UNIT - 2

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LINEAR ALGEBRA - II

1) $y = (3,4)$, $u = (1,2)$

$$\text{projection}_u y = \frac{y \cdot u}{u \cdot u} \cdot u$$

$y \cdot u = (3,4) \cdot (1,2) = 3 + 4(2) = 11$

$u \cdot u = (1,2) \cdot (1,2) = 1 + 4 = 5$

$$\text{projection}_u y = \frac{11}{5}(1,2)$$

orthogonal projection of y on u $= \left(\frac{11}{5}, \frac{22}{5}\right)$

2) $V = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $A = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}$

$$AV = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 9 \\ 5 \end{bmatrix} = \frac{9}{2} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

V is NOT an eigenvector of A

3) $A = \begin{bmatrix} 2 & 1 & 5 \\ -2 & -3 & -2 \\ 3 & 3 & 1 \end{bmatrix}$

$a_1 = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$ $a_2 = \begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix}$ $a_3 = \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$

$u_1 = a_1$

$\|u_1\| = \sqrt{17}$ $\Rightarrow q_1 = \frac{u_1}{\sqrt{17}}$

$$u_2 = a_2 - \frac{a_2 \cdot u_1}{u_1 \cdot u_1} u_1 = \begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix} - \frac{17}{17} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$$

$$q_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$$

$u_3 = a_3 - \text{projection}_{u_1} \cdot a_3 - \text{proj}_{u_2} \cdot a_3$

$$u_3 = \begin{bmatrix} 3/2 \\ 3/2 \\ -1 \end{bmatrix}$$

$$q_3 = \frac{1}{\sqrt{19/2}} \begin{bmatrix} 3/2 \\ 3/2 \\ -1 \end{bmatrix}$$

$R = Q^T A$

4) $A = PDP^{-1}$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

$|A - \lambda I| = 0$ $\lambda = 0, 2, 4$

When $\lambda = 0$: $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

$\lambda = 2$: $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

$\lambda = 4$: $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

$P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 0 & 2 \end{bmatrix}$ $D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$A = PDP^{-1}$ //

5) $A = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ -1 & -1 & 0 \end{bmatrix}$

$a_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$ $a_2 = \begin{bmatrix} -1 \\ 4 \\ 4 \\ -1 \end{bmatrix}$ $a_3 = \begin{bmatrix} 4 \\ -2 \\ 2 \\ 0 \end{bmatrix}$

$u_1 = a_1$ $\|u_1\| = \sqrt{1+1+1+1} = 2$

$$q_1 = \frac{u_1}{2} = \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$u_2 = a_2 - u_1 \cdot a_2$

$$a_2 = \frac{a_2 \cdot u_1}{u_1 u_1} u_1 = \frac{8}{4} a_1 = 2u_1$$

$$u_2 = \begin{bmatrix} -1 \\ 4 \\ 4 \\ -1 \end{bmatrix} - 2\begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$

$$\|u_2\| = \sqrt{18} \qquad q_2 = \frac{1}{\sqrt{18}} \begin{bmatrix} -3 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$

$$u_3 = a_3 - \frac{u_1 a_3}{u_1 \cdot u_1}(u_1) - \frac{u_2 a_3}{u_2 \cdot u_2}(u_2)$$

$$u_3 = \begin{bmatrix} 2 \\ -2 \\ 2 \\ 2 \end{bmatrix} \qquad q_3 = \sqrt{\cancel{4+4+4+4}}\ 4$$

6) $A = \begin{bmatrix} 5 & 7 & 0 \\ 5 & 1 & 0 \end{bmatrix}$

$$A^T \cdot A = \begin{bmatrix} 50 & 40 & 0 \\ 40 & 50 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$|A^T \cdot A - \lambda I| = 0 \rightarrow \begin{bmatrix} 50-\lambda & 40 & 0 \\ 40 & 50-\lambda & 0 \\ 0 & 0 & -\lambda \end{bmatrix}$$

$\lambda = 90, 10, 0$

$\sigma_1 = \sqrt{90} = 3\sqrt{10}$, $\sigma_2 = \sqrt{10}$, $\sigma_3 = 0$

For $\lambda = 90$

$$\begin{bmatrix} -40 & 40 & 0 \\ 40 & -40 & 0 \\ 0 & 0 & -90 \end{bmatrix} \qquad v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\|v_1\| = \sqrt{1+1} = \sqrt{2} \qquad \hat{v}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

For $\lambda = 10$

$$\begin{bmatrix} 40 & 40 & 0 \\ 40 & 40 & 0 \\ 0 & 0 & -10 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\|v_2\| = \sqrt{2} \quad \hat{v}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

