



Academic year 2025-2026 (Odd Semester)
DEPARTMENT OF MATHEMATICS

Date: 06.11.2025	Time: 9.00 AM - 11.00 AM	CIE- I	Max. Marks: 10+50
Semester: III	Branches: CS, CD, CY, IS	UG	Duration: 2 Hrs
Course: LINEAR ALGEBRA AND PROBABILITY THEORY			Course Code: MA231TC

Instructions:

1. Answer all questions.
2. Answer part A in first two pages only.
3. Handbook not permitted; necessary formulas are provided at the end of the question paper.

Sl. No.	PART - A	M	BTL	CO
1	Let S be the set of all 2×2 symmetric matrices, a subspace of $M_{2 \times 2}$. Then the zero vector is _____ and the inverse vector is _____.	2	1	1
2	Given $u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $w = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$. If w can be expressed as a linear combination of u and v then the coefficients of u and v are respectively _____ and _____.	2	1	2
3	Let $U = \text{Span}\{(1,0,1), (0,1,1)\}$ and $W = \{(x,y,z) : x = y\}$ be subspaces of \mathbb{R}^3 . Then $\dim(U \cap W)$ is _____.	2	2	2
4	The orthogonal projection matrix for projecting a vector $[x \ y]^T$ onto the vector $[-1 \ \sqrt{3}]^T$ is _____.	2	2	2
5	Column space of a matrix A of order 4×5 is a subspace of \mathbb{R}^k where k is _____.	1	1	1
6	Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation such that $T(1,0) = (2, -3)$ and $T(0,1) = (3,0)$. Then image of $(2,5)$ is _____.	1	1	1
	PART - B	M	BTL	CO
1	Verify whether the following sets form a subspace of corresponding vector space. i) $S_1 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 : x + y + z = 0, x, y, z \in \mathbb{Z} \right\}$. ii) $S_2 = \{[x \ y]^T \in \mathbb{R}^2 : e^{2x-3y} = 0\}$. iii) $S_3 = \{f(x) \in P_2 : 2f(0) = f(1)\}$. iv) $S_4 = \{(x,y) \in \mathbb{R}^2 : xy = 0\}$. v) $S_5 = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{2 \times 2} : a - d = 0 \right\}$.	10	2	2



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2	<p>(a) Determine the values of λ such that the vectors $(1,1,2,1), (2,1,2,3), (1,4,2,1), (-1,3,5,\lambda)$ are linearly independent.</p> <p>(b) Check whether the polynomial $3x^2 + x + 5$ is in the linear span of the set $S = \{x^3, x^2 + 2x, x^2 + 2, 1 - x\}$ of the vector space of all polynomials over the field \mathbb{R}.</p>	5	3	3
3	<p>Find the basis and dimension for the four fundamental subspace of the matrix $A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$.</p>	10	3	3
4	<p>(a) Verify the following are linear transformation</p> <p>(i) $T_1: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T_1(x, y) = (3x + 2y, 5x - y)$</p> <p>(ii) $T_2: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T_2(x, y) = (xy, x + y)$.</p> <p>(b) Find the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ such that $T(1, 1, 1) = (1, 1, 1)$, $T(1, 2, 3) = (-1, -2, 3)$ and $T(1, 1, 2) = (2, 2, 4)$.</p>	5	2	2
5	<p>(a) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a linear mapping such that $T(x, y) = (x + 2y, 3x + y, x - y)$. Find the basis for Kernel and range space of T. Hence verify the Rank - Nullity theorem.</p> <p>(b) Find the linear transformation matrix which rotates the vector $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ by 30° clockwise and then stretches by a factor of 2. Also find the transformed vector.</p>	6	2	4
		4	3	4

Transformation Matrices:

Scaling: $\begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}$, Rotation: $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$, Projection: $\begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix}$,

Reflection: $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$.

Projection matrix on a single vector u is $\frac{uu^T}{u^T u}$.

- Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	04	26	20	10	6	28	26	-	-	-

*****All The Best *****