

The Most Mathematical Flag: Flag of Nepal

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1 Method of making the shape inside the border

1.1 On the lower portion of a crimson cloth draw a line AB of the required length from left to right.

Let the length of base of flag be l ,
This corresponds to the length of line AB on the lower portion of a crimson cloth.

Let A be origin $(0,0)$ and B be $(l,0)$

A	$(0,0)$
B	$(l,0)$

Join A and B

1.2 From A draw a line AC perpendicular to AB making AC equal to AB plus one third AB. From AC mark off D making line AD equal to line AB. Join B and D.

Let C be $(0, \frac{4l}{3})$ and D be $(0,l)$

C	$(0, \frac{4l}{3})$
D	$(0,l)$

Join B and D

1.3 From BD mark off E making BE equal to AB.

Let E be $(\frac{l(\sqrt{2}-1)}{\sqrt{2}}, \frac{l}{\sqrt{2}})$

E	$(\frac{l(\sqrt{2}-1)}{\sqrt{2}}, \frac{l}{\sqrt{2}})$
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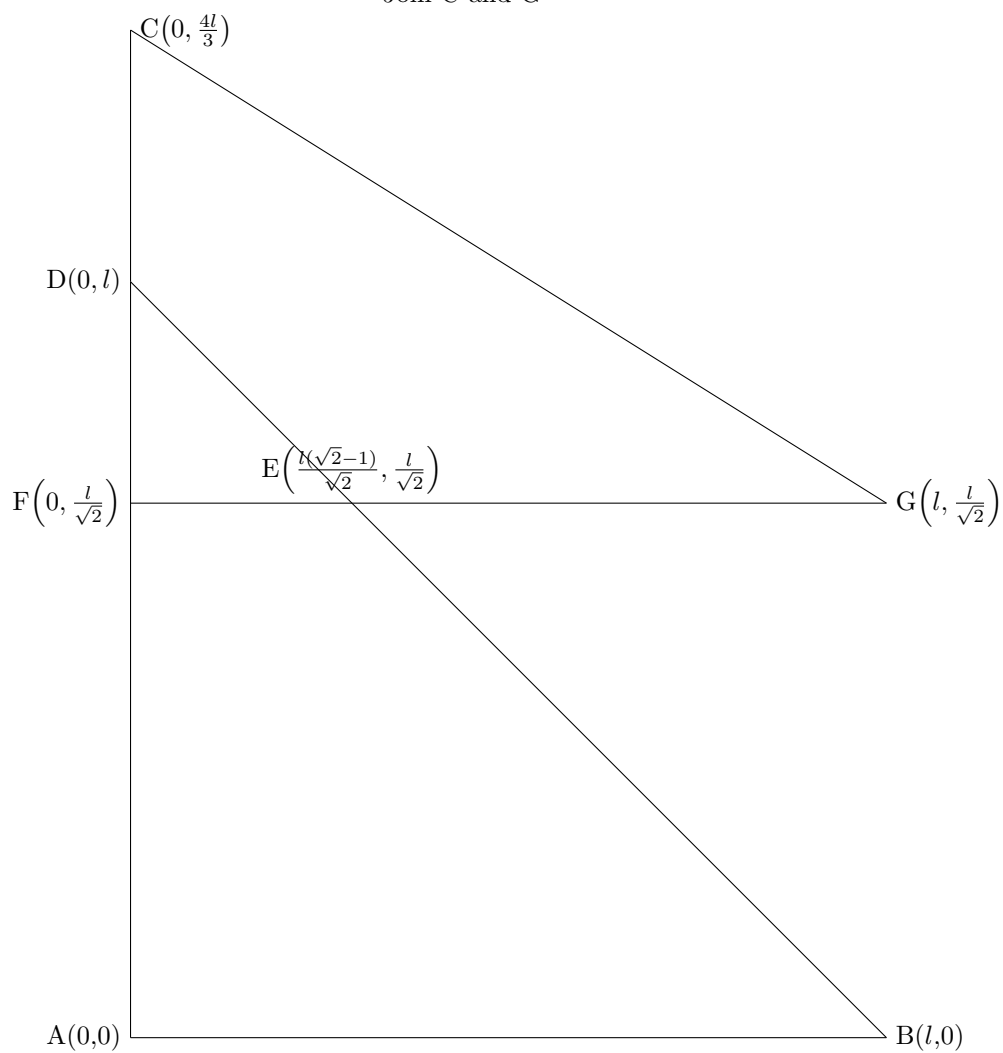
- 1.4 Touching E draw a line FG, starting from the point F on line AC, parallel to AB to the right hand-side.
Mark off FG equal to AB.