For $\lambda = 0$

$$v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$V = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 3\sqrt{10} & 0 & 0 \\ 0 & \sqrt{10} & 0 \end{bmatrix}$$

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$$u_i = \frac{1}{\sigma_i} A v_i$$

$$= \frac{1}{3\sqrt{10}} A\hat{v}_1 = \frac{1}{3\sqrt{10}} \begin{bmatrix} 12/\sqrt{2} \\ 6/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$$

$$u_2 = \frac{1}{\sqrt{10}} A\hat{v}_2 = \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$$

$$U = \begin{bmatrix} \frac{2}{\sqrt{5}} & -2/\sqrt{5} \\ 1/\sqrt{5} & 1/\sqrt{5} \end{bmatrix}$$

7) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ x & y & x \end{bmatrix}$

$(1,1,1) \cdot (x,y,z) = 0$

$x + y + z = 0$ — (1)

$(1,-1,0) \cdot (x,y,z) = 0$

$x - y = 0 \Rightarrow x = y$ — (2)

$y = x$

$x + x + z = 0 \qquad z = -2x$

Let $x = 1$

$y = 1,\ z = -2$

$(1, 1, -2)$

8) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\det(A) = 0$

$= 1(\lambda^2 - 1) - (\lambda - 1) + (1 - \lambda)$

$= \lambda^2 - 2\lambda$

$\lambda(\lambda - 2) = 0$

$\lambda = 0$ or $\lambda = 2$.

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9. $v_1 = (-3, 1, 0, 2, -1)$,

$v_2 = (1, 2, -3, -1, 2)$,

$v_3 = (3, 2, -1, -1, 3)$

$$||v_1|| = \sqrt{(-3)^2 + 1^2 + 0^2 + 2^2 + (-1)^2} = \sqrt{15}$$

$$u_1 = \frac{v_1}{||v_1||} = \frac{1}{\sqrt{15}}(-3, 1, 0, 2, -1)$$

$$v_2 = (v_2 u_1) u_1$$

$$= (1, 2, -3, -1, 2) \cdot \frac{1}{\sqrt{15}}(-3, 1, 0, 2, -1)$$

$$= \frac{-5}{\sqrt{15}}$$

$$w_2 = v_2 - (v_2 \cdot u_1) u_1$$

$$= (1, 2, -3, -1, 2) - \frac{-5}{\sqrt{15}} \cdot \frac{1}{\sqrt{15}}(-3, 1, 0, 2, -1)$$

$$= \left(0, \frac{7}{3}, -3, \frac{-1}{3}, \frac{5}{3}\right)$$

$$||w_2|| = \sqrt{0^2 + \left(\frac{7}{3}\right)^2 + (-3)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{5}{3}\right)^2}$$

$$= \frac{\sqrt{156}}{3} = \frac{2\sqrt{39}}{3}$$

$$u_2 = \frac{w_2}{||w_2||} = \cancel{0}.$$

$$\frac{1}{2\sqrt{39}/3}\left(0, \frac{7}{3}, -3, \frac{-1}{3}, \frac{5}{3}\right)$$

$$= 0, \frac{7}{2\sqrt{39}}, \frac{-9}{2\sqrt{39}}, \frac{-1}{2\sqrt{39}}, \frac{5}{2\sqrt{39}}$$

$$v_3 = (v_2 \cdot u_1) u_1$$

$$v_3 \cdot u_1 = (3, 2, -1, -1, 3) \cdot \frac{1}{\sqrt{15}}(-3, 1, 0, 2, -1)$$

$$= \frac{-12}{\sqrt{15}}$$

$$(v_3 \cdot u_2) u_2 \Rightarrow u_3 u_2 = (3, 2, -1, -1, 3) \cdot \frac{1}{2\sqrt{39}}(0, 7, -9, -1, 5)$$

$$= \frac{1}{2\sqrt{39}}(0 + 14 + 9 + 1 + 15) = \frac{\sqrt{39}}{2}$$

$$w_3 = v_3 - \frac{v_3 \cdot u_1}{\cancel{u_1 \cdot u_1}}(u_1) - (v_3 u_2) u_2$$

$$w_3 = (3, 2, -1, -1, 3) - \left(\frac{-12}{\sqrt{15}} \cdot \frac{1}{\sqrt{15}}\right) \cdot (-3, 1, 0, 2, -1) - \frac{\sqrt{39}}{2} \cdot \frac{1}{2\sqrt{39}}(0, 7, -9, -1, 5)$$

$$w_3 = \left(\frac{3}{5}, \frac{21}{20}, \frac{5}{4}, \frac{17}{20}, \frac{19}{20}\right)$$

$$||w_3|| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{21}{20}\right)^2 + \left(\frac{5}{4}\right)^2 + \left(\frac{17}{20}\right)^2 + \left(\frac{19}{20}\right)^2} = \frac{\sqrt{465}}{10}$$

