Large Sampling

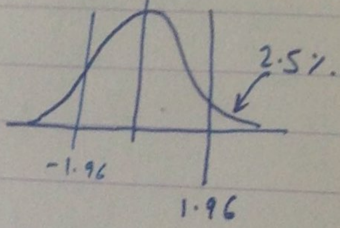
$\bar{x}$ is a Mean of each sample of size n drawn from a population th mean μ and S.D. σ then $\bar{X}$ is ~~uniformly~~ normally distributed with mean μ d S.D. $\frac{\sigma}{\sqrt{n}}$

$$\bar{x} \sim N(\mu, \sigma/\sqrt{n})$$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

e Know that 95% area will lie between ± 1.96

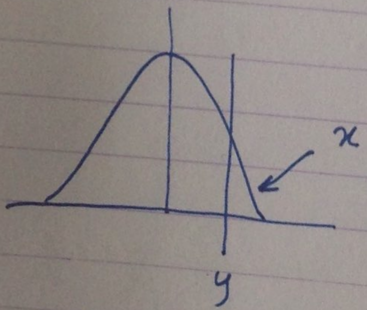
$$P(|z| < 1.96) = 0.05$$

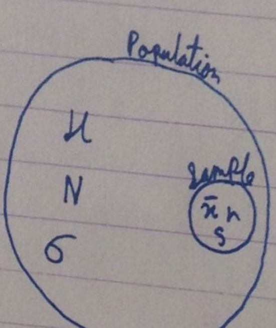


5% is critical level, 95% is confidence level

Critical Value

2 Tailed	10%	5%	2%	1%	$2x$
1 Tailed	5%	2.5%	1%	0.5%	x
Value	1.64	1.96	2.33	2.58	y





$\bar{x}$ – Mean of sample
n – size of sample
s – S.D. of Sample

Interval Estimation

$$\mu \in \left(\bar{x} - c\frac{\sigma}{\sqrt{n}}, \bar{x} + c\frac{\sigma}{\sqrt{n}} \right) \quad \text{Two tailed}$$

① The measurement of weights of random sample of 200 ball bearings showed a mean of 0.824 N and S.D. 0.042. find 95% confidence limits for mean weight ①

→

S.D. = 0.042 at 95%, $c = 1.96$ (two tailed for interval estimation)

$s = 0.042$

$n = 200$

$\bar{x} = 0.824$

Confidence limits for mean

$$\mu \in \left(\bar{x} - \frac{c\sigma}{\sqrt{n}}, \bar{x} + \frac{c\sigma}{\sqrt{n}} \right)$$

$$\left(0.824 - 1.96 \times \frac{0.042}{\sqrt{200}}, 0.824 + 1.96 \times \frac{0.042}{\sqrt{200}} \right)$$

$$(0.8183, 0.8297)$$

② Sample of 65 was drawn in estimating mean annual income of 950 families. The mean and S.D. of sample were 4730₹ and 765₹ respectively. Find 95% confidence interval for population mean.

→

Given : $n = 65$ $N = 950$ $C: 1.96$

$\bar{x} = 4730$

$s = 765$

Since: $\frac{n}{N} = \frac{65}{950} = 0.068 > 0.05$, Approximation of $\sigma = \frac{s}{\sqrt{n}}$ wont work

$$\sigma = \frac{s}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} = \frac{765}{\sqrt{65}} \sqrt{\frac{950-65}{950-1}} = 91.63$$

$$\mu \in (\bar{x} - 1.96 \times 91.63, \bar{x} + 1.96 \times 91.63)$$

$$\in (4550.4, 4909.6)$$

Testing Hypothesis

1) A random sample of 50 items gives a mean 6.2 and variance 10.24. Can it be regarded as drawn from a normal population with mean 5.4 at 5% LOS

→ $n = 50$ $\qquad \mu = 5.4$ (Assume) $\qquad$ 5% LOS

$\bar{x} = 6.2$

s ~~σ~~ $= 10.24$

H_0 : Null Hypothesis $\quad \mu = 5.4$

H_a : Alternate Hypothesis $\quad \mu \neq 5.4$ $\qquad$ for $\neq$ use two tailed

$\therefore$ 5% → 1.96

Always Assume H_0 $\therefore \mu = 5.4$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{6.2 - 5.4}{10.24/\sqrt{50}} = 1.77$$

$Z_{cal} < Z_{table}$

$1.77 < 1.96$

$\therefore Z_{cal}$ falls within acceptable confidence $\therefore$ Accept H_0

$\therefore$ We can say that given sample is drawn from normal population

2) A random sample of 400 members has mean 4.45 cm. Can it be regarded as drawn from a large population whose mean is 5 and variance 4

$n = 400$ $\qquad \mu = 5$

$x = 4.45$ $\qquad \sigma^2 = 4$

$H_0 : \mu = 5$

$H_a : \mu \neq 5$

$$Z_{cal} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \left| \frac{4.45 - 5}{4/\sqrt{400}} \right| = 5.5$$

At 5% LOA $\quad 5.5 > 1.96 \therefore$ Reject H_0

Two Population

$$S.E. = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\bar{X}_1 - \bar{X}_2 = N\left(\mu_1 - \mu_2, \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$$

