K. J. Somaiya College of Engineering, Mumbai-77
(A Constituent College Affiliated to Somaiya Vidyavihar University)
Semester: **September – Jan 2020**
In-Semester Examination

Class: FY B. Tech
Branch: All Branches — **Semester:** I
Full name of the course: Applied Mathematics-I — **Course Code:** 116U06C101
Duration: 1hr.15 min (attempting questions) +20 min (uploading) — **Max. Marks:** 30

Q. No	Questions	Marks
Q1	Choose the correct option from the following MCQ (1 MARK EACH)	10 marks
1.1	If $Z = e^{i\emptyset}$ then which of the following is **FALSE**? (A) $z^{1/5} = \left(cos\frac{1}{5}\emptyset + i\ sin\frac{1}{5}\emptyset\right)$ (B) $\frac{1}{z} = \bar{z}$ (C) $z^2 + \frac{1}{z^2} = 2isin2\emptyset$ (D) $z^2 - \frac{1}{z^2} = 2isin2\emptyset$	
1.2	Find the value of $\tanh\left(\log\sqrt{5}\right)$ (A) $\frac{2}{3}$ (B) $-\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $-\frac{1}{3}$	
1.3	Which of the following is not a correct formula for $\cosh 2x$? (A) $\cosh^2 x + \sinh^2 x$ (B) $2\cosh^2 x - 1$ (C) $1 - 2\sinh^2 x$ (D) $\frac{1+\tanh^2 x}{1-\tanh^2 x}$	
1.4	Roots of $x^{16} + i = 0$ are (A) $cos(4k+1)\frac{\pi}{16} - isin(4k+1)\frac{\pi}{16}$; $k = 0,1 \ldots 16$ (B) $cos(4k+1)\frac{\pi}{16} - isin(4k+1)\frac{\pi}{16}$; $k = 0,1 \ldots 15$ (C) $cos(4k+1)\frac{\pi}{32} - isin(4k+1)\frac{\pi}{32}$; $k = 0,1 \ldots 15$ (D) $cos(4k+1)\frac{\pi}{32} + isin(4k+1)\frac{\pi}{32}$; $k = 0,1 \ldots 16$	

1.5	If $x_n = cos\frac{\pi}{2^n} + i\ sin\frac{\pi}{2^n}$ then find $x_0 x_1 x_2 \ldots x_\infty$ (A) -1 (B) 1 (C) i (D) $-i$	
1.6	The number of non-zero rows in the echelon form of $\begin{bmatrix} 1 & 2 & -4 & 5 \\ 1 & 9 & -4 & 5 \\ -2 & 3 & 8 & -10 \\ -1 & 5 & 4 & -5 \end{bmatrix}$ is (A) 1 (B) 2 (C) 3 (D) 4	
1.7	Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -4 \\ -2 & -3 & -10 \end{bmatrix}$. Then for any $B \in R^3$, the system $AX = B$ will be inconsistent when $rank(A\|B)$ is (A) 0 (B) 1 (C) 2 (D) 3	
1.8	Consider $S_1 = \{(1, -1, 4), (2, -2, 0)\}\ and\ S_2 = \{(1,0,0), (0,0,1), (1,0,1)\}$ (A) Only S_1 is Linearly independent. (B) Only S_2 is Linearly independent. (C) both S_1 and S_2 are Linearly independent (D) both S_1 and S_2 are Linearly Dependent.	
1.9	If $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \\ \frac{i}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$, what is A^{-1} ? (A) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \\ \frac{i}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ \frac{i}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \\ \frac{-i}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} \frac{-1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$	
1.10	Rank of non-zero singular matrix A of order n is (A) n (B) 0 (C) less than n (D) can not say	
Q2	Attempt any **TWO** of the following	
(a)	If $x_n + iy_n = \left(1 + i\sqrt{3}\right)^n$, prove that $x_{n-1}y_n - x_n y_{n-1} = 4^{n-1}\sqrt{3}$.	5 marks

(b)	If $\sin^{-1}(\alpha + i\beta) = x + iy$, show that $\sin^2 x$ and $\cos h^2 y$ are the roots of the equation $\lambda^2 - (\alpha^2 + \beta^2 + 1)\lambda + \alpha^2 = 0$	5 marks
(c)	Prove that $\log\left\{\frac{\cos(x - iy)}{\cos(x + iy)}\right\} = 2\,i\tan^{-1}(\tan x \tanh y)$	5 marks
Q3	Attempt any **Two** of the following	
(a)	Determine the values of α, β, γ when the matrix given by $A = \begin{bmatrix} \alpha & \beta & -\gamma \\ \alpha & -2\beta & 0 \\ \alpha & \beta & \gamma \end{bmatrix}$ is orthogonal.	5 marks
(b)	Reduce $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 6 & 8 & 10 \\ 15 & 27 & 39 & 51 \\ 6 & 12 & 18 & 24 \end{bmatrix}$ to normal form and hence obtain rank of A.	5 marks
(c)	Determine all the possible solutions of the following equations. $2x - y - z + w = 10, x - 2z + 3w = 6, 4x - y - 5z + 7w = 22.$	5 marks