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$$u_3 = \frac{w_3}{||w_3||} = \frac{1}{\sqrt{93}/20}\left(\frac{3}{5}, \frac{21}{20}, \frac{5}{4}, \frac{17}{20}, \frac{19}{20}\right)$$

$$= \frac{2\sqrt{5}}{\sqrt{93}} \cdot \frac{1}{20}(12, 21, 25, 17, 19)$$

$$u_3 = \frac{\sqrt{465}}{930}(12, 21, 25, 17, 19)$$

$$u_1 = \left(\frac{-3}{\sqrt{15}}, \frac{1}{\sqrt{15}}, 0, \frac{2}{\sqrt{15}}, -\frac{1}{\sqrt{15}}\right)$$

$$u_2 = \left(0, \frac{7}{2\sqrt{39}}, \frac{-9}{2\sqrt{39}}, \frac{-1}{2\sqrt{39}}, \frac{5}{2\sqrt{39}}\right)$$

$$u_3 = \frac{1}{2\sqrt{465}}(12, 21, 25, 17, 19)$$

10) $[A - \lambda I] = 0$

$$= \begin{bmatrix} 7-\lambda & -4 & -2 \\ -4 & 1-\lambda & -4 \\ -2 & -4 & 7-\lambda \end{bmatrix} = 0$$

$$(7-\lambda)[(1-\lambda)(7-\lambda) - 16] +$$

$$4(-4(7-\lambda) - 8) - 2(16 + 2 - 2\lambda)$$

$$\lambda^3 - 15\lambda^2 + 27\lambda + 243 = 0$$

$$(\lambda + 3)(\lambda - 9)^2 = 0$$

$$\lambda_1 = -3, \lambda_2 = 9, \lambda_3 = 9$$

when $\lambda = -3$

$$\begin{bmatrix} 10 & -4 & -2 \\ -4 & 4 & -4 \\ -2 & -4 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$V_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

when $\lambda_2, \lambda_3 = 9$

$$\begin{bmatrix} -2 & -4 & -2 \\ -4 & -8 & -4 \\ -2 & -4 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x - 4y - 2z = 0$

$x + 2y + z = 0$

Let $y = 1, z = 0$ $\quad V_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

$x = -2$

Let $y = 0, z = 1$ $\quad V_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$x = -1$

$$P = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$P^{-1} = \frac{1}{|P|} \operatorname{adj}(P)$$

$|P| = 1(1(1) - 0(0)) - (-2)(2-0) +$

$(-1(2(0) - 1)) = 6$

$$P^{-1} = \frac{1}{6}\begin{bmatrix} 1 & 2 & 1 \\ -2 & 2 & 2 \\ -1 & -2 & 5 \end{bmatrix}$$

11.
$$A = \begin{bmatrix} 1 & -2 & 2 \\ 2 & -4 & 1 \\ -2 & 0 & 4 \\ 1 & 0 & 2 \\ 2 & -2 & -1 \end{bmatrix}$$

Let $u_1 = a_1$

$$u_1 = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \\ 2 \end{bmatrix}$$

$\|u_1\| = \sqrt{1^2 + 2^2 + (-2)^2 + 1^2 + 2^2}$

$= \sqrt{14}$

$$q_1 = \frac{1}{\sqrt{14}}\begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \\ 2 \end{bmatrix}$$

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$$u_2 = a_2 - \frac{a_2 u_1}{\|u_1\|^2} u_1$$

$a_2 \cdot u_1 = (-2)(1) + (-4)(2) + (0)(-2)$

$(0)(1) + (-2)(2) = -14$

$$u_2 = \begin{bmatrix} -2 \\ -4 \\ 0 \\ 0 \\ -2 \end{bmatrix} - \frac{-14}{14}\begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

$\|u_2\| = \sqrt{1^2 + 2^2 + 2^2 + 1^2} = \sqrt{10}$

$$q_2 = \frac{1}{\sqrt{10}}\begin{bmatrix} -1 \\ -2 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

$$u_3 = a_3 - \frac{a_3 u_1}{\|u_1\|^2} u_1 - \frac{a_3 u_2}{\|u_2\|^2} u_2$$

$a_3 \cdot u_1 = (2)(1) + (1)(2) + 4(-2) + 2(1) + (-2)$

$= -4$

$a_3 \cdot u_2 = (2)(-1) + (-2) + (4)(-2) + 2 + 0$

$= -10$

$$u_3 = \begin{bmatrix} 2 \\ 1 \\ 4 \\ 2 \\ -1 \end{bmatrix} + \frac{4}{14}\begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \\ 2 \end{bmatrix} + \frac{10}{10}\begin{bmatrix} -1 \\ -2 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 9 \\ -3 \\ 10 \\ 23 \\ -3 \end{bmatrix}$$