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

① Two samples are drawn from different Populations

	Size	Mean	S.D.
I	400	124	14
II	250	120	12

find 95% confidence limits for difference between population Mean

→ $S.E. = \sqrt{\frac{14^2}{124} + \frac{12^2}{120}} = 1.0324$

$\cancel{Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{SE}}$ 95% → 1.96

$$\mu_1 - \mu_2 \in \left((\bar{x}_1 - \bar{x}_2) - 1.96\,S.E.,\ (\bar{x}_1 - \bar{x}_2) + 1.96\,S.E.\right)$$

$$\in (1.9764, 6.02)$$

② Average Marks scored by 32 boys is 72, S.D. = 8 while that of 36 girls is 70 with S.D. 5. Test at 10% LOS whether boys perform better than girls

→ Boys

$n_1 = 32$

$\bar{x}_1 = 72$

$s_1 = 8$

Girls

$n_2 = 36$

$\bar{x}_2 = 70$

$s_2 = 6$

H_0: Boys perform same as girls ($\mu_1 = \mu_2$)

H_a: Boys perform better than girls ($\mu_1 > \mu_2$)

$\mu_1 > \mu_2$ ∴ Use 1 tailed test at 10% LOS → 2.33

$$z = \left| \frac{(x_1 - x_2) - (\mu_1 - \mu_2)}{S.E.} \right|$$

But Here assuming H_0, ie $\mu_1 = \mu_2$, $\mu_1 - \mu_2 = 0$

$$\therefore z = \left| \frac{x_1 - x_2}{S.E.} \right|$$

$$S.E. = \sqrt{\frac{8^2}{32} + \frac{6^2}{36}} = 1.73$$

$$z = \left| \frac{72 - 70}{1.73} \right| = 1.156$$

$z_{cal} < z_\alpha$

$1.156 < 2.33$

∴ Accept H_0 ie Boys perform same as girls

ie. Boys dont perform better than girls

The means of ~~a~~ two samples of sizes 1000 and 2000 are 67.50 and 68. Can the samples be regarded as drawn from the same population with SD = 2.5 LOS → 0.27

→

A	B	Assume
$n = 1000$	$n = 2000$	$x_1 = x_2$
$x_1 = 67.50$	$x_2 = 68$	$\sigma = 2.5$

$H_0: \mu_1 = \mu_2, \sigma_1 = \sigma_2 = 2.5$, $H_a: \sigma_1 \neq \sigma_2, x_1 \neq x_2$

$$\therefore SE = \sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 0.0968$$

$$z = \left(\frac{(x_1 - x_2) - (\mu_1 - \mu_2)}{SE} \right)$$

$$= \frac{x_1 - x_2}{SE} = \frac{67.50 - 68}{0.0968} = 5.168$$

$\neq$ Case :. use two tailed test. At 0.27 LOS $Z_{cal} = 3$

$Z_{cal} > Z_\alpha$

$\therefore$ Reject H_0

$\therefore$ Two were not drawn from the same ~~sample~~. population

Two samples were drawn from different populations gave the result

	Size	Mean	S.D.
I	125	340	25
II	150	380	30

...l the hypothesis at 5% LOS that difference between means ... two populations is 35

$H_0: (\mu_1 - \mu_2) = 35$

$H_a: (\mu_1 - \mu_2) \neq 35$ 2 ~~One~~ tailed

$$= \sqrt{\frac{25^2}{125} + \frac{30^2}{150}} = 3.316$$

$$z = \left| \frac{(x_1 - x_2) - (u_1 - u_2)}{S.E.} \right|$$

Assum $u_1 - u_2 = 35$

$$z = \left| \frac{340 - 380 - 35}{3.31} \right| = 1.507$$

$z_{\alpha} = 1.96$ (5% 2 tailed)

$z_{cal} < z_{\alpha}$ $\therefore$ Accept H_0

$\therefore$ Difference between means is 35

④ Pain reliever breings relief in 3.5 min average with S.D. = 2.1 minute
New Pain reliever was tested on 50 patients, mean was 3.1 minutes, Sample SD = 1.5
at 5% level was new pain reliever effective?

→ Old

u ~~$\bar{x}$~~ $= 3.5$

σ ~~s~~ $= 2.1$ (No use)

New

$\bar{x} = 3.1$

$s = 1.5$

$n = 50$

H_0: $u_1 = x_2 = 3.5$

H_a: $u_2 > u_1$ $\therefore$ One tailed

$$z : \left| \frac{\bar{x} - x_0}{S/\sqrt{n}} \right| = 1.886$$

$z_{cal} < z_{\alpha}$

$1.886 < 1.96$

$\therefore$ Accept H_0

$\therefore$ Not effective

(5) Sample of 100 students has mean height 64 & S.D. = 4.
Can it be regarded that population mean height is more than 66% at
5% LOS

→

$H_0 : \mu_1 = \mu_2$
$H_a : \mu_1 > \mu_2$

one tailed

$\bar{x}_1 = 64$
$s = 4$
$n = 100$

$\mu = 66$

$$Z = \left| \frac{x - \mu}{s/\sqrt{n}} \right| = \frac{2}{4} \times 10 = 5$$

$Z_\alpha = 1.64$ (5% one tailed)

$Z_{cal} > Z_\alpha$

$\therefore$ Reject H_0

$\therefore$ Height can be greater