Uniform distribution

A discrete random variable X follows Uniform distribution

$$X = 1, 2, 3, \ldots n \qquad P(x = x_i) = \frac{1}{n} \quad \forall x_i = 1, 2, 3 \ldots n$$
$$1 < i < n$$

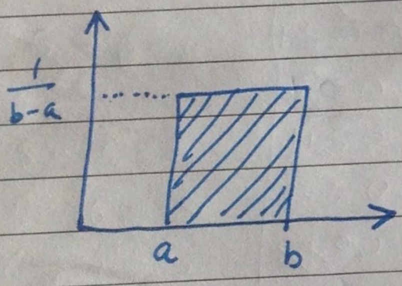
$$E(x) = \sum x_i p_i = \sum_{i=1}^{n} \frac{x_i}{n} = \frac{1}{n} \sum x_i = \frac{1}{n} \frac{n(n+1)}{2} = \frac{n+1}{2}$$

$$E(x^2) = \sum x_i^2 p_i = \frac{1}{n} \sum x_i^2 = \frac{1}{\cancel{n}} \frac{\cancel{n}(n+1)(2n+1)}{6}$$

$$V = E(x^2) - [E(x)]^2 = \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} = (n+1)\left[\frac{2n+1}{6} - \frac{n+1}{4}\right] = \frac{n^2-1}{12}$$

A continuous random variable X follows uniform or rectangular distribution over (a, b)

$$\text{if } f(x) \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & o/w \end{cases}$$



$$E(x) = \int_a^b x_i f_i$$
$$= \frac{1}{b-a} \int_a^b x$$
$$= \frac{1}{b-a} \left[\frac{x^2}{2}\right]_a^b$$
$$= \frac{a+b}{2}$$

$$V(x) = \frac{(b-a)^2}{12}$$

Note : for any subinterval $[c, d]$ $\quad a \leq c \leq d \leq b$

$$P(c \leq x \leq d) = \int_c^d f(x)\,dx = \frac{d-c}{b-a}$$

① In a quiz there are 30 participants. Question is given to all participants. Time allowed is 25 seconds. find probability of participant response within 6 seconds, if it is uniform distribution

→

Let x = Time taken to respond. We want $P(x < 6)$

Total interval is $(0, 25)$ $\therefore f(x) = \frac{1}{b-a} = \frac{1}{25-0}$

$P(x < 6) = \int_0^6 \frac{1}{25} dx = \frac{6}{25} = \underline{0.24}$

For 30 participants, No of participants response within 6 seconds is $30 \times 0.24 = \underline{\underline{7.2}}$

---

② Suppose a random no N is taken from 690 to 850 in uniform distribution find probability $N > 790$

→ $P(x > 790) = P(830 \geq x > 790)$

$= \frac{850 - 790}{830 - 690} = \underline{0.1}$

---

③ If x is uniformly distributed in interval -2 to 2 find ⓐ $P(x < 1)$

ⓑ $P(|x-1| \geq 1/2)$

→

ⓐ $P(x < 1) = \frac{1-(-2)}{2-(-2)} = \underline{\underline{0.75}}$

ⓑ $|x-1| \geq 1/2$ $\therefore -1/2 \leq x-1 \leq 1/2$

$= 1 - |x-1| \leq 1/2$ $\therefore 1/2 \leq x \leq 3/2$

$\therefore P(|x-1| \geq 1/2) = \frac{3/2 - 1/2}{2-(-2)} = \underline{\underline{0.25}}$

$P(|x-1| > 1/2) = 1 - 0.25 = \underline{\underline{0.75}}$

Exercises

① X is a normal variate with mean 10 and S.D. = 4 find $P(|x-14|<1)$, $P(5 \le x \le 18)$, $P(x \le 12)$

→

Normal variation mean = 10

$\sigma = 4$

$$\cancel{P(|x-14|<1) = P(-1 < x-14 < 1)}$$
$$\cancel{= P(13 < x < 15)}$$
$$\cancel{= P(13-}$$

