K. J. Somaiya College of Engineering, Mumbai-77
(A Constituent College Affiliated to Somaiya Vidyavihar University)
Semester: **September – Jan 2020**
In-Semester Examination

Class: FY B. Tech
Branch: All Branches **Semester :** I
Full name of the course: Applied Mathematics-I **Course Code: 116U06C101**
Duration: 1hr.15 min (attempting questions) +20 min (uploading) **Max. Marks:** 30

Q. No	Questions	Marks
Q1	Choose the correct option from the following MCQ (1 MARK EACH)	10 marks
1.1	If $Z = e^{i\emptyset}$ then which of the following is **FALSE**? (A) $z^{1/5} = \left(cos\frac{1}{5}\emptyset + i\ sin\frac{1}{5}\emptyset\right)$ (B) $\frac{1}{z} = \bar{z}$ (C) $z^2 + \frac{1}{z^2} = 2isin2\emptyset$ (D) $z^2 - \frac{1}{z^2} = 2isin2\emptyset$	C
1.2	Find the value of $\tanh\left(\log\sqrt{5}\right)$ (A) $\frac{2}{3}$ (B) $-\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $-\frac{1}{3}$	A
1.3	Which of the following is not a correct formula for $\cosh 2x$? (A) $\cosh^2 x + \sinh^2 x$ (B) $2\cosh^2 x - 1$ (C) $1 - 2\sinh^2 x$ (D) $\frac{1+\tanh^2 x}{1-\tanh^2 x}$	C
1.4	Roots of $x^{16} + i = 0$ are (A) $cos(4k+1)\frac{\pi}{16} - isin(4k+1)\frac{\pi}{16}\ ; k = 0,1 \dots 16$ (B) $cos(4k+1)\frac{\pi}{16} - isin(4k+1)\frac{\pi}{16}\ ; k = 0,1 \dots 15$ (C) $cos(4k+1)\frac{\pi}{32} - isin(4k+1)\frac{\pi}{32}\ ; k = 0,1 \dots 15$ (D) $cos(4k+1)\frac{\pi}{32} + isin(4k+1)\frac{\pi}{32}\ ; k = 0,1 \dots 16$	C

1.5	If $x_n = cos\frac{\pi}{2^n} + i\ sin\frac{\pi}{2^n}$ then find $x_0x_1x_2 \ldots x_\infty$ (A) -1 (B) 1 (C) i (D) $-i$	B
1.6	The number of non-zero rows in the echelon form of $\begin{bmatrix} 1 & 2 & -4 & 5 \\ 1 & 9 & -4 & 5 \\ -2 & 3 & 8 & -10 \\ -1 & 5 & 4 & -5 \end{bmatrix}$ is (A) 1 (B) 2 (C) 3 (D) 4	B
1.7	Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -4 \\ -2 & -3 & -10 \end{bmatrix}$. Then for any $B \in R^3$, the system $AX = B$ will be inconsistent when $rank(A\|B)$ is (A) 0 (B) 1 (C) 2 (D) 3	D
1.8	Consider $S_1 = \{(1,-1,\ 4)\ ,(2,-2,0)\}\ and\ S_2 = \{(1,0,0),(0,0,1),(1,0,1)\}$ (A) Only S_1 is Linearly independent. (B) Only S_2 is Linearly independent. (C) both S_1 and S_2 are Linearly independent (D) both S_1 and S_2 are Linearly Dependent.	A
1.9	If $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \\ \frac{i}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$, what is A^{-1} ? (A) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \\ \frac{i}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ \frac{i}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \\ \frac{-i}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} \frac{-1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$	A
1.10	Rank of non-zero singular matrix A of order n is (A) n (B) 0 (C) $less\ than\ n$ (D) can not say	C
Q2	Attempt any **TWO** of the following	
(a)	If $x_n + iy_n = \left(1 + i\sqrt{3}\right)^n$, prove that $x_{n-1}y_n - x_ny_{n-1} = 4^{n-1}\sqrt{3}$. Solution: $x_n + iy_n = \left(1 + i\sqrt{3}\right)^n = 2^n\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^n$ $= 2^n\left(\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}\right)$ (By De-Moivre's theorem) $\therefore\ x_n = 2^n\ \cos\frac{n\pi}{3}$ and $y_n = 2^n \sin\frac{n\pi}{3}$ (Comparing real and imaginary	2 marks