$$\|u_3\| = \sqrt{\frac{1}{49}(9^2 + (-3)^2 + 10^2 + 23^2 + (-3)^2}$$

$$= \sqrt{\frac{728}{49}} = \sqrt{\frac{104 \cdot 7}{7 \cdot 7}} = \frac{2\sqrt{26}}{7}$$

$$q_3 = \frac{u_3}{\|u_3\|} = \frac{1}{2\sqrt{26}}\begin{bmatrix} 9 \\ -3 \\ 10 \\ 23 \\ -3 \end{bmatrix}$$

$$r_{11} = a_1 \cdot q_1 = \|u_1\| = \sqrt{14}$$

$$r_{12} = a_2 \cdot q_1 = \frac{a_2 \cdot u_1}{\|u_1\|} = \frac{-14}{\sqrt{14}} = -\sqrt{14}$$

$$r_{22} = a_2 \cdot q_2 = \|u_2\| = \sqrt{10}$$

$$r_{13} = a_3 \cdot q_1 = \frac{a_3 \cdot u_1}{\|u_1\|} = \frac{-4}{\sqrt{14}}$$

$$r_{23} = a_3 \cdot q_2 = \frac{a_3 \cdot u_2}{\|u_2\|} = \frac{-10}{\sqrt{10}} = -\sqrt{10}$$

$$r_{33} = a_3 \cdot q_3 = \|u_3\| = \frac{2\sqrt{26}}{7}$$

$$= \begin{bmatrix} 1/\sqrt{14} & -1/\sqrt{10} & 9/2\sqrt{26} \\ 2/\sqrt{14} & -2/\sqrt{10} & -3/2\sqrt{26} \\ -2/\sqrt{14} & -2/\sqrt{10} & 10/2\sqrt{26} \\ 1/\sqrt{14} & 1/\sqrt{10} & 23/2\sqrt{26} \\ 2/\sqrt{14} & 0 & -3/2\sqrt{26} \end{bmatrix}$$

$$R = \begin{bmatrix} \sqrt{14} & -\sqrt{14} & -4/\sqrt{14} \\ 0 & \sqrt{10} & -\sqrt{10} \\ 0 & 0 & 2\sqrt{26}/7 \end{bmatrix}$$

12. $$A^T A = \begin{bmatrix} 4 & 4 & -2 \\ 2 & 2 & -1 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 4 & 2 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 36 & 18 \\ 18 & 9 \end{bmatrix}$$

$$(A^T A - \lambda I) = 0 = \begin{bmatrix} 36-\lambda & 18 \\ 18 & 9-\lambda \end{bmatrix}$$

$$= (36-\lambda)(9-\lambda) - 18^2 = \lambda^2 - 45\lambda$$

$$= \lambda(\lambda - 45) = 0$$

$$\lambda_1 = 45, \ \lambda_2 = 0$$

$$\sigma_1 = \sqrt{45} = 3\sqrt{5}, \ \sigma_2 = 0$$

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 3\sqrt{5} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

For $\lambda_1 = 45$

$$= \begin{bmatrix} -9 & 18 \\ 18 & -36 \end{bmatrix}\begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

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$$-9v_{11} + 18v_{12} = 0$$

$$v_{11} = 2v_{12}$$

$$v_1 = \frac{1}{\sqrt{2^2+1^2}}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{5}}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$\lambda = 0$

$$\begin{bmatrix} 36 & 18 \\ 18 & 9 \end{bmatrix}\begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

$$36v_{21} + 18v_{22} = 0 \ ; \ v_{22} = -2v_{21}$$

$$v_2 = \frac{1}{\sqrt{1^2 + (-2)^2}}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$V = [v_1 \ v_2] = \frac{1}{\sqrt{5}}\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$$

$$u_1 = \frac{1}{3\sqrt{5}} A v_1 = \frac{1}{3\sqrt{5}}\begin{bmatrix} 4 & 2 \\ 4 & 2 \\ -2 & -1 \end{bmatrix}\left(\frac{1}{\sqrt{5}}\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right)$$

$$= \frac{1}{15}\begin{bmatrix} 10 \\ 10 \\ -5 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -4 \end{bmatrix}$$

$$u_3 = \frac{1}{\sqrt{-1^2 + (-1)^2 + (-4)^2}}\begin{bmatrix} -1 \\ -1 \\ -4 \end{bmatrix} = \frac{1}{3\sqrt{2}}\begin{bmatrix} -1 \\ -1 \\ -4 \end{bmatrix}$$

$$AV_2 = \begin{bmatrix} 4 & 2 \\ 4 & 2 \\ -2 & -1 \end{bmatrix}\left(\frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

