Singular Value Decomposition for dimentionality Reduction

SVD is a fundamental matrix factorization technique in linear algebra

Any matrix of size $m \times n$ canbe split into three matrices

$$A = U \Sigma V^T$$

matrix we want to decompose

$m \times m$ orthogonal matrix

$m \times n$ diagonal matrix with "singular" values

$n \times n$ orthogonal matrix

SVD has many applications in data compression numeric computing and mathematics

$$
\underset{A}{\begin{array}{|c|c|c|c|}\hline \; & \; & \; & \; \\ \hline \; & & & \\ \hline \; & & & \\ \hline \; & & & \\ \hline \end{array}}
=
\underset{U}{\begin{array}{|c|c|c|c|}\hline U_1 & U_2 & U_3 & U_4 \\ \hline \end{array}}
\;
\underset{\Sigma}{\begin{array}{|c|c|c|c|}\hline \sigma_1 & 0 & 0 & 0 \\ \hline 0 & \sigma_2 & 0 & 0 \\ \hline 0 & 0 & \sigma_3 & 0 \\ \hline 0 & 0 & 0 & \sigma_4 \\ \hline \end{array}}
\;
\underset{V}{\begin{array}{|c|}\hline V_1 \\ \hline V_2 \\ \hline V_3 \\ \hline V_4 \\ \hline \end{array}}
$$

$$= \boxed{\sigma_1}\, U_1\, \boxed{\quad v_1 \quad} + \boxed{\sigma_2}\, U_2\, \boxed{\quad v_2 \quad} + \cdots$$

the σ are already sorted in ascending value
so

$\sigma_1 \rightarrow$ high $\quad \therefore$ first term $\rightarrow$ more important
$\sigma_2 \rightarrow$ medium $\quad$ second term $\rightarrow$ lesser imp.
$\sigma_3 \rightarrow$ low
$\vdots \quad \vdots$

So we can consider the first k values from m
and the best values with maximum importance

Useful for data compression

PCA using SVD steps

1. Center the data $X_c = X - \bar{X}$

2. Compute SVD of centered data

$$X_c = U \Sigma V^T$$

3. Compute principle components

principle components are given by V

principle scores $Z = X_c \cdot V$ is the transformed data into principle component space

Dimentions can be reduced by selecting k components.

SVD makes PCA easy and efficiant when compared to eigenvalue decomposition

SVD is more stable and reliable than eigenvalue decomposition.

For large datasets, SVD is faster than computing the full covariance matrix.