

## UNIT - V Inferential Statistics

### 1 Confidence Interval

1. A machine fills bottles with soda. A quality control inspector takes a random sample of 100 bottles and finds that the sample mean volume is 498 ml with a population standard deviation of 10 ml. Construct a 95% confidence interval for the true mean volume filled by the machine.
2. A company wants to estimate the average weight of its packaged flour. A random sample of 150 bags shows a sample mean weight of 2.02 kg. The population standard deviation is known to be 0.1 kg. Construct a 99% confidence interval for the true mean weight of the flour bags.
3. A researcher wants to estimate the average sleep duration of university students. A random sample of 9 students yields the following sleep hours per night: 6.5, 7.0, 6.8, 5.9, 7.1, 6.3, 7.4, 6.0, 6.7. Assuming the sleep durations are normally distributed, construct a 95% confidence interval for the population mean sleep duration.
4. A health clinic is researching the average recovery time (in days) for patients after a minor surgery. A random sample of 7 patients gives the following recovery times: 12, 15, 14, 10, 13, 11, 16. Assuming recovery times are approximately normally distributed, construct a 90% confidence interval for the mean recovery time.
5. A survey of 500 voters in a city finds that 285 support a particular candidate. Construct a 95% confidence interval for the true proportion of voters in the city who support this candidate.
6. In a quality control check, a manufacturer finds that 18 out of 120 items produced in a day are defective. Find a 99% confidence interval for the true proportion of defective items produced by the manufacturer.
7. A random sample of 10 metal rods has a sample variance of  $4.5 \text{ mm}^2$ . Assume the population of rod diameters is normally distributed. Construct a 95% confidence interval for the population variance.
8. The breaking strength of a certain type of wire is tested for a sample of 16 wires, yielding a sample standard deviation of 1.2 N. Assume the data are normally distributed. Find a 99% confidence interval for the population variance.
9. A researcher is comparing the variability in lifespans of two brands of batteries. A sample of 12 batteries from Brand A has a variance of  $18.2 \text{ hours}^2$ , and a sample of 10 batteries from Brand B has a variance of  $10.5 \text{ hours}^2$ . Assume both populations are normally distributed. Construct a 90% confidence interval for the ratio of the variances of Brand A to Brand B.

10. Two machines are used to manufacture bolts. A sample of 14 bolts from Machine 1 has a sample variance of  $0.022 \text{ mm}^2$ , and a sample of 16 bolts from Machine 2 has a sample variance of  $0.015 \text{ mm}^2$ . Assume normality in both populations. Find a 98% confidence interval for the ratio of the variances of Machine 1 to Machine 2.

## 2 Type-I and Type-II error

1. A quality control manager believes that the defect rate in a production line is  $p = 0.4$ . To test this belief, a random sample of 12 items is inspected. If 2 or fewer defective items are found, the null hypothesis that  $p = 0.4$  is rejected in favor of the alternative  $p < 0.4$ .

Use the binomial distribution to answer the following:

- (a) Find the probability of committing a Type-I error if the true defect rate is  $p = 0.4$ .
  - (b) Find the probability of committing a Type-II error if the true defect rate is  $p = 0.2$ .
2. The lifetime of a certain type of battery is normally distributed with standard deviation  $\sigma = 10$  hours. A battery manufacturer claims that the mean lifetime is  $\mu = 100$  hours. To test the null hypothesis:

$$H_0 : \mu = 100 \quad \text{vs} \quad H_1 : \mu < 100,$$

a sample of size  $n = 25$  is taken, and  $H_0$  is rejected if the sample mean  $\bar{x} < 96.9$ .