	parts) Now, LHS = $x_{n-1}y_n - x_n y_{n-1}$ $= 2^{n-1}\cos\frac{(n-1)\pi}{3} \times 2^n \sin\frac{n\pi}{3} - 2^n \cos\frac{n\pi}{3} \times 2^{n-1}\sin\frac{(n-1)\pi}{3}$ $= 2^{n-1} \times 2^n \left[\sin\frac{n\pi}{3}\cos\frac{(n-1)\pi}{3} - \cos\frac{n\pi}{3}\sin\frac{(n-1)\pi}{3}\right]$ $= 2^{2n-1}\sin\left[\frac{n\pi}{3} - \frac{(n-1)\pi}{3}\right]$ $= 2^{2n-1}\sin\left[\frac{\pi}{3}\right] = 2^{2n-1}\left(\frac{\sqrt{3}}{2}\right) = 2^{2n-2}\sqrt{3} = 4^{n-1}\sqrt{3} = RHS$	4 marks 5 marks
(b)	If $\sin^{-1}(\alpha + i\,\beta) = x + i\,y$, show that $\sin^2 x$ and $\cos h^2 y$ are the roots of the equation $\lambda^2 - (\alpha^2 + \beta^2 + 1)\lambda + \alpha^2 = 0$ Solution: We have $\sin(x + iy) = \alpha + i\,\beta$ $\sin x \cos iy + \cos x \sin iy = \alpha + i\,\beta$ $\therefore \sin x \cosh y + i \cos x \sinh y = \alpha + i\,\beta$ Equating real and imaginary parts $\sin x \cosh y = \alpha$ and $\cos x \sinh y = \beta$ We know that, in terms of the roots, the quadratic equation is given by $\lambda^2 - (sum\ of\ the\ roots)\lambda + (product\ of\ the\ roots) = 0$ Hence the equation whose roots are $\sin^2 x$ and $\cosh^2 y$ is $\lambda^2 - (\sin^2 x + cosh^2 y)\lambda + (\sin^2 x.cosh^2 y) = 0$ This means we have to prove that $\alpha^2 + \beta^2 + 1 = \sin^2 x + cosh^2 y$ and $\alpha^2 = \sin^2 x.cosh^2 y$ Now, $\alpha^2 + \beta^2 + 1 = sin^2 x cosh^2 y + cos^2 x sinh^2 y + 1$ $= sin^2 x cosh^2 y + (1 - sin^2 x)(\cos h^2 y - 1) + 1$ $= sin^2 x cosh^2 y + \cos h^2 y - 1 - sin^2 x cosh^2 y + sin^2 x + 1$ $= \sin^2 x + cosh^2 y = sum\ of\ the\ roots$ And $\alpha^2 = \sin^2 x.cosh^2 y$ = Product of the roots Hence the equation whose roots are $sin^2 x$, $\cos h^2 y$ is $\lambda^2 - (\alpha^2 + \beta^2 + 1)\lambda + \alpha^2 = 0$	2 marks 3 Marks 5 marks
(c)	Prove that $\log\left\{\frac{\cos(x - i\,y)}{\cos(x + i\,y)}\right\} = 2\,i\tan^{-1}(\tan x \tanh y)$ Solution : $\cos(x + iy) = \cos x \cos iy - \sin x \sin iy$ $= \cos x \cos hy - i \sin x\, sinhy = a - ib$ (say) Similarly $\cos(x - iy) = \cos x \cos hy + i \sin x\, sinhy = a + ib$ (say)	1 mark

