$$V(x) = E(x-m)^2$$

$$= \int_{-\infty}^{\infty} \frac{(x-m)^2}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx$$

let $\frac{x-m}{\sigma} = t$ $\qquad dx = \sigma\, dt$

$$= \int_{-\infty}^{\infty} \frac{\sigma^2}{\sqrt{2\pi}} t^2 e^{-\frac{1}{2}t^2} dt$$

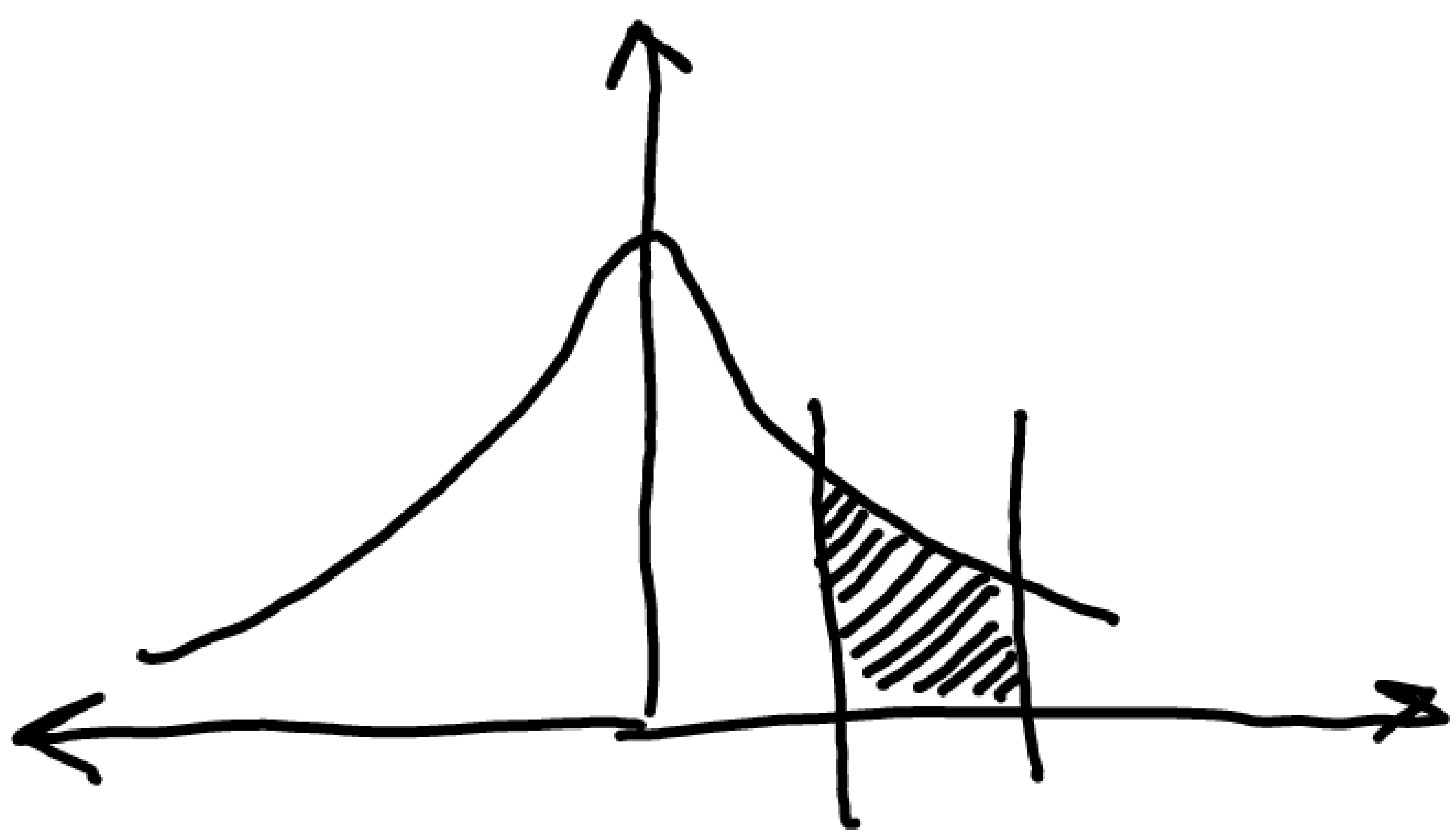
$$= \frac{\sigma^2}{\sqrt{2\pi}} \left[\left(-t e^{-1/2 t^2}\right)_{-\infty}^{\infty} + \int_{-\infty}^{\infty} -e^{-1/2 t^2} dt \right]$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} \left(0 + \sqrt{2\pi} \right) = \sigma^2$$

Hence the variance of a normal distribution is σ^2

① find the probability that a random variable having standered normal distribution will take a value between 0.87 and 1.28

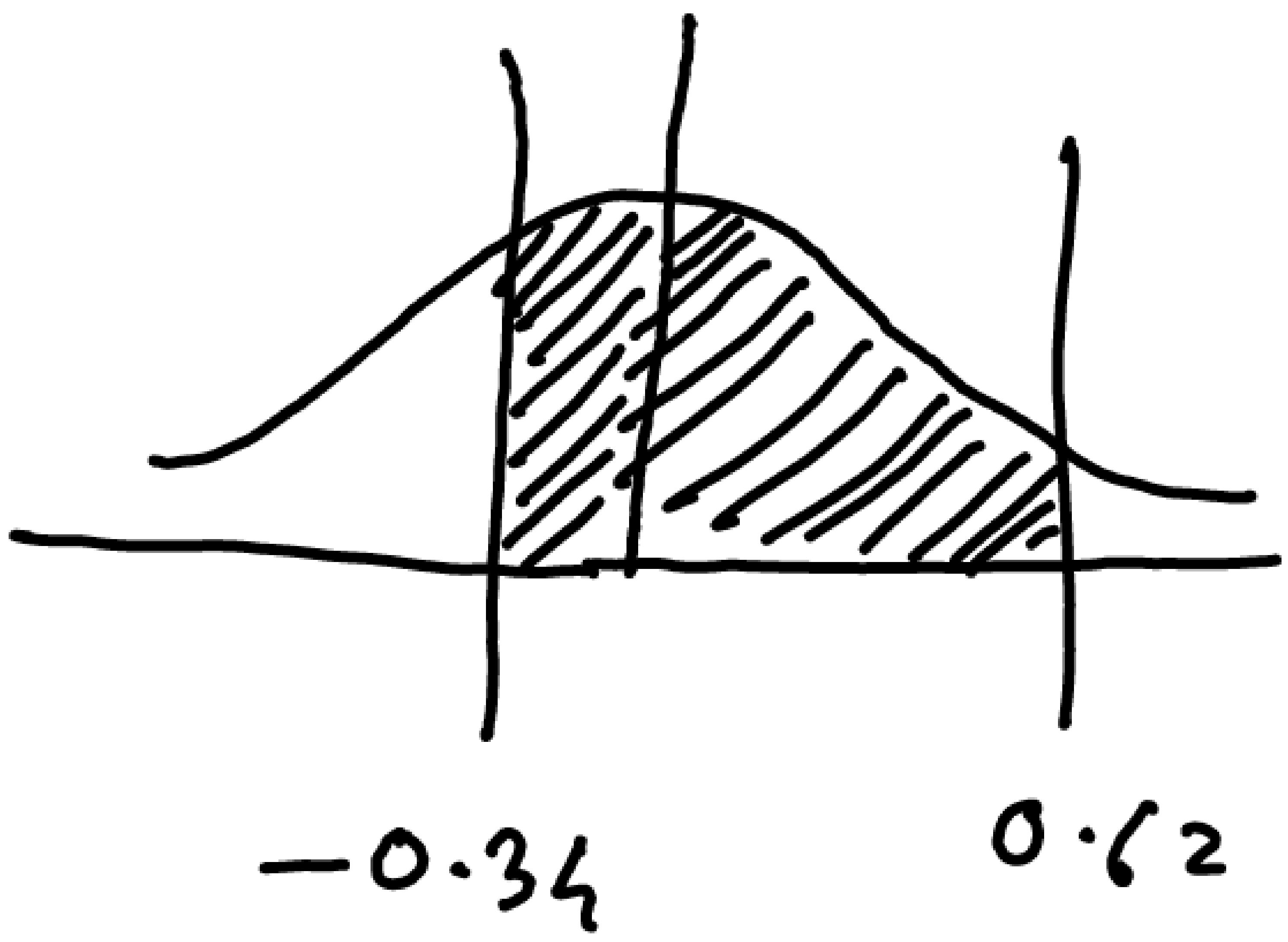
→ $P(0.87 < z < 1.28) =$ Area between 0.87 & 1.28



$= \text{(area from } z=0 \text{ to } z=1.28)$
$- \text{(area from } z=0 \text{ to } z=0.87)$

$= 0.3997 - 0.3078 = 0.0919$

② Find probability that a random variable having standered normal distribution will take value between -0.34 to 0.62



$$\left.\begin{aligned} &= \text{Area from } (0 \text{ to } 0.34) \\ &+ \text{Area from } (0 \text{ to } 0.62) \end{aligned}\right\} \text{Symmetry}$$

