Small Sampling

Small Sample test

If we take large no. of samples of small size $n < 30$ then we apply students T distribution test

n

	σ known	σ unknown
$n \geq 30$	z - test	z - test
$n < 30$	z - test	T - test

Degree of freedom → No of variables you are free to choose

Formulae → 1) σ of parent population is known

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$s = \frac{\sqrt{n-1}}{\sqrt{n}} S$$

2) σ Not known & parent population is normal

Unbiased estimate given

$$T = \frac{\bar{x} - \mu}{S / \sqrt{n}} \qquad S^2 = \frac{\Sigma (x_i - \bar{x})^2}{n - 1}$$

Sample SD find this

$$T = \frac{\bar{x} - \mu}{s / \sqrt{n-1}} \qquad s^2 = \frac{\Sigma (x_i - \bar{x})^2}{n}$$

① Interval estimation. ① Sample of size 10 has mean 40, $s = 10$ construct 99% confidance interval for population mean

$$\hat{\sigma}_x = \frac{s}{\sqrt{n-1}} = \frac{10}{3} = 3.33$$

$$\mu \in (\bar{x} - t_\alpha \hat{\sigma}_x, \ x + t_\alpha \hat{\sigma}_x)$$

t_α for 99% confidance level & $n-1$ dof $\quad$ dof = 10 - 1 = 9

T table search 0.01 at 9 DOF → 3.250 $\quad \therefore t_\alpha = 3.250$

$\therefore \mu \in (40 - 3.25 \times 3.33,\ 40 + 3.25 \times 3.33)$

$\mu \in (29.18,\ 50.82)$

Testing hypothesis

② A soap manufacturing company was distributing soap through large no of retail stores. Before heavy advertizement campaign mean sale per week per shop was 140 dozen. After the campaign sample of 26 shops was taken and mean sale was 147 dozen with standered deviation 16. Can you consider advertizement efficiant

→ $n < 30$ $\therefore$ T test σ unknown $\mu = 140$

H_0 : $\mu = 140$ Campaign not efficiant, so $\mu_2 = 140$

H_a : Campaign was efficiant $\mu_2 > 140$

$$t_{cal} = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{147 - 140}{16/\sqrt{26-1}} = 2.187$$

Assume 5% LOS for 1 tail dof = 26 - 1 = 25

$t_\alpha = 1.708$

Since $t_{cal} > t_\alpha$, reject H_0 $\therefore$ Campaign was efficiant

③ 9 items of sample had values : 45, 47, 50, 52, 48, 47, 49, 53, 51 does Mean of 9 items differ significantly from assumed population mean 47.5.

$$\bar{x} = \frac{\Sigma x_i}{n} = 49.11$$

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n} = 6.0987 \quad \therefore s = 2.46$$

$H_0 : \mu = 47.5$

$H_a : \mu \neq 47.5$

dof = 8

$$t_{cal} = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{1.61}{0.869} = 1.96$$

$t_{5\%} = 2.306$ $t_{cal} < t_\alpha$ $\therefore$ Accept H_0

Difference between Means.

Case 1 $n_1 + n_2 - 2 < 30$

Common, then unbiased estimate of common population S.D.

$$S_p = \sqrt{\frac{\sum(x_{i1} - \bar{x}_1)^2 + (\sum(x_{i2} - \bar{x}_2)^2)}{n_1 + n_2 - 2}}$$

~~Case 2~~

If unbiased S.D of population are different,

$$S_1 = \sqrt{\frac{\sum(x_{i1} - \bar{x}_1)^2}{n_1 - 1}} \qquad S_2 = \sqrt{\frac{\sum(x_{i2} - \bar{x}_2)^2}{n_2 - 1}}$$

$$\therefore \quad S_p = \sqrt{\frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2}}$$

If S.D. of two samples are

$$s_1 = \sqrt{\frac{\sum(x_{i1} - \bar{x}_1)^2}{n_1}} \qquad s_2 = \sqrt{\frac{\sum(x_{i2} - \bar{x}_2)^2}{n_2}}$$

$$S_p = \sqrt{\frac{s_1^2 n_1 + s_2^2 n_2}{n_1 + n_2 - 2}}$$

Then standard error of difference between means

$$S.E. = S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S.E.}$$

① A sample of 8 students of 16 years each show mean blood pressure 118.4 mm while S.D. is 12.17 mm while sample of 10 students of 17 years each show mean blood pressure of 121 mm with S.D. 12.88 mm. Investigator feels that the BP is related to Age. Does data provide enough reason to support the feeling

$\rightarrow$

A - 16 yr

$n_1 = 8$

$\bar{x}_1 = 118.4$

$s_1 = 12.17$

B - 17 yr

$n_2 = 10$

$\bar{x}_2 = 121$

$s_2 = 12.88$

$H_0 : \mu_1 = \mu_2$

$H_a : \mu_1 \neq \mu_2$

$$S_p = \sqrt{\frac{s_1^2 n_1 + s_2^2 n_2}{n_1 + n_2 - 2}} = \sqrt{177.7} = 13.33$$

$$S.E. = S_p \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 6.32$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S.E.} = 0.41$$

