① $n = 6$

$$9(P(x=4)) = P(x=2)$$

$$P(x=4) = {}^6C_4 \; p^6 \, q^2$$

$$P(x=2) = {}^6C_2 \; p^2 \, q^6$$

$$9(\cancel{{}^6C_4} \; p^{\cancel{6}\,4} \cancel{q^2}) = \cancel{{}^6C_2} \, \cancel{p^2} \, \cancel{q^6} q^4$$

$$9p^4 = (1-p)^4$$

$$9p^4 = 1 - 4p + 6p^2 - 4p^3 + p^4$$

$$8p^4 + 4p^3 - 6p^2 + 4p - 1 = 0$$

$$9^{1/4} = \frac{1-p}{p}$$

$$9^{1/4} + 1 = \frac{1}{p} \qquad p = \frac{1}{9^{1/4}+1}$$

Mean $= np = \dfrac{6}{9^{1/4}+1}$ Variance $= npq = \dfrac{6}{9^{1/4}+1}\left(1 - \dfrac{1}{9^{1/4}+1}\right)$

(2) Mean $= 5$ $\qquad np$

Variance $\rightarrow \frac{10}{3}$ $\qquad npq$

$\therefore q = \dfrac{\frac{10}{3}}{5} = \dfrac{2}{3}$ $\qquad \therefore p = 1/3$

$P(x = 2) = {}^5C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^3 =$

$P(x \leq 4) = 1 - P(x = 5)$

$= 1 - {}^5C_5 \left(\frac{1}{3}\right)^5$

$=$

x:	0	1	2	3	4	5	6
f:	5	18	28	12	7	6	4

$$\text{Mean} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{192}{80}$$

$$= 2.4$$

$$= np$$

$$n = 6$$

$$\frac{2.4}{6} = p \therefore p = 0.4$$

$$P(X = x) = {}^6C_x (0.4)^x (0.6)^{n-x}$$

	0	1	2	3	...	6
P:	0.4	.....			.....	0.6

Poisson distribution

$n \to \infty$

$$P(x) = \frac{\lambda^{x} e^{-\lambda}}{x!}$$

$$E(x) = \lambda$$

$$Var(x) = \lambda$$

Proof by expanding

$$\sum P_i x_i = e^{-\lambda} \sum \frac{\lambda^{x}}{x!} x$$

$$= \lambda e^{-\lambda} \sum \frac{\lambda^{x-1}}{(x-1)!} = \lambda e^{-\lambda} e^{\lambda} = \lambda$$

Similarly prove for Var

① If Two independant variates are in poission distribution with mean λ_1, λ_2 then their sum is also poission with mean $\lambda_1 + \lambda_2$

Recurrance relation of poision

$$P(x+1) = \frac{m}{x+1} P(x)$$

① If x follows poisson such that $P(x=1) = 2P(x=2)$ find mean, variance & $P(x=3)$

$\rightarrow$

$$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$P(x=1) = \lambda e^{-\lambda}$$

$$P(x=2) = \frac{\lambda^2 e^{-\lambda}}{2}$$

$$P(x=1) = 2P(x=2)$$

$$\lambda e^{-\lambda} = 2\frac{\lambda^2 e^{-\lambda}}{2}$$

$$\therefore \lambda = 1$$

$\therefore$ Mean $= \lambda = 1$

$\therefore$ Variance $= \lambda = 1$

$$P(x=3) = \frac{\lambda^3 e^{-\lambda}}{3!}$$

$$= \frac{1}{6e} = 0.0613$$

② A hospital switch board recieves average 4 calls in 10 minutes interval

find Probability atleast 2 calls, 3 calls

→

10 minutes → useless information

4 → Expected (mean)

$\lambda = 4$

$P(x \geq 2) = 1 - P(x < 2)$

(at least condition)

$= 1 - P(x = 0) - P(x = 1)$

$$= 1 - \frac{\lambda^0 e^{-4}}{0!} - \frac{\lambda e^{-4}}{1}$$

$= 0.908$

$$P(x = 3) = \frac{\lambda^3 e^{-\lambda}}{x!} = \frac{4^3 e^{-4}}{3!} = 0.195$$

③ If x and y are independant poisson variates with mean m_1 & m_2 find probability that $x + y = k$

→

By property Let $z = X + y$

z is also poisson variate with mean

$m = m_1 + m_2$

$$\therefore P(Z = k) = \frac{(m_1 + m_2)^k \, e^{-(m_1 + m_2)}}{k!}$$

(4) If x, y are independant poisson variates with mean 2 and 3, find the variance of $3X-2Y$

Since X & Y are independant

$$Var(3x-2y) = 9\,Var(x) + 4\,Var(y)$$

$$= 9 \times 2 + 4 \times 3$$

$$= 30$$

Property of Variance

$$V(A \pm B) = V(A) + V(B)$$

$$V(kA) = k^2 V(A)$$

⑤ It is known that the probability of an item being defective is 0.05 These are sent to the market in packets of 20 find no of packets containing

① at least
② Exactly
③ At Most

2 defective in 1 packet in 1000 packets

Using ① Binomial
② Poission

Find Number of packets in 1000 packets

Binomial

$n = 20$ $N = 1000$

$p = 0.05$

$$P(x=2) = {}^{20}C_2 (0.05)^2 (0.95)^{18}$$

$$= 0.1886$$

$E(X) = NP$ (Expected No of such packets)

$$= 1000 \times 0.1886$$

$$= 188.6$$

$$P(x \leq 2) = P(x=0) + P(x=1) + P(x=3)$$

.... -

Similarly

② Using Poisson $n \to \infty$

$\lambda = np = 20 \times 0.05 = 1$

-

$$P(x=2) = \frac{1 e^{-1}}{2!} = 0.1839$$

$E = NP = 183.9$

⑤ In a certain factory there is 1/500 chance for any blade to be defective. The blade are supplied in packet of 10 find approx no of packets with 0, 1, 2 defective pieces in 1000 packets

→

$n = 10$

$p = \frac{1}{500}$

$\lambda = np = \frac{10}{500} = 0.02$

$p(x=0) = \frac{(0.02)^0 e^{-0.02}}{0!} = 0.9802$

$p(x=1) = \frac{(0.02)^1 e^{-0.02}}{1} = 0.0196$

$$P(x=2) = \frac{e^{-0.02} \times (0.02)^2}{2!} = 0.0002$$

Expected No $= NP$

$(x=0) = 1000 \times 0.9802 = 9802$

$(x=1) = 196$

$(x=2) = 1000 \times 0.002 = 2$

7 Fit poisson distibution to the following