Let F be $\left(0, \frac{l}{\sqrt{2}}\right)$ and G be $\left(l, \frac{l}{\sqrt{2}}\right)$

F	$\left(0, \frac{l}{\sqrt{2}}\right)$
G	$\left(l, \frac{l}{\sqrt{2}}\right)$

Join F and G

Join C and G



2 Method of making the moon

2.1 From AB mark off AH making AH equal to one-fourth of line AB and starting from H draw a line HI parallel to line AC touching line CG at point I.

Let H be $(\frac{l}{4}, 0)$

Since $HI \perp AB$, x-co-ordinate of I is $\frac{l}{4}$

To find y-co-ordinate,

Let $C(0, \frac{4l}{3}) = (x_1, y_1)$ and $G(l, \frac{l}{\sqrt{2}}) = (x_2, y_2)$

Equation of line CG is given by,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow y - \frac{4l}{3} = \frac{\frac{l}{\sqrt{2}} - \frac{4l}{3}}{l}(x - 0)$$

$$\Rightarrow y = \frac{4l}{3} + \left(\frac{1}{\sqrt{2}} - \frac{4}{3}\right)\left(\frac{l}{4}\right)$$

$$\Rightarrow y = l\left(\frac{4}{3} + \frac{1}{4}\left(\frac{1}{\sqrt{2}} - \frac{4}{3}\right)\right)$$

$$\Rightarrow y = l\left(\frac{4}{3} + \frac{1}{4\sqrt{2}} - \frac{1}{3}\right)$$

$$\Rightarrow y = l\left(1 + \frac{1}{4\sqrt{2}}\right)$$

$$\Rightarrow y = l\left(\frac{4\sqrt{2} + 1}{4\sqrt{2}}\right)$$

Thus, co-ordinate of I is $\left(\frac{l}{4}, l\left(\frac{4\sqrt{2} + 1}{4\sqrt{2}}\right)\right)$

H	$\left(\frac{l}{4}, 0\right)$
I	$\left(\frac{l}{4}, l\left(\frac{4\sqrt{2} + 1}{4\sqrt{2}}\right)\right)$

Join H and I

2.2 Bisect CF at J and draw a line JK parallel to AB touching CG at point K.

Let J be $\left(0, \frac{l(8+3\sqrt{2})}{12}\right)$

Since $JK \parallel AB$, y-co-ordinate of K is $\frac{l(8+3\sqrt{2})}{12}$

To find x-co-ordinate of K,
Let $C\left(0, \frac{4l}{3}\right) - > (x_1, y_1)$ and $G\left(l, \frac{l}{\sqrt{2}}\right) - > (x_2, y_2)$
Equation of line CG is given by,

$$\begin{aligned}
y - y_1 &= \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \\
\Rightarrow y - \frac{4l}{3} &= \frac{\frac{l}{\sqrt{2}} - \frac{4l}{3}}{l}(x - 0) \\
\Rightarrow \frac{l(8 + 3\sqrt{2})}{12} - \frac{4l}{3} &= x \cdot \frac{3 - 4\sqrt{2}}{3\sqrt{2}} \\
\Rightarrow \frac{l(8 + 3\sqrt{2} - 16)}{12} &= x \cdot \frac{3 - 4\sqrt{2}}{3\sqrt{2}} \\
\Rightarrow x &= \frac{l(6 - 8\sqrt{2})}{12 - 6\sqrt{2}} \\
\Rightarrow x &= \frac{l}{2}
\end{aligned}$$

Thus, co-ordinate of K is $\left(\frac{l}{2}, \frac{l(8+3\sqrt{2})}{12}\right)$

J	$\left(0, \frac{l(8+3\sqrt{2})}{12}\right)$
K	$\left(\frac{l}{2}, \frac{l(8+3\sqrt{2})}{12}\right)$

Join J and K

2.3 Let L be the point where lines JK and HI cut one another.

Let L be $\left(\frac{l}{4}, \frac{l(8+3\sqrt{2})}{12}\right)$

L	$\left(\frac{l}{4}, \frac{l(8+3\sqrt{2})}{12}\right)$
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2.4 Join J and G.

