K. J. Somaiya College of Engineering, Mumbai-77
(Constituent College of Somaiya Vidyavihar University, Mumbai)

Semester: I **Oct 2021-Feb 2022**

In-Semester Examination

Class: F.Y. B. Tech
Branch: All Branches **Semester : I**
Full name of the course: Applied Mathematics-I **Course Code: 116U06C101**
Duration: 1hr.15 min (attempting questions) +20 min (uploading) **Max. Marks: 30**

Q. No	Questions	Marks
Q1	Choose the correct option from the following MCQ (2 Marks Each)	10 Marks
1.1	Which of the following is **Correct**? (A) $\sinh x = \frac{e^x - e^{-x}}{2i}$ (B) $tanhix = itanhx$ (C) $cosech^2 x = coth^2 x - 1$ (D) $cosh^2 x = sinh^2 x - 1$	C
1.2	If $p = cos4\alpha - isin4\alpha$, $q = cos4\beta - isin4\beta$, then $\left(\frac{q}{p}\right)^{\frac{1}{4}} - \left(\frac{p}{q}\right)^{\frac{1}{4}} =$ (A) $2\cos 3(\alpha - \beta)$ (B) $-2isin(\beta - \alpha)$ (C) $2si\,n(\alpha - \beta)$ (D) $-2\cos(\beta - \alpha)$	B
1.3	Real part of $cos^{-1}(i)$ is (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) π (D) 0	B
1.4	For $A = \begin{bmatrix} 3 & -2 & 5 \\ 0 & 1 & -4 \\ -6 & 5 & -14 \end{bmatrix}$ and any column vector $B \in R^3$, the system $AX = B$ will be inconsistent if $rank(A\|B)$ is (A) 0 (B) 1 (C) 2 (D) 3	D
1.5	If A is any square Matrix then which of the following is **correct**? (A) If A is orthogonal then AA^T is not orthogonal. (B) $-i\left(A - A^\theta\right)$ is skew Hermitian. (C) For Hermitian Matrix A, $\overline{\imath A} = -iA^t$ (D) For Unitary Matrix A, AA^θ is not Hermitian.	C

Q2	Attempt any **Two** of the following	
(a)	**Find the roots of** $(\boldsymbol{x}+\mathbf{1})^{\mathbf{7}} = (\boldsymbol{x}-\mathbf{1})^{\mathbf{7}}$ **Soln** - We have $(x+1)^7 = (x-1)^7$ $\therefore \left(\frac{x+1}{x-1}\right)^7 = 1 = \cos 0 + i\sin 0 = cos(2k\pi) + i\sin(2k\pi)$ $\therefore \frac{x+1}{x-1} = [\cos(2k\pi) + i\sin(2k\pi)]^{1/7}$ $= cos\left(\frac{2k\pi}{7}\right) + i\sin\left(\frac{2k\pi}{7}\right)$ $\therefore \frac{x+1}{x-1} = \cos\theta + i\sin\theta$ where $\theta = \left(\frac{2k\pi}{7}\right)$ $for\ k = 0,1,2,3,4,5,6$ by componendo dividend, $\therefore \frac{2x}{2} = \frac{x}{1} = \frac{1+\cos\theta+i\sin\theta}{\cos\theta-1+i\sin\theta}$ Simplifying, we get $\therefore \frac{x}{1} = \frac{2cos^2(\theta/2)+ i\,2\sin(\theta/2)\cos(\theta/2)}{-2sin^2(\theta/2)+ i\,2\sin(\theta/2)\cos(\theta/2)}$ $\therefore x = \frac{\cos(\theta/2)\,[\cos(\theta/2)+ i\ \sin(\theta/2)]}{\sin(\theta/2)[-\sin(\theta/2)+ i\cos(\theta/2)]}$ $\therefore x = \cot(\theta/2)\left\{\frac{\cos(\theta/2)+ i\ \sin(\theta/2)}{\cos[(\pi/2)+(\theta/2)]+ i\sin[(\pi/2)+(\theta/2)]}\right\}$ $\therefore x = cot\ \frac{\theta}{2}\ e^{-i(\pi/2)} = -icot\ \frac{\theta}{2}$ $\therefore x = -icot\left(\frac{k\pi}{7}\right),\quad where\ k = 0,1,2,3,4,5,6$ Since cot 0 is infinite we neglect that term, $\therefore z = \pm icot\left(\frac{k\pi}{7}\right),\quad where\ k = 1,2,3$	2 marks 5 marks
(b)	**If** $\cos(\boldsymbol{u}+\boldsymbol{i}\,\boldsymbol{v}) = \boldsymbol{x}+\boldsymbol{i}\,\boldsymbol{y}$, **Prove that** $(\mathbf{1}+\boldsymbol{x})^{\mathbf{2}} + \boldsymbol{y}^{\mathbf{2}} = (\mathbf{cosh}\,\boldsymbol{v} + \mathbf{cos}\,\boldsymbol{u})^{\mathbf{2}}$ **Soln** – Consider $\cos(u+i\,v) = x+i\,y$ $\cos u\cos iv - \sin u\sin iv = x+iy$ $\therefore \cos u\cos hv - i\sin u\sin hv = x+iy$ Equating real and imaginary parts, $\cos u\cos h\,v = x\quad and\quad \sin u\sin hv = -y$ Now consider, $(1+x)^2 + y^2 = 1+2x+x^2+y^2$ $= 1 + 2\cos u\cosh v + cos^2u\cos h^2v + sin^2u\sin h^2v$ $= 1 + 2\cos u\cosh v + cos^2u\cos h^2v + (1-cos^2u)(\cos h^2v - 1)$ Cancelling and simplifying we get, $= 2\cos u\cosh v + \cos h^2v + cos^2u\quad = (\cosh v + \cos u)^2$	2 marks 5 marks

