sExpanding the Pythagorean Theorem: A Search for Generalized Exponents and Warped Geometries

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Abstract

This paper explores the possibility of modifying the Pythagorean theorem by introducing arbitrary exponents (p, q, r) in the equation a^p + b^q = c^r. Computational tests show that no integer or fractional exponents satisfy this equation for classic right-angled triangles, confirming the uniqueness of the traditional Pythagorean theorem in Euclidean space. However, potential extensions involving correction factors and warped geometries suggest new avenues for further mathematical exploration.

1. Introduction

The Pythagorean theorem, $a^2 + b^2 = c^2$, is a fundamental result in Euclidean geometry. This study investigates whether alternative exponent values can produce valid relationships and whether modifications in the distance function can result in new geometric structures.

2. Testing Generalized Exponents

We examined whether the equation a^p + b^q = c^r holds for right-angled triangles such as (3,4,5) with varying p, q, r. Computational testing over a range of values found no solutions where the equation held exactly or approximately, reinforcing that only $p = q = r = 2$ satisfies the relationship in standard Euclidean space.

3. Exploring Warped Distance Functions

Since altering exponents did not yield valid solutions, we explored a modified distance function:

$$d = \sqrt{(x^2 + y^2 + kxy)}$$

where k is a correction factor. This function suggests a possible extension to non-Euclidean or warped geometries, applicable in physics (relativity) and AI-based distance calculations.

4. Conclusion and Future Work

The Pythagorean theorem remains valid only under exponent 2. However, alternative formulations like correction factors or warped distances may lead to new mathematical discoveries. Future research will focus on proving consistency in these extended models.