

# UNIT - 5

[AHYUSH]

## 1. CONFIDENCE INTERVAL

$$1) n = 100, \bar{x} = 498 \text{ ml}$$

$$\sigma = 10 \text{ ml}, \alpha = 1 - 0.95 = 0.05$$

$$Z_{\alpha/2} = 1.96$$

$$\text{Margin of error} = Z_{\alpha/2} \left( \frac{\sigma}{\sqrt{n}} \right)$$

$$= 1.96 \left( \frac{10}{\sqrt{100}} \right) = 1.96$$

$$\text{Lower limit} = 498 - 1.96 = 496.04$$

$$\text{Upper limit} = 498 + 1.96 = 499.96$$

$$2) n = 150, \bar{x} = 2.02 \text{ kg},$$

$$\sigma = 0.1 \text{ kg} \quad Z_{\alpha/2} = 2.576$$

margin of error (ME)

$$= 2.576 \left( \frac{0.1}{\sqrt{150}} \right) = 0.021$$

$$\text{lower limit} = 2.02 - 0.021$$

$$= 1.999 \text{ kg}$$

$$\text{Upper limit} = 2.02 + 0.021$$

$$= 2.041 \text{ kg}$$

$$3) n = 9$$

$$\text{mean} = \frac{6.5 + 7 + 6.8 + 5.9 + 7.1 + 6.3 + 7.4 + 6 + 6.7}{9}$$

$$= 6.633 \text{ hrs}$$

$$S.d = 0.505 \text{ hrs}$$

degree of freedom df = n-1

$$\rightarrow t_{\alpha/2} \left( \frac{s.d}{\sqrt{n}} \right) = 9-1 = 8$$

$$ME = 2.306 \left( \frac{0.505}{\sqrt{9}} \right) = 0.388$$

$$\text{Lower limit} = 6.633 - 0.388$$

$$= 6.245 \text{ hrs}$$

$$\text{Upper limit} = 6.633 + 0.388$$

$$= 7.021 \text{ hrs}$$

$$4) n = 7$$

$$\text{mean } \bar{x} = \frac{12 + 15 + 14 + 10 + 13 + 11 + 16}{7}$$

$$= \frac{91}{7} = 13$$

$$\text{standard deviation : } s = 2.16 \text{ kg}$$

$$\text{Degree of freedom} \Rightarrow df = n-1$$

$$= 7-1 = 6$$

$$\text{confidence level} = 90\%.$$

$$t_{\alpha/2} = 1.943$$

$$ME = t_{\alpha/2} \left( \frac{s}{\sqrt{n}} \right) = 1.943 \left( \frac{2.16}{\sqrt{7}} \right)$$

$$= 1.586$$

$$\text{Lower limit} = 13 - 1.586 = 11.414$$

$$\text{Upper limit} = 13 + 1.586 = 14.586$$

$$5) n = 500, x = 285, \hat{p} = \frac{x}{n} = \frac{285}{500}$$

$$\text{confidence level} = 95\%. \quad = 0.57$$

$$Z_{\alpha/2} = 1.96$$

$$ME = Z_{\alpha/2} \sqrt{\hat{p}(1-\hat{p})}$$

$$= 1.96 \sqrt{\frac{0.57(1-0.57)}{500}} = 0.0434$$

$$\text{Lower limit} = 0.57 - 0.0434$$

$$= 0.5266$$

$$\text{Upper limit} = 0.57 + 0.0434$$

$$= 0.6134$$

$$6) n = 120, x = 18, p = \frac{18}{120} = 0.15$$

$$\text{confidence level} = 99\%.$$

$$\alpha = 0.01, \frac{\alpha}{2} = 0.005$$

$$Z_{\alpha/2} = Z_{0.005} = 2.576$$

$$\text{standard error} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.15(1-0.15)}{120}}$$