13.) $y \cdot u = (3)(2) + (4)(2) = 6 + 8 = 14$

$\|u\|^2 = u_1^2 + u_2^2 = 4 + 4 = 8$

y projection on $u = \frac{14}{8} u = \frac{7}{4}(2,2)$

$= \left(\frac{7}{4} \times 2, \frac{7}{4} \times 2\right) = \left(\frac{7}{2}, \frac{7}{2}\right)$

14.) $v = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$Av = \begin{bmatrix} 5 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$

$= -3 \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 3 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\lambda = 3$

15.) $x_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ $x_2 = \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix}$ $x_3 = \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix}$

$u_1 = x_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

$u_2 = x_2 - \frac{x_2 \cdot u_1}{\|u_1\|^2} u_1$

$x_2 \cdot u_1 = (3)(1) + (-3(-1) + 2(0) = 6$

$\|u_1\|^2 = 1^2 + (-1)^2 + 0^2 = 2$

$u_2 = \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix} - \frac{6}{2} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

$u_3 = x_3 - \frac{x_3 \cdot u_1}{\|u_1\|^2} u_1 - \frac{x_3 u_2}{\|u_2\|^2} u_2$

$x_3 \cdot u_1 = (5)(1) + (1)(-1) + 3(0) = 4$

$x_3 \cdot u_2 = (5)(0) + (1)(0) + 3 \times 2 = 6$

$\|u_2\|^2 = 4$

$u_3 = \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix} - \frac{4}{2} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} - \frac{6}{4} \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$

$\|u_1\| = \sqrt{2}$

$\|u_2\| = \sqrt{4} = 2$

$\|u_3\|^2 = 3^2 + 3^2 + 0^2 = 18$

$\|u_3\| = \sqrt{18} = 3\sqrt{2}$

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$q_1 = \frac{u_1}{\|u_1\|} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

$q_2 = \frac{u_2}{\|u_2\|} = \frac{1}{2} \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$q_3 = \frac{u_3}{\|u_3\|} = \frac{1}{3\sqrt{2}} \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$

$Q = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix}$

$R_{11} = \|u_1\| = \sqrt{2}$

$R_{12} = x_2 \cdot q_1 = \frac{x_2 \cdot u_1}{\|u_1\|} = \frac{6}{\sqrt{2}} = 3\sqrt{2}$

$R_{13} = x_3 q_1 = \frac{x_3 \cdot u_1}{\|u_1\|} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

$R_{22} = \|u_2\| = 2$

$R_{23} = x_3 \cdot q_2 = \frac{x_3 \cdot u_1}{\|u_1\|} = \frac{6}{\sqrt{2}} = 3$

$R_{33} = \|u_3\| = 3\sqrt{2}$

$R = \begin{bmatrix} \sqrt{2} & 3\sqrt{2} & 2\sqrt{2} \\ 0 & 2 & 3 \\ 0 & 0 & 3\sqrt{2} \end{bmatrix}$

16. $A = PDP^{-1}$

$(A - \lambda I) = \begin{bmatrix} -1-\lambda & 2 & -1 \\ 1 & -1-\lambda & 2 \\ 1 & -1 & 2-\lambda \end{bmatrix} = 0$

$(1-\lambda)(\lambda-1)^2 = 0$

$\lambda_1 = 1, \lambda_2 = 1$

For $\lambda = 1$

$\begin{bmatrix} -2 & 2 & -1 \\ 1 & -2 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ $R_1 \to R_1 + 2R_2$

$R_3 \to R_3 - R_2$

$$\sim \begin{bmatrix} 0 & -2 & 3 \\ 1 & -2 & 2 \\ 0 & 1 & -1 \end{bmatrix} R_2 \to R_2 + 2R_3$$

$$\sim \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} \xrightarrow[R_2 \leftrightarrow R_3]{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

has only 1 free variable so, only 1 linearly independent eigenvector $\lambda = 1$

17. Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix}$ $v_2 = \begin{bmatrix} -2 \\ -4 \\ 0 \\ 0 \end{bmatrix}$

$v_3 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}$

$$u_1 = \frac{v_1}{\|v_1\|} = \frac{1}{\sqrt{1^2+2^2+(-2)^2+1^2}} \begin{pmatrix} 1 \\ 2 \\ -2 \\ 1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ 2 \\ -2 \\ 1 \end{pmatrix}$$

