SAMPLING DISTRIBUTIONS AND ESTIMATION

1. Mean: $\mu = \frac{\Sigma X}{N} = \frac{69+71+73+75}{4} = 72$ kgs

Variance $\sigma^2 = \frac{\Sigma(X-\mu)^2}{N}$

$= \frac{(69-72)^2+(71-72)^2+(73-72)^2+(75-72)^2}{4}$

$= \frac{20}{4} = 5$

$\sigma = \sqrt{5} \approx 2.236$ kgs

$n = 2 \quad N^n = 4^2 = 16$

sample	$\bar{x}$	sample	$\bar{x}$
(69,69)	69	(73,69)	71
(69,71)	70	(73,71)	72
(69,73)	71	(73,73)	73
(69,75)	72	(73,75)	74
(71,69)	70	(75,69)	72
(71,71)	71	(75,71)	73
(71,73)	72	(75,73)	74
(71,75)	73	(75,75)	75

$\mu_{\bar{x}} = \frac{69+70+71+72+70+\dots+74+75}{16}$

$= 72$ kgs

$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n} = \frac{5}{2} = 2.5$

$\sigma_{\bar{x}} = \sqrt{2.5} = 1.581$ kg

$\cancel{\sigma_{\bar{x}}^2 = 94}$

$n = 2 \quad {}^4C_2 = 6$

sample	$\bar{x}$
(69,71)	70
(69,73)	71
(69,75)	72
(71,73)	72
(71,75)	73
(73,75)	74

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$\mu_{\bar{x}} = \frac{70+71+72+72+73+74}{6} = 72$ kgs

$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n} \times \frac{N-n}{N-1} = \frac{5}{2} \times \frac{4-2}{4-1} \approx \frac{5}{3} \approx 1.667$

$\sigma_{\bar{x}} = \sqrt{5/3} \approx 1.291$ kgs

$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n} \times \frac{N-n}{N-1} \Rightarrow \left[\frac{5}{3} = \frac{5}{2} \times \frac{2}{5}\right]$

2. $\mu = \frac{65+69+73+77}{4} = 71$ kgs

$\sigma^2 = \frac{(65-71)^2+(69-71)^2+(73-71)^2+(77-71)^2}{4}$

$= \frac{80}{4} = 20$

$\sigma = \sqrt{20} \approx 4.472$ kgs

$n = 2 \quad N^n = 4^2 = 16$

$\mu_{\bar{x}} = 71$ kgs $\quad \sigma^2_{\bar{x}} = \frac{\sigma^2}{n} = \frac{20}{2} = 10$

$\sigma_{\bar{x}} = \sqrt{10} = 3.162$ kgs

$\mu_{\bar{x}} = \mu \; (71 = 71)$ & $\cancel{\sigma^2_{\bar{x}} = \sigma}$

${}^4C_2 = 6$

$\mu_{\bar{x}} = 71$ kgs

$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n} \times \frac{N-n}{N-1} = \frac{20}{2} \times \frac{4-2}{4-1} \approx 6.667$

$\sigma_{\bar{x}} = \sqrt{20/3} \approx 2.582$ kgs

3. $\mu = \frac{50+60+70+80}{4} = \frac{260}{4} = 65$ kgs

$\sigma^2 = \frac{(50-65)^2+(60-65)^2+(70-65)^2+(80-65)^2}{4}$

$= \frac{500}{4} = 125$

$\sigma = \sqrt{125} = 11.180$ kgs

$n = 2 \quad N^n = 4^2 = 16$

$\mu_{\bar{x}} = 65$ kgs

$\sigma^2_{\bar{x}} = \sigma^2/n = 125/2 = 62.5$

$\sigma_{\bar{x}} = \sqrt{62.5} \approx 7.906$ kgs

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$\mu_{\bar{x}} = \mu (65 = 65).$

$n = 2 \quad {}^4C_2 = 6$

$\mu_{\bar{x}} = 65$ kg

$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n} \times \frac{N-n}{N-1} = \frac{125}{2} \times \frac{4-2}{4-1}$