$$= 0.0326$$

$$ME = Z_{\alpha/2} \times SE$$

$$= 2.576 \times 0.0326$$

$$= 0.0839$$

$$\text{Lower limit} = 0.15 - 0.0839 \\ = 0.0661$$

$$\text{Upper limit} = 0.15 + 0.0839 \\ = 0.2339$$

$$7.7 \quad n=10, df=n-1=9$$

$$S^2 = 4.5 \text{ mm}^2$$

$$\alpha = 0.05$$

$$X^2_{1-\alpha/2, df} = X^2_{0.975, 9} = 2.70$$

$$X^2_{\alpha/2, df} = X^2_{0.025, 9} = 19.02$$

$$\text{Lower bound} = \frac{(n-1)S^2}{X^2_{\alpha/2, df}} = \frac{(10-1) \times 4.5}{19.02}$$

$$\approx 2.129 \text{ mm}^2$$

$$\text{Upper bound} = \frac{(n-1)S^2}{X^2_{1-\alpha/2, df}} = \frac{(10-1) \times 4.5}{2.70}$$

$$= 15 \text{ mm}^2$$

$$8.7 \quad n=16, df=n-1=15$$

$$\text{std deviation : } s = 1.2 N$$

$$S^2 = 1.44 N^2$$

$$\alpha = 0.01$$

$$X^2_{1-\alpha/2, df} = X^2_{0.995, 15} = 4.60$$

$$X^2_{\alpha/2, df} = X^2_{0.005, 15} = 3.280$$

$$\text{Lower bound} = \frac{(n-1)S^2}{X^2_{\alpha/2, df}} = \frac{(16-1) \times 1.44}{3.280}$$

$$= \frac{21.6}{32.8} = 0.659 N^2$$

$$\text{Upper bound} = \frac{(n-1)S^2}{X^2_{1-\alpha/2, df}}$$

$$= \frac{(16-1) \times 1.44}{4.60} = \frac{21.6}{4.6} = 4.696 N^2$$

$$9.7 \quad n_A = 12, df_A = 11, S^2_A = 18.2$$

$$n_B = 10, df_B = 9, S^2_B = 10.5$$

confidence level = 90%  $\alpha = 0.10$

$$F_{\alpha/2, df_A, df_B} = F_{0.05, 11, 9} = 3.10$$

$$F_{1-\alpha/2, df_A, df_B} = F_{0.95, 11, 9} = 0.345$$

$$F_{\alpha/2, df_B, df_A} = F_{0.05, 9, 11} = 2.90$$

$$\frac{S^2_A}{S^2_B} = \frac{18.2}{10.5} = 1.733$$

$$\text{Lower bound} = 1.733$$

$$= \frac{1.733}{3.10} = 0.559$$

$$\text{Upper bound} = 1.733 \times 2.90 \\ = 5.026$$

$$10.7 \quad n_1 = 14, S_1^2 = 0.022 \text{ mm}^2$$

$$df_1 = n_1 - 1 = 13$$

$$n_2 = 16, S_2^2 = 0.015 \text{ mm}^2$$

$$df_2 = n_2 - 1 = 15$$

$$\alpha = 1 - 0.98 = 0.02$$

$$\alpha/2 = 0.01$$

$$F_{\alpha/2, df_1, df_2} = F_{0.01, 13, 15} = 3.84$$

$$F_{1-\alpha/2, df_1, df_2} = F_{0.99, 13, 15} = \frac{1}{F_{0.01, 15, 13}}$$

$$\frac{\sigma_1^2}{\sigma_2^2} = \left( \frac{S_1^2}{S_2^2} \frac{1}{F_{\alpha/2, df_1, df_2}}, \frac{S_1^2}{S_2^2} \frac{1}{F_{1-\alpha/2, df_1, df_2}} \right)$$

$$= \left( \frac{0.022}{0.015} \frac{1}{3.84}, \frac{0.022}{0.015} \frac{1}{0.235} \right)$$