(c)	**If $a\cos\alpha + b\cos\beta + c\cos\gamma = a\sin\alpha + b\sin\beta + c\sin\gamma = 0$, Prove that $a^3\cos 3\alpha + b^3\cos 3\beta + c^3\cos 3\gamma = 3abc\cos(\alpha+\beta+\gamma)$** **Soln -** Given $a\,cos\alpha + b\cos\beta + c\cos\gamma = a\sin\alpha + b\sin\beta + c\sin\gamma = 0$ $\therefore (a\,cos\alpha + b\cos\beta + c\cos\gamma) + i\,(a\sin\alpha + b\sin\beta + c\sin\gamma) = 0$ $\therefore a(\cos\alpha + i\sin\alpha) + b(\cos\beta + i\sin\beta) + c\,(\cos\gamma + i\sin\gamma) = 0$ Let $x = a\,(cos\,\alpha + i\sin\alpha), y = b(\cos\beta + i\sin\beta), z = c(\cos\gamma + i\sin\gamma)$ $\therefore x + y + z = 0 \qquad \therefore (x+y+z)^3 = 0$ $\therefore x^3 + y^3 + z^3 + 3(x+y+z)(xy+yz+zx) - 3xyz = 0$ $\therefore x^3 + y^3 + z^3 = 3\,xyz$ $\therefore a^3(\cos\alpha + i\sin\alpha)^3 + b^3(\cos\beta + i\sin\beta)^3 + c^3(\cos\gamma + i\sin\gamma)^3$ $= 3a(\cos\alpha + i\sin\alpha).b.(\cos\beta + i\sin\beta).c.(\cos\gamma + i\sin\gamma)$ $\therefore$ By De Moivre's Theorem, $a^3(\cos 3\alpha + i\sin 3\,\alpha) + b^3(\cos 3\beta + i\sin 3\beta) + c^3(\cos 3\gamma + i\sin 3\gamma)$ $= 3abc[cos(\alpha+\beta+\gamma) + i\sin(\alpha+\beta+\gamma)]$ $(a^3\cos 3\alpha + b^3\cos 3\beta + c^3\cos 3\gamma) + i(a^3\sin 3\alpha + b^3\sin 3\beta + c^3\sin 3\gamma)$ $= 3abc[cos(\alpha+\beta+\gamma) + i\sin(\alpha+\beta+\gamma)]$ Equating imaginary parts, we get the required result.	3 marks 5 marks
Q3	Attempt any **Two** of the following	
(a)	**Determine the values of a, b, c For orthogonal matrix** $\frac{1}{9}\begin{bmatrix} a & 1 & b \\ c & b & 7 \\ 1 & a & c \end{bmatrix}$ **Soln-** If A is the orthogonal, then by definition, we have $AA^T = I$ $\Rightarrow \frac{1}{9}\begin{bmatrix} a & 1 & b \\ c & b & 7 \\ 1 & a & c \end{bmatrix} \times \frac{1}{9}\begin{bmatrix} a & c & 1 \\ 1 & b & a \\ b & 7 & c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\Rightarrow \frac{1}{81}\begin{bmatrix} 1+b^2+a^2 & ac+8b & 2a+bc \\ ac+8b & c^2+b^2+49 & 8c+ab \\ 2a+bc & 8c+ab & 1+a^2+c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Equating the corresponding diagonal elements, we get $1 + b^2 + a^2 = 81 \Rightarrow b^2 + a^2 = 80$ $c^2 + b^2 + 49 = 81 \Rightarrow c^2 + b^2 = 32$ $1 + a^2 + c^2 = 81 \Rightarrow a^2 + c^2 = 80$ We get $c = \pm 4, b = \pm 4\ and\ a = \pm 8$ And equating the corresponding non – diagonal elements, we get $ac + 8b = 0 \Rightarrow ac = -8b, \qquad 2a + bc = 0 \Rightarrow bc = -2a,$ $8c + ab = 0 \Rightarrow ab = -8c$ Hence the required values are $a = 8, b = -4, c = 4$ or $a = 8, b = 4, c = -4$ Or $a = -8, b = 4, c = 4\ or\ a = -8, b = -4, c = -4$	2 marks 4 marks 5 marks

