\phi'(s))$$

$$\therefore L^{-1}(\phi(s)) = -\frac{1}{t}\left(e^{-t} - e^{t}\right)$$

$$= \frac{2}{t}\sinh t$$

(89) $\tan^{-1}\left(\frac{2}{s^2}\right) = \phi(s)$

$$\phi'(s) = \frac{-\frac{2}{s^3}}{\left(\frac{2}{s^2}\right)^2 + 1}$$

$$= \frac{-\frac{2}{s^3}}{\frac{4}{s^4}+1} = -\frac{2}{s^3}\left(\frac{s^4}{s^4+4}\right)$$

$$= \frac{-2s}{s^4+4} = \frac{-2s}{s^4+2s^2+4-4s^2}$$

$$= -\frac{2s}{(s^2+2)^2-(2s)^2}$$

$$= \frac{-2s}{(s^2+2-2s)(s^2+2+2s)}$$

$$= \frac{1}{2}\left(\frac{1}{s^2+2+2s} - \frac{1}{s^2+2-2s} \right)$$

$$= \frac{1}{2}\left(\frac{1}{(s+1)^2+1} - \frac{1}{(s-1)+1} \right)$$

$$L^{-1}(\phi'(s)) = \frac{1}{2}\left(e^{-t}\sin t - e^{t}\sin t \right)$$

$$= -\sin t \sinh t$$

$$L^{-1}(\phi(s)) = +\frac{\sin t \sinh t}{t}$$

(90) $\tan^{-1}\left(\frac{s+a}{b}\right) = \phi(s)$

$$\phi'(s) = \frac{1/b}{1+\frac{(s+a)^2}{b^2}}$$

$$= \frac{b}{b^2+(s+a)^2}$$

$$L^{-1}(\phi'(s)) = e^{-at}\sin bt$$

$$L^{-1}(\phi(s)) = -\frac{1}{t}e^{-at}\sin bt$$

(91) $\log\sqrt{\frac{s^2+1}{s^2}} = \frac{1}{2}\left(\log(s^2+1) - \log s^2\right) = \phi(s)$

$$\phi'(s) = \frac{1}{2}\left(\frac{2s}{s^2+1} - \frac{2s}{s^2}\right)$$

$$L^{-1}(\phi'(s)) = (\cos t - 1)$$

$$L^{-1}(\phi(s)) = \frac{1}{t}(1-\cos t)$$

(92) $\cot^{-1}(s+1) = \phi(s)$

$$\phi'(s) = -\frac{1}{(s+1)^2+1}$$

$$L^{-1}(\phi'(s)) = -e^{-t}\sin t$$

$$L^{-1}(\phi(s)) = \frac{1}{t}e^{-t}\sin t$$

(93) $\log(s^2+4) = \phi(s)$

$$\phi'(s) = \frac{2s}{s^2+4} \qquad L^{-1}(\phi'(s)) = 2\cos(2t)$$

$$L^{-1}(\phi(s)) = \frac{-2}{t}\cos(2t)$$

(94) $\dfrac{s^2}{(s^2+a^2)^2}$

$$L^{-1}\left[\frac{s}{(s^2+a^2)} \cdot \frac{s}{(s^2+a^2)}\right]$$

Let $\phi_1(s) = \dfrac{s}{s^2+a^2}$

$$f_1(s) = L^{-1}\left(\frac{s}{s^2+a^2}\right) = \cos as$$

$$\phi_2(s) = \frac{s}{s^2+a^2}$$

$$f_2(s) = L^{-1}\left(\frac{s}{s^2+a^2}\right) = \cos as$$

$$L^{-1}[\,] = \int_0^t \cos as \cos(t-a)s \, ds = \int_0^t \cos as \cos t \cos as + \cos a \sin \sin as \, ds$$

