

# UNIT - 5

AAVUSHI

## 1. CONFIDENCE INTERVAL

1.  $n = 100$ ,  $\bar{x} = 498 \text{ ml}$   
 $\sigma = 10 \text{ ml}$ ,  $\alpha = 1 - 0.95 = 0.05$

$Z_{\alpha/2} = 1.96$

margin of error  $= Z_{\alpha/2} \left( \frac{\sigma}{\sqrt{n}} \right)$   
 $= 1.96 \left( \frac{10}{\sqrt{100}} \right) = 1.96$

lower limit  $= 498 - 1.96 = 496.04$

upper limit  $= 498 + 1.96 = 499.96$

2.  $n = 150$ ,  $\bar{x} = 2.02 \text{ kg}$ ,  
 $\sigma = 0.1 \text{ kg}$ ,  $Z_{\alpha/2} = 2.576$

margin of error (ME)

$= 2.576 \left( \frac{0.1}{\sqrt{150}} \right) = 0.021$

lower limit  $= 2.02 - 0.021$   
 $= 1.999 \text{ kg}$

upper limit  $= 2.02 + 0.021$   
 $= 2.041 \text{ kg}$

3.  $n = 9$

mean  $= \frac{6.5 + 7 + 6.8 + 5.9 + 7.1 + 6.3 + 7.4 + 6 + 6.7}{9}$

$= \frac{59.7}{9} = 6.633 \text{ hrs}$

S.d  $= 0.505 \text{ hrs}$

degree of freedom  $df = n - 1$

$\rightarrow t_{\alpha/2} \left( \frac{sd}{\sqrt{n}} \right) = 9 - 1 = 8$

ME  $= 2.306 \left( \frac{0.505}{\sqrt{9}} \right) = 0.388$

lower limit  $= 6.633 - 0.388$   
 $= 6.245 \text{ hrs}$

upper limit  $= 6.633 + 0.388$   
 $= 7.021 \text{ hrs}$

4.  $n = 7$

mean  $\bar{x} = \frac{12 + 15 + 14 + 10 + 13 + 11 + 16}{7}$   
 $= \frac{91}{7} = 13$

standard deviation  $s = 2.16 \text{ kg}$

Degree of freedom  $\Rightarrow df = n - 1$   
 $= 7 - 1 = 6$

confidence level  $= 90\%$

$t_{\alpha/2} = 1.943$

ME  $= t_{\alpha/2} \left( \frac{s}{\sqrt{n}} \right) = 1.943 \left( \frac{2.16}{\sqrt{7}} \right)$   
 $= 1.586$

Lower limit  $= 13 - 1.586 = 11.414$

Upper limit  $= 13 + 1.586 = 14.586$

5.  $n = 500$ ,  $x = 285$ ,  $\hat{p} = \frac{x}{n} = \frac{285}{500}$

Confidence level  $= 95\% = 0.95$

$Z_{\alpha/2} = 1.96$

ME  $= Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$= 1.96 \sqrt{\frac{0.57(1-0.57)}{500}} = 0.0434$

Lower limit  $= 0.57 - 0.0434$   
 $= 0.5266$

Upper limit  $= 0.57 + 0.0434$   
 $= 0.6134$

6.  $n = 120$ ,  $x = 18$ ,  $p = \frac{18}{120} = 0.15$

Confidence level  $= 99\%$

$\alpha = 0.01$ ,  $\frac{\alpha}{2} = 0.005$

$Z_{\alpha/2} = Z_{0.005} = 2.576$

standard error  $= \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.15(1-0.15)}{120}}$

$= 0.0326$

ME  $= Z_{\alpha/2} \times SE$

$= 2.576 \times 0.0326$

$= 0.0839$



$$\text{Lower limit} = 0.15 - 0.0839 \\ = 0.0661$$

$$\text{Upper limit} = 0.15 + 0.0839 \\ = 0.2339$$

$$7.7 \ n=10, \text{df} = n-1 = 9$$

$$S^2 = 4.5 \text{ mm}^2$$

$$\alpha = 0.05$$

$$\chi^2_{1-\alpha/2, \text{df}} = \chi^2_{0.975, 9} = 2.70$$

$$\chi^2_{\alpha/2, \text{df}} = \chi^2_{0.025, 9} = 19.02$$

$$\text{Lower bound} = \frac{(n-1)S^2}{\chi^2_{\alpha/2, \text{df}}} = \frac{(10-1) \times 4.5}{19.02}$$

$$= 2.129 \text{ mm}^2$$

$$\text{Upper bound} = \frac{(n-1)S^2}{\chi^2_{1-\alpha/2, \text{df}}} = \frac{(10-1) \times 4.5}{2.70}$$