(b)	**Check whether following vectors are linearly dependent? If so find the relation between them** $\boldsymbol{X_1 = [1\ 2\ 1\ ],\ X_2 = [2\ 1\ 4],\ X_3 = [4\ 5\ 6],\ X_4 = [1\ 8 - 3]}$ **Soln-** consider the matrix equation $k_1X_1 + k_2X_2 + k_3X_3 + k_4X_4 = 0$ ...(i) $\therefore k_1[1\ 2\ 1] + k_2[2\ 1\ 4] + k_3[4\ 5\ 6] + k_4[1\ \ 8 - 3] = 0$ $\therefore k_1 + 2k_2 + 4k_3 + k_4 = 0,\quad 2k_1 + k_2 + 5k_3 + 8k_4 = 0,$ $k_1 + 4k_2 + 6k_3 - 3k_4 = 0$ $\therefore \begin{bmatrix}1 & 2 & 4 & 1\\ 2 & 1 & 5 & 8\\ 1 & 4 & 6 & -3\end{bmatrix}\begin{bmatrix}k_1\\ k_2\\ k_3\\ k_4\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$	2 marks
	Applying $R_2 - 2R_1, R_3 - R_1$, we get $\begin{bmatrix}1 & 2 & 4 & 1\\ 0 & -3 & -3 & 6\\ 0 & 2 & 2 & -4\end{bmatrix}\begin{bmatrix}k_1\\ k_2\\ k_3\\ k_4\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$ Applying $3R_3 + 2R_2$ we get $\begin{bmatrix}1 & 2 & 4 & 1\\ 0 & -3 & -3 & 6\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}k_1\\ k_2\\ k_3\\ k_4\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$ Rank A (r) = 2, n = 4. r < n Hence Non-trivial solution and linearly dependent. $\therefore k_1 + 2k_2 + 4k_3 + k_4 = 0,\quad k_2 + k_3 - 2k_4 = 0$,	4 marks
	Let $k_4 = t\ \ and\ k_3 = s,\quad \therefore k_2 = 2t - s$ $\therefore k_1 + 4t - 2s + 4s + t = 0\qquad \therefore k_1 = -5t - 2s$ Putting the values of k_1, k_2, k_3, k_4 in (i) we get, $(-5t - 2s)X_1 + (2t - s\ )X_2 + sX_3 + tX_4 = 0$ Hence, X_1, X_2, X_3, X_4 are linearly dependent	5 marks
(c)	**Test for consistency the following equations and find solution if consistent** $\boldsymbol{5x_1 - 3x_2 - 7x_3 + x_4 = 10}$ $\boldsymbol{-x_1 + 2x_2 + 6x_3 - 3x_4 = -3}$ $\boldsymbol{x_1 + x_2 + 4x_3 - 5x_4 = 0}$ **Soln-** The system of linear equations can be written in the matrix form as $AX\ =\ B$ i.e., $\begin{bmatrix}5 & -3 & -7 & 1\\ -1 & 2 & 6 & -3\\ 1 & 1 & 4 & -5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} = \begin{bmatrix}10\\ -3\\ 0\end{bmatrix}$	1 mark
	The augmented matrix is $[A{:}B] = \begin{bmatrix}5 & -3 & -7 & 1 & 10\\ -1 & 2 & 6 & -3 & -3\\ 1 & 1 & 4 & -5 & 0\end{bmatrix}$ Applying R_{13}, we get $[A{:}B] = \begin{bmatrix}1 & 1 & 4 & -5 & 0\\ -1 & 2 & 6 & -3 & -3\\ 5 & -3 & -7 & 1 & 10\end{bmatrix}$	

	Applying $R_2 + R_1$ *and* $R_3 - 5R_1$, we get $[A:B] = \begin{bmatrix} 1 & 1 & 4 & -5 & 0 \\ 0 & 3 & 10 & -8 & -3 \\ 0 & -8 & -27 & 26 & 10 \end{bmatrix}$ Applying $3R_3 + 8R_2$, we get $[A:B] = \begin{bmatrix} 1 & 1 & 4 & -5 & 0 \\ 0 & 3 & 10 & -8 & -3 \\ 0 & 0 & -1 & 14 & 6 \end{bmatrix}$ $\therefore$ rank of A = rank of $[A:B] = 3 < 4$ (Number of variables) $\therefore (4-3) = 1$ variable can be assigned as an arbitrary value The reduced form of the linear equations can be written as $x_1 + x_2 + 4x_3 - 5x_4 = 0$ $3x_2 + 10x_3 - 8x_4 = -3$ $-x_3 + 14x_4 = 6$ Let $x_4 = k$ (arbitrary) $\Rightarrow x_3 = 14k - 6$ $\Rightarrow 3x_2 + 10(14k - 6) - 8k = -3$ $\Rightarrow 3x_2 = 57 - 132k \Rightarrow x_2 = 19 - 44k$ Also $x_1 + 19 - 44k + 4(14k - 6) - 5k = 0 \Rightarrow x_1 = 5 - 7k$ Thus the solution is $x_1 = 5 - 7k, x_2 = 19 - 44k,$ $x_3 = 14k - 6, x_4 = k$	3 marks 5 marks