$= 0.1331 + 0.2324$

$= 0.3655$

(3) For a normal variate with Mean 2.5 & S.D. 3.5 find the probability that

(1) $2 \leq x \leq 4.5$ (2) $-1.5 \leq x \leq 5.5$

$$Z = \frac{X - m}{\sigma}$$

$x = 2$ $\quad Z = \frac{2 - 2.5}{3.5} = -0.14$

$x = 4.5$ $\quad Z = \frac{4.5 - 2.5}{3.5} = 0.57$

$P(2 \leq x \leq 4.5) = P(-0.14 \leq z \leq 0.57)$

$= \text{Area}(z \to 0.14) + \text{Area}(z \to 0.57)$

$= 0.0557 + 0.2157$

$= 0.2714$

Similarly do 2nd

$x : -1.5$ $\qquad$ $z : -1.142$

$x : 5.5$ $\qquad$ $z : 0.8$

$$P(-1.5 \le x \le 5.5) = P(-1.142 \le z \le 0.8)$$

$$= 0.3739 + 0.2881$$

$$= 0.6610$$

Quartile deviation of a normal distribution

$$Q.D. = \frac{Q_3 - Q_1}{2} = \frac{2}{3}\sigma$$

$$Q_1 = m - \frac{2}{3}\sigma$$

$$Q_2 = m + \frac{2}{3}\sigma$$

Normal Approximation to the binomial distribution

$X \sim B(n, p)$ then $Z = \dfrac{X - np}{\sqrt{npq}} = \dfrac{X - \mu}{\sigma}$

is a SNV

Normal distribution can be used in place of binomial distribution if np and $np > R$

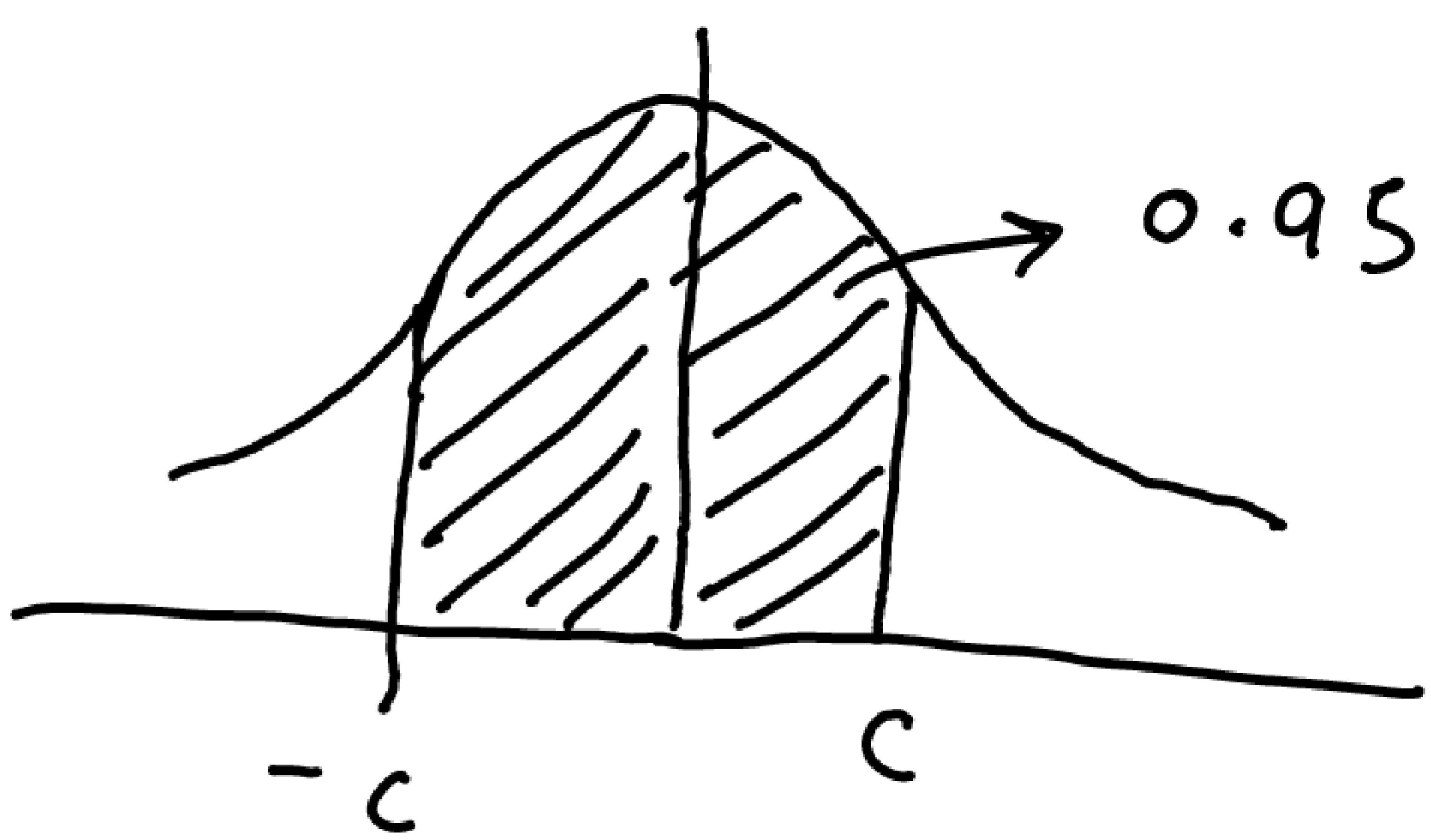
④ Find c, such that

$$\left.\begin{array}{l} P(-c < z < c) = 0.95 \\ P(|z| > c) = 0.01 \end{array}\right\} \rightarrow \text{SNV}$$

$$\left.\begin{array}{l} P(X > c) = 0.02 \\ P(X < c) = 0.05 \end{array}\right\}$$

Given $m = 120$, $\sigma = 10$

→



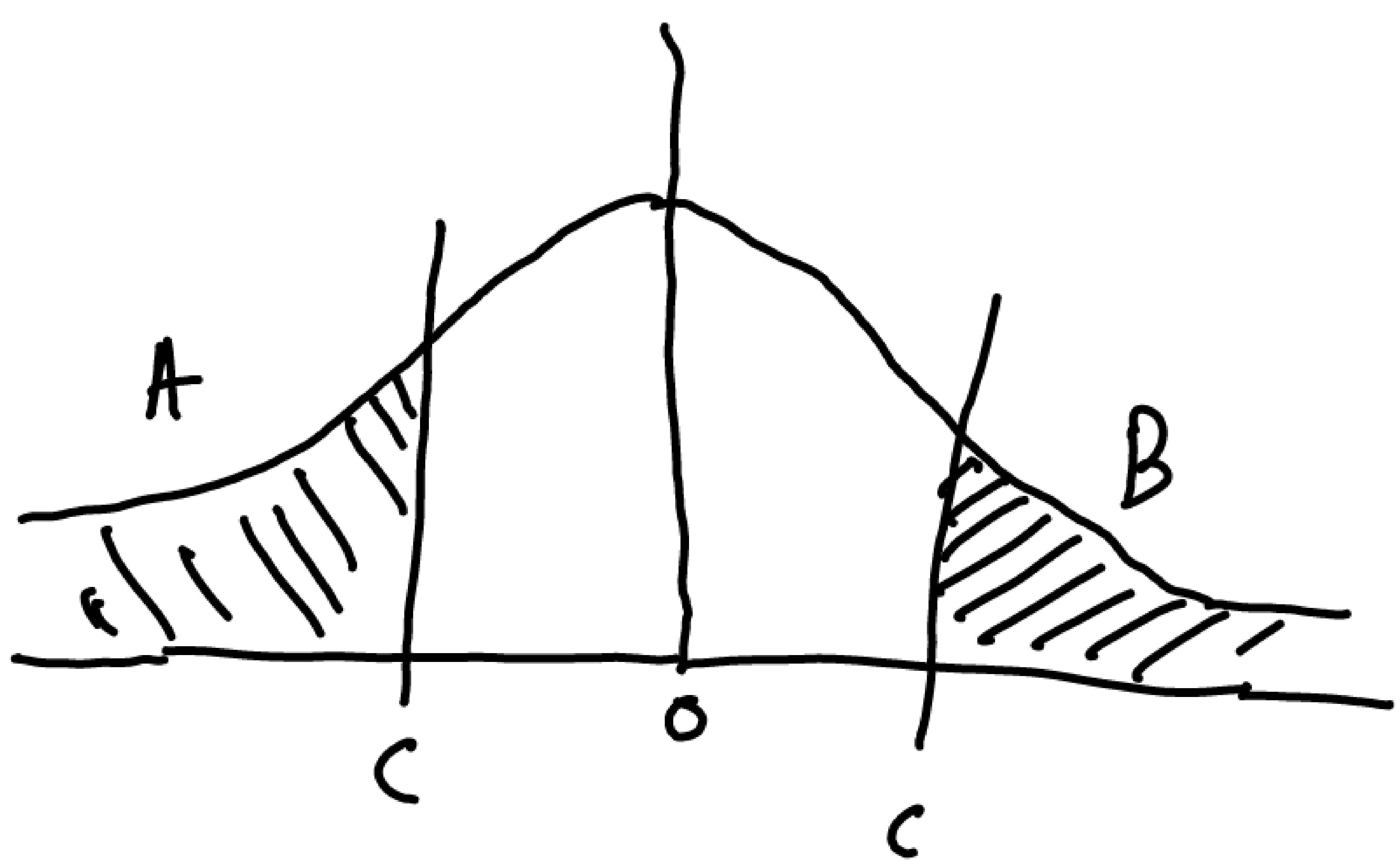
By Symmetry $2P(0 < x < c) = 0.95$

$\therefore P(0 < x < c) = 0.475$

$= 1.96$

(use table in reverse manner)