$w_2 = v_2 - (v_2 \cdot u_1) u_1$

$$v_2 \cdot u_1 = \frac{1}{\sqrt{10}} \left((-2) + (-4 \cdot 2) + 0(-2) + (0)(1) \right)$$

$$= \frac{-10}{\sqrt{10}} = -\sqrt{10}$$

$$w_2 = \begin{pmatrix} -2 \\ -4 \\ 0 \\ 0 \end{pmatrix} - (-\sqrt{10}) \frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ 2 \\ -2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ -2 \\ -2 \\ 1 \end{pmatrix}$$

$$u_2 = \frac{w_2}{\|w_2\|} = \frac{1}{\sqrt{-1^2+(-2)^2+(-2)^2+1^2}}$$

$$\times \begin{pmatrix} 1 \\ 2 \\ -2 \\ 1 \end{pmatrix} \quad u_2 = \frac{1}{\sqrt{10}} \begin{pmatrix} -1 \\ -2 \\ -2 \\ 1 \end{pmatrix}$$

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$$w_3 = v_3 - \frac{v_3 u_1}{\|u_1\|^2} \cdot u_1 - \frac{v_3 \cdot u_2}{\|u_2\|^2} \cdot u_2$$

$$v_3 \cdot u_1 = \frac{1}{\sqrt{10}} (2 \cdot 1 + 1 \cdot 2 + 1 \cdot (-2) + 0 \cdot 1) = \frac{2}{\sqrt{10}}$$

$$v_3 \cdot u_2 = \frac{1}{\sqrt{10}} (2(-1) + (1) \cdot -2 + 1(-2) + 0 \cdot 1) = \frac{-6}{\sqrt{10}}$$

$$w_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \end{pmatrix} - \frac{2}{\sqrt{10}} u_1 - \frac{-6}{\sqrt{10}} u_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

$$- \frac{2}{10} \begin{pmatrix} 1 \\ 2 \\ -2 \\ 1 \end{pmatrix} + \frac{6}{10} \begin{pmatrix} -1 \\ -2 \\ -2 \\ 1 \end{pmatrix}$$

$$w_3 = \frac{1}{5} \begin{pmatrix} 6 \\ -3 \\ 13 \\ 2 \end{pmatrix}$$

$$\|w_3\| = \frac{1}{5} \sqrt{6^2 + (-3)^2 + 13^2 + 2^2} = \frac{\sqrt{218}}{5}$$

$$u_3 = \frac{w_3}{\|w_3\|} = \frac{1}{\sqrt{218}} \begin{pmatrix} 6 \\ -3 \\ 13 \\ 2 \end{pmatrix}$$

18) $A = \begin{pmatrix} 6 & -3 & 0 \\ -3 & 6 & 0 \end{pmatrix}$

$$AA^T = \begin{pmatrix} 6 & -3 & 0 \\ -3 & 6 & 0 \end{pmatrix} \begin{pmatrix} 6 & -3 \\ -3 & 6 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 45 & -36 \\ -36 & 45 \end{pmatrix}$$

$(AA^T - \lambda I) = 0$

$$= \begin{pmatrix} 45-\lambda & -36 \\ -36 & 45-\lambda \end{pmatrix} = (45-\lambda)^2 - (36)^2$$

$45 - \lambda = \pm 36$

$\lambda_1 = 45 - 36 = 9$, $\lambda_2 = 45 + 36 = 81$

$\sigma_1 = \sqrt{9} = 3$, $\sigma_2 = \sqrt{81} = 9$

$$\Sigma = \begin{pmatrix} 9 & 0 & 0 \\ 0 & 3 & 0 \end{pmatrix}$$

For $\lambda_2 = 81$

$$\begin{pmatrix} -36 & -36 \\ -36 & -36 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

$x_1 + x_2 = 0 \quad v = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$u_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

For $\lambda_1 = 9$

$\begin{pmatrix} 36 & -36 \\ -36 & 36 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$x_1 - x_2 = 0 \quad v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$u_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$U = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$

$V_1 = \frac{1}{\sigma_2} A^T u_1 = \frac{1}{9} \begin{pmatrix} 6 & -3 \\ -3 & 6 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$

$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

$V_2 = \frac{1}{\sigma_2} A^T u_2 = \frac{1}{3} \begin{pmatrix} 6 & -3 \\ -3 & 6 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$

$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$V = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$A = U \Sigma V^T$

$= \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 9 & 0 & 0 \\ 0 & 3 & 0 \end{pmatrix}$

$\begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(9) $A^T A = \begin{bmatrix} 6 & -3 & 6 \\ -2 & 1 & -2 \end{bmatrix} \begin{bmatrix} 6 & -2 \\ -3 & 1 \\ 6 & -2 \end{bmatrix}$