2.5 Let M be the point where line JG and HI cut one another.

Let $J\left(0, \frac{l(8+3\sqrt{2})}{12}\right) - > (x_1, y_1)$ and $G\left(l, \frac{l}{\sqrt{2}}\right) - > (x_2, y_2)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\begin{aligned}
\Rightarrow y - \frac{l(8+3\sqrt{2})}{12} &= \frac{\frac{l}{\sqrt{2}} - \frac{l(8+3\sqrt{2})}{12}}{l} \cdot x \\
\Rightarrow y &= \frac{l(8+3\sqrt{2})}{12} + \left(\frac{1}{\sqrt{2}} - \frac{(8+3\sqrt{2})}{12} \right) \left(\frac{l}{4} \right) \\
\Rightarrow y &= \frac{l(8+3\sqrt{2})}{12} + \frac{12-8\sqrt{2}-6}{12\sqrt{2}} \left(\frac{l}{4} \right) \\
\Rightarrow y &= \frac{l(8+3\sqrt{2})}{12} + \frac{6-8\sqrt{2}}{12\sqrt{2}} \left(\frac{l}{4} \right) \\
\Rightarrow y &= \frac{l(8+3\sqrt{2})}{12} + \frac{l(6-8\sqrt{2})}{48\sqrt{2}} \\
\Rightarrow y &= \frac{l(32\sqrt{2}+24+6-8\sqrt{2})}{48\sqrt{2}} \\
\Rightarrow y &= \frac{l(24\sqrt{2}+30)}{48\sqrt{2}} \\
\Rightarrow y &= \frac{l(12\sqrt{2}+15)}{24\sqrt{2}}
\end{aligned}$$

Thus, co-ordinate of M is $\left(\frac{l}{4}, \frac{l(4\sqrt{2}+5)}{8\sqrt{2}} \right)$

M	$\left(\frac{l}{4}, \frac{l(4\sqrt{2}+5)}{8\sqrt{2}} \right)$
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2.6 With centre M and with a distance shortest from M to BD mark off N on the lower portion of line HI.

Shortest distance from M to BD is $\frac{l}{8}$.

Thus, co-ordinate of N is $\left(\frac{l}{4}, \frac{l(3\sqrt{2}+5)}{8\sqrt{2}} \right)$

N	$\left(\frac{l}{4}, \frac{l(3\sqrt{2}+5)}{8\sqrt{2}} \right)$
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2.7 Touching M and starting from O, a point on AC, draw a line from left to right parallel to AB.

Let point O be $\left(0, \frac{l(12\sqrt{2}+15)}{24\sqrt{2}} \right)$

O	$\left(0, \frac{l(12\sqrt{2}+15)}{24\sqrt{2}} \right)$
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2.8 With centre L and radius LN draw a semi-circle on the lower portion and let P and Q be the points where it touches the line OM respectively.

Let $L\left(\frac{l}{4}, \frac{l(8+3\sqrt{2})}{12}\right) = (h, k)$,
Radius(r) of semi-circle is given by,

$$\begin{aligned} r &= \sqrt{\left(\frac{l(9\sqrt{2}+15)}{24\sqrt{12}} - \frac{l(8+3\sqrt{2})}{12}\right)^2} \\ &= \frac{l(9\sqrt{2}+15-16\sqrt{2}-12)}{24\sqrt{2}} \\ &= \frac{l(3-7\sqrt{2})}{24\sqrt{2}} \end{aligned}$$

Now, Equation of semi-circle is given by,

$$y = k - \sqrt{r^2 - (x-h)^2}$$

$$y = \frac{l(8+3\sqrt{2})}{12} - \sqrt{\left(\left(\frac{l(3-7\sqrt{2})}{24\sqrt{2}}\right)^2 - \left(x - \frac{l}{4}\right)^2\right)}$$

Plotting Equation in graph.