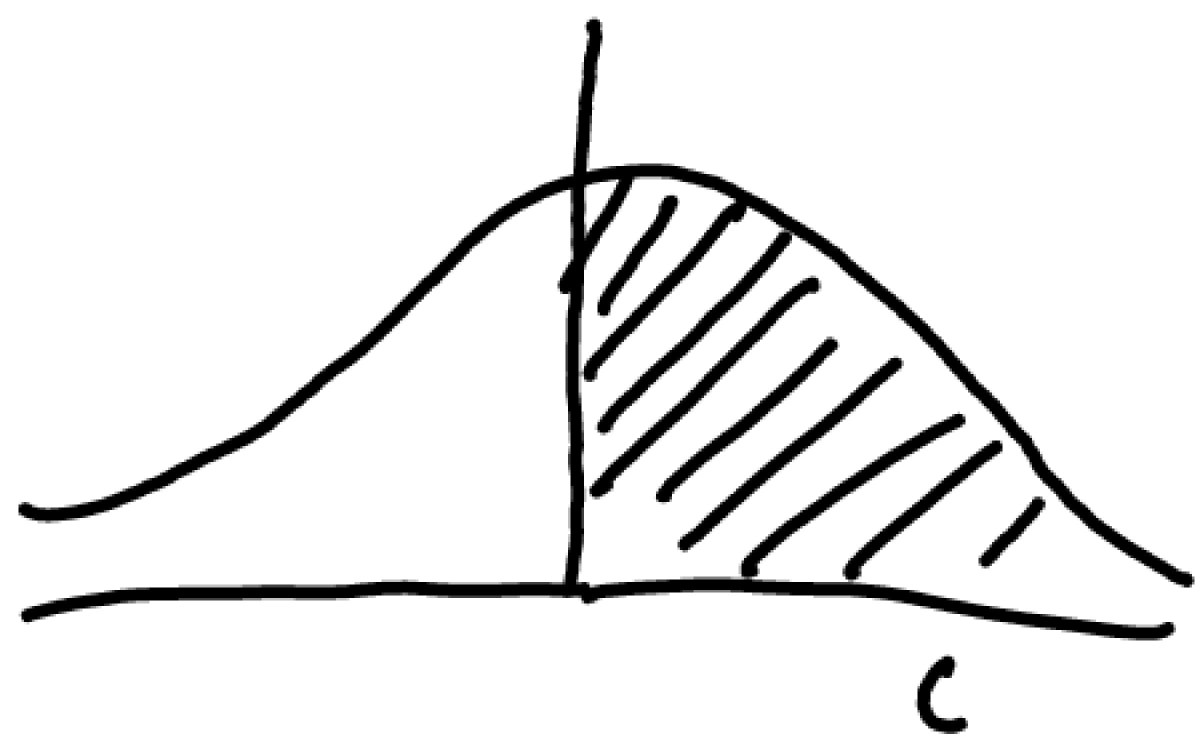
ii) $P(|z| > c) = 0.01$



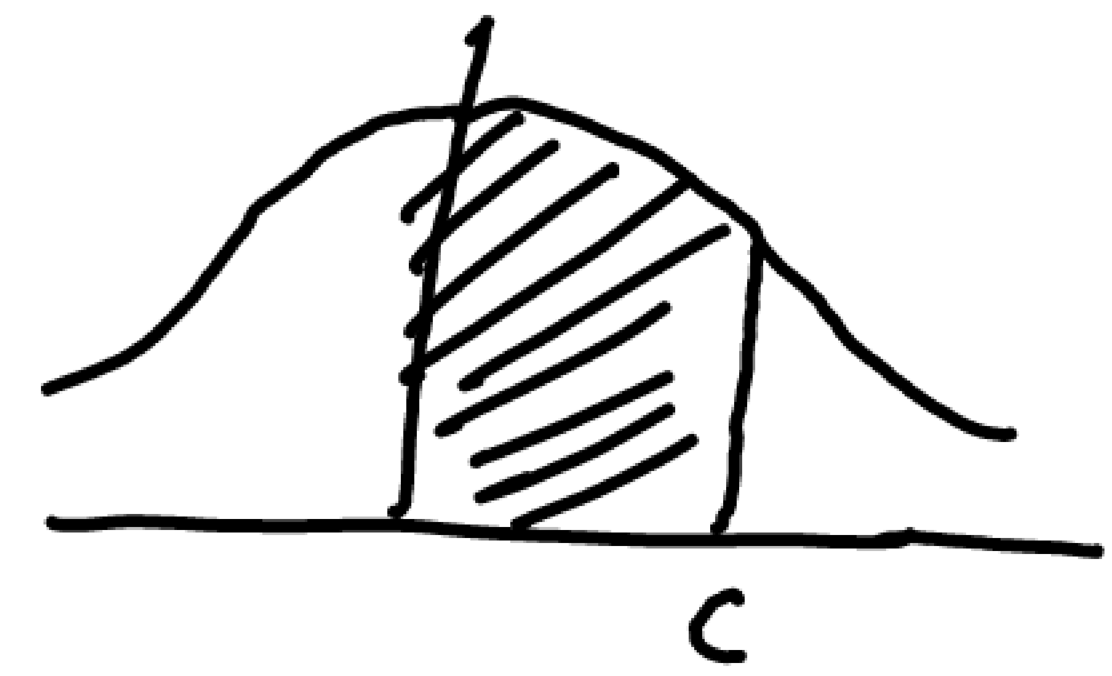
$P(|z| > c) = 0.01$

$\therefore A + B = 0.01$

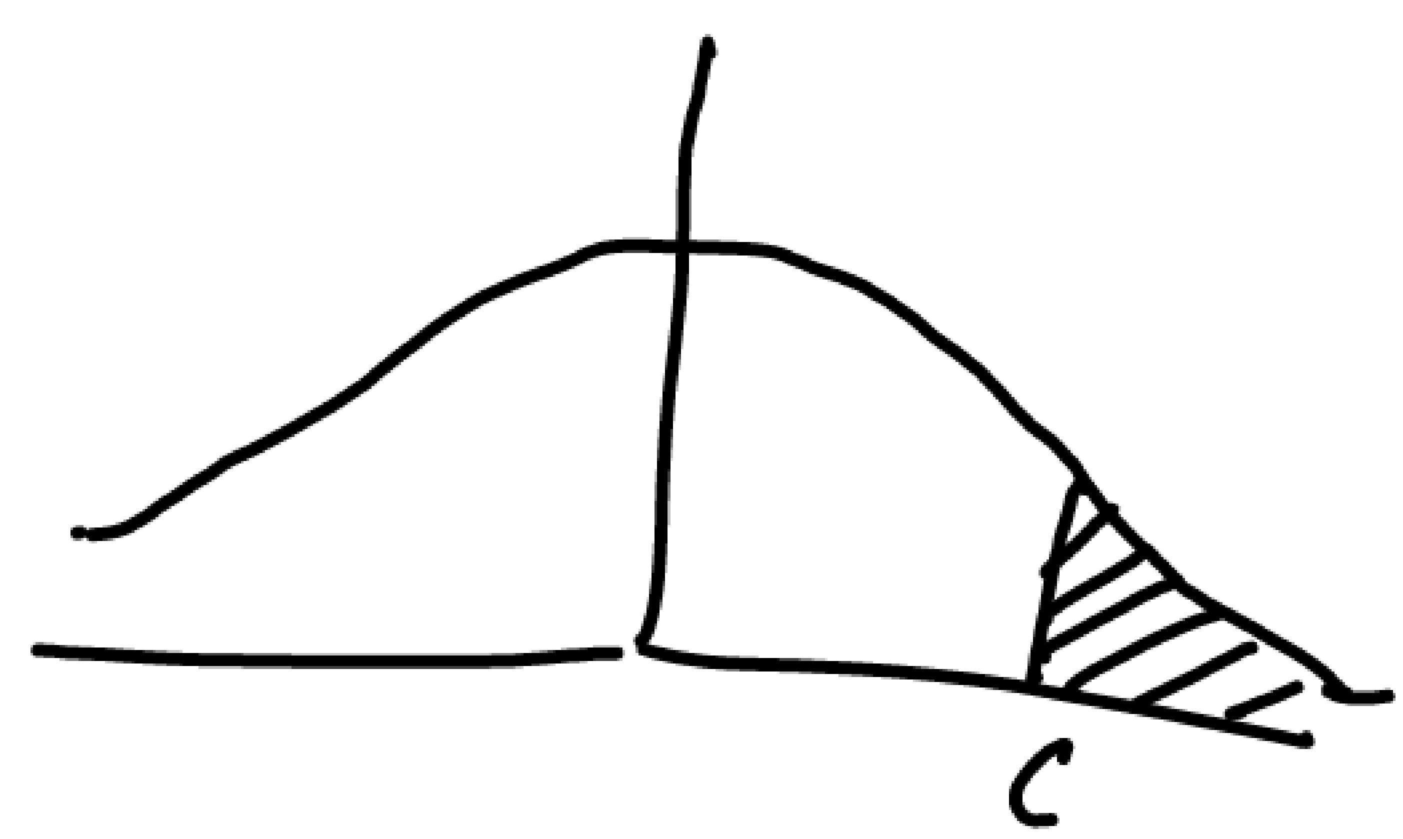
$\therefore B = 0.005$



$-$



$=$



$\therefore \quad 0.5 - 0.005 = 0.495$

$\therefore c = 2.58$ Using table

iii) Convert x to z

$$z = \frac{x - \sigma}{m}$$

$\sigma \to 10$

$m \to 120$

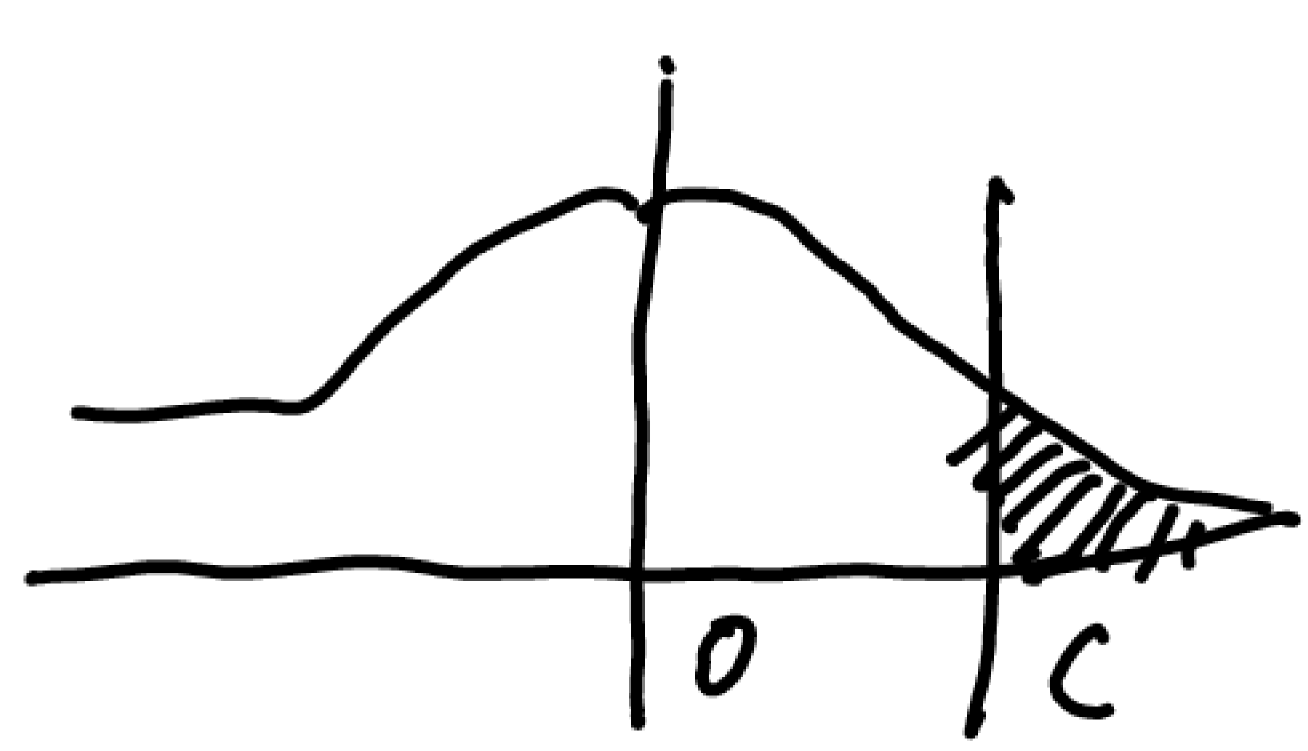
