# UNIT – 4

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## TUTORIAL – 1 BINOMIAL DISTRIBUTION

1 a) Yes, fixed trials, 2-outcomes, independent $p = 0.5$

b) Not binomial – probability changes after each draw

c) Yes, fixed trials, independent.

d) Not binomial – Poisson model fits better.

2. $p = 0.1 \quad n = 10, \quad q = 1 - 0.1 = 0.9$

$^{10}C_x (0.1)^x (0.9)^{10-x}$.

a) $P(X \leq 2) = P(x=0) + P(x=1) + P(x=2)$

$^{10}C_0 (0.1)^0 (0.9)^{10} + {}^{10}C_1 (0.1)^1 (0.9)^9 + {}^{10}C_2 (0.1)^2 (0.9)^8$

$= 0.3487 + 0.3874 + 0.1937 = 0.9298$

b) $P(X \geq 8) = P(x=9) + P(x=10)$

$= {}^{10}C_9 (0.1)^9 (0.9)^1 + {}^{10}C_{10} (0.1)^{10} (0.9)^0 = 9 \times 10^{-9} + 1 \times 10^{-10}$

$= 9.1 \times 10^{-9} \approx 0$

c) $P(X=4) = {}^{10}C_4 (0.1)^4 (0.9)^6 = 1.1 \times 10^{-2} = 0.0112$

d) $P(5 \leq X \leq 7) = P(x=5) + P(x=6) + P(x=7)$

$= {}^{10}C_5 (0.1)^5 (0.9)^5 + {}^{10}C_4 (0.1)^6 (0.9)^4 + {}^{10}C_7 (0.1)^7 (0.9)^3$

$= 0.00149 + 0.00014 + 0.000008 = 0.00164$

3: $n = 10, p = 0.4, q = 0.6 : {}^{10}C_x (0.4)^x (0.6)^{10-x}$

a) $P(x=3) = {}^{10}C_3 (0.4)^3 (0.6)^7 = 0.2149$

b) $P$ (atleast 1 not occupied) $= 1 - P$ (all occupied)

$P(x=10) = 0.4^{10} = 0.0001049$

$= 1 - 0.0001049 = 0.9998$

c) $E(x) = np = 10 \times 0.4 = 4$

4. $p = 0.2, \quad q = 0.8$

a) $n = 5 \qquad {}^5C_x (0.2)^x (0.8)^{5-x}$

$P(x=1) = {}^5C_1 (0.2)^1 (0.8)^4 = 0.409 = 0.41$

b) $n = 20$, $P(x=4)$ $\quad {}^{20}C_4 (0.2)^4 (0.8)^{16} = 0.218$

c) $P(x>4) = 1 - P(x \leq 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$= 1 - 0.63 = 0.37.$

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5) a) $p = 61.22\% = 0.6122$, $n = 20$

$q = 0.3878$

$P(x \geq 1) = 1 - P(x=0)$

$P(x=0) = (1-p)^{20} \Rightarrow (0.3878)^{20}$

$= 3 \times 10^{-9}$

$P(x \geq 1) = 1 - 3 \times 10^{-9} = 1$

b) $P(x \geq 3) = 1 - (P(x=0) + P(x=1) + P(x=2))$

$= 1 - [(0.3878)^{20} + 20 \times (0.6122)(0.3878)^{19} + 10 \times 19 \times (0.6122)^2 (0.3878)^{18}]$

$P(x \geq 3) = 1 - (0 + 0 + 0.00003)$

$= 0.99997$

c) Mean $= np = 20 \times 0.6122 = 12.244$

Variance $\sigma^2 = npq = 20 \times 0.6122 \times 0.3878$

$= 4.748$

$\sigma = \sqrt{4.748} = 2.179$

6) $p = 1 - 0.1 = 0.9$ $\quad n = 125$, $q = 0.1$

a) $x \leq 120$ $\quad$ mean $= 120 \times 0.9 = 112.5$

Variance $\sigma^2 = npq = 125(0.9)(0.1) = 11.25$

$\sigma = \sqrt{11.25} = 3.354$

$P(x \leq 120) = \dfrac{120.5 - 112.5}{3.354} = P(Z \leq 2.38)$

$= 0.5 + \phi(2.38) = 0.5 + 0.4913$

$= 0.9913$

b) $x < 120 = x \leq 119 = \frac{119.5 - 112.5}{3.354} = P(Z \leq 209)$

$= 0.5 + 0.48169 = 0.98169$

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7) $p = 0.3, q = 0.7, n = 20$