	Now $\log(a+ib) = \frac{1}{2}\log(a^2+b^2) + i\tan^{-1}\left(\frac{b}{a}\right)$ $\log(a-ib) = \frac{1}{2}\log(a^2+b^2) - i\tan^{-1}\left(\frac{b}{a}\right)$ $\therefore \log\left\{\frac{\cos(x-iy)}{\cos(x+iy)}\right\} = \log[\cos(x-iy)] - \log[\cos(x+iy)]$ $= \log(a+ib) - \log(a-ib)$ $= \frac{1}{2}\log(a^2+b^2) + i\tan^{-1}\left(\frac{b}{a}\right) - \frac{1}{2}\log(a^2+b^2) + i\tan^{-1}\left(\frac{b}{a}\right)$ $= 2i\ \tan^{-1}\left(\frac{b}{a}\right)$ $= 2i\ \tan^{-1}\left(\frac{\sin x sinh y}{\cos x \cos hy}\right) = 2\,i\tan^{-1}(\tan x \tanh y)$	2 marks 3 marks 5 marks
Q3	Attempt any **Two** of the following	
(a)	Determine the values of α,β,γ when the matrix given by $A = \begin{bmatrix} \alpha & \beta & -\gamma \\ \alpha & -2\beta & 0 \\ \alpha & \beta & \gamma \end{bmatrix}$ is orthogonal. Solution : Given $A = \begin{bmatrix} \alpha & \beta & -\gamma \\ \alpha & -2\beta & 0 \\ \alpha & \beta & \gamma \end{bmatrix}$ $\therefore A^t = \begin{bmatrix} \alpha & \alpha & \alpha \\ \beta & -2\beta & \beta \\ -\gamma & 0 & \gamma \end{bmatrix}$ Since A is orthogonal, $AA^t = I$ $\therefore \begin{bmatrix} \alpha & \beta & -\gamma \\ \alpha & -2\beta & 0 \\ \alpha & \beta & \gamma \end{bmatrix}\begin{bmatrix} \alpha & \alpha & \alpha \\ \beta & -2\beta & \beta \\ -\gamma & 0 & \gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\therefore \begin{bmatrix} \alpha^2+\beta^2+\gamma^2 & \alpha^2-2\beta^2 & \alpha^2+\beta^2-\gamma^2 \\ \alpha^2-2\beta^2 & \alpha^2+4\beta^2 & \alpha^2-2\beta^2 \\ \alpha^2+\beta^2-\gamma^2 & \alpha^2-2\beta^2 & \alpha^2+\beta^2+\gamma^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Comparing the elements we get , $\alpha^2+\beta^2+\gamma^2 = 1, \alpha^2-2\beta^2 = 0, \alpha^2+\beta^2-\gamma^2 = 0\ and\ \alpha^2+4\beta^2 = 1$ Solving these equations we will get $\alpha = \pm\frac{1}{\sqrt{3}}, \beta = \pm\frac{1}{\sqrt{6}}, \gamma = \pm\frac{1}{\sqrt{2}}$	1 marks 3 marks 5 marks
(b)	Reduce $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 6 & 8 & 10 \\ 15 & 27 & 39 & 51 \\ 6 & 12 & 18 & 24 \end{bmatrix}$ to normal form and hence obtain rank of A.	5 marks

	Solution: $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 6 & 8 & 10 \\ 15 & 27 & 39 & 51 \\ 6 & 12 & 18 & 24 \end{bmatrix}$ By $R_2 - 4R_1$, $R_3 - 15R_1$, $R_4 - 6R_1$ $A \sim \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -6 & -12 & -18 \\ 0 & -18 & -36 & -54 \\ 0 & -6 & -12 & -18 \end{bmatrix}$ By $C_2 - 3C_1, C_3 - 5C_1, C_4 - 7C_1$ $A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -6 & -12 & -18 \\ 0 & -18 & -36 & -54 \\ 0 & -6 & -12 & -18 \end{bmatrix}$ By $\frac{-1}{6} R_2$ $A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & -18 & -36 & -54 \\ 0 & -6 & -12 & -18 \end{bmatrix}$ By $R_3 + 18R_2$, $R_4 + 6R_2$ $A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ By $C_3 - 2C_2, C_4 - 3C_2$ $A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$ $\therefore$ Rank of $A = 2$	
(c)	Determine all the possible solutions of the following equations. $2x - y - z + w = 10, x - 2z + 3w = 6, 4x - y - 5z + 7w = 22.$ Solution: Writing the given system in matrix form $\begin{bmatrix} 2 & -1 & -1 & 1 \\ 1 & 0 & -2 & 3 \\ 4 & -1 & -5 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ 22 \end{bmatrix}$ The augmented matrix $[A:B] = \begin{bmatrix} 2 & -1 & -1 & 1 & 10 \\ 1 & 0 & -2 & 3 & 6 \\ 4 & -1 & -5 & 7 & 22 \end{bmatrix}$ Applying $2R_2 - R_1,\ R_3 - 2R_1,\ we\ get$	5 marks

	$[A:B] \sim \begin{bmatrix} 2 & -1 & -1 & 1 & 10 \\ 0 & 1 & -3 & 5 & 2 \\ 0 & 1 & -3 & 5 & 2 \end{bmatrix}$ Applying $R_3 - R_2$, we get $[A:B] \sim \begin{bmatrix} 2 & -1 & -1 & 1 & 10 \\ 0 & 1 & -3 & 5 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ Hence $\rho(A) = \rho[A:B] = 2$, therefor the system is consistent Number of unknowns = 4, system has $(n - r) = 4 - 2 = 2$ linearly independent solution. The reduced form of the linear equations can be written as $2x - y - z + w = 10$ ……………………….(i) $y - 3z + 5w = 2$ …………………………(ii) Let $z = t$ and $w = s$ where t and s are arbitrary constants then (ii) ⇒ $y = 2 + 3t - 5s$ (i) ⇒ $x = 6 + 2t - 2s$ Hence $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 6 + 2t - 2s \\ 2 + 3t - 5s \\ t \\ s \end{bmatrix}$ The system has infinite solutions as 't' and 's' vary.	