$x : c \quad \therefore z = \dfrac{c - 120}{10} = A$

$P(z > A) = 0.5 - 0.02 = 0.48$

$\therefore A = 2.06$

$\therefore c = (2.06 + 120) \times 10$

$= 140.6$



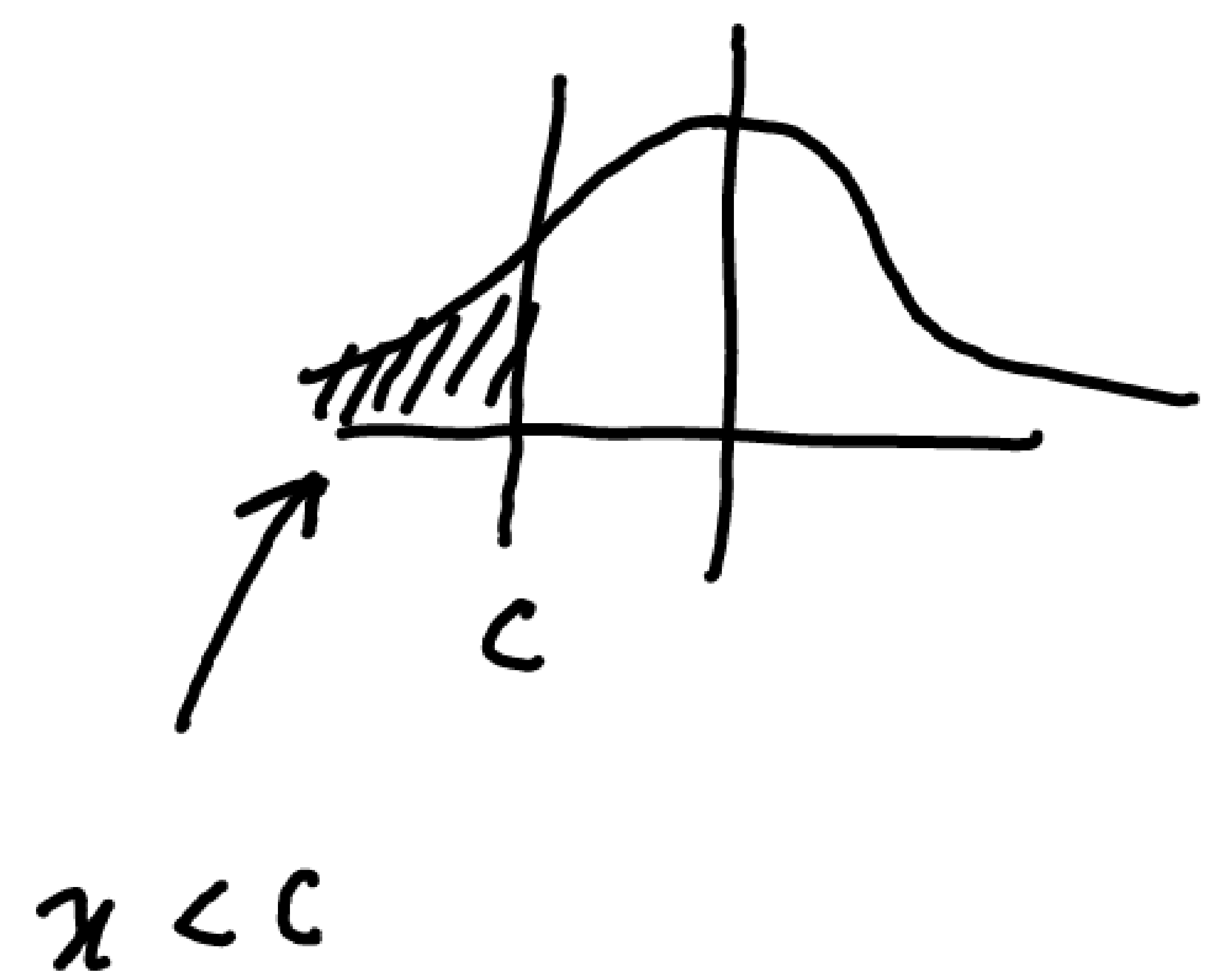
iv) $P(x < c) = 0.05$

$P(z < A) = 0.5 - 0.05 = 0.45$

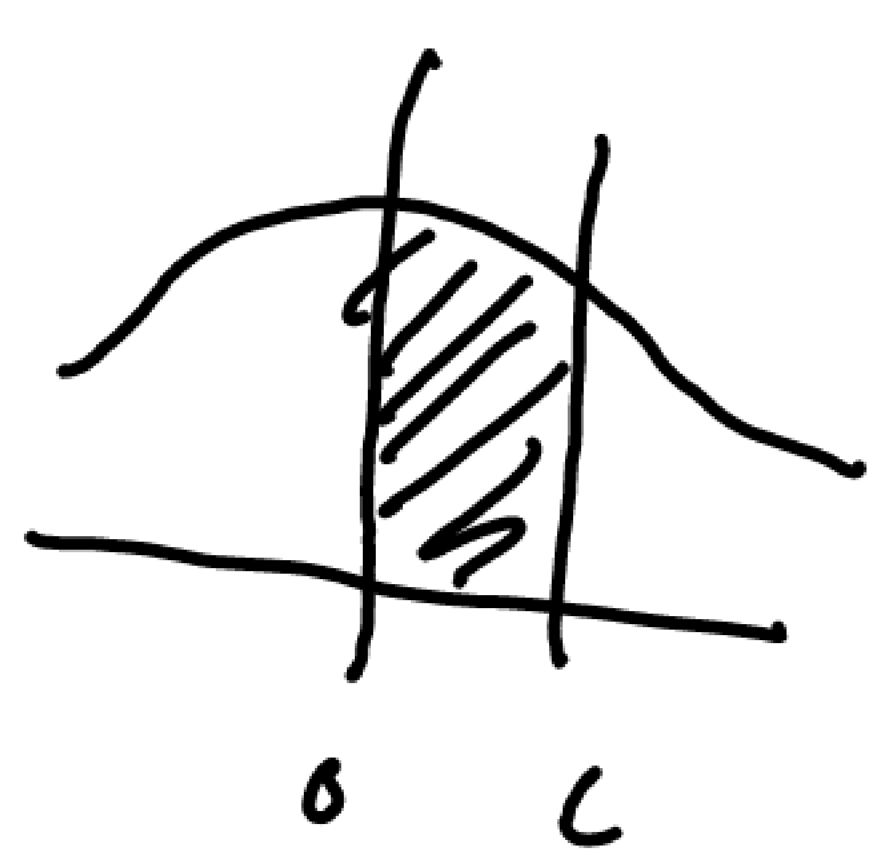
$\therefore A = 1.65$

$\frac{c-120}{16} = 1.63$

$\therefore c = 136.5$

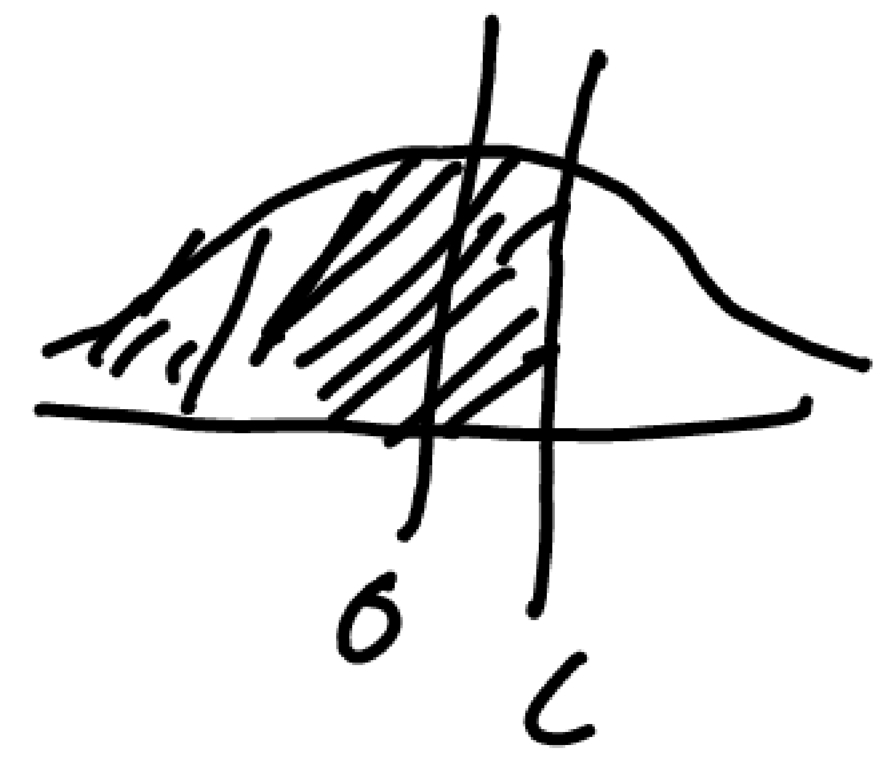


Why Not This ?