$= \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}$

$(81 - \lambda)(9 - \lambda) - (-27)^2 = 0$

$729 - 90\lambda + \lambda^2 - 729 = 0$

$\lambda^2 - 90\lambda = 0, \ \lambda_1 = 90, \lambda_2 = 0$

~~From~~ $\sigma_1 = \sqrt{90} = 3\sqrt{10}$

$\sigma_2 = 0$

$\Sigma = \begin{bmatrix} 3\sqrt{10} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

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For $\lambda = 90$

$(A^T A - 90I) = \begin{bmatrix} -9 & -27 \\ -27 & -81 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{matrix} 0 \\ 0 \end{matrix}$

$-9x - 27y = 0$

$x = -3y$

~~For $\lambda_1 = 90$~~

$V_1 = \frac{1}{\sqrt{(-3)^2 + 1^2}} \begin{bmatrix} -3 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{10}} \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

For $\lambda_2 = 0$

$\begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$81x - 27y = 0 \Rightarrow y = 3x$

$V_2 = \frac{1}{\sqrt{1^2 + 3^2}} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \frac{1}{\sqrt{10}} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$V = \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}$

$u_i = \frac{1}{\sigma_i} A v_i$

$= \frac{1}{3\sqrt{10}} A v_1 = \frac{1}{3\sqrt{10}} \begin{bmatrix} 6 & -2 \\ -3 & 1 \\ 6 & -2 \end{bmatrix} \frac{1}{\sqrt{10}} \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

$= \frac{1}{3} \begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix}$

$u_2 = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

$u_3 = \frac{1}{3\sqrt{5}} \begin{bmatrix} -4 \\ 2 \\ 5 \end{bmatrix}$

$$V = \begin{bmatrix} -2/3 & 1/\sqrt{5} & -4/3\sqrt{5} \\ 1/3 & 2/\sqrt{5} & 2/3\sqrt{5} \\ -2/3 & 0 & 5/3\sqrt{5} \end{bmatrix}$$

20) $a_1 = \begin{bmatrix} -6 \\ 1 \\ 4 \\ 0 \\ 5 \end{bmatrix}$ $a_2 = \begin{bmatrix} 1 \\ -3 \\ 2 \\ 1 \\ 2 \end{bmatrix}$

$a_3 = \begin{bmatrix} 0 \\ 2 \\ -2 \\ -5 \\ -1 \end{bmatrix}$

$u_1 = a_1 = \begin{bmatrix} -6 \\ 1 \\ 4 \\ 0 \\ 5 \end{bmatrix}$

$\|u_1\|^2 = (-6)^2 + 1^2 + 4^2 + 0^2 + 5^2 = 78$

$\|u_1\| = \sqrt{78}$

$q_1 = \frac{1}{\sqrt{78}} \begin{bmatrix} -6 \\ 1 \\ 4 \\ 0 \\ 5 \end{bmatrix}$

$u_2 = a_2 - \frac{a_2 \cdot u_1}{\|u_1\|^2} u_1$

$a_2 \cdot u_1 = (1)(-6) + (-3)(1) + (2)(4) +$
$(1)(0) + (2)(5) = 9$

$u_2 = \begin{bmatrix} 1 \\ -3 \\ 2 \\ 1 \\ 2 \end{bmatrix} - \frac{9}{78} \begin{bmatrix} -6 \\ 1 \\ 4 \\ 0 \\ 5 \end{bmatrix} = \frac{1}{26} \begin{bmatrix} 44 \\ -81 \\ 40 \\ 26 \\ 37 \end{bmatrix}$

$\|u_2\|^2 = \frac{1}{26^2} (44^2 + (-81)^2 + 40^2 + 26^2 +$
$37^2) = \frac{6071}{338}$

$\|u_2\| = \sqrt{\frac{6071}{338}}$

$q_2 = \frac{1}{\|u_2\|} u_2 = \frac{26}{\sqrt{12142}} \frac{1}{26} \begin{bmatrix} 44 \\ -81 \\ 40 \\ 26 \\ 37 \end{bmatrix}$

$= \frac{1}{\sqrt{12142}} \begin{bmatrix} 44 \\ -81 \\ 40 \\ 26 \\ 37 \end{bmatrix}$

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$u_3 = a_3 - \frac{a_3 u_1}{\|u_1\|^2} u_1 - \frac{a_3 u_2}{\|u_2\|^2} \cdot u_2$

$a_3 \cdot u_1 = 0(-6) + 2 \cdot 1 + (-2) \cdot 4 + (-5) \cdot 0 + (-1) \cdot 5$
$= -11$