$= 41.667$

$\sigma_{\bar{x}} = \sqrt{125/3} = 6.455$ kgs

4. a) $\mu = \frac{152+156+160+164}{4} = 158$

$\sigma^2 = \frac{(152-158)^2 + (156-158)^2 + (160-158)^2 + (164-158)^2}{4}$

$= \frac{80}{4} = 20$

$\sigma = \sqrt{\sigma^2} = \sqrt{20} = 4.472$

$\mu_{\bar{x}} = \mu = 158$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{20}{2}} \approx 3.162$

b) $\mu = 158, \ \sigma = \sqrt{20} \approx 4.472$

$\mu_{\bar{x}} = \mu = 158$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{20}{2}} \times \sqrt{\frac{4-2}{4-1}}$

$= 2.582$

5. a) $\mu = \frac{125+150+175+200}{4} = 162.5$

$\sigma^2 = \frac{(125-162.5)^2 + (150-162.5)^2 + (175-162.5)^2 + (200-162.5)^2}{4}$

$= 781.25$

$\sigma = \sqrt{\sigma^2} = \sqrt{781.25} \approx 27.95$

$\mu_{\bar{x}} = \mu = 162.5$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{781.25}{2}} = 19.76$

b) $\mu = 162.5, \ \sigma = \sqrt{781.25} \approx 27.95$

$\mu_{\bar{x}} = \mu = 162.5$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{781.25}{2}} \times \sqrt{\frac{4-2}{4-1}}$

$= 16.12$

6. $\mu_{\bar{x}} = \mu = 128$

$\sigma = 22, \ n = 36$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{22}{\sqrt{36}} = \frac{22}{6} = 3.667$

$Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$

i) $P(118 < \bar{x} < 138)$

$= P\left(\frac{118-128}{3.667} < Z < \frac{138-128}{3.667}\right)$

$= P(-2.727 < Z < 2.727)$

$P \approx 0.9934 - 0.0032$

$= 0.9902$

ii) $P(\bar{X} > 138 \text{ or } \bar{X} < 118)$

$= 1 - P(118 < \bar{x} < 138)$

$= 1 - 0.9902 = 0.0098$

iii) $P(113 < \bar{x} < 118 \text{ or } 138 < \bar{x} < 143)$

$P(Z < -2.727) - P(Z < -4.091) + P(Z < 4.091) - P(Z < 2.727)$

$= P(-4.091 < Z < -2.727) + P(2.727 < Z < 4.091)$

$= (0.0032 - 0) + (1 - 0.9934)$

$= 0.0032 + 0.0066 = 0.0098$

7. $\mu = 36.6$ mph, $\sigma = 1.7$ mph

$n = 20$

$\mu_{\bar{x}} = \mu = 36.6, \ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.7}{\sqrt{20}} = 0.380$

i) $P(35 < \bar{x} < 40) = P\left(\frac{35-36.6}{0.380} < Z < \frac{40-36.6}{0.380}\right) = P(-4.21 < Z < 8.95)$

$P = 1 - 0 = 1.0$

ii) $P(\bar{x} > 36) = P\left(Z > \frac{36-36.6}{0.380}\right)$

$= P(Z > -1.58) = 1 - 0.057$
$= 0.943$

iii) $P(\bar{x} < 37) = P\left(Z < \frac{37-36.6}{0.380}\right)$

$= P(Z < 1.05)$

$= 0.853$

8. $N = 3000$, $\mu = Rs\ 68$, $\sigma = Rs\ 3$, $n = 25$

a) $\mu_{\bar{x}} = \mu = Rs\ 68$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{25}} = \frac{3}{5} = ₹0.6$

b) $\mu_{\bar{x}} = \mu = Rs\ 68$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}}$

$= 0.6 \times \sqrt{\frac{3000-25}{3000-1}} = Rs\ 0.599$

i) $P(66.8 < \bar{x} < 68.3)$

$= P\left(\frac{66.8-68}{0.6} < Z < \frac{68.3-68}{0.6}\right)$

$= P(-2 < Z < 0.5)$

$= 0.6915 - 0.0228 = 0.6687$

ii) $P(\bar{x} < 66.4) = P\left(Z < \frac{66.4-68}{0.6}\right)$

$= P(Z < -2.67) \simeq 0.0038$

$= 80 \times 0.0038 = 0.3$ samples

9. $\mu = 170$ cm, $\sigma = 6$ cm, $n = 25$

i) $\mu_{\bar{x}} = \mu = 170$ cm

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{25}} = 1.2$ cm

ii) $P(\bar{x} < 168) = P\left(Z < \frac{168-170}{1.2}\right)$

$= P(Z < -1.67) = 0.0475$

iii) $z_1 = \frac{169-170}{1.2} = -0.82$

$z_2 = \frac{172-170}{1.2} = 1.67$

$P(169 < \bar{x} < 172)$

$= P(-0.82 < Z < 1.67)$

$= 0.9525 - 0.2033$

$= 0.7492$

10) $\mu_{\bar{x}} = \mu = 72$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{64}} = 1$