Because $0 < x < c$ is Not the case

Also c has to be less than 0 as area is less than 0.5



Hence this is also Not possible

Make sure to draw diagram & visualize before using formula

⑤ Monthly salary X is normally distributed with mean 3000 Rs & S.D. 250 Rs what is the minimum salary of a worker so that he belongs to top 5%

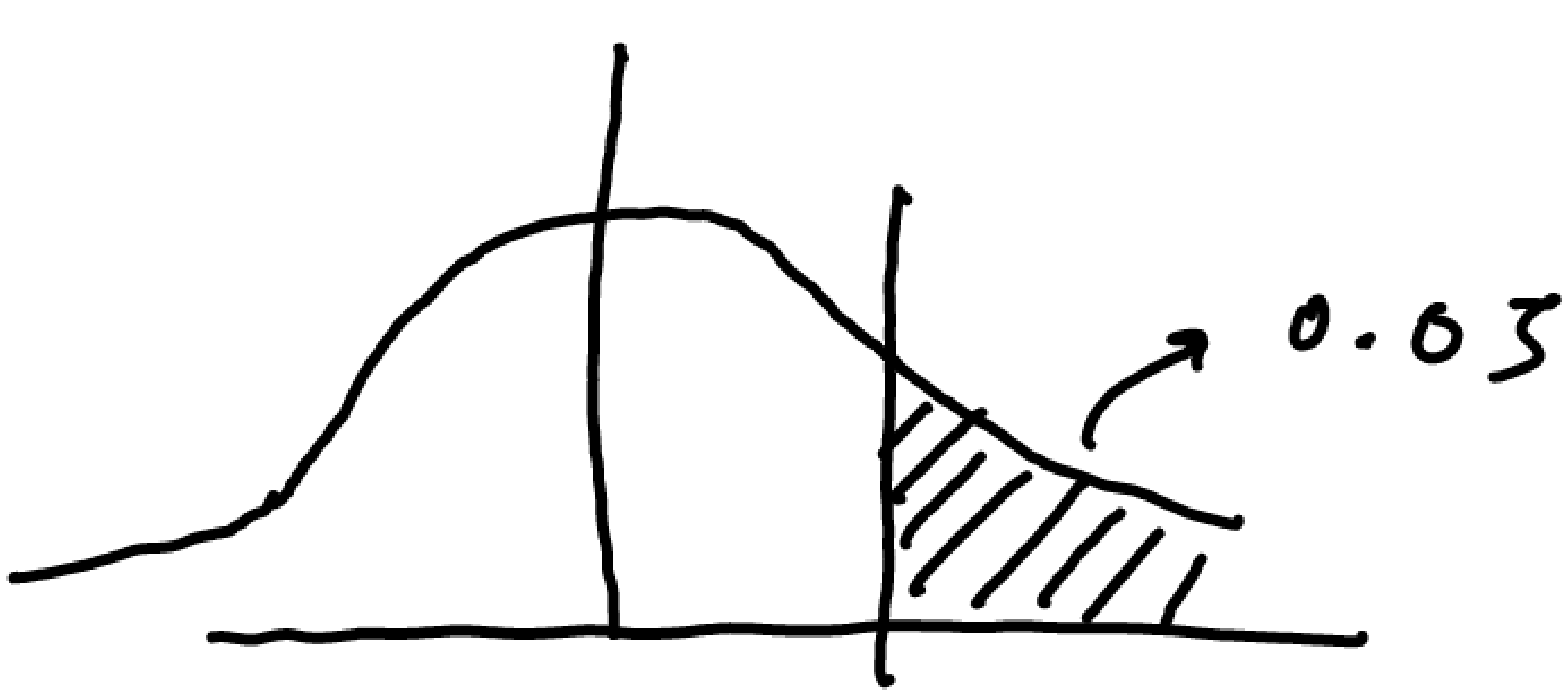
→

x = monthly salary of worker

x_1 = minimum salary of top 5%

Given $P(x \geq x_i) = 5\% = 0.05$

$$P\left(z > \frac{x_1 - m}{\sigma}\right) = 0.05$$



$$= 0.5 - 0.05 = 0.45$$

$$\frac{x_1 - m}{\sigma} = 1.65$$

$$\frac{x_i - 300}{250} = 1.65$$

$$x_1 = 3412.5$$

⑥ Scores of an exam are normally distributed mean = 527 and standered deviation 112 what is ⓐ probability of indivisual scoring Above 500 ⓑ Score for reaching 5% Top