$\therefore t = 0.41$ (calculated)

for 95% confidence, dof = $n_1 + n_2 - 2 = 16$

t Table search(0.05, 16) = 2.12

$t_{cal} < t_\alpha$

$\therefore$ ~~Reject~~ Null Hypothesis Accept

$\therefore \mu_1 = \mu_2 \therefore$ data doesn't provide enough data

(2) 6 guinea pigs injected with 0.5 mg medication. On an average took 15.4 seconds to fall asleep with unbiased S.D 2.2 sec while 6 other guinea pigs injected with 1.5 mg medication took on avg 11.2 sec to fall asleep with unbiased SD. 2.6 sec use 5% LOS to test that difference in doses has no effect.

H_0 $\quad \mu_1 = \mu_2$

H_a $\quad \mu_1 \neq \mu_2$

$n_1 = 6$ $\qquad n_2 = 6$

$\bar{x}_1 = 15.4$ $\qquad \bar{x}_2 = 11.2$

$s_1 = 2.2$ $\qquad s_2 = 2.6$

$$S_p = \sqrt{\frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1+n_2-2}} = 2.408$$

$$S.E. = S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 0.98$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S.E.} = |4.27| \qquad t = 4.27$$

5% LOS, DOF = 12-2 = 10

$t_\alpha = 2.12$

$4.27 > t_\alpha$ $\quad \therefore$ Reject H_0

$\therefore$ Difference has effect

③ Height of 6 sailors in Inches are 63, 65, 68, 69, 71, 72
& Height of 10 solders are 61, 62, 65, 66, 69, 69, 70, 71, 72, 73
Discuss solders on average taller than sailors

$n_1 = 6$

63
65
68
69
71
72

$$S_1 = \sqrt{\frac{\Sigma(x_1 - \bar{x})^2}{n_1 - 1}} =$$

$$\bar{x}_1 = \frac{\Sigma x_i}{n} = 68$$

$n_2 = 10$

61
62
65
66
69
69
70
71
72
73

$$S_2 = \sqrt{\frac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1}}$$

$=$

$$\bar{x}_2 = 67.8$$

$H_0 = \mu_1 = \mu_2$
$H_1 = \mu_1 \neq \mu_2$ } two tailed

$$S_p = \sqrt{\frac{S_1^2(n_1-1) + (S_2^2(n_2-1)}{n_1+n_2-2}} = 3.9$$

$$SE = S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 0.099$$

t cal
5% LON, DOF = 14 for 2 tailed ∴ Chek Table for 2.5%
$t_\alpha = 2.145$

$t_{cal} < t_\alpha$ ∴ Accept H_0
∴ Solders Not Taller to Sailors

Case 2 Samples are Not Independant

$\mu_1 = \mu_2 = \mu = 0$ ie $\mu = 0$

① Certain injection given to 12 patients resulted in following change of blood pressure

x : 5 2 8 -1 3 0 6 -2 1 5

Can it be concluded that injection will be in general accompanied by ~~increase~~ change in blood pressure

$H_0 = u = 0$
$H_a : u \neq 0$

$\rightarrow \bar{x} = \frac{\sum x_i}{n} = 2.58$

$\sum (x - \bar{x})^2 = \cancel{242}\ 104.94$

$\sum (x - \bar{x})^2 = \sum x^2 - 2\bar{x} x - \bar{x}^2$

$= \sum x^2 - 2\bar{x} \sum x + \bar{x}^2 n$

$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} = 2.95$ (standard dev)

$t_{cal} = \frac{\bar{x} - \mu}{s / \sqrt{n-1}} = 2.9$

dof $= n - 1 = 11$ at 5% LOS $= 2.201$

$t_{cal} > t_\alpha$ ∴ Reject H_0

ie. Change is present

② 10 boys were tested in statistics. Then they were given training. Second test was given. Test of marks given below give evidance that students are benifitted

T_1 :	70	68	56	75	80	90	68	75	56	58
T_2 :	68	70	52	73	75	78	80	92	54	55

→

$T_1 - T_2$: 2   -2   +4   +2   5   12   -12   -17   2   3

Take the difference between the two parameters

$H_0 : \mu = 0$

$H_a : \mu \neq 0$

$\bar{x} = -0.1$

$|\bar{x}| = 0.1$

$\Sigma(x_i - \bar{x})$: ~~3.61~~ ~~4.41~~

$x_i - \bar{x}$	$(x_i - \bar{x})^2$
1.9	3.61
-2.1	4.41
3.9	15.21
1.9	3.61
4.9	24.01
11.9	141.61
-12.1	146.41
-17.1	292.41
1.9	3.61
2.9	8.41

$\Sigma(\bar{x} - \bar{x})^2 = 643.3$

$$s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}$$

$$= \sqrt{\frac{643.3}{10}} = 8.02$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = 0.037$$

t_α = 5% LOA = 2.26

DOF : 10 - 1 = 9

$t_{cal} < t_\alpha$

∴ Accept H_0 i.e student Not benifitted