Exponential

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & o/w \end{cases}$$

$$\int_0^\infty f(x)\, dx = 1$$

$$E(x) = \int_0^\infty \lambda e^{-\lambda x} \cdot x \quad dx$$

$$\lambda x = t$$

$$dx = \frac{dt}{\lambda}$$

$$E(x) = \frac{1}{\lambda^r} \int_0^\infty e^{-t}\, t^r\, dt$$

$$= \frac{1}{\lambda^r} \Gamma(r+1) = \frac{r!}{\lambda^r}$$

Property

If X is exponentially distributed then

$$P(x > s+t \mid x > s) = P(x > t)$$

Proof $\dfrac{e^{-s+t}}{e^{-s}} = e^{-t}$

① Length of shower has exponential distribution parameter 2 ($\therefore \lambda = 2$) time measured in minutes what is probability that shower will last more than 3 minutes? if shower has already lasted for 2 min what is probability it will last for 1 more minute?

$\rightarrow$ $P(x > 3) =$

$$= \int_3^{\infty} f(x)\, dx$$

$$= \int_3^{\infty} \lambda e^{-\lambda x}\, dx$$

$$= \int_3^\infty \lambda e^{-\lambda} \, dx = \lambda \int_3^\infty e^{-\lambda} \, dx = \lambda \left[\frac{e^{-\lambda}}{-\lambda} \right]_3^\infty$$

$$= \frac{\lambda}{-\lambda} \left[e^{-\infty} - e^{-3\lambda} \right] = e^{-3\lambda} = e^{-6}$$

$$P(x > 3 \mid x > 2) = P(x > 2+1 \mid x > 2)$$

$t = 1$, $s = 2$ By property

$$= P(x > 1)$$

$$= \int_1^\infty (\lambda e^{-\lambda}) \, dx$$

$$= e^{-1}$$

② The milage of car has exp. mean = 40000
find $P(x \geq 20000)$, P at most 30000

$$\text{mean} = \frac{1}{\lambda} = 40000$$

$$\therefore \lambda = \frac{1}{40000}$$

$$P(x \geq 20000) = \; = \lambda \int_{20000}^{\infty} e^{-\lambda x} dx$$

$$= e^{-\lambda 20000}$$

$$= e^{-\frac{20000}{40000}} = \underline{\underline{e^{-0.5}}}$$

$$P(x \leq 30000) = \int_{0}^{30000} \frac{1}{40000} e^{-1/40000 x}$$

easy integrat

$$= -\left[0.47 - 1\right]$$

$$= 0.53$$