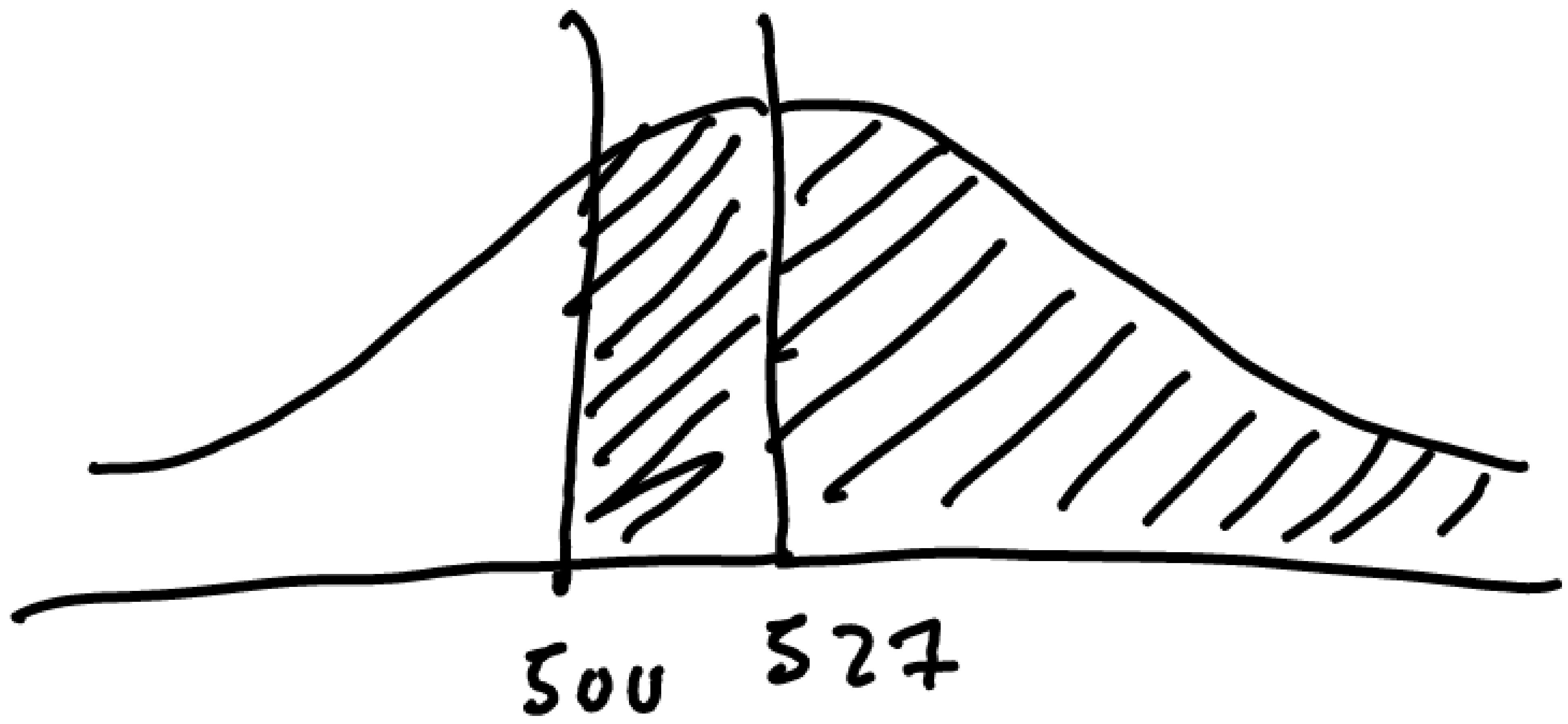
$\rightarrow$

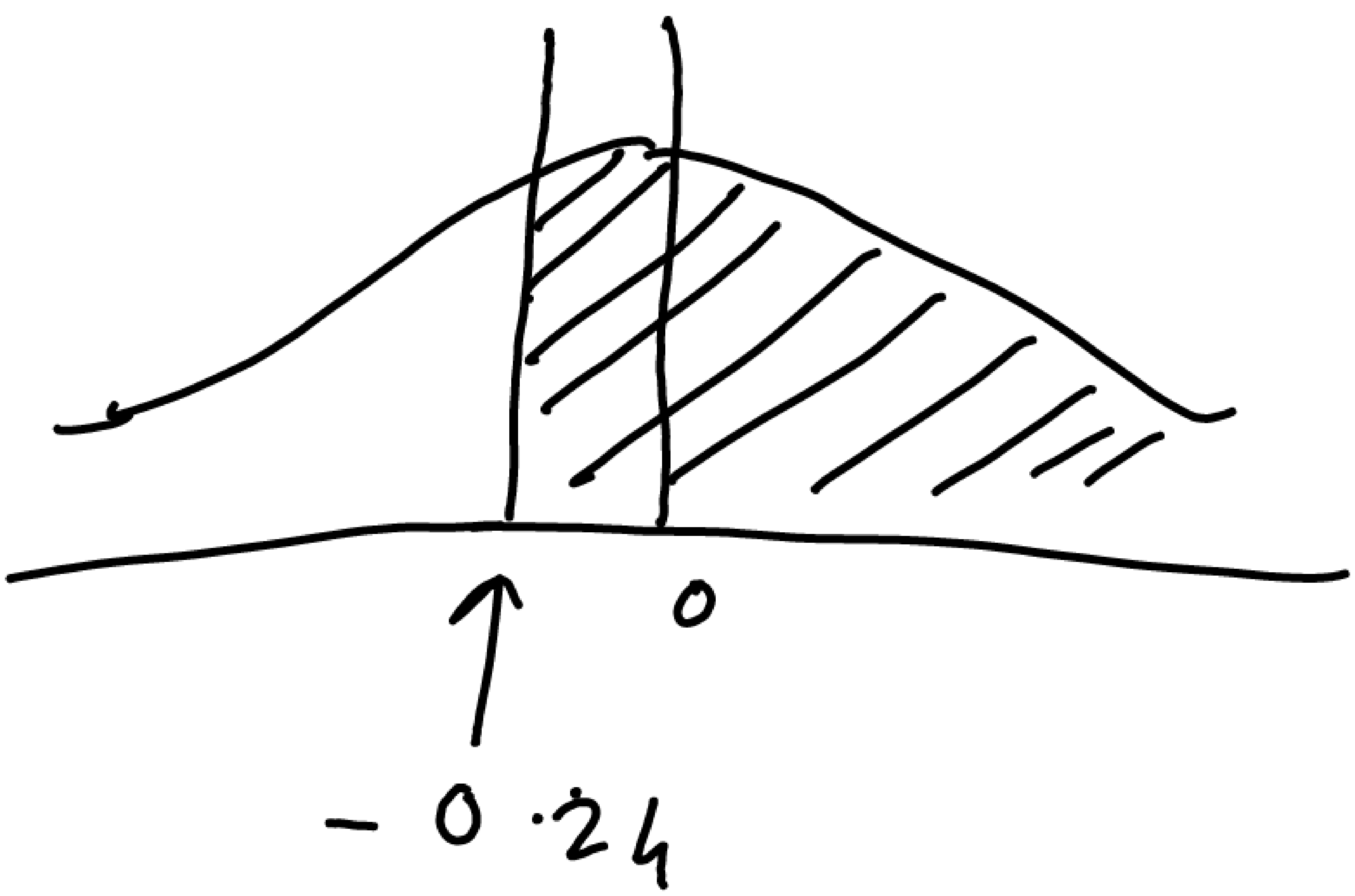
$$z = \frac{x - m}{\sigma}$$

$$P(x > 500) = ?$$

$x > 500$ means $z > \frac{500 - m}{\sigma}$

$$z > \frac{500 - 527}{112} = -0.241$$

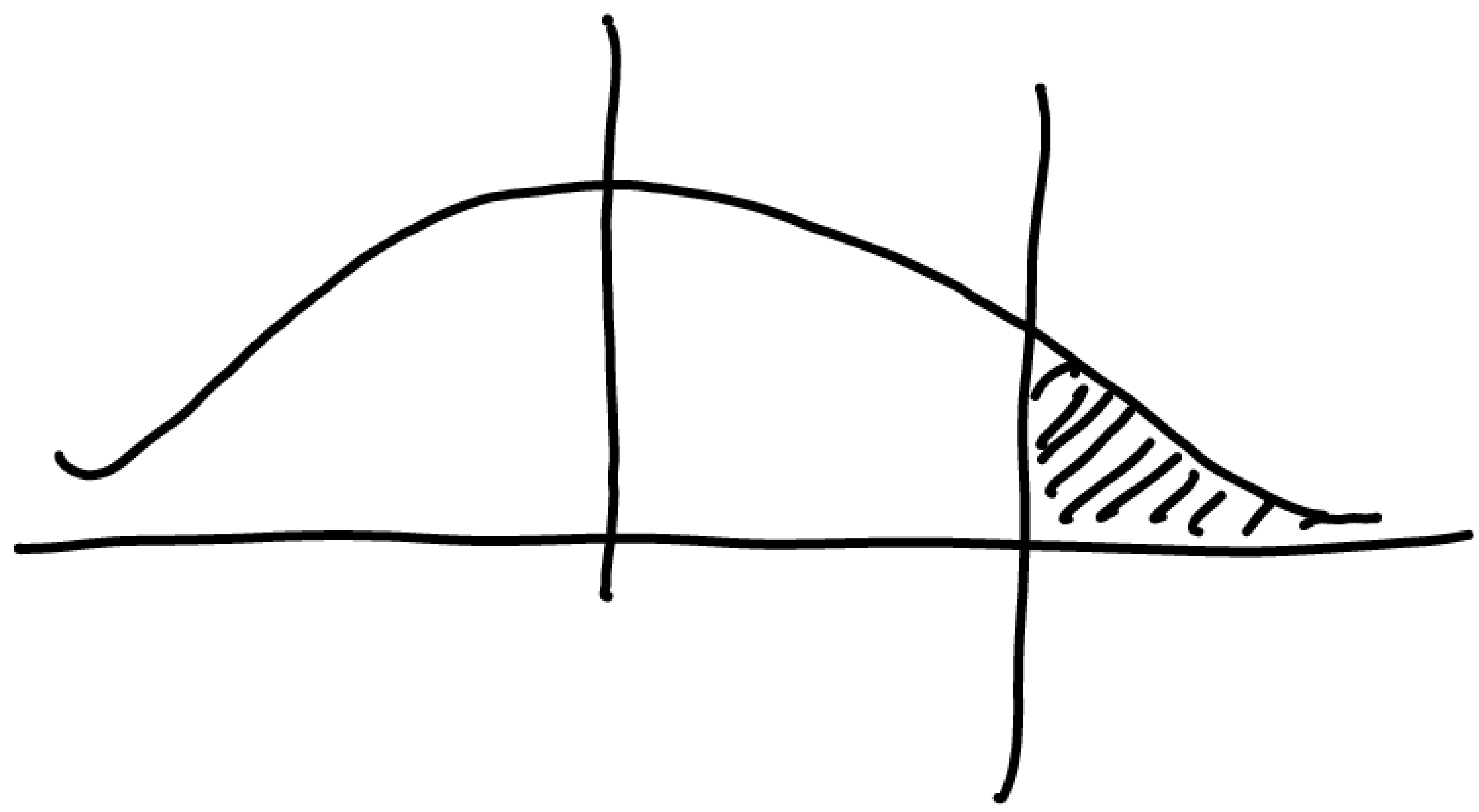




$= 0.5 + P(0 < z < 0.24)$

$= 0.5948$

Highest 5% $P(z > \;?\;) = 0.95$



0.05

$Z \to 1.645$ Inverse table

$\therefore X = Z\sigma + m$

$= 711.24$

⑦ diatemeter of can tops are normal distributed $\sigma = 0.05$ at what mean diameter the machine must be set so not more than 5 % of tops can be produced having diameter exceeding 3

$\longrightarrow$

$$P[x > 3] = \overset{0.5-}{0.05} \quad 5\%$$

$$z = \frac{x - m}{\sigma}$$

$$P\left(z > \frac{3 - m}{\sigma}\right) = \overset{0.5-}{0.05}$$

$$\therefore \quad \frac{3 - m}{0.05} = 1.65$$

$$\therefore m = 2.9175$$

⑧ If X_1, X_2 are two normal variales with

mean $m_1 = 30$ $m_2 = 25$

$\sigma_1^2 = 16$ $\sigma_2^2 = 12$

$Y = 3X_1 - 2X_2$ find $P(60 \le y \le 80)$

$m_y = 3m_1 - 2m_2$

$= 90 \quad 50 = \underline{\underline{40}}$

$\sigma_y^2 = 3 \times 16 + 2 \times 12$

$= 72$

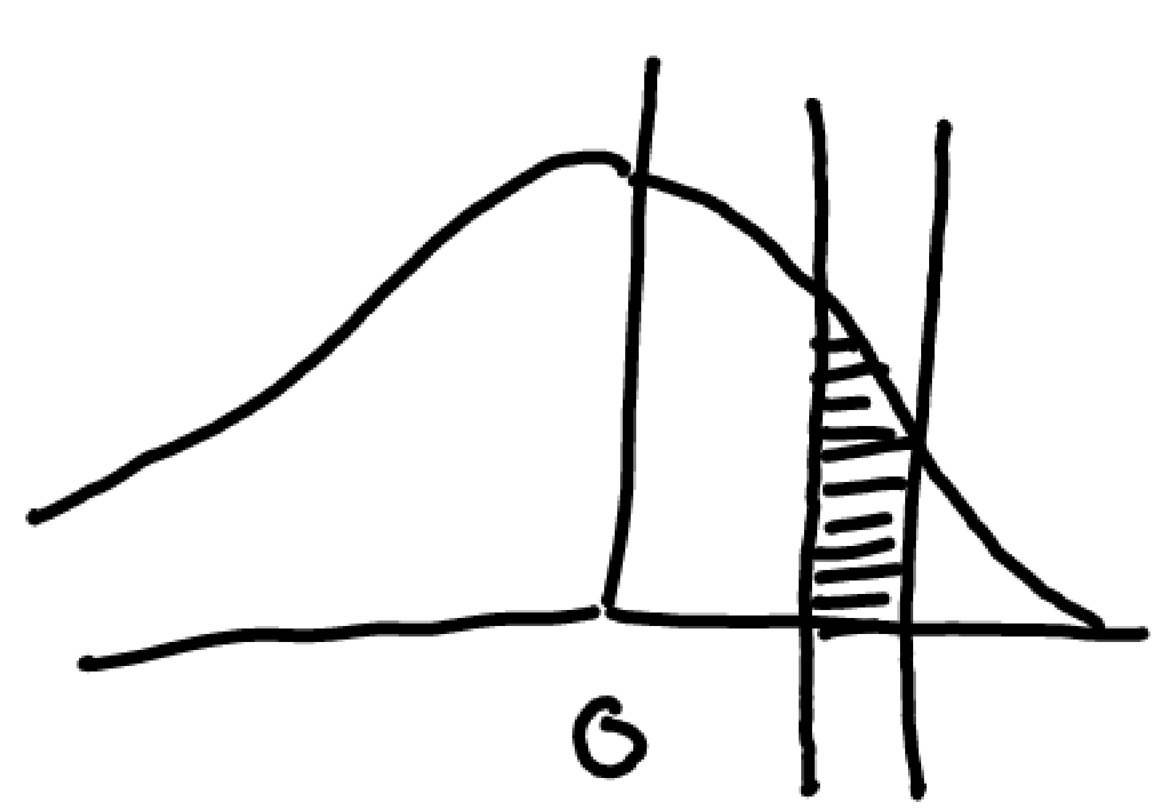
$\sigma_y = \underline{\underline{8.485}}$