$$= (0.3819, 6.2415)$$

AAJU SHI PRIYA

## Q. TYPE - I & TYPE - II ERROR

a)  $\alpha > \beta = 0.4, n=12$

$$x \leq 2$$

$$P(X=k) = {}^n C_k p^k (1-p)^{n-k}$$

$$P(X=0) = {}^{12} C_0 (0.4)^0 (0.6)^{12} \\ = 0.00218$$

$$P(X=1) = {}^{12} C_1 (0.4)^1 (0.6)^{11} \\ = 0.01743$$

$$P(X=2) = {}^{12} C_2 (0.4)^2 (0.6)^{10} \\ \approx 0.05634$$

$$\alpha = 0.00218 + 0.01743 + 0.05634 \\ = 0.07595$$

b)  $p = 0.2$

$$x \geq 3$$

$$\beta = P(X \geq 3 | p = 0.2) \\ = 1 - P(X \leq 2 | p = 0.2)$$

$$P(X \leq 2 | p = 0.2)$$

$$P(X=0) = {}^{12} C_0 (0.2)^0 (0.8)^{12} \\ = 0.06872$$

$$P(X=1) = {}^{12} C_1 (0.2)^1 (0.8)^{11} \\ = 0.20615$$

$$P(X=2) = {}^{12} C_2 (0.2)^2 (0.8)^{10} \\ = 0.28347$$

$$P(X \leq 2) = 0.06872 + 0.20615 + \\ 0.28347 \\ = 0.55834$$

$$\beta = 1 - 0.55834 = 0.44166$$

2) a)  $\alpha = P(\bar{X} < 96.9 | \mu = 100, n=25)$

$$\mu_0 = 100, \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{25}} = 2$$

$$\bar{x} = 96.9 \quad \text{LAHYUSHI PRIYA}$$

$$2 = \frac{\bar{x} - \mu_0}{\sigma_{\bar{X}}} = \frac{96.9 - 100}{2} = -1.55$$

$$\alpha \neq P(Z < -1.55) = 0.0606$$

b)  $\mu = 95$ , acceptance region  $\bar{x} \geq 96.9$

$$Z = \frac{\bar{x} - \mu}{\sigma_{\bar{X}}} = \frac{96.9 - 95}{10/\sqrt{25}} = 0.95$$

