χ^2 chi square distribution

① Uses of χ^2 test

A) To test the independance of attributes

ⓐ Used to test weather there is association between 2 or more attributes.

ⓑ Used to test if characteristic is dependant upon another characteristic

B) To test Goodness of fit

ⓐ Enables to acertain how well the theorotical distribution like binomial poission or normal fit the observed frequencies.

c) To test equality of several proportions

ⓐ Used to test if the porportions $P_1 P_2 P_3 P_4$ in different population are equal.

② Condition for χ^2

i) frequency of any cell is less than or equal to five then it than its combined frequency with neighbouring frequency is > 5 & DOF are reduced accordingly

③ Yates correction

in 2×2 Table DOF is $(2-1)(2-1) = 1$ if all cell frequencies are > 5 then use

$$\chi^2 = \sum \frac{(|O - E| - 0.5)^2}{E}$$

$$x = \frac{\sum \text{row} \sum \text{column}}{\sum \text{total}}$$

Independance of Attributes

① Investigate Association between the darkness of eye color in father & son from data

	← Color of father's eye →		
↑ Color of son's eye ↓	Dark	Not Dark	Total
Dark	48	90	138
Not Dark	80	782	862
Total	128	872	1000

→

H_0 : There is no association (Ratio Same)
H_a : There is association (Ratio Not Same)

Expected frequency $x_{\text{Dark Dark}} = \frac{128 \times 138}{1000} = 17.669 \approx 18$

18	120	138
110	752	862
128	872	1000

O	E	$(O-E)^2$	$(O-E)^2/E$
48	18	900	50
90	120	900	7.5
80	110	900	8.181
782	752	900	1.19

$$X^2_{cal} = \sum \frac{(O-E)^2}{E} = 66.87$$

X^2_{table} at 5% LOS & $(r-1)(c-1)$ dof $= 3.84$
$(2-1)(2-1) = 1$ dof

$X^2_{cal} > X^2_{table}$ Reject H_0

$\therefore$ There is association

② Two batches of 12 animals each are giving test of inoculation One was inoculated & other was not No of dead & survival animals are given. Can inoculation be regarded effective against disease at 5% LOS

	Dead	Survived	Total
Inoculated	2	10	12
Not inoculated	8	4	12
Total	10	14	24

H_0 : No association
H_a : Association

Expected $x = \frac{12 \times 10}{24} = 5$

E	
5	7
5	4

Yate Correction

O	E	$(\lvert O-E\rvert - 0.5)^2$	$\frac{(\lvert O-E\rvert - 0.5)^2}{E}$
2	5	6.25	1.25
10	7	6.25	0.89
8	5	6.25	1.5625
4	7	6.25	0.8928

$\chi_{cal} \; \Sigma \frac{(\lvert O-E\rvert - 0.5)^2}{E} = 4.28$

χ_{tab} at 5% LOS & 1 DOF, = 3.84

$\chi_{cal} > \chi_{tab}$

∴ Reject Null Hypothesis
∴ Vaccine effective

Goodness of Fit

① The following table gives no of accidents during the week. Find if accidents are uniformly distributed over week. 13, 15, 9, 11, 12, 10, 14

→ H_0 : Fit present (Always) Accidents occur uniformly
H_a : Fit not present

x :	13	15	9	11	12	10	14

Total = 84

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

If H_0 is true, then there will be 12 accidents per day $\therefore E = \frac{84}{7} = 12$ for all values

E/χ^2 $\sum \frac{(O-E)^2}{E}$

$$= \frac{(13-12)^2}{12} + \frac{(15-12)^2}{12} + \cdots$$

$$\chi^2 = 2.333$$

at 5% LOS & 6 DOF, $\chi_{tab} = 12.59$

$\chi^2 < \chi_{tab}$, Accept H_0

$\therefore$ Uniformly distributed

② Theory predicts that proportion of beans in 4 groups ABCD should be 9:3:3:1. In an experiment in 1600 beans, number in 4 groups were 882, 313, 287, 118 does experimental result support theory.