a) $P(x \geq 10) = 1 - P(x \leq 9)$

$^{20}C_x (0.3)^x (0.7)^{20-x}$

$^{20}C_9 (0.3)^9 (0.7)^{11} = 0.9520$

$P(x \geq 10) = 1 - 0.9520 = 0.048$

b) $P(x \leq 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$= (0.7)^{20} + {}^{20}C_1 (0.3)(0.7)^{19} + {}^{20}C_2 (0.3)^2 (0.7)^{18} +$

$^{20}C_3 (0.3)^3 (0.7)^{17} + {}^{20}C_4 (0.3)^4 (0.7)^{16}$

$= 0.2375$

c) $P(x = 5) = {}^{20}C_5 (0.3)^5 (0.7)^{15}$

$= 0.1789$

8) $p = 0.7, q = 0.3, n = 10$

a) $P(x \leq 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3) +$

$P(x=4)$

$^{10}C_x (0.7)^x (0.3)^{10-x}$

$(0.3)^{10} + {}^{10}C_1 (0.7)^1 (0.3)^9 + {}^{10}C_2 (0.7)^2 (0.3)^8 +$

$^{10}C_3 (0.7)^3 (0.3)^7$

$= 0.0474$

b) $n = 20 \Rightarrow {}^{20}C_x (0.7)^x (0.3)^{20-x}$

$P(x \leq 9) = P(x=0) + P(x=1) + P(x=2) + P(x=3) +$

$P(x=4) + P(x=5) + P(x=6) + P(x=7) + P(x=8) +$

$P(x=9) = (0.3)^{20} + 20(0.7)(0.3)^{19} + 10 \times 19 (0.7)^2 (0.3)^{18} +$

$\frac{20 \times 19 \times 18}{2 \times 3} (0.7)^3 (0.3)^{17} + {}^{10}C_5 (0.7)^5 (0.3)^{15} + {}^{10}C_4 (0.7)^4 (0.3)^{16} +$

$^{10}C_6 (0.7)^6 (0.3)^{14} + {}^{10}C_7 (0.7)^7 (0.3)^{13} + {}^{10}C_8 (0.7)^8$

$(0.3)^{12} + {}^{10}C_9 (0.7)^9 (0.3)^{11} = 0.2375$

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9. $p = 0.9$, $n = 20$, $q = 0.1$

$^{20}C_x (0.9)^x (0.1)^{20-x}$

a) $P(x=18) = {}^{20}C_{18} (0.9)^{18} (0.1)^2$

$= 0.2852$

b) $P(X \geq 15) = 1 - P(X \leq 14) =$

~~$1 - {}^{20}C_{14} (0.9)^{14} (P(X=0) + P(X=1) + P(X=2) +$~~

~~$P(X=3)$~~

$P(X=15) + P(X=16) + P(X=17) + P(X=18) + P(X=19) +$

$P(X=20) = 0.9887$.

c) $P(X \geq 2) = 1 - [P(Y=0) + P(Y=1)]$

$= 1 - [(0.9)^{20} + {}^{20}(0.1)(0.9)^{19})$

$= 1 - (0.1216 + 0.2702) = 1 - 0.3918$

$= 0.6083$

## TUTORIAL SHEET-2 POISSON DISTRIBUTION

1) $P(x=0) = e^{-\lambda} = 0.05$

$\lambda = -\ln(0.05) = 2.9957$

Variance $= 2.9957$

2) $\lambda = 10$

a) $P(x=5) = \frac{e^{-10} 10^x}{x!} = \frac{e^{-10} 10^5}{5!} = 0.0378$

b) $P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$

$e^{-10} + e^{-10} \cdot 10 + \frac{e^{-10} 10^2}{2} + \frac{e^{-10} 10^3}{6} = 0.00005 + 0.00045 + 0.00227 + 0.00757$