$$\beta = P(Z \geq 0.95) = 1 - P(Z < 0.95) \\ = 1 - 0.8289 = 0.1711$$

3) a)  $\beta = 0.6, n=50$

$$x \leq 24.5$$

$$\mu_x = np = 50 \times 0.6 = 30$$

$$\sigma_x = \sqrt{np(1-p)} = \sqrt{50 \times 0.6 \times 0.4} \\ = 3.464$$

$$Z = \frac{x - \mu_x}{\sigma_x} = \frac{24.5 - 30}{\sqrt{12}} = -1.5877$$

$$P(Z \leq -1.5877) \approx 0.056$$

b)  $p = 0.3, x \geq 24.5$

$$\mu_x = np = 50 \times 0.3 = 15$$

$$\sigma_x = \sqrt{np(1-p)} = \sqrt{50 \times 0.3 \times 0.7} = 3.24$$

$$Z = \frac{x - \mu_x}{\sigma_x} = \frac{24.5 - 30}{\sqrt{12}} = -1.5877$$

$$4) a) |\bar{x} - 100| > 1.96 \cdot \frac{5}{\sqrt{25}}$$

$$\frac{|\bar{x} - \mu_0|}{\sigma_{\bar{X}}} > 1.96$$

$$\mu_0 = 100, \sigma = 5, n = 25$$

$$|z| > 1.96$$

$$\alpha = P(|z| > 1.96)$$

$$P(|z| > 1.96) = 2 \cdot P(z < -1.96)$$

$$= 2 \cdot P(z > 1.96) = 0.02$$

$$= 2 \times 0.025 = 0.05 = 5\%$$

b)  $\mu_1 = 998.5$

$$|\bar{x} - 1000| \leq 1.96 \cdot \frac{s}{\sqrt{n}}$$

$$1.96 \cdot \frac{5}{\sqrt{25}} = 1.96$$

$$\bar{x} > 1000 + 1.96 \text{ or } \bar{x} < 1000 - 1.96$$

$$\bar{x} > 1001.96 \text{ or } \bar{x} < 998.04$$

$$998.04 \leq \bar{x} \leq 1001.96$$

$$\alpha = 0.05$$

$$\beta = P(998.04 \leq \bar{x} \leq 1001.96 | \mu_1 = 998.5)$$

$$\frac{\sigma}{\sqrt{n}} = 1 : z_1 = \frac{998.04 - 998.5}{1} = -0.46$$

$$z_2 = \frac{1001.96 - 998.5}{1} = 3.46$$

$$\beta = P(z_1 \leq z \leq z_2) = P(-0.46 \leq z \leq 3.46)$$

$$\beta = P(z \leq 3.46) - P(z \leq -0.46)$$

$$= 0.9997 - 0.3228$$

$$= 0.6769 = 67.69\%$$

### 3. Z-TEST

i)  $H_0: \mu = 150, H_1: \mu \neq 150$

$$\sigma = 5, n = 36, \bar{x} = 148.5, \alpha = 0.05$$

$$\Rightarrow z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{148.5 - 150}{5 / \sqrt{36}} = -1.8$$

$$\alpha = 0.05, z_{\alpha/2} = \pm 1.96$$

$$z = -1.8 \text{ is b/w } -1.96 \text{ & } 1.96$$

Fail to reject null hypothesis

$H_0: \mu = 150$  at 5% level of significance

b)  $P = 2 \times P(z < |z|)$   
 $= 2 \times P(z < -1.8) = 2 \times 0.0359$   
 $= 0.0718$

2.  $H_0: \mu = 25, H_1: \mu < 25$

$$\sigma = 2, n = 49, \bar{x} = 24.4, \alpha = 0.01$$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{24.4 - 25}{2 / \sqrt{49}} = -2.1$$

$$\alpha = 0.01, z_\alpha = -2.33$$

$$z = -2.1$$

We fail to reject null hypothesis

$H_0: \mu = 25$  at 1% level of significance.

$$3. H_0: \mu = 5000, H_1: \mu \neq 5000$$

$$n = 100, \bar{x} = 4950, s = 120,$$

$$\alpha = 0.05, n \geq 30,$$

$$z = \frac{\bar{x} - \mu}{s / \sqrt{n}} = \frac{4950 - 5000}{120 / \sqrt{100}} = -4.167$$

$$z_{\alpha/2} = \pm 1.96$$

$$z = -4.167$$

$H_0: \mu = 5000$  at 5% level of significance

$$4. H_0: \mu = 2, H_1: \mu < 2$$

$$n = 64, \bar{x} = 1.98, s = 0.08, \alpha = 0.01$$

$$z = \frac{\bar{x} - \mu}{s / \sqrt{n}} = \frac{1.98 - 2}{0.08 / \sqrt{64}} = -2$$

$$\alpha = 0.01, z_\alpha = -2.33$$

$$z = -2$$

We fail to reject null hypothesis

$H_0: \mu = 2$  at 1% level of significance

$$5. H_0: p = 0.6, H_1: p \neq 0.6$$

$$\alpha = 0.05$$

$$n = 200, x = 108, \hat{p} = \frac{x}{n} = \frac{108}{200} = 0.54$$

$$\text{standard error : } SE = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.6(1-0.6)}{200}} = 0.03464$$

$$z = \frac{\hat{p} - p_0}{SE} = \frac{0.54 - 0.6}{\sqrt{\frac{0.6(1-0.6)}{200}}} = -1.732$$

$$z = \pm 1.96$$

$$z = -1.732$$

$$2 \times P(z < -1.732)$$

$$\text{p-value} = 2 \times P(z < -1.732) = 2 \times 0.0416 = 0.0832$$

6)  $H_0: \mu = 0.05$ ,  $H_1: \mu > 0.05$   
 $n = 400, x = 28$

$$\hat{p} = \frac{x}{n} = \frac{28}{400} = 0.07 \quad \alpha = 0.01$$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.07 - 0.05}{\sqrt{\frac{0.05(1-0.05)}{400}}} = 1.835$$