- (a) Calculate the probability of Type-I error.
  - (b) Calculate the probability of Type-II error if the true mean is  $\mu = 95$ .
3. A fabric manufacturer believes that the proportion of orders for raw material arriving late is  $p = 0.6$ . If a random sample of 50 orders shows that 24 or fewer arrived late, the hypothesis that  $p = 0.6$  should be rejected in favor of the alternative  $p < 0.6$ . Assume the normal approximation to the binomial distribution is appropriate.
- (a) Find the probability of committing Type-I error if the true proportion is  $p = 0.6$ .
  - (b) Find the probability of committing Type-II error if the true proportion is  $p = 0.3$ .
4. A packaging machine is set to fill bags with 1000 grams of flour. The amount dispensed per bag follows a normal distribution with known standard deviation  $\sigma = 5$  grams. To test:

$$H_0 : \mu = 1000 \quad \text{vs} \quad H_1 : \mu \neq 1000,$$

a sample of size  $n = 25$  is taken, and the rejection region is defined as

$$|\bar{x} - 1000| > 1.96 \cdot \frac{5}{\sqrt{25}}.$$

- (a) What is the probability of Type-I error?
- (b) What is the probability of Type-II error if the true mean is  $\mu = 998.5$ ?

### 3 z-test

1. A snack company claims that the average weight of a bag of chips is 150 grams. The weights are normally distributed with a known standard deviation of  $\sigma = 5$  grams. A quality control officer randomly selects a sample of 36 bags and finds that the average weight is  $\bar{x} = 148.5$  grams.
  - (a) Test the hypothesis  $H_0 : \mu = 150$  against  $H_1 : \mu \neq 150$  at the 5% level of significance.
  - (b) What is the  $p$ -value of the test?
2. An engine manufacturer claims that the average fuel consumption of its new model is 25 km/litre. Assume fuel consumption follows a normal distribution with a known standard deviation  $\sigma = 2$  km/litre. A consumer organization tests 49 cars and finds the sample mean fuel consumption to be  $\bar{x} = 24.4$  km/litre. Test the hypothesis  $H_0 : \mu = 25$  against  $H_1 : \mu < 25$  at the 1% level of significance.
3. A manufacturer claims that the average lifetime of a type of LED bulb is 5000 hours. A consumer group tests a random sample of 100 bulbs and finds a sample mean lifetime of  $\bar{x} = 4950$  hours and sample standard deviation  $s = 120$  hours. Test the hypothesis  $H_0 : \mu = 5000$  against  $H_1 : \mu \neq 5000$  at the 5% level of significance using the  $z$ -test.
4. A soft drink company claims that its bottles contain 2 liters of soda on average. A random sample of 64 bottles is taken and the sample mean is found to be  $\bar{x} = 1.98$  liters with a sample standard deviation of  $s = 0.08$  liters. Test the null hypothesis  $H_0 : \mu = 2$  against the alternative  $H_1 : \mu < 2$  at the 1% level of significance using the  $z$ -test.
5. A political analyst claims that 60% of voters support a new education policy. To test this claim, a random sample of 200 voters is surveyed, and 108 indicate their support. Test the hypothesis  $H_0 : p = 0.60$  against the alternative  $H_1 : p \neq 0.60$  at the 5% level of significance using a  $z$ -test. Also calculate the  $P$ -value.
6. A factory claims that no more than 5% of its products are defective. In a recent quality inspection of 400 items, 28 defective items were found. Test the null hypothesis  $H_0 : p = 0.05$  against the alternative  $H_1 : p > 0.05$  at the 1% level of significance using a  $z$ -test. Find the  $p$ -value and state your conclusion.

### 4 t-test

1. A dietitian claims that the average sodium content in a certain brand of soup is 500 mg. A random sample of 9 cans has a mean sodium content of 485 mg and a sample standard deviation of 20 mg. Test the hypothesis  $H_0 : \mu = 500$  against  $H_1 : \mu \neq 500$  at the 5% level of significance.

2. A battery manufacturer claims that its batteries last at least 100 hours on average. A consumer organization tests 12 batteries and finds the sample mean to be 96.5 hours with a sample standard deviation of 8 hours. Test the hypothesis  $H_0 : \mu = 100$  against  $H_1 : \mu < 100$  at the 1% level of significance.
3. An energy drink company claims that their product increases alertness scores. The average baseline alertness score is 70. A sample of 10 individuals who consumed the drink has a mean score of 74 with a sample standard deviation of 5. Test the hypothesis  $H_0 : \mu = 70$  against  $H_1 : \mu > 70$  at the 5% level of significance.