ii) $z_1 = \frac{70-72}{1} = -2$

$z_2 = \frac{74-72}{1} = 2$

$P(70 < \bar{x} < 74) = P(-2 < Z < 2)$

$= 0.97725 - 0.02275$

$= 0.9545$

iii) $\bar{x} = 75$

$z = \frac{75-72}{1} = 3$

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$P(\bar{x} > 75) = P(Z > 3)$

$= 1 - P(Z < 3)$

$= 1 - 0.99865 = 0.0035$

10 i) $\mu_{\hat{p}} = 0.30$ $n = 150$

$\sigma_{\hat{p}} = \sqrt{\frac{0.3(1-0.3)}{150}} = 0.03742$

ii) $z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{0.27-0.30}{0.03742}$

$= -0.8016$

$P(Z \le 0.27) = P(Z \le -0.8016)$

$\simeq 0.2112$

iii) $x = 51$ $\hat{p} = \frac{51}{150} = 0.34$

$P(\hat{p} \geq 0.34)$

$$Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{0.34 - 0.3}{0.03742} = 1.069$$

$P(\hat{p} \geq 0.34) = P(Z > 1.069)$

$= 1 - P(Z < 1.069)$

$= 0.8575$

$P(\hat{p} \geq 0.34) = 1 - 0.8575 = 0.1425$

iv) $\hat{p}_1 = 0.32$, $\hat{p}_2 = 0.35$

For $\hat{p}_1$: $Z_1 = \frac{0.32 - 0.30}{0.03742} = 0.5345$

For $\hat{p}_2$: $Z_2 = \frac{0.35 - 0.30}{0.03742} = 1.336$

$P(0.32 < \hat{p} < 0.35)$

$= P(0.5345 < Z < 1.336)$

$= \phi(1.336) - \phi(0.5345)$

$= 0.9156 - 0.7035 = 0.2121$

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12> i> $\mu_{\hat{p}} = p = 0.175$

$$\sigma_{\hat{p}} = \sqrt{\frac{0.175(1-0.175)}{200}} = 0.02687$$

ii> $x = 32$, $\hat{p} = \frac{32}{200} = 0.16$

$P(x < 32) = P(\hat{p} < 0.16)$

$$Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{0.16 - 0.175}{0.02687} = -0.5582$$

$P(Z < -0.5582) = 0.2884$

iii> $Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{0.20 - 0.175}{0.02687}$

$= 0.9304$

$P(\hat{p} > 0.20) = P(Z > 0.9304)$

$= 1 - P(Z < 0.9304)$

$= 1 - 0.8239 = 0.1761$

iv) $x_1 = 34$ $\hat{p}_1 = \frac{34}{200} = 0.17$

$x_2 = 44$, $\hat{p}_2 = \frac{44}{200} = 0.22$

$P(34 < x < 44) \Rightarrow P(0.17 < \hat{p} < 0.22)$

For $\hat{p}_1 = 0.17$

$$Z_1 = \frac{0.17 - 0.175}{0.02687} = -0.186$$

$\hat{p}_2 = 0.22$

$$Z_2 = \frac{0.22 - 0.175}{0.02687} = 1.674$$

$P(-0.186 < Z < 1.674)$

$= 0.9529 - 0.426 = 0.5269$

13> i) $\mu_{\hat{p}} = p = 0.12$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.12(1-0.12)}{250}}$$

$= 0.02055$

ii) $x = 25$, $\hat{p} = \frac{25}{250} = 0.10$

$$Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{0.10 - 0.12}{0.02055} = -0.9732$$

$P(\hat{p} < 0.10) = P(Z < -0.9732)$

$= 0.1653$

ii) $P(x < 25) = P(x \leq 24.5)$

$\mu_x = np = 30$

$$Z = \frac{x - \mu_x}{\sigma_x} = \frac{24.5 - 30}{5.138} = -1.07045$$

$P(Z < -1.07) = 0.1423$

iii) $\mu_{\hat{p}} = 0.12$ $\sigma_{\hat{p}} = 0.02055$

$$Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{0.15 - 0.12}{0.02055}$$

$= 1.46$

$P(Z \leq 1.46) = 0.9279$

$P(Z > 1.46) = 1 - 0.9279$

$= 0.0721$

iv) $P(28 \le X \le 38)$

$= P(27.5 \le X \le 38.5)$

$\mu_x = 30, \sigma_x = 5.138$

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For $x = 27.5$

$$Z_1 = \frac{27.5 - 30}{5.138} = \frac{-2.5}{5.138} = -0.49$$

For $x = 38.5$

$$Z_2 = \frac{38.5 - 30}{5.138} = 1.65$$

$P(-0.49 \le Z \le 1.65)$.