z-value is approx 2.33

$$z \approx 1.835$$

$$P(z > 1.835) \approx 0.0332$$

#### 4. T-TEST

$$1) \mu_0 = 500 \text{ mg}, n = 9, \bar{x} = 485 \text{ mg}$$

$$S = 20 \text{ mg}, \alpha = 0.05$$

$$H_0: \mu = 500 \text{ } \& H_1: \mu \neq 500$$

$$t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{485 - 500}{20/\sqrt{9}} = -2.25$$

$$df = n - 1 = 9 - 1 = 8$$

$$\alpha = 0.05$$

$$t \text{ values are } \pm t_{\alpha/2, df} = \pm t_{0.0258} \\ = \pm 2.306$$

$$t = -2.25$$

$$(-2.306 < -2.25 < 2.306)$$

we do not reject null hypothesis

$$2. H_0: \mu = 100 ; H_1: \mu < 100$$

$$\alpha = 0.01, n = 12, \bar{x} = 96.5$$

$$t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{96.5 - 100}{8/\sqrt{12}} = -1.516$$

$$df = n - 1 = 12 - 1 = 11$$

$$\alpha = 0.01, -t_{\alpha, df} = -t_{0.01, 11} \\ = -2.718$$

$$t = -1.516$$

$$(-1.516 > -2.718)$$

we do not reject  $H_0$ . Avg battery life is less than 100 hrs at 1% level of significance

3.  $H_0: \mu = 70, H_1: \mu > 70$   
 $\alpha = 0.05, n = 10, \bar{x} = 74, S = 5$

$$t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{74 - 70}{5/\sqrt{10}} = 2.53$$

$$df = n - 1 = 10 - 1 = 9$$

$$t_{\alpha, df} = t_{0.05, 9} = 1.833$$

$$t = 2.53 > 1.833$$

We reject  $H_0$ .

#### 5. $\chi^2$ TEST

$$1) H_0: \sigma^2 = 0.04, H_1: \sigma^2 \neq 0.04$$

$$\alpha = 0.05 \quad S^2 = 0.06, n = 10$$

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2} = \frac{(10-1)0.06}{0.04} = 13.5$$

$$df = n - 1 = 10 - 1 = 9$$

$$\chi^2_{1-\alpha/2} = \chi^2_{0.975} \quad \chi^2_{\alpha/2} = \chi^2_{0.025}$$

$$\chi^2_{0.975} = 2.70 \quad \chi^2_{0.025} = 19.02$$

$$\chi^2 < 2.70 \text{ or } \chi^2 > 19.02$$

$$\chi^2 = 13.5 \quad (2.7 < 13.5 < 19.02)$$

do not reject  $H_0$ .

$$2. H_0: \sigma^2 \leq 0.0025, H_1: \sigma^2 > 0.0025$$

$$\alpha = 0.05, n = 20, S^2 = 0.004,$$

$$\sigma^2 = 0.0025$$

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2} = \frac{(20-1)0.004}{0.0025} = 30.4$$

$$df = n - 1 = 20 - 1 = 19$$

$$\chi^2_{\alpha} = \chi^2_{0.05} \approx 30.14$$

$$\chi^2 > 30.14$$

$$30.4 > 30.14$$

Reject  $H_0$

$$3. H_0: \sigma^2 \leq 0.01, H_1: \sigma^2 > 0.01$$

$$\alpha = 0.05$$

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2} \quad n = 12, S^2 = 0.015, \sigma^2 = 0.01$$