Now Solving Equation of semi-circle and line OM,

$$\text{Let the point P} = \left(\frac{l}{4} - l\sqrt{\frac{11-3\sqrt{2}}{192}}, \frac{l(12\sqrt{2}+15)}{24\sqrt{2}}\right),$$

$$\text{Let the point Q} = \left(\frac{l}{4} + l\sqrt{\frac{11-3\sqrt{2}}{192}}, \frac{l(12\sqrt{2}+11.25)}{24\sqrt{2}}\right)$$

2.9 With centre M and radius MQ draw a semi-circle on the lower portion touching P and Q.

Let $M\left(\frac{l}{4}, \frac{l(4\sqrt{2}+5)}{8\sqrt{2}}\right) = (h, k)$,
Radius(r) of semi-circle is given by,

$$\begin{aligned} r = MQ &= \sqrt{\left(\frac{l}{4} + l\sqrt{\frac{11-3\sqrt{2}}{192}} - \frac{l}{4}\right)^2} \\ &= l\sqrt{\frac{11-3\sqrt{2}}{192}} \end{aligned}$$

Now, Equation of semi-circle is given by,

$$y = k - \sqrt{r^2 - (x-h)^2}$$

$$y = \frac{l(12\sqrt{2}+15)}{24\sqrt{2}} - \sqrt{\left(l\sqrt{\frac{11-3\sqrt{2}}{192}}\right)^2 - \left(x - \frac{l}{4}\right)^2}$$

2.10 With centre N and radius NM draw an arc touching PNQ at R and S. Join RS. Let T be the point where RS and HI cut one another.

$$\text{Let } N\left(\frac{l}{4}, \frac{l(3\sqrt{2}+5)}{8\sqrt{2}}\right) = (h,k),$$

Radius(r) of semi-circle is given by,

$$\begin{aligned} r = MN &= \sqrt{\left(\frac{l(4\sqrt{2}+5)}{8\sqrt{2}} - \frac{l(3\sqrt{2}+5)}{8\sqrt{2}}\right)^2} \\ &= \frac{l(-3\sqrt{2}+15+4\sqrt{2}-15)}{8\sqrt{2}} \\ &= \frac{l\sqrt{2}}{8\sqrt{2}} \\ &= \frac{l}{8} \end{aligned}$$

Now Equation of semi-circle is given by,

$$y = k - \sqrt{r^2 - (x - h)^2}$$

$$y = \frac{l(3\sqrt{2}+5)}{8\sqrt{2}} + \sqrt{\frac{l^2}{64} - \left(x - \frac{l}{4}\right)^2}$$

Solving Equations of two semi-circles,

$$\begin{aligned} y &= \frac{r_1^2 - r_2^2 - k_1^2 + k_2^2}{2(k_2 - k_1)} \\ &= \frac{l(26+15\sqrt{2})}{8(7\sqrt{2}-3)} \\ x &= \frac{l}{4} \pm \sqrt{\frac{l^2}{64} \left(1 - \left(\frac{3}{14-3\sqrt{2}}\right)^2\right)} \end{aligned}$$

Let the point R be $\left(\frac{l}{4} + \sqrt{\frac{l^2}{64} \left(1 - \left(\frac{3}{14-3\sqrt{2}}\right)^2\right)}, \frac{l(26+15\sqrt{2})}{8(7\sqrt{2}-3)}\right)$, S be

$$\left(\frac{l}{4} - \sqrt{\frac{l^2}{64} \left(1 - \left(\frac{3}{14-3\sqrt{2}}\right)^2\right)}, \frac{l(26+15\sqrt{2})}{8(7\sqrt{2}-3)}\right)$$

Let the point T be $\left(\frac{l}{4}, \frac{l(26+15\sqrt{2})}{8(7\sqrt{2}-3)}\right)$

2.11 With center T and radius TS draw a semi-circle on the upper portion of PNQ touching at two points.

Equation of semi-circle is given by,

$$y = \frac{l(26+15\sqrt{2})}{8(7\sqrt{2}-3)} + \sqrt{\frac{l^2}{64} \left(1 - \left(\frac{3}{14-3\sqrt{2}}\right)^2\right) - \left(x - \frac{l}{4}\right)^2}$$

2.12 With center T and radius TM draw an arc on the upper portion of PNQ touching at two points.

Equation of semi-circle is given by,

$$y = \frac{l(26+15\sqrt{2})}{8(7\sqrt{2}-3)} + \sqrt{\left(\frac{l(26+15\sqrt{2})}{8(7\sqrt{2}-3)} - \frac{l(4\sqrt{2}+5)}{8\sqrt{2}}\right)^2 - \left(x - \frac{l}{4}\right)^2}$$

2.13 Eight equal and similar triangles of the moon are to be made in the space lying inside the semi-circle of No.(2.11) and outside the arc of No.(2.12) of his Schedule.