$= 0.9505 - 0.3121 = 0.6384$

14) $p = 0.22, n = 300$

i) $\mu_{\hat{p}} = p = 0.22$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.22(1-0.22)}{300}}$$

$= 0.0239$

ii) $P(X < 60)$

$np = 300 \times 0.22 = 66 \ge 10$

$n(1-p) = 300 \times 0.78 = 234 \ge 10$

$\sigma_x = \sqrt{np(1-p)} = \sqrt{66 \times 0.78}$

$= 7.175$

$$Z = \frac{X - \mu_x}{\sigma_x} = \frac{59.5 - 66}{7.175} = -0.91$$

$P(Z < -0.91) = 0.1814$

iii) $P(\hat{p} > 0.25)$ $\mu_{\hat{p}} = 0.22$

$\sigma_{\hat{p}} = 0.0239$

$$Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{0.25 - 0.22}{0.0239}$$

$= 1.26$

$P(Z > 1.26) = 1 - P(Z \le 1.26)$

$= 1 - 0.8962 = 1038$

iv) $P(58 \le X \le 75)$

$\mu_x = 66, \sigma_x = 7.175$

For $x = 57.5$

$$Z_1 = \frac{57.5 - 66}{7.175} = -1.18$$

For $x = 75.5$

$$Z_2 = \frac{75.5 - 66}{7.175} = 1.32$$

$P(-1.18 \le Z \le 1.32)$

$= 0.9066 - 0.1190$

$= 0.7876$

15 i) $p = 0.09, n = 180$

$\mu_{\hat{p}} = p = 0.09$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.09 \times (1-0.09)}{180}}$$

$= 0.02133$

ii) $np = 180 \times 0.09 = 16.2 \ge 10$

$n(1-p) = 180 \times 0.91 = 163.8 \ge 10$

$\mu_x = np = 16.2$

$\sigma_x = \sqrt{np(1-p)} = \sqrt{16.2 \times 0.91}$

$= 3.8395$

$$Z = \frac{X - \mu_x}{\sigma_x} = \frac{9.5 - 16.2}{3.8395}$$

$= -1.743$

iii) $\mu_{\hat{p}} = 0.09, \sigma_{\hat{p}} = 0.02133$

$$Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{0.12 - 0.09}{0.02133} = 1.406$$

$P(Z > 1.406) = 1 - 0.920 = 0.08$

iv) $P(14 \le X \le 20)$

$$Z_1 = \frac{13.5 - 16.2}{3.843} = -0.702$$

$$Z_2 = \frac{20.5 - 16.2}{3.843} = 1.119$$

$P(-0.702 \le Z \le 1.119)$

$= 0.868 - 0.241 = 0.627$

16) $n_1 = 32$, $n_2 = 50$, $\mu = 540$, $\sigma = 50$

$\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2 = 540 - 540 = 0$

$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{50^2}{32} + \frac{50^2}{50}}$
$= 11.32$

i) $P|\bar{x}_1 - \bar{x}_2| > 20$

$= P(\bar{x}_1 - \bar{x}_2 > 20) + P(\bar{x}_1 - \bar{x}_2 < -20)$

$2 \times P(\bar{x}_1 - \bar{x}_2 > 20)$

$z = \frac{20-0}{11.32} = 1.767$

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$P(z > 1.767) = 1 - 0.961 = 0.039$

$2 \times 0.039 = 0.078$

ii) $P(5 < |\bar{x}_1 - \bar{x}_2| < 10)$

$z_1 = \frac{5-0}{11.32} = 0.442$, $z_2 = \frac{10-0}{11.32} = 0.883$

$P(5 < D < 10) = P(z \le 0.883) - P(z \le 0.442)$

$= 0.811 - 0.670 = 0.141$

$2 \times 0.141 = 0.282$

17. i) $\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

$\sigma_1 = 40$, $n_1 = 36$, $\sigma_2 = 40$, $n_2 = 49$

$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{40^2}{36} + \frac{40^2}{49}} = 8.78$