$$\chi^2 = \frac{(12-1)0.015}{0.01} = 16.5$$

$$df = n - 1 = 12 - 1 = 11$$

$$\chi^2 \alpha = \chi^2_{0.05} = 19.68$$

$$(16.5 < 19.68)$$

We do not reject null hypothesis

## 6. F-TEST

$$H_0: \sigma_A^2 = \sigma_B^2$$

$$F = S_B^2 / S_A^2 \quad S_B^2 \text{ is larger}$$

$$n_A = 10, S_A^2 = 0.002 \text{ cm}^2, n_B = 11,$$

$$S_B^2 = 0.005 \text{ cm}^2, \alpha = 0.10$$

$$df_B = n_B - 1 = 10$$

$$df_A = n_A - 1 = 9$$

$$F_{0.10, 9, 10} \text{ is } F_{0.05, 10, 9} = 3.259$$

$$F = \frac{S_B^2}{S_A^2} = \frac{0.005}{0.002} = 2.5$$

$2.5 < 3.259$  we don't  
reject null hypothesis.

$$2) F = S_y^2 / S_x^2$$

$$n_x = 8, S_x^2 = 1.5, n_y = 10, S_y^2 = 2.8$$

$$\alpha = 0.01$$

$$df_y = n_y - 1 = 9, df_x = n_x - 1 = 7$$

$$F_{0.01, 9, 7} \approx 6.71$$

$$F = \frac{S_y^2}{S_x^2} = \frac{2.8}{1.5} \approx 1.867$$

$$1.867 < 6.71$$

Do not reject null hypothesis

$$3) n_A = 8, S_A^2 = 20, n_B = 7, S_B^2 = 12$$

$$\alpha = 0.05$$

$$df_A = n_A - 1 = 7, df_B = n_B - 1 = 6$$

$$F_{0.05, 7, 6} \approx 4.21$$

AAVUSHI  
PRIYA

$$F = \frac{S_A^2}{S_B^2} = \frac{20}{12} \approx 1.667$$

$$1.667 < 4.21$$

∴ At 5% significant level,  
not enough evidence to conclude  
method A has greater variance  
than method B.

$$4) n_A = 10$$

$$\bar{x}_A = \frac{10.2 + 10.4 + 10.1 + 10.5 + 10.3 + 10.2 + 10.4 + 10.3 + 10.1 + 10.2}{10} \\ = 10.27 \text{ mm}$$

$$S_A^2 \approx 0.019 \text{ mm}^2$$

$$n_B = 12$$

$$\bar{x}_B = \frac{10.7 + 10.9 + 10.8 + 10.6 + 11.0 + 10.9 + 10.7 + 10.8 + 10.9 + 11.1 + 10.8 + 10.7}{12} \\ = 10.825 \text{ mm}$$

