Conditional Probability

S is sample space, A is event of sample space S, B is event already occured then $P(A|B) = \dfrac{n(A \cap B)}{n(B)}$

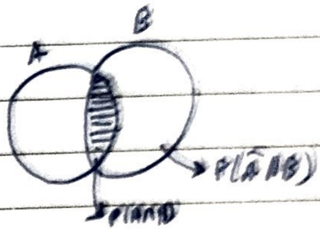
① Data on leadership of a certain magazine shows that proportion of male reader under 35 is 0.40 and over 35 is 0.20 if proportion of reader under 35 is 0.70, find proportion of subscribers that are female over 35. Also find prob that randomly selected male is under 35.

→ Let A : reader of magazine is male
B : reader is over 35 years

Given: $P(A \cap B) = 0.2$
$P(A \cap \bar{B}) = 0.4$
$P(\bar{B}) = 0.7$

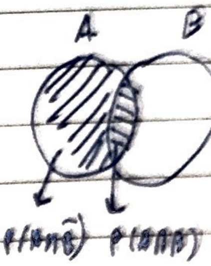
$\therefore P(B) = 1 - P(\bar{B}) = 0.3$

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$
$= 0.3 - 0.2$
$= \underline{0.1}$



$P(\bar{B}|A) = \dfrac{P(\bar{B} \cap A)}{P(A)} = \dfrac{0.4}{P(A)}$

$P(A) = P(A \cap B) + P(A \cap \bar{B})$
$= 0.2 + 0.4$
$= 0.6$

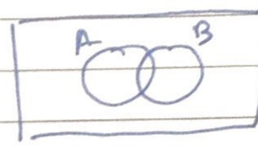


$\therefore P(\bar{B}|A) = \dfrac{0.4}{0.6} = 0.66$

② 60% employees are graduate, of these 10% are in sales those who did not graduate, 80% are in sales. what is P that employee selected at random is in sales? what is P that employee is neither in sales nor graduate

→

A : Employee graduate
B : Employee in sales



$P(A) = 0.6$

~~P(B) = 0.8~~

~~P(A∩B)~~

$P(B|A) = 0.1$

$P(B|\bar{A}) = 0.8$

$P(B) = P(A \cap B) + P(\bar{A} \cap B)$

$P(B|A) = \frac{P(B \cap A)}{P(A)}$ $\quad \therefore P(B \cap A) = P(A)(P(B|A)) = 0.6 \times 0.1 = 0.06$

$P(B|\bar{A}) = \frac{P(B \cap \bar{A})}{P(\bar{A})}$ $\quad P(B \cap \bar{A}) = P(\bar{A})\,P(B|\bar{A}) = 0.4 \times 0.8 = \cancel{0.48}\ 0.32$

$\therefore P(B) = P(A \cap B) + P(\bar{A} \cap B)$

$= 0.06 + 0.32 = \cancel{0.54}\ \ 0.38$

$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$ (De Morgan's law)

$= 1 - (P(A) + P(\bar{A} \cap B))$

$= 1 - (0.4 + 0.48) = 0.12$

Baye's theorem

If $E_1\ E_2\ E_3 \ldots E_m$ are mutually disjoint events $E_i \bigcap_{i=1}^{n} E_j = \phi$ with probability $E_i \neq 0\ \forall i = 1 \ldots n$ then for any event A which is subset of $\bigcup E_i$ such that $P(A) > 0$ then $P(E_i|A) = \frac{P(E_i)\,P(A|E_i)}{\sum_{i=1}^{n} P(E_i)\,P(A|E_i)}$

$$P(E_i|A) = \frac{P(E_i)\,P(A|E_i)}{P(A)}$$

① Suppose that a product is produced in three factories X Y & Z. It is known that factory X produces thrice as many times as factory Y & Z. Y & Z produce same no of times. Assume 3% of items produced by x & z are defective while 5% produced by Y are defective. An item is selected at random
i) what is probability item is defective
ii) If an item is found defective, it is produced by which factory?

⟶

Let no of items produced by y & z = n.
∴ x produces 3n
Total items is 5n. ∴ Probability that an item is produced by x is 3/5 Event → E_1
y is 1/5 E_2
z is 1/5 E_3

Let A denote event that the item is defective

$P(E_1) = 3/5$ $\qquad P(A|E_1) = 0.03$ $\qquad \cancel{P(A|A) = 0.03}$
$P(E_2) = 1/5$ $\qquad P(A|E_2) = 0.05$ $\qquad \cancel{P(E_2|A) = 0.03}$
$P(E_3) = 1/5$ $\qquad P(A|E_3) = 0.03$ $\qquad \cancel{P(E_3|A) = 0.05}$

$\cancel{P(A|E_i) = \frac{P(E_i)P(E_i|A)}{P(A)}}$

$$P(E_i|A) = \frac{P(E_i)\,P(A|E_i)}{P(A)}$$ — Bays' Theorem

$$P(A) = P(A\cap E_1) + P(A\cap E_2) + P(A\cap E_3)$$
$$= P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + P(A|E_3)P(E_3)$$
$$= \frac{3\times 0.03}{5} + \frac{1}{5}\times 0.05 + \frac{1}{5}\times 0.03$$

0.036

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(2) A factory produces certain type of output by three types of machines. Respective daily production figures are

Machine I : 3000 units
Machine II : 2500 units
Machine III : 4500 units

Past experiance shows 1% output of MI is defective while for other two machines are 1.2% & 2%. Item selected at random is defective. What is the probability that it comes from M_1, M_2, M_3

$\rightarrow$ $P(E_i) = \dfrac{3000}{10000} = 0.30$ $(3000 + 2500 + 4500 = 10000)$

Similarly $P(E_2) = 0.25$
$P(E_3) = 0.45$

E_i = Item produced by machines
A = output is defected.