$= 0.0103$

c) $\lambda = 20$

$P(X=15) = \frac{e^{-20} 20^{15}}{15!} = 0.0516$

d) $\lambda = 5$

$P(x=5) = \frac{e^{-5} 5^5}{5!} = 0.17547$

3) $\lambda = 1$

a) $P(X \geq 2) = 1 - P(X \leq 2) = 1 - [P(X=0) + P(X=1)]$

$P(X=0) = e^{-1}$ $\quad P(X=1) = e^{-1} = 1 - (2e^{-1})$

$= 0.2642.$

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b) $P(X \geq 1) > 0.95 = 1 - P(X \leq 1)$

$= 1 - P(X=0) = 1 - e^{-\lambda} > 0.95$

$e^{-\lambda} < 0.05$

$\lambda = -\ln(0.05) = 2.996$

Volume = $2.996 \times 16 = 48$ cubic light year

4) a) $\mu = 1.61$ $\quad P(X>1) = 1 - P[X < 1]$

$= 1 - [P(X=0) + P(X=1) = 1 - (e^{-1.61} - e^{-1.61}(1.61))$

$= 1 - 0.199\,(+1.61) = 1 - 0.5434 = 0.4566.$

b) $\lambda = 5 \times 1.61 = 8.05$

$P(X=0) = e^{-8.05}$

5) $\lambda = 16 \times 0.05 = 0.5$

a) $P(X=0) = e^{-0.5} = 0.60653$

b) $P(\text{all 10 claim}) = (0.6065)^{10} = 0.0067$

c) $P(X \geq 1) \Rightarrow q = 1 - 0.6065 = 0.3935$

$P(X=0) = (0.6065)^{10} = 0.0067$

$P(X=1) = 10(0.3935)(0.6065)^9 = 0.0437$

$P(W \leq 1) = 0.0067 + 0.0437 = 0.0504$

6) $\lambda = 0.25$

a) $P(X \geq 2) = 1 - [P(X=0) + P(X=1)]$

$= 1 - (e^{-0.25} + 0.25 e^{-0.25}) = 0.026$

b) $\lambda = 5 \times 0.25 = 1.25$

$P(X \geq 0) = e^{-1.25} = 0.287$

c) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= e^{-1.25} + 1.25 e^{-1.25} + \frac{e^{-1.25}(1.25)^2}{2} = 0.8688$

# TUTORIAL - 3 — Exponential Distribution

1. $\lambda = 2$ per min

a) $P(T > 0.5) = e^{-2 \times 0.5} = e^{-1}$
$= 0.3679$

b) $P(T < \frac{1}{6}) = 1 - e^{-2(\frac{1}{6})}$
$= 1 - 0.7165 = 0.2835$
$1 - P(0) = 1 - e^{-2(\frac{1}{6})} = 0.1353$

c) $P(1 < T < 2) = e^{-2(1)} - e^{-2(2)}$
$= 0.1170$

2. $\lambda = \frac{1}{15}$

a) $P(T > 30) = e^{-\frac{1}{15} \times 30}$
$= 0.1353$

b) $P(T \leq 10) = 1 - e^{-\frac{1}{15} \times 10} = 0.4866$

c) $P(5 < T < 10) = e^{-\frac{1}{15} \times 5} - e^{-\frac{1}{15} \times 10}$
$= 0.2031$

d) $1 - e^{-\frac{t}{15}} = 0.90$
$e^{-t/15} = 0.10$
$t = -15 \ln(0.10)$
$= 34.54$ mins

3. a) $\lambda = P(T > 10000)$
$= e^{-0.0003 \times 10000} = e^{-3}$
$= 0.9498$

b) $P(T \leq 7000)$
$1 - e^{-0.0003 \times 7000} = 1 - 0.1225$
$= 0.8775$

4) $\lambda = \frac{1}{10} = 0.1$

a) $P(T > 60) = e^{-0.1(60)} = e^{-6}$
$= 0.00025$

b) $P(T \leq 10 \mid T > 60) = P(T < 10)$
$= 1 - e^{-0.1(10)} = 1 - e^{-1} = 0.6321$

c) $e^{-0.1x} = 0.10$
$x = \frac{-\ln(0.1)}{0.1} = 23.025$

d) $1 - e^{-0.1x} = 0.90$
$1 - 0.9 = e^{-0.1x}$
$x = \frac{-\ln(0.1)}{0.1} = 23.03$