$$S_B^2 = 0.017 \text{ mm}^2$$

$$\alpha = 0.10$$

$$df_A = n_A - 1 = 9, df_B = n_B - 1 = 11$$

$$F_{0.05, 9, 11} \approx 2.90$$

$$F = \frac{S_A^2}{S_B^2} = \frac{0.019}{0.017} = 1.118$$

$$F_{0.05, 9, 11} = \frac{1}{F_{0.05, 11, 9}} \approx \frac{1}{3.10} = 0.32$$

$$0.32 < 1.5 < 2.90$$

At 10% significant level, we fail  
to reject null hypothesis

$$5) H_0: \sigma_A^2 \leq \sigma_B^2$$

$$H_1: \sigma_A^2 > \sigma_B^2, \alpha = 0.01$$

$$n_A = 15, n_B = 15$$

$$\bar{x}_A = 83.87, \bar{x}_B = 77.2$$

$$S_A^2 = 14.98, S_B^2 = 4.4$$

$$F = \frac{S_A^2}{S_B^2} = \frac{14.98}{4.4} = 3.405$$

$$df_A = n_A - 1 = 14; df_B = n_B - 1 = 14$$

$$F_{0.01, 14, 14} = 3.70$$

AAYUSHI  
PRIYA

$3.405 < 3.70 \therefore$  we fail to reject null hypothesis

7. GOODNESS-OF-FIT

$$\forall p = 0.5, N = 240, m = 4$$

$$P(X=k) = nC_k p^k (1-p)^{n-k}$$

$$E_k = N \times P(X=k)$$

$$k=0$$

$$E_0 = 240 \cdot 4C_0 (0.5)^0 (0.5)^4 \\ = 15$$

$$k=1$$

$$E_1 = 240 \cdot 4C_1 (0.5)^1 (0.5)^3 = 60$$

$$k=2$$

$$E_2 = 240 \times 4C_2 (0.5)^2 (0.5)^2 = 90$$

$$k=3$$

$$E_3 = 240 \times 4C_3 (0.5)^3 (0.5)^1 = 60.$$

$$k=4$$

$$E_4 = 240 \times 4C_4 (0.5)^4 (0.5)^0 = 15$$

No. of heads	Observed (O <sub>i</sub> )	Expected (E <sub>i</sub> )	$\frac{(O_i - E_i)^2}{E_i}$
0	9	15	$(9-15)^2/15 = 2.4$
1	32	60	$(32-60)^2/60 = 13.07$
2	72	90	$(72-90)^2/90 = 3.6$
3	85	60	$(85-60)^2/60 = 10.42$
4	42	15	$(42-15)^2/15 = 48.6$

$$\chi^2_{\text{calc}} = 2.4 + 13.07 + 3.6 + 10.42 + 48.6 = 78.09$$

$$df = k - p - 1 = 5 - 0 - 1 = 4$$

At 5% level of significance

$$\alpha = 0.05 \quad df = 4$$

$$\chi^2_{\text{crit}} = 9.488$$

$$\chi^2_{\text{calc}} = 78.09 > \chi^2_{\text{crit}} = 9.488$$

We reject null hypothesis

$$2Y_N = 80$$

$$\bar{x} = \frac{30.0 + 28.1 + 16.2 + 6.3}{80} = 0.975$$

$$P(X=k) = \frac{e^{-\lambda} \lambda^k}{k!}$$

$$E_k = N \times P(X=k)$$

$$k=0$$

$$E_0 = 80 \times e^{-0.975} (0.975)^0 = 30.17$$

$$k=1$$

$$E_1 = 80 \times e^{-0.975} \frac{0!}{(0.975)^1} = 29.38$$

$$k=2$$

$$E_2 = 80 \times e^{-0.975} \frac{0!}{(0.975)^2} = 14.34$$

$$k=3$$

$$E_3 = 80 \times e^{-0.975} \frac{0!}{(0.975)^3} = 4.66$$

$$\text{Observed } (O_i) : 16 + 6 = 22$$

$$\text{Expected } (E_i) : 14.34 + 4.66 = 19$$

No. of defective	Observed (O <sub>i</sub> )	Expected (E <sub>i</sub> )	$\frac{(O_i - E_i)^2}{E_i}$
0	80	30.17	0.0009
1	28	29.38	0.0657
$\geq 2$	22	19	0.4734

$$\chi^2_{\text{calc}} = 0.0009 + 0.0657 + 0.4734 \\ = 0.5403$$

$$df = k - p - 1 = 3 - 1 - 1 = 1$$

$$\chi^2_{\text{crit}} = 3.841$$

$$3.841 > 0.5403$$

we do not reject null hypothesis ⑦

3)  $N=150, k=6$

[AAYUSHI PRIYA]

$$E_i = \frac{150}{6} = 25$$

sector (xi)	observed (oi)	Expected (Ei)	$(O_i - E_i)^2 / E_i$
1	20	25	$(20-25)^2 / 25 = 1$
2	21	25	$(21-25)^2 / 25 = 0.64$
3	27	25	$(27-25)^2 / 25 = 0.16$
4	29	25	$(29-25)^2 / 25 = 0.64$
5	26	25	$(26-25)^2 / 25 = 0.04$
6	27	25	$(27-25)^2 / 25 = 0.16$

$$\chi^2_{\text{calc}} = 1 + 0.64 + 0.16 + 0.64 + 0.04 \\ + 0.16 = 2.64$$

$$df = 6 - 1 = 5$$

$$\alpha = 0.05 \quad \chi^2_{\text{crit}} = 11.07$$

$$2.64 < 11.07$$

we don't reject null hypothesis