$$\text{Let } r_1 = \left| \frac{l(26+15\sqrt{2})}{8(7\sqrt{2}-3)} - \frac{l(4\sqrt{2}+5)}{8\sqrt{2}} \right|, r_2 = \left| \frac{l}{8} \sqrt{\left(1 - \left(\frac{3}{14-3\sqrt{2}}\right)^2\right)} \right|,$$

$$(h, k) - \left(\frac{l}{4}, \frac{l(26+15\sqrt{2})}{8(7\sqrt{2}-3)} \right)$$

S.N.	x	y
1	$h + r_2 \cos 0$	$k + r_2 \sin 0$
2	$h + r_1 \cos 11.25$	$k + r_1 \sin 11.25$
3	$h + r_2 \cos (2 \cdot 11.25)$	$k + r_2 \sin (2 \cdot 11.25)$
4	$h + r_1 \cos (3 \cdot 11.25)$	$k + r_1 \sin (3 \cdot 11.25)$
5	$h + r_2 \cos (4 \cdot 11.25)$	$k + r_2 \sin (4 \cdot 11.25)$
6	$h + r_1 \cos (5 \cdot 11.25)$	$k + r_1 \sin (5 \cdot 11.25)$
7	$h + r_2 \cos (6 \cdot 11.25)$	$k + r_2 \sin (6 \cdot 11.25)$
8	$h + r_1 \cos (7 \cdot 11.25)$	$k + r_1 \sin (7 \cdot 11.25)$
9	$h + r_2 \cos (8 \cdot 11.25)$	$k + r_2 \sin (8 \cdot 11.25)$
10	$h + r_1 \cos (9 \cdot 11.25)$	$k + r_1 \sin (9 \cdot 11.25)$
11	$h + r_2 \cos (10 \cdot 11.25)$	$k + r_2 \sin (10 \cdot 11.25)$
12	$h + r_1 \cos (11 \cdot 11.25)$	$k + r_1 \sin (11 \cdot 11.25)$
13	$h + r_2 \cos (12 \cdot 11.25)$	$k + r_2 \sin (12 \cdot 11.25)$
14	$h + r_1 \cos (13 \cdot 11.25)$	$k + r_1 \sin (13 \cdot 11.25)$
15	$h + r_2 \cos (14 \cdot 11.25)$	$k + r_2 \sin (14 \cdot 11.25)$
16	$h + r_1 \cos (15 \cdot 11.25)$	$k + r_1 \sin (15 \cdot 11.25)$
17	$h + r_1 \cos (16 \cdot 11.25)$	$k + r_1 \sin (16 \cdot 11.25)$

Join 1,2,3,...,17



3 Method of Making the Sun

- 3.1 Bisect line AF at U, and draw a line UV parallel to AB line touching line BE at V.**

Let the point U be $\left(0, \frac{l}{2\sqrt{2}}\right)$ and V be $\left(l\left(\frac{2\sqrt{2}-1}{2\sqrt{2}}\right), \frac{l}{2\sqrt{2}}\right)$

- 3.2 With center W, the point where HI and UV cut one another and radius MN draw a circle.**

Let the point W be $\left(\frac{l}{4}, \frac{l}{2\sqrt{2}}\right)$

Equation of circle is given by,

$$\left(x - \frac{l}{4}\right)^2 + \left(y - \frac{l}{2\sqrt{2}}\right)^2 = \frac{l^2}{64}$$