$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1 - \bar{x}_2}} = \frac{15-0}{8.78} = 1.71$

$P(z > 1.71) = 1 - P(z \le 1.71)$

$= 1 - 0.9564 = 0.0436$

ii) $z_1 = \frac{5-0}{8.78} = 0.57$

$z_2 = \frac{12-0}{8.78} = 1.37$

$P(z \le 0.57) = 0.7157$

$P(z \le 1.37) = 0.9147$

$P(0.57 \le z \le 1.37)$

$= 0.9147 - 0.7157 = 0.199$

18. i) $\mu_{\bar{x}_A - \bar{x}_B} = \mu_A - \mu_B = 82 - 79 = 3$

$\sigma_{\bar{x}_A - \bar{x}_B} = \sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}$

$\sigma_A = 6$, $n_A = 30$, $\sigma_B = 8$, $n_B = 30$

$\sigma_{\bar{x}_A - \bar{x}_B} = \sqrt{\frac{6^2}{30} + \frac{8^2}{30}} = 1.826$

ii) $z = \frac{(\bar{x}_A - \bar{x}_B) - (\mu_A - \mu_B)}{\sigma_{\bar{x}_A - \bar{x}_B}}$

$\mu_A - \mu_B = 3$ $\quad \sigma_{\bar{x}_A - \bar{x}_B} = 1.826$

$z = \frac{5-3}{1.826} = 1.095$

$P(\bar{x}_A - \bar{x}_B > 5)$

$= P(z > 1.095)$

$= 1 - P(z \le 1.095)$

$= 1 - 0.8633 = 0.1367$

iii) $\mu_A - \mu_B = 3$ $\quad \sigma_{\bar{x}_A - \bar{x}_B} = 1.826$

$z_1 = \frac{0-3}{1.826} = \frac{-3}{1.826} = -1.643$

$z_2 = \frac{4-3}{1.826} = 0.548$

$P(Z \le -1.643) = 0.0503$

$P(Z \le 0.548) = 0.7082$

$= P(-1.643 \le Z \le 0.548)$ AAYUSHI ARYA

$= 0.7082 - 0.0503 = 0.6579$

19) i) $E(\bar{x}_1 - \bar{x}_2) = \mu_1 - \mu_2 = 3.2 - 2.8$
$= 0.4$ mins

$Var(\bar{x}_1 - \bar{x}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$

$= \frac{0.6^2}{40} + \frac{0.5^2}{35} = 0.016143$

$SD(\bar{x}_1 - \bar{x}_2) = \sqrt{Var(\bar{x}_1 - \bar{x}_2)} = \sqrt{0.016143}$
$= 0.127$ mins

ii) $P(\bar{x}_1 - \bar{x}_2 \ge 0.7)$ $D = \bar{x}_1 - \bar{x}_2$

$\mu_D = 0.4$, $\sigma_D = 0.127$

$Z = \frac{x - \mu_D}{\sigma_D} = \frac{0.7 - 0.4}{0.127} = 2.362$

$P(Z \ge 2.362) = 1 - P(Z < 2.362)$
$= 1 - 0.9909 = 0.0091$

iii) $P(\bar{x}_1 - \bar{x}_2 < 0)$

$Z = \frac{x - \mu_D}{\sigma_D} = \frac{0 - 0.4}{0.127} = -3.15$

$P(Z < -3.15) = 0.0008$

20) i) $E(\bar{x}_A - \bar{x}_B) = \mu_A - \mu_B = 52 - 49$
$= 3$ mpg

$Var(\bar{x}_A - \bar{x}_B) = \frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B} = \frac{4^2}{36} + \frac{5^2}{36} = 1.1389$