$z = \dfrac{y - m}{\sigma}$

$P(60 \le y \le 80) =$

$P(2.35 \le z \le 4.714)$

$= A(4.714)$
$\quad - A(2.35)$



$= 0.0730$

⑨ If x & y are normally distributed with $m_1 = 52$, $\sigma_1 = 3$, $m_2 = 50$, $\sigma_2 = 2$ } find probability that randomly chosen x & y will differ by 1.7 or more.

$\rightarrow$ $P(|x-y| \geq 1.7)$ (| mod| as $x-y$ and $y-x$ both cases must be considered)

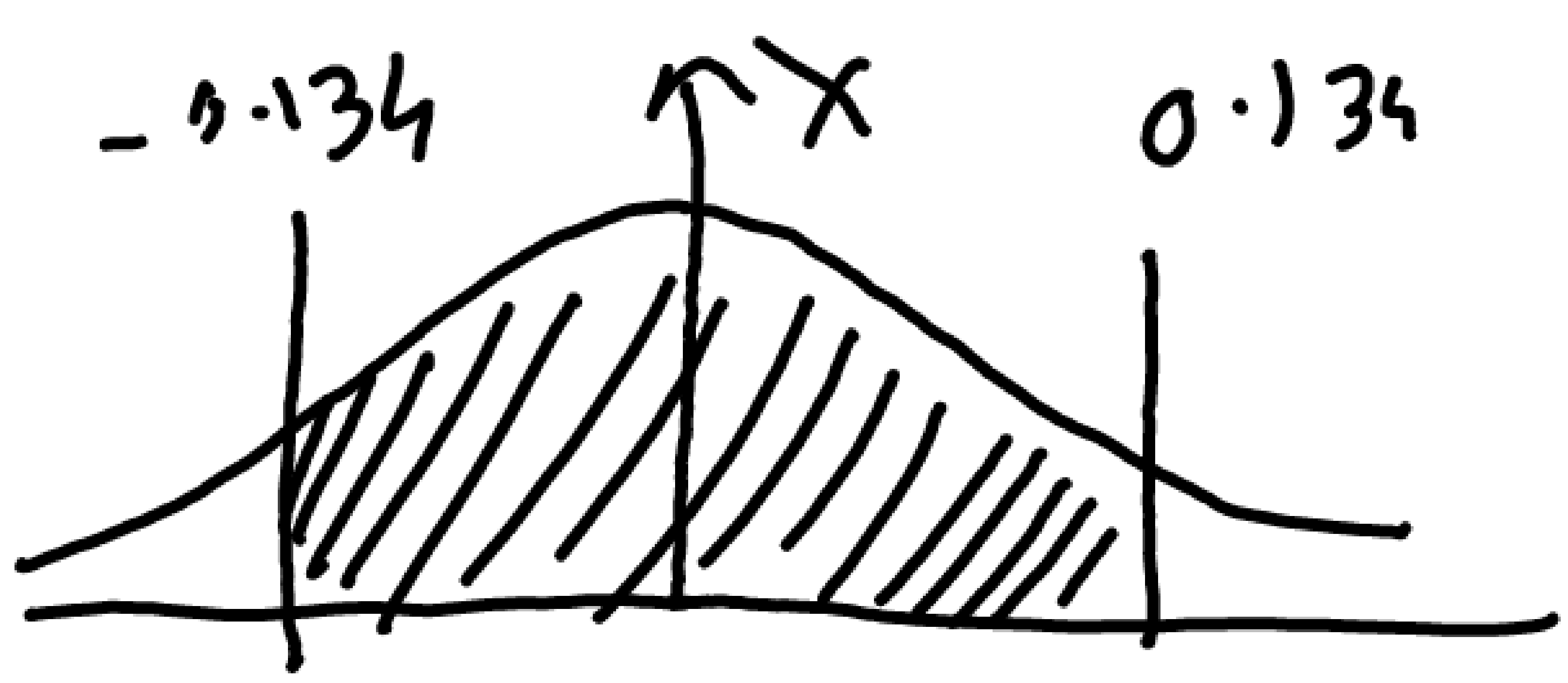
Let $u = x - y$

$m = 52 - 50 = 2$

$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} = 3.608$

$P(|U| \geq 1.7)$

$= P\left(|Z| \geq \frac{1.7 - 2}{2.23}\right) = P(|Z| \geq -0.134)$



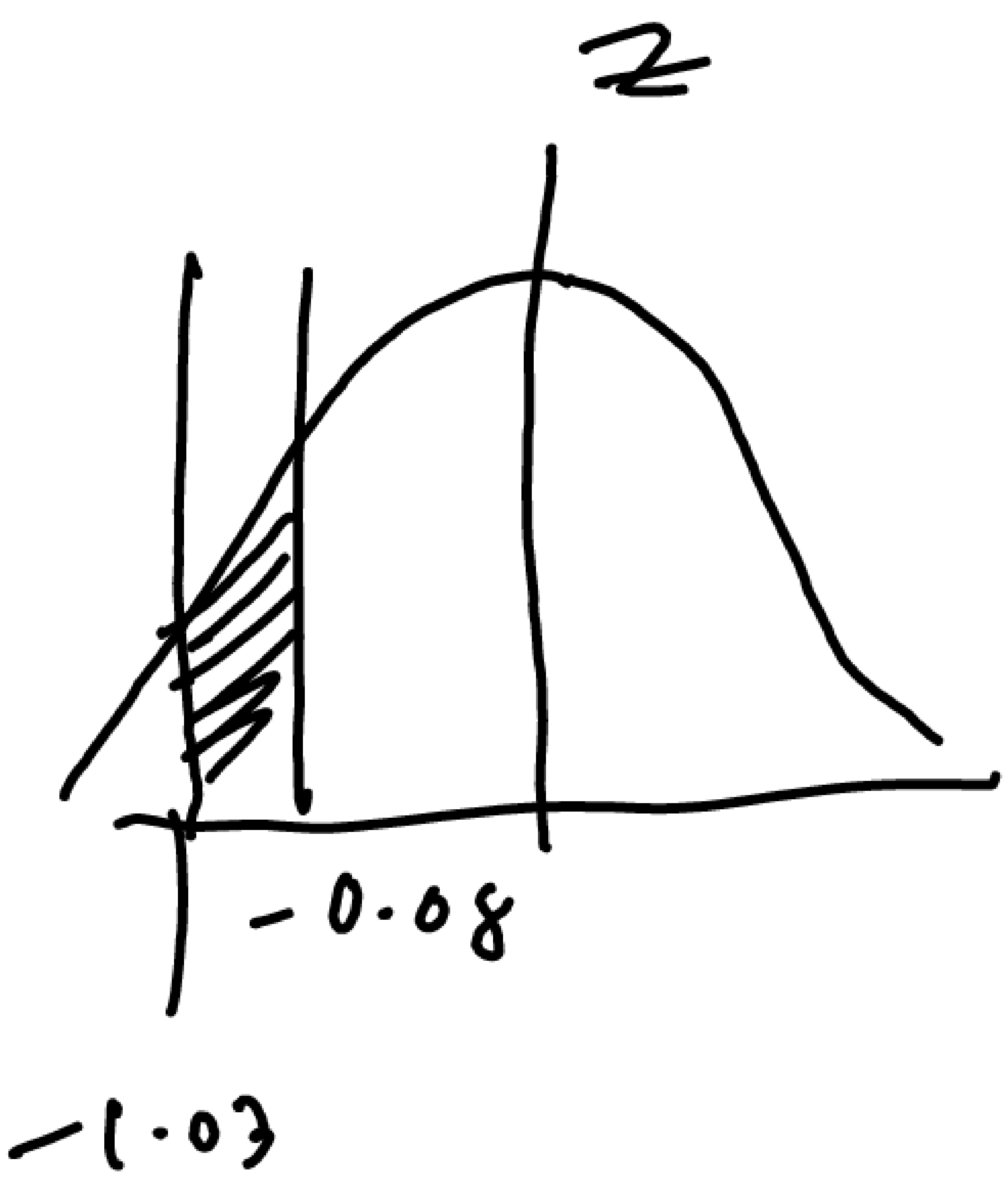
convert to $\longrightarrow$ Z

$$= 1 - P(|U| < 1.7)$$

$$= 1 - P(-1.7 < U < 1.7)$$

$$= 1 - P(-1.03 < z < 0.08)$$

$$= 1 - [P(0 < z < 1.03 - P(0 < z < 0.05)]$$



(10) $m_1 = 8 \quad \sigma_1 = 2$

$m_2 = 12 \quad \sigma_2 = 4\sqrt{3}$

find α such that $P[(2x-y) \leq 2\alpha] = P[(x+2y) \geq 3\alpha]$

let

$u = 2x - y \qquad v = x + 2y$

$m_u = 4 \qquad m_v = 32$

$\sigma_u = 64 \qquad \sigma_v = 196$

(using linearity property)