## 5 $\chi^2$ -test

1. A machine is supposed to produce metal rods with a variance of  $0.04 \text{ cm}^2$ . A sample of 10 rods shows a variance of  $0.06 \text{ cm}^2$ . Test at the 5% significance level whether the variance differs from  $0.04 \text{ cm}^2$ .
2. A manufacturer claims that the variance of the diameter of ball bearings is at most  $0.0025 \text{ cm}^2$ . To verify this claim, a random sample of 20 ball bearings is taken, and the sample variance is found to be  $s^2 = 0.004 \text{ cm}^2$ . Using a significance level of 5%, test the null hypothesis  $H_0 : \sigma^2 \leq 0.0025$  against  $H_1 : \sigma^2 > 0.0025$ . Conduct a right-tailed Chi-square test.
3. The variance of thickness in paper sheets is claimed to be no more than  $0.01 \text{ mm}^2$ . A sample of 12 sheets shows a variance of  $0.015 \text{ mm}^2$ . Test at 5% significance level if the variance exceeds  $0.01 \text{ mm}^2$ .

## 6 $F$ -test

1. Two different machines produce bolts. From 10 bolts from machine A, variance is  $0.002 \text{ cm}^2$ , and from 11 bolts from machine B, variance is  $0.005 \text{ cm}^2$ . Test at 10% significance level whether the two machines have equal variances.
2. A company claims that the variance of strength from supplier X is less than that from supplier Y. From 8 samples of X, variance is 1.5, and from 10 samples of Y, variance is 2.8. Test the claim at 1% significance level.
3. A researcher wants to check if the variance of two teaching methods differs. Sample variances from method A (8 samples) and method B (7 samples) are 20 and 12 respectively. Test at the 5% significance level if method A has greater variance than method B.
4. Machine A and Machine B produce bolts. The diameters (in mm) of bolts from each machine are measured as follows:
  - Machine A: 10.2, 10.4, 10.1, 10.3, 10.5, 10.2, 10.4, 10.3, 10.1, 10.2
  - Machine B: 10.7, 10.9, 10.8, 10.6, 11.0, 10.9, 10.7, 10.8, 10.9, 11.1, 10.8, 10.7

At the 10% significance level, test whether the variances of diameters from the two machines are equal.

5. Two teaching methods are evaluated by measuring the test scores of students:

- Method A: 78, 85, 80, 90, 88, 82, 84, 79, 81, 87, 83, 86, 80, 85, 88
- Method B: 75, 80, 79, 77, 76, 78, 75, 79, 74, 80, 77, 78, 76, 75, 79

At the 1% significance level, test if the variance of scores for Method A is greater than that of Method B.

## 7 Goodness-of-fit

1. A set of four identical coins were tossed 240 times and the results are recorded below:

Number of Heads	0	1	2	3	4
Frequency	9	32	72	85	42

Test the hypothesis that the data follows a binomial distribution with  $p = 0.5$  (i.e., fair coins). Use a 5% level of significance.

2. Define  $X$  as the number of defective bulbs found in a box of 20 bulbs selected from a production line. Eighty boxes are inspected, and the number of defective bulbs per box is recorded as follows:

Number of Defectives ( $X$ )	0	1	2	3
Frequency	30	28	16	6

Assume that the number of defective bulbs in a box follows a Poisson distribution. Test the goodness of fit of the data with a binomial distribution at the 5% level of significance.

3. A spinner with six equal sectors is spun 150 times, and the following outcomes are recorded:

Sector ( $x$ )	1	2	3	4	5	6
Frequency ( $f$ )	20	21	27	29	26	27

Test at the 0.05 level of significance whether the spinner is fair using the chi-square goodness-of-fit test.