$a_3 \cdot u_2 = 0 \cdot 44 + 2(-81) + (-2)40 + (-5)26 +$
$-1 \cdot 37 = -409$

$\frac{a_3 \cdot u_1}{\|u_1\|^2} = \frac{-11}{78}$

$\frac{a_3 \cdot u_2}{\|u_2\|^2} = \frac{-409}{12142/676} = -22.76$

$r_{11} = \sqrt{78}$, $r_{12} = \frac{9}{\sqrt{78}}$, $r_{13} = \frac{-11}{\sqrt{78}}$

$r_{22} = \sqrt{\frac{12142}{676}}$, $r_{23} = \frac{-409}{\sqrt{12142/676}}$

$r_{33} = \|u_3\|$

$$Q = \begin{bmatrix} -6/\sqrt{78} & 44/\sqrt{12142} \\ 1/\sqrt{78} & -81/\sqrt{12142} \\ 4/\sqrt{78} & 40/\sqrt{12142} \\ 0 & 26/\sqrt{12142} \\ 5/\sqrt{78} & 37/\sqrt{12142} \end{bmatrix}$$

$$R = \begin{bmatrix} \sqrt{78} & 9/\sqrt{78} & -11/\sqrt{78} \\ 0 & \sqrt{12142/676} & -409/\sqrt{\frac{12142}{676}} \\ 0 & 0 & 4499 \end{bmatrix}$$

21) $V_1 = \begin{bmatrix} 3 \\ 2 \\ -2 \\ 1 \\ 3 \end{bmatrix}$ $V_2 = \begin{bmatrix} 6 \\ 0 \\ 4 \\ -1 \\ 4 \end{bmatrix}$ $V_3 = \begin{bmatrix} 6 \\ -4 \\ 4 \\ 2 \\ -1 \end{bmatrix}$

$u_1 = V_1$

$\|u_1\|^2 = 3^2 + 2^2 + (-2)^2 + 1^2 + 3^2 = 27$

$\|u_1\| = \sqrt{27} = 3\sqrt{3}$

$e_1 = \frac{1}{3\sqrt{3}} \begin{bmatrix} 3 \\ 2 \\ -2 \\ 1 \end{bmatrix}$

~~$u_2 = V_2 - \frac{V_2 \cdot u_1}{\|u_1\|^2} u_1$~~

~~$V_2 \cdot u_1 = 6 \cdot 3 + 0 \cdot 2 + 4(-2) + (-1 \cdot 1) + 4 \cdot 3 = 21$~~

$$u_2 = v_2 - \frac{v_2 \cdot u_1}{\|u_1\|^2} \cdot u_1$$

$v_2 \cdot u_1 = 6\cdot 3 + 0\cdot 2 + 4(-2) + (-1\cdot 1) + 4\cdot 3 = 21$

22) $(A-\lambda I) = \begin{bmatrix} 5-\lambda & -4 & -2 \\ -4 & 5-\lambda & 2 \\ -2 & 2 & 2-\lambda \end{bmatrix} = 0$

$-\lambda^3 + 12\lambda^2 - 21\lambda + 10 = 0$

$(\lambda-1)^2(\lambda-10) = 0$

$\lambda_1 = 10,\ \lambda_2 = 1\ \&\ \lambda_3 = 1$

For $\lambda_1 = 10$

$\begin{bmatrix} -5 & -4 & -2 \\ -4 & -5 & 2 \\ -2 & 2 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$V_1 = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$

For $\lambda_2 = \lambda_3 = 1$

$\begin{bmatrix} 4 & -4 & -2 \\ -4 & 4 & 2 \\ -2 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$2x - 2y - z = 0$

$\lambda = 1$ Let $u_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$u_3 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} - \frac{\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}}{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1/2 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$

$e_1 = \frac{1}{\sqrt{2^2+(-2)^2+(-1)^2}}\begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$

$e_2 = \frac{1}{\sqrt{1^2+1^2+0^2}}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$e_3 = \frac{1}{\sqrt{1^2+(-1)^2+4^2}}\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix} = \frac{1}{3\sqrt{2}}\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$

$P = \begin{bmatrix} 2/3 & 1/\sqrt{2} & 1/3\sqrt{2} \\ -2/3 & 1/\sqrt{2} & -1/3\sqrt{2} \\ -1/3 & 0 & 4/3\sqrt{2} \end{bmatrix}$

$P^{-1} = P^T = \begin{bmatrix} 2/3 & -2/3 & -1/3 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/3\sqrt{2} & -1/3\sqrt{2} & 4/3\sqrt{2} \end{bmatrix}$

$D = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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