$$P(U \leq 2\alpha) = P(V \geq 3\alpha)$$

$$P\left(z \leq \frac{2\alpha - 4}{8}\right) = P\left(z \geq \frac{3\alpha - 32}{14}\right)$$

using symmetry

$$= P\left(z \leq -\left(\frac{3\alpha - 32}{14}\right)\right)$$

$$\therefore \frac{2\alpha - 4}{8} = -\left(\frac{3\alpha - 32}{14}\right)$$

$$\therefore \alpha = 6$$

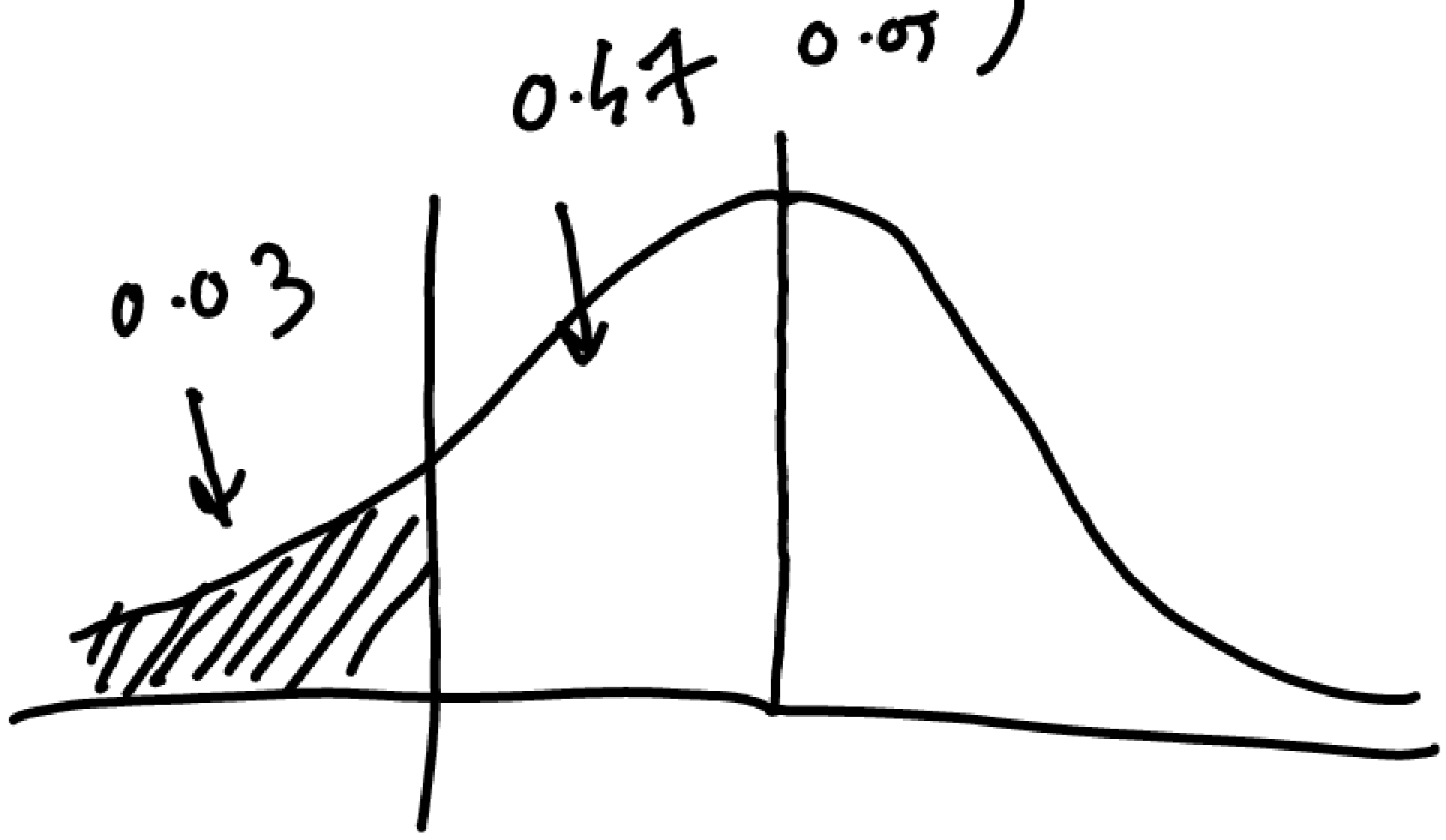
$$\boxed{\begin{aligned} P(Z \geq c) &= 1 - P(Z \leq c) \\ &= P(Z \leq -c) \end{aligned}}$$

(10) If actual amount of coffee which machine puts into 6 ounce jar is a random variable having normal dist. $\sigma = 0.05$ ounce if only 3% of jars are to contain less than 6 ounce of coffee what must be mean fill.

$$P(x < 6) = 3\%$$

$$\therefore P\left(z < \frac{6-m}{0.05}\right) = 0.03$$

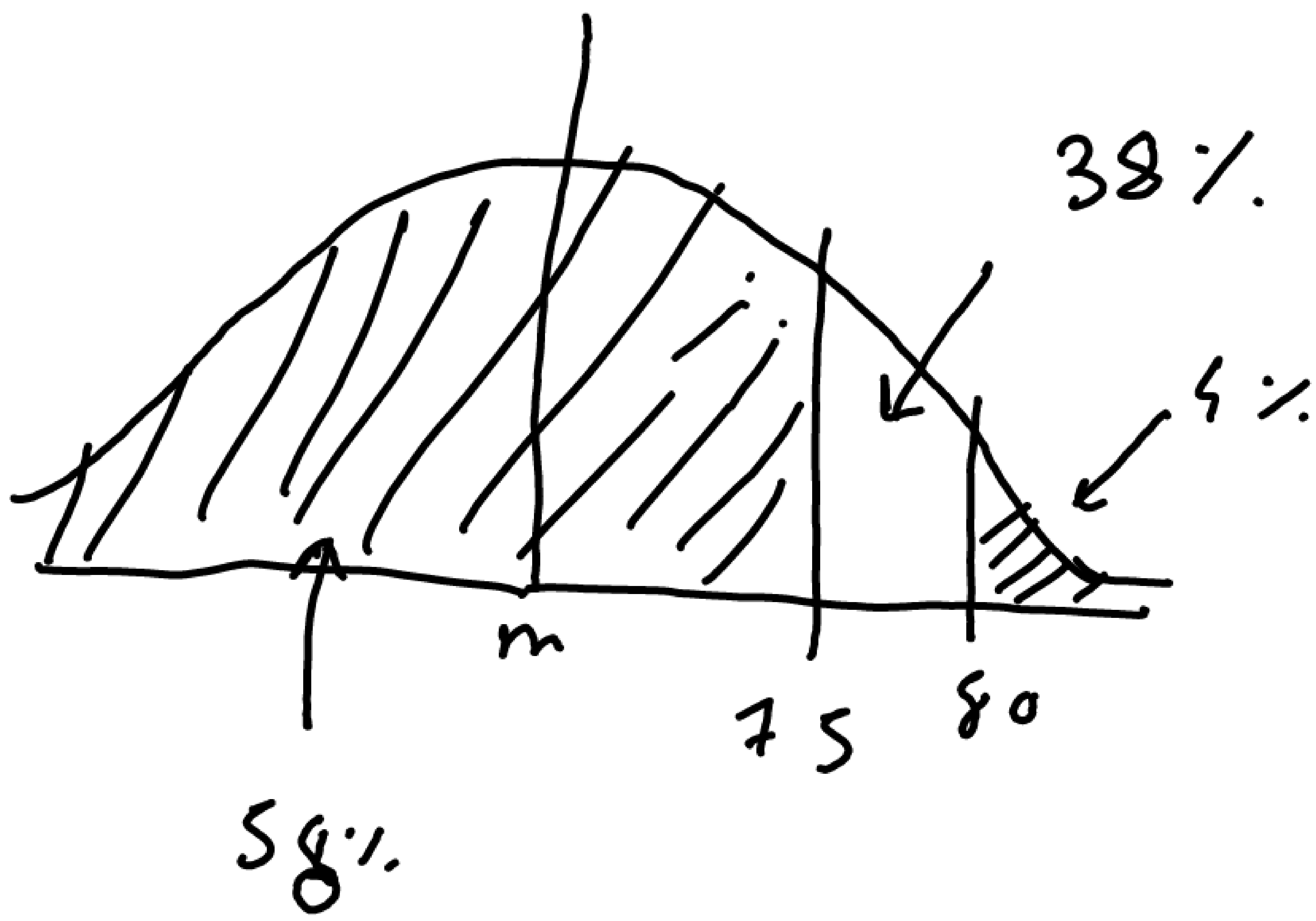
$$P\left(-\infty \le z \le \frac{6-m}{0.05}\right) = 0.5 - 0.03 = 0.47$$



put -ve sign as $\frac{6-m}{0.05}$ is on LHS

$$\therefore \frac{6-m}{0.05} = -1.89 \quad \therefore m = 6.09$$

(11) Find mean & s.d. of marks where 58% of candidates get below 75, 4% above 80 and rest between



as curve is symmetric, 50% is left half

$\therefore P(m < x < 75) = 8\%$

$\therefore P\left(0 < z < \frac{75-m}{\sigma}\right) = 0.08$

$\therefore \frac{75-m}{\sigma} = 0.02$ — (1)

$P(z > 80) = 4\% = 0.04$

$P\left(0 < z < \frac{80-m}{\sigma}\right) = 0.5 - 0.04$

$= 0.46$

$\therefore \frac{80-m}{\sigma} = 1.76$ — 2

from 1 & 2

$\therefore m = 74.35$

$\therefore \sigma = 3.21$