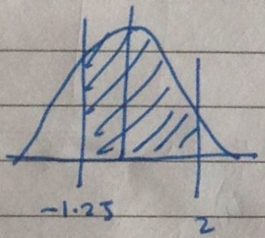
→ first move to z then expand x.

when $x = 14$ → $z = \frac{14-10}{4} = 1$

$\therefore |x-14| \to |z|$

$$P(|x-14| \le 1) = P(|z|<1)$$
$$= 2(A(z=0 \text{ to } z=1)$$
$$= 2 \times 0.3413$$
$$= 0.6826$$

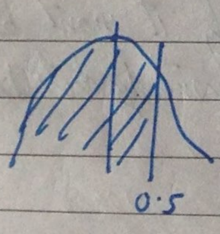
② $P(5 \le x \le 18) = P\left(\frac{5-10}{4} \le z \le \frac{18-10}{4}\right)$



$$= P(-1.25 \le z \le 2)$$
$$= A(0-1.25) + A(0-2)$$
$$= 0.3944 + 0.4772$$
$$= 0.8716$$

③ $P(x \le 12) = P\left(z \le \frac{12-10}{4}\right)$

$$= P(z \le 0.5)$$



$$= 0.5 + 0.1915$$
$$= 0.6915$$

② Marks of students are in normal distribution mean of physics, chemistry and mathematics is 51, 53, 46 S.D. is 15, 12, 16 respectively find probability of total marks i) above 180 & ii) below 80 or 80

→

Total marks is sum of three indivisual marks

$x_1 = 51$ $\quad\quad$ $\sigma_1 = 15$

$x_2 = 53$ $\quad\quad$ $\sigma_2 = 12$

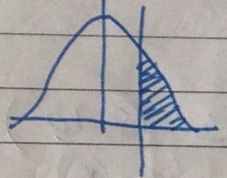
$x_3 = 46$ $\quad\quad$ $\sigma_3 = 16$

$y = x_1 + x_2 + x_3$ $\quad\quad$ $\sigma^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2$

$= 51 + 53 + 46$ $\quad\quad$ $\sigma^2 = 15^2 + 12^2 + 16^2$

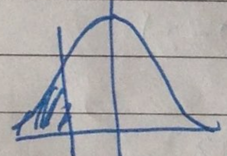
$= 150$ $\quad\quad$ $\therefore \sigma = 25$

$$P(y > 180) = P\left(z > \frac{180-150}{25}\right)$$



$$= P(z > 1.2)$$
$$= 0.5 - P(0 < z < 1.2)$$
$$= 0.5 - 0.3849$$
$$= 0.1151$$

$$P(y < 80) = P\left(z < \frac{80-150}{25}\right)$$



$$= P(z < -2.8)$$
$$= 0.5 - P(0 < z < 2.8)$$
$$= 0.5 - 0.4978$$
$$= 0.0022$$

③ The amount of time a watch will run without having to be reset is 120 days at mean. find probability that the watc will be set in 24 days or less and probability it wont b need to set at least 180 days. Here time is exponentialy distribu

→

$$\lambda = \frac{1}{120} \quad \text{(mean)}$$

$$\therefore f(x) = \frac{1}{120} e^{-x/120}$$

$$P(x \le 24) = \int_0^{24} \lambda e^{-\lambda x} dx = -\left[e^{-\lambda x}\right]_0^{24}$$

$$= -\left[e^{-24/120} - 1\right]$$

$$= 0.181$$

$$P(x \ge 180) = \int_{180}^{\infty} \lambda e^{-\lambda x} dx = -\left[e^{-\lambda x}\right]_{180}^{\infty}$$

$$= +\left[e^{-180/120}\right]$$

$$= 0.223$$

④ Prove the memoryless property of exponential distribution

$$P(x > s+t \mid x > s) = P(x > t) \quad \forall s, t > 0$$

$\rightarrow$ $P(x > s+t \mid x > s) = \dfrac{P(x > s+t \cap x > t)}{P(x > s)}$ Bayes theorem

$$\boxed{\begin{array}{l} P(T > t) = e^{-\lambda t} \\ f(x) = \lambda e^{-\lambda t} \end{array}}$$

$$= \frac{\cancel{\lambda} e^{-\lambda(s+t)}}{\cancel{\lambda} e^{-\lambda(s)}} = e^{-\lambda t}$$

$$= P(x > t)$$

Hence proved

⑤ If x is exponentially distributed then prove that the probability that x exceeds its expected value is less than 0.5