e) $1 - e^{-0.1x} = 0.5$
$e^{-0.1x} = 0.5$
$x = \frac{-\ln(0.5)}{0.1}$
$= 6.93$

5. $\lambda = \frac{1}{400} = 0.0025$

a) $P(T < 100) = 1 - e^{-0.0025(100)} = 0.2212$
$= e^{-0.25}$

b) $P(T > 500) = e^{-0.0025(500)}$
$= 0.2865$
$= e^{-0.0025(100)}$

c) $P(T \leq 100) = 1 - e$
$= 0.2212$

d) $P(T > 100) = e^{-0.25}$
$= 0.7788$
All 10 survive $= (0.7788)^{10}$
$= 0.0821$
$P(\text{at least 1 fail}) = 1 - 0.0821$
$= 0.9179$

e) $P(T < 800)$
$= 1 - e^{-0.0025(800)}$
$= 1 - e^{-2} = 0.8647$
All 10 fail $= (0.8647)^{10}$
$= 0.2337$

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# TUTORIAL-3

6. $\lambda = \frac{1}{10} = 0.1$

a) $P(N>3) = P(T \leq 3)$

$= 1 - [P(0)+P(1)+P(2)+P(3)]$

$= 1 - (e^{-3} + 3e^{-3} + \frac{9e^{-3}}{2} + \frac{27e^{-3}}{6})$

$= 1 - 13e^{-3} = 0.3528$

b) $P(T>30) = e^{-0.1 \times 30}$

$= 0.04979$

c) $P(T>60x) = e^{-0.1 \times 60x} = 0.01$

$e^{-6x} = 0.01$

$x = \frac{-\ln(0.01)}{6}$

$= 0.767$ hrs.

$= 46.05$ mins

d) $P(T>120) = e^{-0.1 \times 120}$

$= 6.14 \times 10^{-6}$

e) $(e^{-3})^4 = e^{-12}$

7. $\lambda = \frac{1}{3.5} = 0.2857$

a) $P(T<2) = 1 - e^{-0.2857 \times 2}$

$= 0.435$

b) $P(T>7) = e^{-0.2857 \times 7}$

$= 0.185$

c) $e^{-2857t} = 0.10$ [1-0.9]

$t = \frac{-\ln(0.10)}{0.2857} = 8.05$

$P(T<t) = 0.9$

$t = 3.69$ days

d) $P(T<10 \mid T>3)$

$= P(T<7)$

$= 1 - e^{-0.2857 \times 7}$

$= 0.865$

# TUTORIAL-4 NORMAL DISTRIBUTION

1. a) $P(X<11)$

$\mu = 5$, S.D. $= 4$

$\frac{x-5}{\sigma} = \frac{11-5}{4} = 1.5$

$P(x<1.5) = 0.5 + \phi(1.5)$

$= 0.5 + 0.4331 = 0.93319$

b) $P(X>0) = z = \frac{0-5}{\mu} = -1.25$

$P(z<1.25) = 0.5 + \phi(1.25)$

$= 0.5 + 0.39435 = 0.89435$

c) $P(3<x<7) \Rightarrow z_1 = \frac{3-5}{4} = -0.5$

$z_2 = \frac{7-5}{4} = 0.5$

$P(-0.5<x<0.5) = 0.19146 \times 2$

$= 0.38292$

d) $P(-2<x<9)$

$z_1 = \frac{-2-5}{4} = -1.75$

$z_2 = \frac{9-5}{4} = 1$

$P(-1.75<z<1)$

$= 0.34134 + 0.45994$

$= 0.80128$

e) $P(2<x<8)$

$z_1 = \frac{2-5}{4} = -0.75$

$z_2 = \frac{8-5}{4} = 0.75$

$P(-0.75<x<0.75)$

$= 2 \times 0.27337$

$= 0.54674$

2. $\mu = 6000$, $\sigma = 100$

a) $P(X<6250)$

$z = \frac{6250-6000}{100} = 2.5$

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$P(z < 2.5) = 0.5 + \phi(2.5)$

