Conditional prob.

If X & Y is two random variables when $f(x,y)$ assigns point (x,y) is called 2 D random variable

2 D PMF

If X & Y are 2 D discrete variables P_{ij} satisfies condition i) $P(x_i, y_j) \geq 0 \ \forall i,j$

ii) $\sum_{j=1}^{\infty} \sum_{i=1}^{\infty} P(x_i y_j) = 1$

Then P is joint PMF & probability distribution of x, y is given by

$x \backslash Y$	y_1	y_2	y_3	.....	y_m	Total
x_1	P_{11}	P_{12}	P_{13}	....	P_{1m}	P_1
x_2	P_{21}	P_{22}	P_{23}	....	P_{2m}	P_2
x_3	P_{31}	P_{32}	P_{33}	.....	P_{3m}	P_3
$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	
x_n	P_{n1}	P_{n2}	P_{n3}		P_{nm}	P_n
Total	P_1'	P_2'	P_3'	....	P_m'	1

Marginal probability distribution of x is given by

X: x_1 x_2 .... x_n

P: P_1 P_2 .... P_n

$$P_1 = \sum_{j=1}^{n} P_{ij}$$

Y: y_1 y_2 .... y_m

P: P_1' P_2' .... P_m'

$$P_1 = \sum_{j=1}^{m} P_{ij} \quad , \quad P_1' = \sum_{i=1}^{n} P_{ij}$$

$$P(x=x_i \mid y=y_i) = \frac{P(x=x_i \quad y=y_j)}{P(y=y_i)}$$

① The joint PMF of X & Y $P(X=x, Y=y) = \frac{x+3y}{24}$

$x, y = 1, 2$

find joint probability distribution and marginal probability distribution of x & y.

→ Joint P.D.

y \ x	1	2	
1	1/6	5/24	9/24
2	7/24	1/3	15/24
Total	11/24	13/24	1

9/24 (By substituting values)

Marginal P.D.

X :	1	2
P(x):	11/24	13/24

y :	1	2
P(y):	9/24	15/24

② Joint PMF of x_1 & x_2 is given by $P(X=x_1, X_2=x_2) = \frac{1}{27}(x_1 + 2x_2)$

$x_1, x_2 \in (0,1,2)$

find PMF of X_1 & X_2 in terms of x_1 & x_2

→ We have to find marginal probability

$$P(X=x_1) = \sum_{x_2=0}^{2} P(x_1, x_2)$$

$$= \frac{1}{27}\left(x_1 + (x_1+2) + (x_1+4)\right)$$

$$= \frac{x_1+2}{9}$$

$$P(X_2 = x_2) = \sum_{x_1=0}^{2} P(x_1, x_2)$$

$$= \frac{1}{27}\left(2x_2 + (1+2x_2) + (2+2x_2)\right)$$

$$= \frac{1}{27}(3 + 6x_2)$$

$$= \frac{(1+2x_2)}{9}.$$

③ from a box containing 2 white, 4 black and 3 red balls 3 balls are removed. If X denote no. of white balls drawn and Y denotes no. of red balls drawn, find joint probability distribution of X, Y., $P(X \le 1)$, $P(X \le 1, Y \le 2)$, $P(Y \le 2 \mid X \le 1)$, $P(X+Y \le 2)$, $P(Y \le 2)$, Marginal prob & check if X & Y are independant

→

x \ y	0	1	2	3	Σ
0	1/21	3/14	1/7	1/84	0.41
1	1/7	2/7	1/14	0	0.5
2	1/21	1/28	0	0	0.083
Σ	0.23	0.53	0.21	0.0119	1

$P(0, 0) =$ 0 white balls drawn, 0 red balls drawn
ie 2 black balls drawn

$$= \frac{{}^4C_3}{{}^9C_3} = 0.0476$$

$P(1, 0) =$ 1 white ball & 0 red ball ie 1 white 0 red, 2 black

$$= \frac{{}^3C_1 \cdot {}^4C_2}{{}^9C_3} = 0.214$$

$P(1, 1) =$ 1 white ball & 1 red ball

$$= \frac{{}^3C_1\, {}^4C_1\, {}^2C_1}{{}^9C_3}$$

$P(1, 3) =$ 1 white & 3 red. Can't exist

	0	1	2	3
	✓	✓	✓	
	✓	✓	✓	

i) $P(x \le 1) = P(x=0) + P(x=1) \quad \cancel{= 0.261} \quad 0.91$

$= 0.41 + 0.5$

ii) $P(x \le 1, y \le 2) = \sum_{j=0}^{2} P(x=0, y=j) + \sum_{j=0}^{2} (P(x=1), y=j)$

$= 1/21 + 3/14 + 1/7$

$+ 4/7 + 2/7 + 1/14 = 0.904$

iii) $P(y \le 2 | x \le 1) = \dfrac{P(y \le 2, x \le 1)}{P(X \le 2)} = \dfrac{0.904}{0.91} = 0.993$

iv) $P(y \le 2) = 0.23 + 0.53 + 0.21$

$= 0.97$

v) $P(x+y \le 2) = \sum_{j=0}^{2} P(x=0, y=j)$

$+ \sum_{j=0}^{1} P(x=1, y=j)$

$+ P(x=2, y=0)$

	0	1	2	3
0	✓	✓	✓	
1	✓	✓		
2	✓			

$= P(0,0) + P(0,1) + P(0,2)$

$+ P(1,0) + P(1,1)$

$+ P(2,0)$ ~~#~~

$= 0.880$

vi) Marginal prob.