$\rightarrow$ Expected value of x is $1/\lambda$

$$P(x > 1/\lambda) = e^{-\lambda \times 1/\lambda} = e^{-1} = 0.367 < 0.5$$

⑥ X is in uniform distribution with mean 1 and variance 4/3 find $P(x < 0)$

$\rightarrow$

$$\frac{a+b}{2} = 1 \qquad \frac{(b-a)^2}{12} = 4/3$$

$$\therefore a+b = 2 \quad -① \qquad \therefore (b-a)^2 = 16$$

$$\therefore b-a = \pm 4$$

Since $b > a$, $b - a = 4$ $-②$

from ① & ② $b = 3$

$a = -1$

$$P(x \leq 0) = \int_a^0 \frac{1}{b-a}\, dx$$

$$= \frac{-a}{b-a} = \frac{1}{4} = 0.25$$

⑦ X is a uniform distribution over the range $(a, a+1)$ find a such that $P(0.25 < x < a) = 0.25$

$$P(0.25 < x < a) = \int_{0.25}^{a} f(x)\, dx = \frac{1}{a+1-a}(a - 0.25)$$

$$\therefore a - 0.25 = 0.25$$

$$\therefore a = 0.5$$

⑧ X is a uniform distribution over the range $(2, b)$ such that $P(3 < x < 6) = 0.3$, find mean and variance of x.

$$P(3 < x < 6) = \int_6^3 f(x)\, dx = \frac{1}{b-a}(6-3) = 0.3$$

$$\therefore \frac{1}{b-a} = 0.1$$

range given $2, b$ $\therefore a = 2$ $\therefore b = 12$

$$\text{Mean} = \frac{a+b}{2} = 7$$

$$\text{Variance} = \frac{(b-a)^2}{12} = 8.33$$

⑨ X is Uniform distribution of conference timing over (a,b) such that mean is $15/2$ variance is $25/12$ find the probability that any conference lasts for at least 5 hours and not exceeds 7, find a & b (range of conference timings.

→ $\frac{15}{2} = \frac{a+b}{2}$, $\frac{25}{12} = \frac{(b-a)^2}{12}$

$\therefore a+b = 15$ - ① $b-a = 5$ - ② $\therefore a = \cancel{5}, b = 10$

$P(5 < x < 7) = \frac{7-5}{b-a} = \frac{7-5}{5} = \frac{2}{5} = \underline{\underline{0.4}}$

⑩ x has interval $(0,5)$ find ⓐ PDF of x
ⓑ $P(x>3)$
ⓒ $P(2 < x < 3.5)$

$a = 0, b = 5$ PDF $= f(x) = \frac{1}{b-a} = \frac{1}{5}$

$P(x>3) = \int_3^5 \frac{1}{b-a} dx = \frac{5-3}{5} = 0.4$

$P(2 < x < 3.5) = \int_2^{3.5} \frac{1}{b-a} d(x) = \frac{3.5-2}{5} = 0.3$

⑪ find the P→Time (in hours) required to repair a machine is exponentially distributed exceeds 2 hours if $\lambda = 0.5$

→ $P(x>2) = e^{-\lambda 2} = e^{-0.5\times 2} = e^{-1} = 0.367$

Uniform distribution

Discrete

mean → $\frac{n+1}{2}$

Var → $\frac{n^2-1}{12}$

$f(x) \rightarrow \frac{1}{n}$

Continuous

mean → $\frac{a+b}{2}$

Var → $\frac{(b-a)^2}{12}$

$f(x) \rightarrow \frac{1}{b-a}$