$= 0.5 + 0.49379 = 0.99379$

b) $P(5800 < x < 5900)$

$z_1 = \frac{5800 - 6000}{100} = -2$

$z_2 = \frac{5900 - 6000}{100} = -1$

$P(-2 < x < -1)$

$= 0.47725 - 0.34134$

$= 0.13591$

c) $z = -1.645$

$x = 6000 + (-1.645)(100)$

$= 5835$

3) $\mu = 0.5, \sigma = 0.05$

a) $P(x > 0.62) \Rightarrow z = \frac{0.62 - 0.5}{0.05}$

$P(x > 2.4) = 0.5 - 0.49180$

$= 0.00820$

b) $z(0.47 < x < 0.63)$

$z_1 = \frac{0.47 - 0.5}{0.05} = -0.6$

$z_2 = \frac{0.63 - 0.5}{0.05} = 2.6$

$P(-0.6 < x < 2.6)$

$= 0.22575 + 0.49534$

$= 0.72109$

c) $z_{0.9} = 0.5 + 1.28(0.05)$

$= 0.564$

4) $\mu = 60, \sigma = 4$

a) $P(X \geq 70) = \frac{70 - 60}{4} = 2.5$

$P(z > 2.5) = 0.5 - 0.49379$

$= 0.00621$

b) $P(x < 58) = \frac{58 - 60}{4}$

$= -0.5$

$P(z < -0.5) = 0.5 - 0.19146$

$= 0.30854$

c) Avg time $= \frac{8000}{60}$

$= 133.33$ sec

5. $\mu = 7000, \sigma = 600$

a) $P(X < 5000) = \frac{5000 - 7060}{600}$

$= -3.33$

$P(X < -3.33) = 0.5 - 0.49957$

$= 0.00043$

b) $z_{0.9} = -1.645$

$x = 7000 - 1.645(600)$

$= 6016$

c) $P(X > 7000) = 0.5$

$= (0.5)^3 = \frac{1}{8}$

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6. $z = \frac{x - 12}{0.5}$

$P(x > 13) = \frac{13 - 12}{0.5} = 2$

$P(z > 2) = 0.5 - 0.47725$

$= 0.02275$

b) $z_{0.999} = 3.09$

$13 = 12 + 3.090\sigma$

$\sigma = 0.324$

c) $13 = \mu + 3.09(0.5)$

$\mu = 11.455$

7. $\mu = 170, \sigma = 30$

a) $P(x < 230)$

$z = \frac{230 - 170}{30} = 2$

$P(Z>2) = 0.5 - 47725$
$= 0.02275$

b) $\mu = np = 300(0.02275)$
$= 6.84$

$\sigma = \sqrt{np(1-p)} = \sqrt{6.684}$
$\simeq 2.58$

8) $\mu = 24, \sigma = 3.8$

a) $P(z>30) = \frac{30-24}{3.8} = 1.58$
$= 0.5 - 0.44295 = 0.05705$

b) $z = \frac{15-24}{3.8} = -2.37$
$P(Z<-2.37) = 0.5 - 0.4911$
$= 0.0089 \simeq 99.11\%$ late

c) $z = \frac{25-24}{3.8} = 0.26$
$P(z>0.26) = 0.5 - 0.1025$
$= 0.3974$

d) $z = 1.04$
$x = 24 + 1.04(3.8)$
$= 27.95$

e) $p = 0.0571$
$P(x-2) = {}^3C_2\, p^2(1-p)$
$= \frac{3\times 2}{2}(0.0571)^2(1-0.0571)$
$= 0.0092$

TUTORIAL SHEET-5

1 a) $\mu = 7, \sigma = 1$
$z = \frac{6-7}{1} = 1$
$P(z<-1) = 0.1587$

b) $n = 9, \sigma_x = \frac{1}{\sqrt{9}} = \frac{1}{3}$
$z_1 = \frac{6.4-7}{1/3} = -1.8$
$z_2 = \frac{7.2-7}{1/3} = 0.6$
$P(-1.8<z<0.6) = 0.6898$ ← $0.7257 - 0.0359$

c) $z = 1.04$
right: 0.15 left area = 0.85
$x = \mu + z\sigma_{\bar{x}}$
$x = 7 + (1.04 \times \frac{1}{3})$
$x = 7 + 0.3466 = 7.3466$
$\simeq 7.35$

2. $\mu = 1.01, \sigma = 0.003, n = 9$
$\sigma_{\bar{x}} = \frac{0.003}{3} = 0.001$
$z_1 = \frac{1.009-1.01}{0.001} = -1$
$z_2 = \frac{1.012-1.01}{0.001} = 2$
$P(-1<z<2) = \phi(2) - \phi(1)$
$= 0.8186$