- 3.3 With center W and radius LN draw a circle.**

Equation of circle is given by,

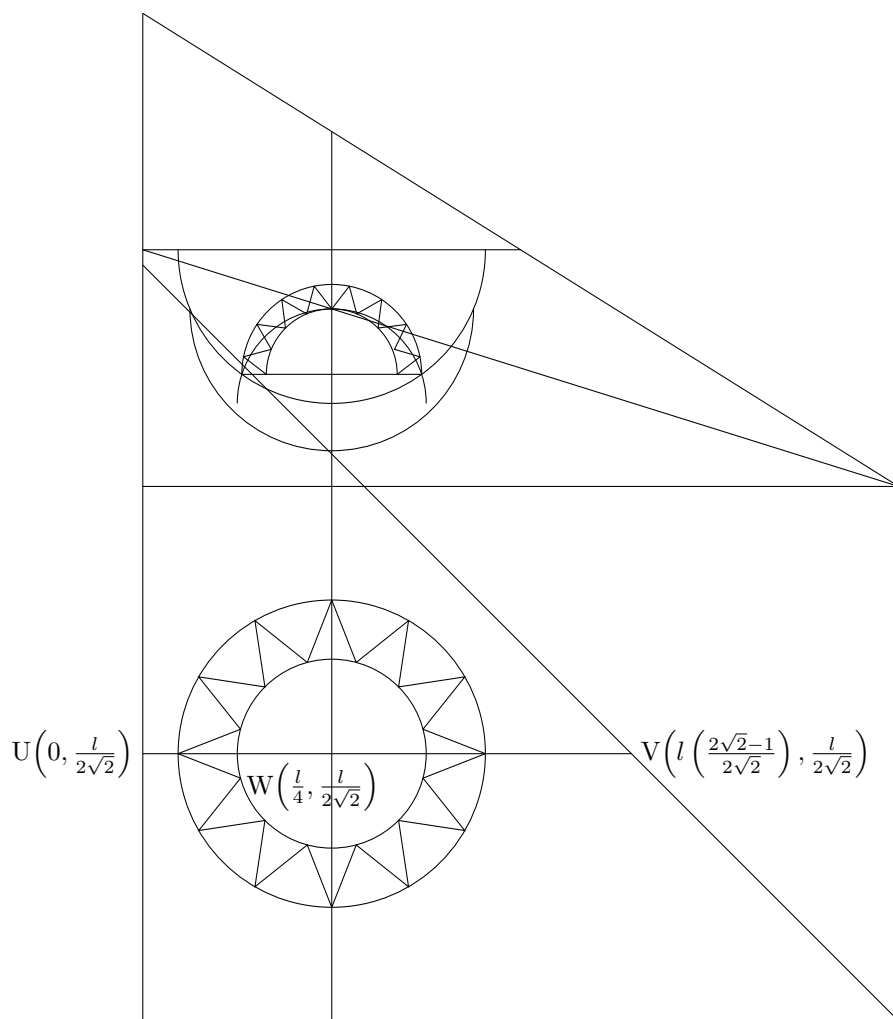
$$\left(x - \frac{l}{4}\right)^2 + \left(y - \frac{l}{2\sqrt{2}}\right)^2 = \left(l\left(\frac{7\sqrt{2}-3}{24\sqrt{2}}\right)\right)^2$$

3.4 Twelve equal and similar triangles of the sun are to be made in the space enclosed by the circle of No.(3.2) and No.(3.3) with the two apexes of two triangles touching line HI.

$$\text{Let } r_1 = \left| \frac{l}{8} \right|, r_2 = \left| l \left(\frac{7\sqrt{2}-3}{24\sqrt{2}} \right) \right|, (h, k) = \left(\frac{l}{4}, \frac{l}{2\sqrt{2}} \right)$$

SN.	x	y
1	$h + r_2 \cos 0$	$k + r_2 \sin 0$
2	$h + r_1 \cos 15$	$k + r_1 \sin 15$
3	$h + r_2 \cos (2 \cdot 15)$	$k + r_2 \sin (2 \cdot 15)$
4	$h + r_1 \cos (3 \cdot 15)$	$k + r_1 \sin (3 \cdot 15)$
5	$h + r_2 \cos (4 \cdot 15)$	$k + r_2 \sin (4 \cdot 15)$
6	$h + r_1 \cos (5 \cdot 15)$	$k + r_1 \sin (5 \cdot 15)$
7	$h + r_2 \cos (6 \cdot 15)$	$k + r_2 \sin (6 \cdot 15)$
8	$h + r_1 \cos (7 \cdot 15)$	$k + r_1 \sin (7 \cdot 15)$
9	$h + r_2 \cos (8 \cdot 15)$	$k + r_2 \sin (8 \cdot 15)$
10	$h + r_1 \cos (9 \cdot 15)$	$k + r_1 \sin (9 \cdot 15)$
11	$h + r_2 \cos (10 \cdot 15)$	$k + r_2 \sin (10 \cdot 15)$
12	$h + r_1 \cos (11 \cdot 15)$	$k + r_1 \sin (11 \cdot 15)$
13	$h + r_2 \cos (12 \cdot 15)$	$k + r_2 \sin (12 \cdot 15)$
14	$h + r_1 \cos (13 \cdot 15)$	$k + r_1 \sin (13 \cdot 15)$
15	$h + r_2 \cos (14 \cdot 15)$	$k + r_2 \sin (14 \cdot 15)$
16	$h + r_1 \cos (15 \cdot 15)$	$k + r_1 \sin (15 \cdot 15)$
17	$h + r_2 \cos (16 \cdot 15)$	$k + r_2 \sin (16 \cdot 15)$
18	$h + r_1 \cos (17 \cdot 15)$	$k + r_1 \sin (17 \cdot 15)$
19	$h + r_2 \cos (18 \cdot 15)$	$k + r_2 \sin (18 \cdot 15)$
20	$h + r_1 \cos (19 \cdot 15)$	$k + r_1 \sin (19 \cdot 15)$
21	$h + r_2 \cos (20 \cdot 15)$	$k + r_2 \sin (20 \cdot 15)$
22	$h + r_1 \cos (21 \cdot 15)$	$k + r_1 \sin (21 \cdot 15)$
23	$h + r_2 \cos (22 \cdot 15)$	$k + r_2 \sin (22 \cdot 15)$
24	$h + r_1 \cos (23 \cdot 15)$	$k + r_1 \sin (23 \cdot 15)$
25	$h + r_2 \cos (24 \cdot 15)$	$k + r_2 \sin (24 \cdot 15)$

