

⑤

$$Z = 2x_1 - 2x_2 + 4x_3 - 5x_4$$

$$x_1 + 4x_2 - 2x_3 + 8x_4 + S_1 + 0S_2 = 2$$

$$-x_1 + 2x_2 + 3x_3 + 4x_4 + 0S_1 + S_2 = 1$$

Variables 4, constraint 2 Putting 2 Variables to 0

$$x_1 = 0, x_2 = 0 \quad \begin{cases} -2x_3 + 8x_4 = 2 \\ 3x_3 + 4x_4 = 1 \end{cases} \quad \begin{cases} x_3 = 0 \\ x_4 = 1/4 \end{cases}$$

$$x_1 = 0, x_3 = 0 \quad \begin{cases} 4x_2 + 8x_4 = 2 \\ 2x_2 + 4x_4 = 1 \end{cases} \quad \begin{cases} x_2 = \\ x_4 = \end{cases} \text{ unbound}$$

$$x_1 = 0, x_4 = 0 \quad \begin{cases} 4x_2 - 2x_3 = 2 \\ 2x_2 + 3x_3 = 1 \end{cases} \quad \begin{cases} x_2 = 1/2 \\ x_3 = 0 \end{cases}$$

$$x_2 = 0, x_3 = 0 \quad \begin{cases} x_1 + 8x_4 = 2 \\ -x_1 + 4x_4 = 1 \end{cases} \quad \begin{cases} x_1 = 2 \\ x_4 = 0 \end{cases}$$

$$x_2 = 0, x_4 = 0 \quad \begin{cases} x_1 - 2x_3 = 2 \\ -x_1 + 3x_3 = 1 \end{cases} \quad \begin{cases} x_1 = 8 \\ x_3 = 3 \end{cases}$$

$$x_3 = 0, x_4 = 0 \quad \begin{cases} x_1 + 4x_2 = 2 \\ -x_1 + 2x_2 = 1 \end{cases} \quad \begin{cases} x_1 = 0 \\ x_2 = 1/2 \end{cases}$$

⑥ Variables 3, constraint 2  $\therefore$  Put 1 Variable to 0

$$x_1 = 0 \quad \begin{cases} 2x_2 + 3x_3 = 9 \\ 2x_2 + 2x_3 = 15 \end{cases} \quad \begin{cases} x_2 = 13.5 \\ x_3 = -6 \end{cases}$$

$$x_2 = 0 \quad \begin{cases} x_1 + 3x_3 = 9 \\ 3x_1 + 2x_3 = 15 \end{cases} \quad \begin{cases} x_1 = 3.85 \\ x_3 = 1.71 \end{cases}$$

$$x_3 = 0 \quad \begin{cases} x_1 + 2x_2 = 9 \\ 3x_1 + 2x_2 = 15 \end{cases} \quad \begin{cases} x_1 = 3 \\ x_2 = 3 \end{cases}$$



13)  $2 - 100x_1 - 50x_2 - 50x_3 + 0s_1 + 0s_2 + 0s_3 = 0$   
 $4x_1 + 3x_2 + 2x_3 + s_1 + 0s_2 + 0s_3 = 10$   
 $3x_1 + 8x_2 + x_3 + 0s_1 + s_2 + 0s_3 = 8$   
 $4x_1 + 2x_2 + x_3 + 0s_1 + 0s_2 + s_3 = 6$   
 $x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	RHS	ratio
Z	-100	-50	-50	0	0	0	0	—
$s_1$	4	3	2	1	0	0	10	10/4
$s_2$	3	8	1	0	1	0	8	8/3
$s_3$	4	2	1	0	0	1	6	6/4

Since all rows in Z not nonnegative, select  $x_1$  as  $6/4$  is smallest  
 $x_1$  enter  $s_3$  leave

Z	0	0	-25	0	0	26	150	—
$s_1$	0	1	①	1	0	-1	4	4
$s_2$	0	13/2	3/4	0	1	-3/4	14/4	14/3
$x_1$	1	1/2	1/4	0	0	1/4	6/4	6

All rows not non negative select  $x_3$

$x_3$  enter  $s_1$  leave

Z	0	25	0	25	0	1	250
$x_3$	0	1	1	1	0	-1	4
$s_2$	0	23/4	0	-3/4	1	0	1/2
$x_1$	1	1/4	0	-1/4	0	1/2	1/2

All row non negative

∴ found solution

$Z = 250$   
 $x_1 = 1/2$   
 $x_2 = 0$   
 $x_3 = 4$

$s_1 = 0$   
 $s_2 = 1/2$   
 $s_3 = 0$



⑪ S.F.

$$Z - 3x_1 - 2x_2 - 5x_3 + 0s_1 + 0s_2 + 0s_3 = 0$$

$$x_1 + 2x_2 + x_3 + s_1 = 430$$

$$3x_1 + 0x_2 + 2x_3 + s_2 = 460$$

$$x_1 + 4x_2 + 0x_3 + s_3 = 420$$

$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$$

Initial Basic feasible solution  $s_1 = 430, s_2 = 460, s_3 = 420$

	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	RHS	ratio
Z	-3	-2	-5	0	0	0	0	0
$s_1$	1	2	1	1	0	0	430	430
$s_2$	3	0	2	0	1	0	460	230
$s_3$	1	4	0	0	0	1	420	$\infty$