X:	0	1	2
P	0.41	0.5	0.83

y:	0	1	2	3
P :	0.23	0.53	0.21	0.0119

vii) $P(X, y) = (P(x))(P(y))$

$P(0,1) = 3/14$ Not valid.

$P(x=0) = 35/84$

$P(y=1) = 15/28$

2D Continuous Probability

Let X, Y be two dimensional continuous random variable and let $f_{xy}(x,y)$ be function of x, y such that

ⓐ $f_{xy}(x,y) \geq 0$

ⓑ $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{xy}(x,y)\,dx\,dy = 1$

ⓒ $\int_a^b \int_c^d f_{xy}(x,y)\,dx\,dy = P(a \leq x \leq b, c \leq y \leq d)$

Marginal PDF of x is

$$f_x(x) = \int_{-\infty}^{\infty} f_{xy}(x,y)\,dy$$

$f_x(x) \geq 0$

$\int_{-\infty}^{\infty} f_x\,dx = 1$

Marginal PDF of y is

$$f_y(y) = \int_{-\infty}^{\infty} f_{xy}(x,y)\,dx$$

$f_y(y) \geq 0$

$\int_{-\infty}^{\infty} f_y\,dx = 1$

Example

① 2D random variable x, y has joint PDF

$$f_{xy}(x,y) \begin{cases} 15e^{-3x-5y} & x>0\ y>0 \\ 0 & o/w \end{cases}$$

find ⓐ ~~$P(x>1)$~~ $P(1<x<2, 0.2<y<0.3)$

ⓑ $P(x<2, y>0.2)$

ⓒ Marginal probability distribution

ⓐ $P(1<x<2, 0.2<y<0.3) = \int_{y=0.2}^{0.3}\int_{x=1}^{2} f(x,y)\,dx\,dy$

$$= \int_{0.2}^{0.3}\int_1^2 15e^{-3x-5y}\,dx\,dy$$

$$= \int_{0.2}^{0.3} 15e^{-5y}\int_1^2 e^{-3x}\,dx\,dy = \int_{0.2}^{0.3} 15e^{-5y}\left(\frac{e^{-6x}-e^{-3x}}{-3}\right)$$

$$= \frac{15}{(-5)(-3)} \cdot \left(e^{-0.15}y - e^{-1}\right)\left(e^{-6} - e^{-3}\right)$$

$$= \cancel{0.0528} \quad 0.00684$$

ii) $P(x<2,\ y>0.2)$

$$\int_{0.2}^{\infty} \int_{-\infty}^{2} f(x,y)\, dx\, dy$$

$$= \left[\frac{e^{-5y}}{-5}\right]_{0.2}^{\infty} \left[e^{-3x}\right]_{-\infty}^{2}$$

$$= -0.367$$

iii) Marginal probability

$$f_x(x) = \int_{-\infty}^{\infty} f_{xy}(x,y)\, dx$$

$$= 15e^{-5y} \int_{-\infty}^{\infty} e^{-3x}\, dx$$

$$= +5e^{-5y} \quad \forall y>0, \quad 0 \quad o/w$$

$$f_y(x) = \int_{-\infty}^{\infty} f_{xy}(x,y)\, dy$$

$$= 15e^{-3x} \left[\frac{e^{-5y}}{-5}\right]_0^{\infty}$$

$$f_y(x) \begin{cases} \to +3e^{-3x} & x>0 \\ \to 0 & o/w \end{cases}$$

② $f_{XY}(x,y) = \begin{cases} cx(x-y) & 0<x<2, \; -x<y<x \\ 0 & \text{o/w} \end{cases}$