Join 1,2,3,...25



4 Method of Making the Border.

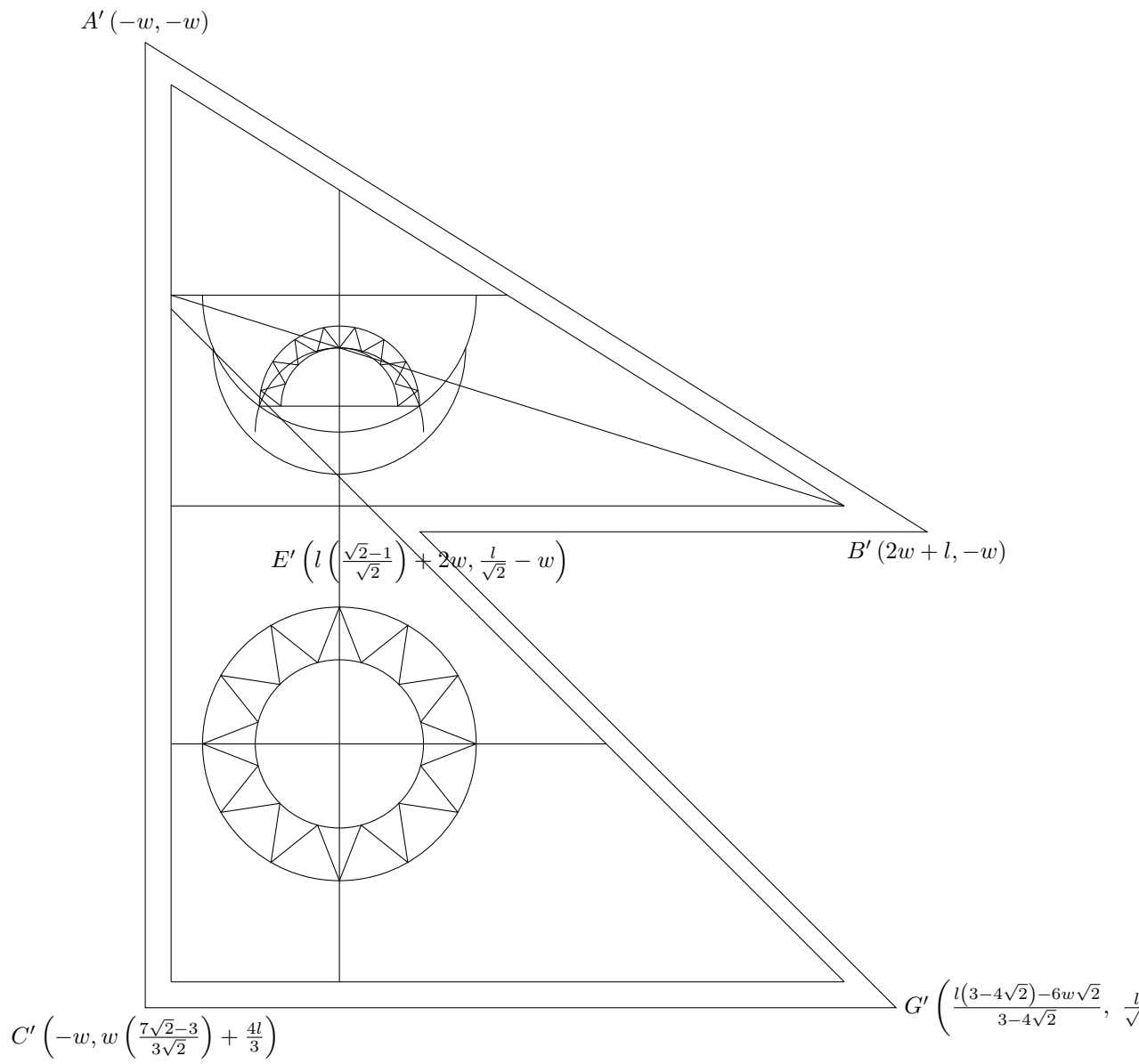
4.1 The width of the border will be equal to the width of TN.