All entries in Z row not non negative  $x_3$  is key column

All ratios are not +ve as present chose smallest +ve entry i.e. 230 as key row. Key element 2  $x_3$  enters,  $s_2$  leaves  $R_3 \rightarrow R_3/2$   $R_1 \rightarrow R_1 + 5R_2$   
 $R_2 \rightarrow R_2 - R_1$   $R_3 \rightarrow R_3$

Key = 2

1								
Z	9/2	-2	0	0	+ $s_1$	0	1150	-575
$s_1$	-1/2	2	0	1	-1/2	0	200	*100
$x_3$	3/2	0	1	0	1/2	0	230	115
$s_3$	1	4	0	0	0	1	420	105

All entries in Z row not Non Negative.  $x_2$  is Key column

$s_3$  leaves  $x_2$  enters Key element 1/2

$$R_1 \rightarrow R_1/2 \quad R_2 \rightarrow R_2 + 2R_1 \quad R_3 \rightarrow R_3 - 4R_1$$

Z	4	0	0	1	2	0	1350
$x_2$	-1/4	1	0	1/2	-1/4	0	100
$x_3$	3/2	0	1	0	1/2	0	230
$s_3$	2	0	0	-2	1	1	20

All rows in Z row +ve hence values found

No. of rows = 2 hence no alternate solution

$$\begin{aligned} x_2 &= 100 & s_3 &= 20 & Z &= 1350 \\ x_3 &= 230 & s_2 &= 0 \\ x_1 &= 0 & s_1 &= 0 \end{aligned}$$



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$$Z = 5x_1 - 2x_2 + 3x_3 - MA_1$$

$$2x_1 + 2x_2 - x_3 - S_1 + A_1 = 2$$

$$3x_1 - 4x_2 + S_2 = 3$$

$$x_2 + 3x_3 + S_3 = 5$$

$$x_1, x_2, x_3 \geq 0$$

$$Z \leftarrow (5+2M)x_1 + (-2+2M)x_2 + (3-M)x_3 + MS_1 + 2M = 0$$

Initial Basic feasible solution  $A_1 = 2$   
 $S_2 = 3$   
 $S_3 = 5$

Basic	$x_1$	$x_2$	$x_3$	$S_1$	$S_2$	$S_3$	$A_1$	RHS	Ratio
Z	$-5-2M$	$+2-2M$	$-3+M$	M	0	0	0	$-2M$	-
$A_1$	$(2)^*$	2	-1	-1	0	0	1	2	1
$S_2$	3	-4	0	0	1	0	0	3	1
$S_3$	0	1	3	0	0	1	0	5	$\infty$

Most negative value is  $x_1$ ,  $x_1$  enter  $A_1$  leave

Z	0	7	$-8\frac{1}{2}$	$-5\frac{1}{2}$	0	0	$\frac{1}{2}$	5	<del>1/2</del>
$x_1$	1	1	$-1/2$	$-1/2$	0	0	$1/2$	1	2
$S_2$	0	-7	$+3\frac{1}{2}$	$+3\frac{1}{2}$	1	0	$1/2$	0	0
$S_3$	0	1	$(3)^*$	0	0	1	$1/2$	5	$5/3$



$x_3$  enter basic  $s_3$  leaves

$z$	0	$5\frac{3}{6}$	0	$-5/2$	0	$1/6$	$85/6$	-ve
$x_1$	1	$7/6$	0	$-1/2$	0	$1/6$	$11/6$	-ve
$s_2$	0	$-13/2$	0	$3/2$	1	$-1/2$	$-5/2$	-ve
$x_3$	0	$1/3$	1	0	0	$1/3$	$1/3$	-ve

No feasible solution



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Max  $Z \rightarrow Z'$  Minimize

$$Z = x_1 - 2x_2 - x_3 \quad \text{MA}_2 \quad \text{two}$$

$$x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3 + s_1 + A_1 = 1$$

$$\frac{3}{2}x_1 + 2x_2 + x_3 + s_2 + A_2 = 8$$

$$Z' = (3M+1)x_1 + (-2-2M)x_2 + (-1+M)x_3 + 8M = 0$$

Initial basis  $s_1=1, A_2=8$

Basis	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$A_2$	RHS	ratio
$Z$	$+ \frac{3}{2}M+1$	$+2-2M$	$+1-M$	0	M	0	$-8M$	
$s_1$	1	$\frac{1}{2}$	$\frac{1}{2}$	1	0	0	1	2
$A_2$	$\frac{3}{2}$	2	1	0	-1	1	8	$\frac{4}{3}$

$x_2$  enters

$s_1$  leaves

$Z$	$\frac{5}{2}M+3$	0	$-1+M$	$+4+4M$	M	0	
$x_2$	2	1	1	2	0	0	2
$A_2$	$-\frac{5}{2}$	0	1	-2	-1	1	4

All entries non Negative but  $A_2$  in basis with positive value

Pseudo solution