3. $\mu = 50, \sigma = 5, n = 16$
$\sigma_{\bar{x}} = \frac{5}{4} = 1.25$
$z = \frac{1.96(5)}{1.25} = 7.84$
$P(-7.84<z<7.84)$
$= 2 \times \phi(7.84)$
$= 0.3159$

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4. $\sigma_{\bar{x}_1} = 2, n_1 = 36$
$2 = \frac{\sigma}{\sqrt{36}}$ $\sigma = 12$
$1.2 = \frac{12}{\sqrt{n_2}}$ $\sqrt{n_2} = 10$
$n_2 = 10^2 = 100$

5. $Var(\bar{x}) = \frac{\sigma^2}{n}$

a) Old var $= \frac{5.6^2}{64} = 0.49$

New var $= \frac{5.6^2}{196} = 0.16$

b) Initial variance

$Var_1 = \frac{5.6^2}{784} = \frac{31.36}{784} = 0.04$

$Var_2 = \frac{5.6^2}{49} = 0.64$

6. $\mu = 240, \sigma = 15, n = 40$

$\sigma_{\bar{x}} = \frac{15}{\sqrt{40}} \simeq 2.37$

Lower limit $= \mu - 2\sigma_{\bar{x}} = 240 - 2(2.37)$
$= 235.26$

Upper limit $= \mu + 2\sigma_{\bar{x}} = 240 + 2(2.37)$
$= 244.74$

$235.26 \leq \bar{x} \leq 244.74$

Yes, 236 fall within interval

7. $\mu = \frac{1+2+3}{3} = 2$

$\sigma^2 = \frac{(1-2)^2 + (2-2)^2 + (3-2)^2}{3}$

$= \frac{2}{3}$

$\sigma_{\bar{x}} = \sqrt{\frac{2}{3 \times 36}} = 0.136$

$E(X^2) = 1^2 \cdot \frac{1}{3} + 2^2 \cdot \frac{1}{3} + 3^2 \cdot \frac{1}{3} = \frac{14}{3}$

$= 4.6667$

$\sigma^2 = E(X^2) - \mu^2$
$= 4.6667 - 2^2$
$= 0.6667$

$\sigma = \sqrt{0.6667} \simeq 0.8165$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.8165}{\sqrt{36}} \simeq 0.1361$

$2.1 < \bar{x} < 2.5$

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For $\bar{x} = 2.1$

$z_1 = \frac{2.1-2}{0.1361} = 0.73$

For $\bar{x} = 2.5$

$z_2 = \frac{2.5-2}{0.1361} = \frac{0.5}{0.1361} \simeq 3.67$

$P(Z < 0.73) \simeq 0.7673$

$P(0.73 < Z < 3.67)$

$= 0.9999 - 0.7673$

$= 0.2326$

TUTORIAL SHEET: 6 SAMPLING DIST. OF DIFF. OF MEAN

1. $\mu_1 = 72cm, \sigma_1 = 10cm, n_1 = 64$

$\mu_2 = 28cm, \sigma_2 = 5cm, n_2 = 100$

$\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2 = 72 - 28 = 44$

$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

$= \sqrt{\frac{10^2}{64} + \frac{5^2}{100}} = \sqrt{\frac{100}{64} + \frac{25}{100}}$

$= \sqrt{1.5625 + 0.25} = \sqrt{1.8125}$

$\simeq 1.3463$

$z = \frac{44.2 - 44}{1.3463} = 0.1485$

$\simeq 0.15$

$P(z \leq 0.15) = 0.5596$

2. $\mu_1 = 75, \sigma_1 = 8, n_1 = 16$

$\mu_2 = 70, \sigma_2 = 12, n_2 = 9$

$\mu_d = 75 - 70 = 5$

a) $P(\bar{x}_1 - \bar{x}_2 \geq 4)$

$z = \frac{4-5}{4.472} = -0.22$

$P(Z > -0.22) = 0.5885$

b) $z_1 = \frac{3.5-5}{4.472} = -0.34$

$z_2 = \frac{5.5-5}{4.472} = 0.11$

$P(-0.34 < Z < 0.11) = \phi(0.11) -$

$\phi(0.34) = 0.1759$

3. $\mu = 540, \sigma = 50$

$n_1 = 32, n_2 = 50$

$\mu_d = 0 \Rightarrow \mu_1 = \mu_2 = 540$

$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{50^2}{32} + \frac{50^2}{50^2}} = 11.32$