Let distance $TN = \left(\frac{l(3\sqrt{2}+5)}{8\sqrt{2}} - \frac{l(26+15\sqrt{2})}{8(7\sqrt{2}-3)} \right)$ be w ,

Let the points

$$A' = (-w, -w), B' = (2w + l, -w), E' = \left(l \left(\frac{\sqrt{2}-1}{\sqrt{2}} \right) + 2w, \frac{l}{\sqrt{2}} - w \right), G' = \left(\frac{l(3-4\sqrt{2})-6w\sqrt{2}}{3-4\sqrt{2}}, \frac{l}{\sqrt{2}} - w \right), C' = \left(-w, w \left(\frac{7\sqrt{2}-3}{3\sqrt{2}} \right) + \frac{4l}{3} \right)$$

Now Join A', B', E', G' and C'



Point	Co-ordinate
A	$(0, 0)$
B	$(l, 0)$
C	$(0, \frac{4l}{3})$
D	$(0, l)$
E	$(\frac{l(\sqrt{2}-1)}{\sqrt{2}}, \frac{l}{\sqrt{2}})$
F	$(0, \frac{l}{\sqrt{2}})$
G	$(l, \frac{l}{\sqrt{2}})$
H	$(\frac{l}{4}, 0)$
I	$(\frac{l}{4}, l(\frac{4\sqrt{2}+1}{4\sqrt{2}}))$
J	$(0, \frac{l(8+3\sqrt{2})}{12})$
K	$(\frac{l}{2}, \frac{l(8+3\sqrt{2})}{12})$
L	$(\frac{l}{4}, \frac{l(8+3\sqrt{2})}{12})$
M	$(\frac{l}{4}, \frac{l(4\sqrt{2}+5)}{8\sqrt{2}})$
N	$(\frac{l}{4}, \frac{l(3\sqrt{2}+5)}{8\sqrt{2}})$
O	$(0, \frac{l(12\sqrt{2}+15)}{24\sqrt{2}})$
P	$(\frac{l}{4} - l\sqrt{\frac{11-3\sqrt{2}}{192}}, \frac{l(12\sqrt{2}+15)}{24\sqrt{2}})$
Q	$(\frac{l}{4} + l\sqrt{\frac{11-3\sqrt{2}}{192}}, \frac{l(12\sqrt{2}+15)}{24\sqrt{2}})$
R	$(\frac{l}{4} + \sqrt{\frac{l^2}{64} \left(1 - \left(\frac{3}{14-3\sqrt{2}}\right)^2\right)}, \frac{l(26+15\sqrt{2})}{8(7\sqrt{2}-3)})$
S	$(\frac{l}{4} - \sqrt{\frac{l^2}{64} \left(1 - \left(\frac{3}{14-3\sqrt{2}}\right)^2\right)}, \frac{l(26+15\sqrt{2})}{8(7\sqrt{2}-3)})$
T	$(\frac{l}{4}, \frac{l(26+15\sqrt{2})}{8(7\sqrt{2}-3)})$
U	$(0, \frac{l}{2\sqrt{2}})$
V	$(l(\frac{2\sqrt{2}-1}{2\sqrt{2}}), \frac{l}{2\sqrt{2}})$
W	$(\frac{l}{4}, \frac{l}{2\sqrt{2}})$
A'	$(-w, -w)$
B'	$(2w + l, -w)$
E'	$(l(\frac{\sqrt{2}-1}{\sqrt{2}}) + 2w, \frac{l}{\sqrt{2}} - w)$
G'	$(\frac{l(3-4\sqrt{2})-6w\sqrt{2}}{3-4\sqrt{2}}, \frac{l}{\sqrt{2}} - w)$
C'	$(-w, w(\frac{7\sqrt{2}-3}{3\sqrt{2}}) + \frac{4l}{3})$