$SE(\bar{x}_A - \bar{x}_B) = \sqrt{Var(\bar{x}_A - \bar{x}_B)} = \sqrt{\frac{41}{36}}$
$= 1.068$ mpg

ii) $P(\bar{x}_A - \bar{x}_B > 4)$ $\mu_D = 3, \sigma_D = 1.068$

$Z = \frac{x - \mu_D}{\sigma_D} = \frac{4 - 3}{1.068} = 0.936$

$P(Z > 0.936) = 1 - P(Z \le 0.936)$
$= 1 - 0.8255 = 0.1745$

iii) $P(\bar{x}_A - \bar{x}_B < 1)$

$Z = \frac{x - \mu_D}{\sigma_D} = \frac{1 - 3}{1.068} = -1.873$

21) $E(\hat{p}_B - \hat{p}_A) = p_B - p_A$
$= 0.09 - 0.05 = 0.04$

$Var(\hat{p}_B - \hat{p}_A) = \frac{p_B(1-p_B)}{n_B} + \frac{p_A(1-p_A)}{n_A}$

$= \frac{0.09(0.91)}{250} + \frac{0.05(0.95)}{500}$

$= 0.0004226$

$SD(\hat{p}_B - \hat{p}_A) = \sqrt{0.0004226}$
$= 0.02056$

$P(D \le -0.02)$

$Z = \frac{x - \mu_D}{\sigma_D} = \frac{-0.02 - 0.04}{0.02056}$
$= -2.918$

$P(Z \le -2.918) = 0.00175$

22) $\mu_{\hat{p}_{2006} - \hat{p}_{2017}} = p_{2006} - p_{2017}$
$= 0.95 - 0.72 = 0.23$

$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$

$= \sqrt{\frac{0.95(1-0.95)}{250} + \frac{0.72(1-0.72)}{250}}$

$= 0.03156$

$Z = \frac{(\hat{p}_{2006} - \hat{p}_{2017}) - \mu_{\hat{p}_{2006} - \hat{p}_{2017}}}{\sigma_{2006-2017}}$

$= \frac{0.20 - 0.23}{0.03156} = -0.9505$

$P(Z \ge -0.9505) = 0.829$

23) $p_T = 0.35$, $n_T = 40$,
$p_D = 0.20$, $n_D = 60$

$\mu_{\hat{p}_T - \hat{p}_D} = p_T - p_D$
$= 0.35 - 0.20 = 0.15$

$$\sigma_{\hat{p}_T-\hat{p}_D} = \sqrt{\frac{P_T(1-P_T)}{n_T} + \frac{P_D(1-P_D)}{n_D}}$$

$$= \sqrt{\frac{0.35(0.65)}{40} + \frac{0.20(0.80)}{50}}$$

$$= 0.09427$$

$$z = \frac{(\hat{P}_T - \hat{P}_D) - \mu_{\hat{P}_T-\hat{P}_D}}{\sigma_{\hat{P}_T-\hat{P}_D}}$$

$$= \frac{0 - 0.15}{0.09427} = -1.591$$

$P(z < -1.591) \simeq 0.056$

24) i) $P_{2023} = 0.90$, $n_{2023} = 120$

$P_{2015} = 0.70$, $n_{2015} = 150$

$\mu = P_{2023} - P_{2015} = 0.90 - 0.70$
$= 0.20$

$$\sigma^2 = \frac{P_{2023}(1-P_{2023})}{n_{2023}} + \frac{P_{2015}(1-P_{2015})}{n_{2015}}$$

$$= \frac{0.90(0.10)}{120} + \frac{0.70(0.30)}{150}$$

$= 0.00075 + 0.0014 = 0.00215$

$\sigma = \sqrt{0.00215} = 0.04637$

ii) $P(\hat{p}_{2023} - \hat{p}_{2015} \geq 0.15)$

$$z = \frac{(p_{2023} - p_{2015}) - \mu}{\sigma}$$

$$= \frac{0.15 - 0.20}{0.04637} = -1.078$$

$P(z \geq -1.08) = 0.8599$

25) i) P_s ~~student~~ $= 0.60$, $n_s = 180$

$P_h = 0.5$, $n_h = 200$

$\mu_{ps-ph} = 0.60 - 0.50 = 0.10$

$$\sigma^2_{\hat{p}_s-\hat{p}_n} = \frac{P_s(1-P_s)}{n_s} + \frac{P_h(1-P_h)}{n_h}$$

$$= \frac{0.60(0.40)}{180} + \frac{0.50(0.50)}{200}$$

$= 0.002583$

$\sigma = \sqrt{0.002583} = 0.05083$

ii) $(\hat{p}_{s}$ ~~science~~ $- \hat{p}_h < 0.05)$

$$z = \frac{(\hat{p}_s - \hat{p}_h) - \mu}{\sigma}$$

$$= \frac{0.05 - 0.10}{0.05083} = -0.983$$

$P(z < -0.98) = 0.1635$

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