→ $E = \frac{9\times1600}{16} : \frac{3\times1600}{16} : \frac{3\times1600}{16} : \frac{1\times1600}{16}$

$= \cancel{14400}$:

$= 900 : 300 : 300 : 100$

H_0 : Proportion
H_a : Not

$$\chi = \sum \frac{(O-E)^2}{E} = \frac{(900-882)^2}{900} + \cdots$$

$= 4.726$

3 dof, 5% LOS, $\chi^2_{tab} = 9.466$

$\chi^2_a < \chi^2_{tab}$ $\therefore$ Accept H_0
$\therefore$ Proportional

3 Figures given below are A observed frequencies of some distribution ; B frequencies of normal distribution having same mean S.D. and total frequency as in A

A : 1 12 66 220 495 792 924 792 495 220 66, 12, 1

B : 2 15 66 210 484 799 963 799 484 210 66, 15, 2

Apply χ^2 of goodness of test

→

A is observed
B is Expected However dof = $n-1-1-1-1$

Same Mean, Same SD, Same Total freq

2 & 1 less than 5
Use Pooling Method

1 12 . . . 12, 1 ⟹ 13 . . . 13
2 15 . . . 15, 2 ⟹ 17 . . . 17

$$\chi^2 = \sum \frac{(O-E)^2}{E} = 3.84$$

H_0 : Fit
H_a : No Fit

dof = $11-1-1-1$ ~~-1~~ = 8

$\chi^2_{5\%}$ = ~~14.067~~ 15.51

$\chi^2_\alpha < \chi^2_{table}$ ∴ Accept fit. fit good

Explaination : DOF

originally 13 classes since they are reduced by pooling to 11 (Grouping twice) Also three constraints induced in B (S.D., $\bar{x}$, & freq) DOF is reduced by 3

∴ DOF = 13 − 2 − 3 = 8

Note $n-1$ is taken when mean is same (−1 for mean)

(4)

The following mistakes per page were observed in a book

No of Mistakes	0	1	2	3	4
No of Pages	211	90	19	5	0

Fit Poison distribution & Test goodness of fit

→

H_0 : Fit is good (Mistakes follow poison distribution)

H_a : Fit is not good

for Poison distribution $P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!}$

$\lambda = \text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = 0.44$

$P(X=x) =$ 0.~~643~~ 0.644 0.283 0.06 0.009 0.0010

Expecte = n.P

= 209.3 91.975 19.5 2.925 0.325

209 92 20 3 ~~0~~ 1

If we take last one 0, then sum ≠ 325 ∴ Consider 1
Here It doesn't matter as we pool

Pool as ~~3 & 4~~ less than 5
3 & 1

O	E	$(O-E)^2$	$(O-E)^2/E$
211	209	4	0.019
90	92	4	0.043
19+5+0=24	20+3+1 = 24	0	0

$\sum \frac{(O-E)^2}{E} = 0.0624$

χ^2 5% .05, 5-2-2=1 dof = 3.841

$\chi^2_{cal} < \chi^2_{table}$ ∴ Accept H_0 ∴ Good fit

Explanation 5-2 for pooling, Reduce further for two constraint ($\sum x_i f_i$ must be same, $\sum f_i$ same) Reduce 3-2=1

Hypothesis with several Proportions

(5) Sample of three shipments ABC gave following items

	A	B	C	Total
Defective	5	8	9	22
Nondefective	35	42	51	128
Total	40	50	60	150

Test weather proportion of defective items is same in three shipments at 5% LOS

→

H_0: Yes proportion is same $P_1 = P_2 = P_3$

H_a: Not same $P_1 \neq P_2 \neq P_3$

(eg. $\frac{x}{40} = \frac{22}{15}$ $\therefore x \simeq 6$)

O	E	$(O-E)^2$	$(O-E)^2/E$
5	6	1	1/6
8	7	1	1/7
9	9	0	0
35	34	1	1/34
42	43	1	1/43
51	51	0	0

$\Sigma (O-E)^2/E = 0.3621$

(Note : Total sum must equal 22 or 128 hence for last cases subtract from total)

$\therefore X_{cal} = 0.3621$

DOF = $(r-1)(c-1) = 2$ (Not total no of O)

X_{table} at 5% LOS = 5.99

$\therefore$ Accept H_0 $\therefore$ Same proportion

⑥ 5 dice were thrown 192 times. No of times 4, 5, 6 were obtained are as given

~~5 4 3 2 1 0~~

Appearance	6	46	70	45	20	2
No of dice	5	4	3	2	1	0

Calculate X^2

$\rightarrow$

Dice must follow uniform distribution ∴ use Binomial

$P(X=0) = {}^5C_0 \cancel{(1/6)^5} \; 5/6$ ← Only 4 to 6 asked ∴ $p = 0.5$ and 1/6

$P(x=0) = {}^5C_0 (0.5)^5 (0.5)^0 = 0.03125$

$P(x=1) = \vdots = 0.156$

$= 0.3125$

$= 0.3125$

$=$ ~~0.03125~~ 0.156

$= 0.03125$

$F = Np$ $\quad N = 192$

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
6	6	0	2.67
46	30	16	2.33
70	60	100	2.4
45	60	225	1.67
20	30	100	8.33
2	6	16	0

$\Sigma = 18.6$

$X^2 = 18.6$