$$= \cos t \int_0^t \cos^2 as + \sin t \int_0^t \sin as \cos as$$

$$= \cos t \int_0^t \frac{1}{2} + \frac{\cos 2as}{2}$$

$$+ \sin t \int_0^t \frac{1}{2} \sin 2as$$

$$= \cos t \ \frac{1}{2} t + \frac{1}{4}\left[\sin 2as\right]_0^t \cos at$$

$$+ \sin t \ \frac{1}{4}\left[\cos 2as\right]_0^t$$

$$= \frac{1}{2} \cos at \, t + \frac{1}{4a} \sin 2at \cos at$$

$$+ \frac{1}{4a} \sin at \left(\cos 2at - 1\right)$$

$$= \frac{1}{2} \cos at \cdot t + \frac{1}{4a} \sin at \cos 2at$$

(95) $$\frac{s^2+2s+3}{(s^2+2s+5)(s^2+2s+2)}$$

$$= \frac{s^2+2s+5 \;-\; 2}{(s^2+2s+5)(s^2+2s+2)}$$

$$= \frac{1}{(s^2+2s+2)} - \frac{2}{(s^2+2s+5)(s^2+2s+2)}$$

$$L^{-1}[\,] = e^{-at}\sin t - e^{-2at}\int_0^t \sin(t-s)\sin(2s)\,ds$$

$$= e^{-at}\sin t - e^{-2at}\int_0^t \sin 2s \sin t \cos s$$

$$- \sin 2s \cos t \sin s$$

Done in class

96

$$\phi_1 = \phi_2 = \frac{s+2}{s^2+4s+8}$$

$$\mathcal{L}^{-1}(\phi(s)) = e^{-2t}\cos 2t$$

$$\mathcal{L}^{-1}[\;] = \int_0^t e^{-4s}\cos 2s\,\cos(2s-2t)\,ds$$

$$\int e^{at}\cos bt = \frac{e^{at}}{a^2+b^2}\left[a\sin bt + b\cos bt\right)$$

97

$$\frac{1}{(s+3)(s^2+2s+2)}$$

$$\phi_1(s) = \frac{1}{s+3}$$

$$L^{-1}(\phi_1(s)) = e^{-3t}$$

$$\phi_2(s) = \frac{1}{s^2+2s+2} = \frac{1}{(s+1)^2+1}$$

$$L^{-1}(\phi_2(s)) = e^{-t} \sin t$$

$$L[\;] = \int_0^t f(u)\, f(t-u)\, du$$

$$= \int_0^t e^{-u} \sin u \; e^{-3(t-u)} \, du$$

$$= e^{-3t} \int_0^t e^{+2u} \sin u \, du$$

$$= e^{-3t}\left[\frac{1}{2^2+1^2} e^{+2u}\left[2\sin u - \cos u\right]\right]$$

$$= e^{-3t}\left[\frac{1}{5} e^{+2u}\left[2\sin u - \cos u\right]\right]_0^t$$

$$= e^{-3t}\left[\frac{1}{5} e^{+2t}\left[2\sin t - \cos t\right] + \frac{1}{5}\right]$$

$$= \frac{1}{5} e^{-t}(2\sin t - \cos t) + e^{-3t}$$

98

$$\frac{1}{(s-2)^4(s+3)}$$

$$\phi_1(s) = \frac{1}{(s-2)^4}$$

$$\phi_2(s) = \frac{1}{(s+3)}$$

$$L^{-1}(\phi_1(s)) = e^{2t}\frac{t^3}{\Gamma 4}$$

$$L^{-1}(\phi_2(s)) = e^{-3t}$$

$$L^{-1}[\,] = \frac{1}{6}\int_0^t e^{2u}u^3 e^{-3(t-u)}\,du$$

$$= \frac{1}{6}e^{-3t}\int_0^t e^{5u}u^3$$

(a9) $\frac{1}{s}\log\left(1+\frac{1}{s^2}\right)$

$$\phi_1(s) = 1/s \qquad L^{-1}(\phi_1(s)) = e^t$$

$$\phi_2(s) = \log\left(\frac{s^2+1}{s^2}\right) = \log(s^2+1) - 2\log s$$

$$\phi_2'(s) = \frac{2s}{s^2+1} - \frac{2}{s}$$

$$L^{-1}(\phi'(s)) = -t\, L^{-1}(\phi(s))$$

$$\frac{L^{-1}(\phi_2'(s))}{-t} = L^{-1}(\phi(s)) = \frac{1}{-t}\left[2\cos t - 2\right]$$

$$\therefore 2 = e^t \int_0^t e^{-u} \frac{\cdot 2}{-u} (\cos u - 4)$$

74 $\frac{1}{s^3+1}$

77 $\frac{5s^2-15s-11}{(s+1)(s-2)^2}$