Semester: October 2021 – February 2022 Examination: ESE Examination		
Programme code: 01 **Programme: B.TECH**	**Class:** FY	**Semester:** I **(SVU 2020)**
Name of the Constituent College: **K. J. Somaiya College of Engineering**	**Name of the Department** COMP/ETRX/EXTC/IT/MECH	
Course Code: 116U06C101	**Name of the Course: Applied Mathematics - I**	
Duration : 1 Hour 45 Minutes	**Maximum Marks : 50**	
Instructions: **1)Draw neat diagrams 2) Assume suitable data if necessary**		

Question No.		**Max Marks**
Q1 (A)	Choose One correct Option for the following Questions **(2 marks Each)**	**10**
(i)	$\int_{\log 1}^{\log 2} sech^2 x \quad dx =$ (a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{-2}{5}$ (d) $\frac{2}{3}$	
(ii)	The first order partial derivative of $f(x,y,z) = e^{3x} \cos y \ z^3$ w.r.t. x is (a) $\frac{\partial f}{\partial x} = 3e^{3x}$ (b) $\frac{\partial f}{\partial x} = 3e^{3x} \cos y \ z^3$ (c) $\frac{\partial f}{\partial x} = 3e^{3x} + \cos y \ z^3$ (d) $\frac{\partial f}{\partial x} = 3e^{3x} + \cos y + z^3$	
(iii)	The trace and determinant of a 2×2 matrix are 5 and -50 respectively. What are the Eigenvalues of the matrix? (a) $5, 10$ (b) $5, -10$ (c) $-5, 10$ (d) $-5, -10$	
(iv)	Which of the following statements is **TRUE**? (I) The diagonal elements of a Hermitian matrix are all real. (II) The diagonal elements of a Skew-Hermitian matrix are all real (III) The diagonal elements of a skew symmetric matrix are zero. (a) Only (I) (b) (I) and (II) (c) (I) and (III) (d) (I), (II)and (III)	
(v)	If $u = x^3 \cos^{-1}\left(\frac{x}{y}\right)$, the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ at (3,3) is (a) 81 (b) 0 (c) 27 (d) 27π	
Q1 (B)	Solve the following questions. **(2 marks Each)**	**10**
(i)	Solve the equation $17 \cosh x + 18 \, sinhx = 1$ for real values of x	
(ii)	Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 1 \\ 4 & 3 & 2 \end{bmatrix}$	
(iii)	A square symmetric matrix of order 3 has eigenvalues $3, -3, 9$. Two of the eigenvectors are $[2, 2, -1]'$ and $[2, -1, 2]'$. Find the third Eigen vector. **OR** Find the Minimal Polynomial of the matrix $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$	

(iv)	If $z = x^2 + y^2, x = cost, y = sint$, find $\frac{dz}{dt}$ at $t = \pi$ using composite rule. **OR** Find $\frac{\partial(u,v)}{\partial(x,y)}$ for $u = xy,\ v = x + y$	
(v)	If A and B are Hermitian matrices then prove that $(AB + BA)$ is Hermitian and $(AB - BA)$ is skew – Hermitian.	
Q. 2	Solve the following questions. **(5 marks Each)**	**10**
(a)	Solve $x^5 = 1 + i$ and find the continued product of all the roots.	
(b)	Solve the following equations by Gauss – Seidel method (4 Iterations) $28x + 4y - z = 32,\ 2x + 17y + 4z = 35,\quad x + 3y + 10z = 24$ **OR** Show that if λ ≠ 0, the system of equations $\begin{matrix} 2x_1 + x_2 = a \\ x_1 + \lambda x_2 - x_3 = b \\ x_2 + 2x_3 = c \end{matrix}$ has a unique solution for every choice of a, b, c. If λ= 0, determine the relation satisfied by a, b, c such that the system is consistent.	
Q. 3	Solve any **Two** of the following questions. **(5 marks Each)**	**10**
(a)	Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonisable. Find the diagonal form D and the diagonalising matrix M	
(b)	Use Cayley Hamilton Theorem to find $2A^5 - 3A^4 + A^2 - 4I$ where $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$	
(c)	If $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$, find 5^A.	
Q. 4	Solve the following questions. **(5 marks Each)**	**10**
(a)	If $u = \frac{1}{3}\log\left(\frac{x^3+y^3}{x^2+y^2}\right)$, find the value of **(i)** $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$. **(ii)** $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$.	
(b)	Find the extreme values of the function $x^3 + y^3 - 63(x + y) + 12xy$ **OR** If $x = e^u cosec\ v, y = e^u \cot v\ and\ z$ is a function of x and y, prove that $\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = e^{-2u}\left[\left(\frac{\partial z}{\partial u}\right)^2 - sin^2 v\left(\frac{\partial z}{\partial v}\right)^2\right]$	