Evaluate c, $f_X(x)$, $f_{Y/X}(y/x)$

i)

$$\int_0^2 \int_{-x}^{x} cx(x-y)\, dy\, dx$$

$$\int_0^2 \left[cx^2y - \frac{cxy^2}{2}\right]_{-x}^{x} dx$$

$$= \int_0^2 \left(cx^3 - \frac{cx^3}{2} + x^3c + \frac{x^3}{2}c\right) dx$$

$$= 2\left[\frac{cx^4}{4}\right]_0^2$$

$$= \frac{c}{8}(16) = 2c = 1$$

$\therefore c = 1/8$

ii) $f_X(x) = \int_{-x}^{x} cx(x-y)\, dy$

$$= cx\left[xy - \frac{y^2}{2}\right]_{-x}^{x}$$

$$= cx\left(x^2 - \frac{x^2}{2} + x^2 + \frac{x^2}{2}\right)$$

$f_X(x) = 2cx^3 = x^3/4$

iii) $f_{Y/X}(y/x) = \dfrac{f_{XY}(x,y)}{f(x)} = \dfrac{1/8\, x(x-y)}{x^3/4} = \dfrac{x-y}{x^2}$

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Grade: AA/ AB / BB / BC / CC / CD / DD
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(3) If $f_{xy}(x,y) = xy^2 + \frac{x^2}{8}$ $0 \le x \le 2$

$\& \; 0 \le y \le 1$, 0 o/w

find
$P(x>1)$, $P(y<1/2)$, $P(x>1, y<1/2)$, ~~$P(y<1/2 \mid x>1)$~~
$P(x<y)$, $P(x+y \le 1)$, $P(x>1 \mid y<1/2)$

(a) $\int_0^1\int_1^2 \left(xy^2 + \frac{x^2}{8}\right) dx\,dy = \frac{19}{24}$

(b) $\int_0^{1/2}\int_0^2 \left(xy^2 + \frac{x^2}{8}\right) dx\,dy = \int_0^{1/2} \left[\frac{x^2}{2}y^2 + \frac{x^3}{24}\right]_0^2 dy$

$= \int_0^{1/2} 2y^2 + \frac{1}{3}\, dy = \left[\frac{2y^3}{3} + \frac{y}{3}\right]_0^{1/2}$

$= \frac{1}{12} + \frac{1}{6} = \frac{1}{4}$

(c) $\int_0^{1/2} \left[2y^2 + 1/3 - \frac{y^2}{2} - \frac{1}{24}\right]_1^2 dx$

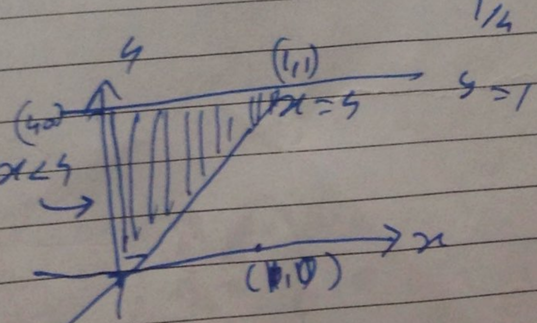
$\left(\int_0^{1/2}\int_1^2 f(x,y)\,dx\,dy\right)$

$= 5/24$

(d) ~~$P(y<1/2 \mid x>1)$~~ $P(x>1 \mid y<1/2) = \frac{P(x>1, y<1/2)}{P(y<1/2)}$

$= \frac{5/24}{1/4} = \frac{5}{6}$

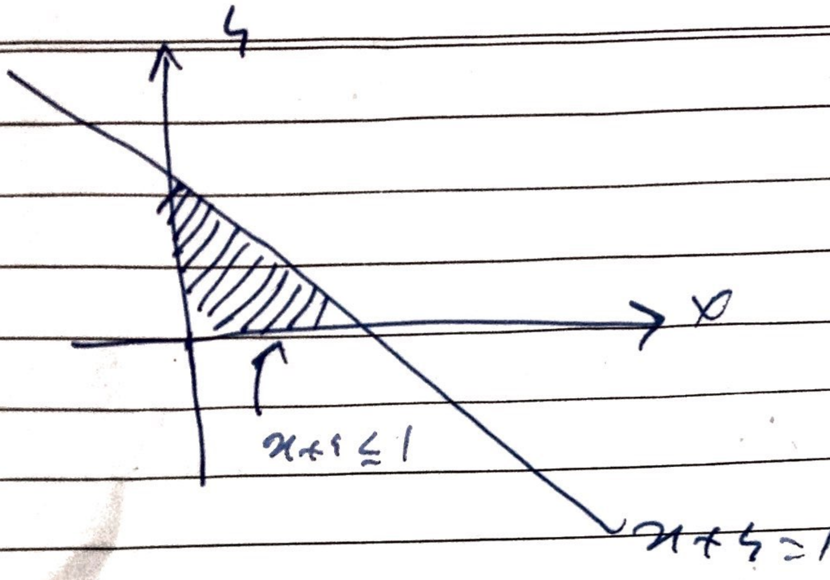
(e) $P(x>y)$



$\int_0^1\int_x^1 \left(xy^2 + \frac{x^2}{8}\right) dy\,dx$

$= \frac{53}{\ldots}$

(f)



$$\int_0^1 \int_0^{1-x} f(x,y) \, dy \, dx$$

$$= \frac{13}{480}$$