a) $z = \frac{20}{11.32} = 1.77$

$P(Z > 1.77) = 0.0385$

b) $z_1 = \frac{5}{11.32} = 0.44$

$z_2 = \frac{10}{11.32} = 0.88$

$P(0.44 < Z < 0.88)$

$= 0.8106 - 0.67 = 0.1406$

4. Boys: $\mu_1 = 15, \sigma_1 = 7, n_1 = 100$

Girls: $\mu_2 = 10, \sigma_2 = 6, n_2 = 50$

$\mu_d = 15 - 10 = 5$

$\sigma_d = \sqrt{\frac{49}{100} + \frac{36}{50}} = 1.1$

$P(\bar{x}_1 - \bar{x}_2 \leq 3)$ $z = \frac{3-5}{1.1} = -1.82$

$P(Z \leq -1.82) = 0.03452$

5. $\mu_1 = 32, \mu_2 = 22$

$\sigma_1^2 = 60, \sigma_2^2 = 70$

$n_1 = 10, n_2 = 14$

$\mu_d = 32 - 22 = 10$

$\sigma_d = \sqrt{\frac{60}{10} + \frac{70}{14}} = \sqrt{6+5} = \sqrt{11}$

$= 3.317$

$P(\bar{x}_1 - \bar{x}_2 \geq 5)$

$z = \frac{5-10}{3.317} = -1.51$

$P(Z > -1.51) = 0.934$

6. $\mu_1 = 175, \mu_2 = 165$

$\sigma_1^2 = \sigma_2^2 = 64$

$\mu_d = 165 - 175 = -10$

$\sigma_d = \sqrt{\frac{64}{n} + \frac{64}{n}}$

$P(\bar{x}_{girls} - \bar{x}_{boys} > 0)$

$z = \frac{0 - (-10)}{\sigma_d}$

$P(Z \geq x) = 0.0062$

Aayushi Priya

## TUTORIAL-7

## SAMPLING DIST PROPORTION

1. $p = 0.43, q = 0.57, n = 50$

a) $0.45 < \hat{p} < 0.50$

$\sigma_{\hat{p}} = \sqrt{\frac{0.43 \times 0.57}{50}} = \sqrt{0.004902} = 0.07$

$z_1 = \frac{0.45 - 0.43}{0.07} = 0.29$

$z_2 = \frac{0.5 - 0.43}{0.07} = 1$

$P(0.25 < Z < 1) = 0.8413 - 0.6126$

$= 0.2287$

b) $z = \frac{0.50 - 0.43}{0.07} = 1$

$P(Z > 1) = 0.1112$

2. $p = 0.87, q = 0.13, n = 100$

$P(\hat{p} < 0.85)$

$\sigma_{\hat{p}} = \sqrt{\frac{0.85 \times 0.13}{100}} = 0.0336$

AAYUSHI PRIYA

$$Z = \frac{0.85 - 0.87}{0.0336} = -0.60$$

$P(Z < -0.6) = 0.20$

3. $p = 0.38$, $q = 0.62$, $n = 900$

a) $np = 900(0.38) = 342 > 5$

$nq = 900(0.62) = 558 > 5$

∴ condition satisfied

b) $P(|\hat{p} - 0.38| < 0.05)$

$$\sigma_{\hat{p}} = \sqrt{\frac{p \cdot q}{n}} = \sqrt{\frac{0.38 \times 0.62}{900}}$$

$$\sigma_{\hat{p}} = \sqrt{\frac{0.2356}{900}} = 0.0162$$

$$Z = \frac{0.05}{0.0162} = 3.09$$

$P(-3.09 < Z < 3.09)$

$= 2 \times \phi(3.09) = 0.9980$

4. $n = 121$, $x = 102$

a) $\hat{p} = \frac{102}{121} = 0.84$

b) $np = 121(0.9) = 108.9 > 5$

$nq = 121(0.1) = 12.175$

c) $\sigma_{\hat{p}} = \sqrt{\frac{0.9 \times 0.1}{121}} = 0.0273$

$$Z = \frac{0.84 - 0.9}{0.0273} = -2.20$$

$P(Z < -2.20) = 0.0139$

d) $p < 0.90$, probability is very small, claim is not reasonable