E_i	$P(E_i)$	$P(A \mid E_i)$	$P(A \cap E_i)$ $P(E_i)\,P(A \mid E_i)$
E_1	0.30	0.012	0.0036
E_2	0.25	0.02	0.005
E_3	0.45	0.02	0.009

$P(A) = \Sigma = 0.0176$

	$P(E_i \mid A) = P(A \cap E_i) / P(A)$
E_1	0.204
E_2	0.284
E_3	0.511

③ There are two bags A & B. A contains n white and 2 black balls & B contains 2 white and n black balls. One ~~ball~~ is selected at random and 2 balls are drawn without replacing. If both are white and P(A) was chosen is 6/7 find value of m. n.

→ Let Probability that A is chosen is P(A) B is chosen is P(B)

Given:

$E_1 \rightarrow$ A is chosen $\quad \cancel{P(A) = 6/7} \quad P(A) = 1/2$

$E_2 \rightarrow$ B is chosen $\quad \therefore \cancel{P(B) = 1/7} \quad P(B) = 1/2$

$E_3 \rightarrow$ Both balls are white

$$P(E_1 \mid E_3) = 6/7$$

A → n white + 2 Black = n+2

B → 2 white + n Black = n+2

= n+2 $\quad$ = n+2

$$P(E_3 \mid E_1) = \frac{{}^{n}C_2}{{}^{n+2}C_2}$$

(Bag A contains n white balls & n+2 total balls So Prob will be No of ways to choose 2 balls from n balls upon no of ways to chose ~~n+2~~ balls from n+2 balls)

$$P(E_3 \mid E_2) = \frac{{}^{2}C_2}{{}^{n+2}C_2}$$

$$P(E_1 \mid E_3) = \frac{P(E_1)\,P(E/E_1)}{P(E_1)\,P(E/E_1) + P(E_2)\,P(E/E_2)}$$

$$= \frac{(1/2)\left(\frac{{}^{n}C_2}{{}^{n+2}C_2}\right)}{(1/2)\left(\frac{{}^{n}C_2}{{}^{n+2}C_2}\right) + (1/2)\left(\frac{{}^{n}C_2}{{}^{n+2}C_2}\right)} = \frac{n(n-1)}{n(n-1)+2} = \frac{6}{7}$$

$$\therefore n^2 - n - 12 = 0$$

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④ 3 white 5 black are contents of a vessel, from which 4 balls are transfered to another vessel. A ball is drawn and is found to be white what is the probability that out of 4 balls transferred, 3 are white & 1 is black.

Let E_1 = Event where 3 white & 1 black
E_2 = ball drawn is white

To find $P(E_1|E_2)$

$P(E_1)$ = 3 white & 1 black ball from random selection

$$= \frac{{}^3C_3 \times {}^5C_1}{{}^8C_4} \qquad \frac{\text{one selection}}{\text{total}} = 1/14$$

$P(E_2|E_1)$ = Probability of choosing a white ball from 3 white and 1 black

$$= \frac{{}^3C_1}{{}^4C_1} = \frac{3}{4}$$

$P(E_2)$ = One White from total selection

$$= \frac{{}^3C_1}{{}^8C_1} = 3/8$$

$$P(E_1|E_2) = \frac{P(E_1)\ P(E_2|E_1)}{P(E_2)} = \frac{(1/14)(3/4)}{(3/8)}$$

$$= 0.14$$

Alternative

$E_1 = 0W + 1B$	$P(E_1) = {}^5C_4 / {}^8C_4 = 1/14$	$P(A\vert E_1) = 0$
$E_2 = 1W + 3B$	$P(E_2) = {}^3C_1 \times {}^5C_3 = 3/7$	$P(A\vert E_2) = 1/4$
$E_3 = 2W + 2B$	$P(E_3) = {}^3C_2 \times {}^5C_2 = 3/7$	$P(A\vert E_3) = 2/4$
$E_4 = 3W + 1B$	$P(E_4) = {}^3C_3 \times {}^5C_1 = 1/14$	$P(A\vert E_4) = 3/4$

$$P(E_4|A) = \frac{P(E_4)\ P(A|E_4)}{\sum^4 P(E_i)\ P(A|E_i)}$$

(5) In MCQ, student knows answer or guess. P is the probability he knows answer. Assume that student who guesses is correct is 1/5 in probability where 5 is no of choices. what is prob. that student who knows the answer & ans is correct

→

E_1 = Student ~~guesses~~ knows answer
E_2 = student gets answer correct.
E_3 = Student guesses.
To find ~~$P(E_2 | E_1)$~~ $P(E_1 | E_2)$

$$P(E_1) = P$$

$$P(E_3) = 1 - P$$

$$P(E_2 | E_1) = 1$$
$$P(E_2 | E_3) = 1/5$$

$$P(E_2) = P(E_2|E_1) \cdot P(E_1) + P(E_2|E_3) \cdot P(E_3)$$
$$= P + (1-P)\, 1/5$$
$$= \frac{4}{5} P + \frac{1}{5}$$

(Either student knows & gets correct or with probability he knows or student guesses right with probability he guess)

$$P(E_1 | E_2) = \frac{P(E_1)\, P(E_2|E_1)}{P(E_2)}$$

$$= \frac{(P) \cdot (1)}{\frac{4}{5}P + \frac{1}{5}} = \frac{5P}{4P+1}$$