$$= 15 \text{ mm}^2$$

$$8.7 \ n=16, \text{df} = n-1 = 15$$

$$\text{std deviation} : s = 1.2 \text{ N}$$

$$s^2 = 1.44 \text{ N}^2$$

$$\alpha = 0.01$$

$$\chi^2_{1-\alpha/2, \text{df}} = \chi^2_{0.995, 15} = 4.60$$

$$\chi^2_{\alpha/2, \text{df}} = \chi^2_{0.005, 15} = 32.80$$

$$\text{Lower bound} = \frac{(n-1)S^2}{\chi^2_{\alpha/2, \text{df}}} = \frac{(16-1) \times 1.44}{32.80}$$

$$= \frac{21.6}{32.8} = 0.659 \text{ N}^2$$

$$\text{Upper bound} = \frac{(n-1)S^2}{\chi^2_{1-\alpha/2, \text{df}}}$$

$$= \frac{(16-1) \times 1.44}{4.60} = \frac{21.6}{4.6} = 4.696 \text{ N}^2$$

$$9.7 \ n_A = 12, \text{df}_A = 11, S_A^2 = 18.2$$

$$n_B = 10, \text{df}_B = 9, S_B^2 = 10.5$$

$$\text{confidence level} = 90\%, \alpha = 0.10$$

$$F_{\alpha/2, \text{df}_A, \text{df}_B} = F_{0.05, 11, 9} = 3.10$$

$$F_{1-\alpha/2, \text{df}_A, \text{df}_B} = F_{0.95, 11, 9} = 0.345$$

$$F_{\alpha/2, \text{df}_B, \text{df}_A} = F_{0.05, 9, 11} = 2.90$$

$$\frac{S_A^2}{S_B^2} = \frac{18.2}{10.5} = 1.733$$

$$\text{Lower bound} = 1.733$$

$$\frac{1}{3.10} = 0.3226$$

$$\text{Upper bound} = 1.733 \times 2.90 \\ = 5.026$$

$$10.7 \ n_1 = 14, S_1^2 = 0.022 \text{ mm}^2$$

$$\text{df}_1 = n_1 - 1 = 13$$

$$n_2 = 16, S_2^2 = 0.015 \text{ mm}^2$$

$$\text{df}_2 = n_2 - 1 = 15$$

$$\alpha = 1 - 0.98 = 0.02$$

$$\alpha/2 = 0.01$$

$$F_{\alpha/2, \text{df}_1, \text{df}_2} = F_{0.01, 13, 15} = 3.84$$

$$F_{1-\alpha/2, \text{df}_1, \text{df}_2} = F_{0.99, 13, 15} = \frac{1}{F_{0.01, 15, 13}} \\ = \frac{1}{4.25} = 0.235$$

$$\frac{\sigma_1^2}{\sigma_2^2} = \left( \frac{S_1^2}{S_2^2} \frac{1}{F_{\alpha/2, \text{df}_1, \text{df}_2}}, \frac{S_1^2}{S_2^2} \frac{1}{F_{1-\alpha/2, \text{df}_1, \text{df}_2}} \right)$$

$$= \left( \frac{0.022}{0.015} \frac{1}{3.84}, \frac{0.022}{0.015} \frac{1}{0.235} \right) \\ = (0.3819, 6.2415)$$

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## 2. TYPE-I & TYPE-II ERROR

1)  $a) p = 0.4, n = 12$

$$x \leq 2$$

$$P(X=k) = {}^nC_k p^k (1-p)^{n-k}$$

$$P(X=0) = {}^{12}C_0 (0.4)^0 (0.6)^{12} = 0.00218$$

$$P(X=1) = {}^{12}C_1 (0.4)^1 (0.6)^{11} = 0.01743$$

$$P(X=2) = {}^{12}C_2 (0.4)^2 (0.6)^{10} = 0.05634$$

$$\alpha = 0.00218 + 0.01743 + 0.05634 = 0.07595$$

b)  $p = 0.2$

$$x \geq 3$$

$$\beta = P(X \geq 3 | p = 0.2)$$

$$= 1 - P(X \leq 2 | p = 0.2)$$

$$P(X \leq 2 | p = 0.2)$$

$$P(X=0) = {}^{12}C_0 (0.2)^0 (0.8)^{12} = 0.06872$$

$$P(X=1) = {}^{12}C_1 (0.2)^1 (0.8)^{11} = 0.20615$$

$$P(X=2) = {}^{12}C_2 (0.2)^2 (0.8)^{10} = 0.28347$$

$$P(X \leq 2) = 0.06872 + 0.20615 + 0.28347 = 0.55834$$

$$\beta = 1 - 0.55834 = 0.44166$$

2) a)  $\alpha = P(\bar{x} < 96.9 | \mu = 100)$   
 $n = 25$

$$\mu_0 = 100, \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{25}} = 2$$

~~$\bar{x} = 96.9$~~  [AHYUSHI PRIYA]  
 $z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{96.9 - 100}{2} = -1.55$

$$\alpha = P(Z < -1.55) = 0.0606$$

b)  $\mu = 95$ , acceptance region  $\bar{x} \geq 96.9$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{96.9 - 95}{10/\sqrt{25}} = 0.95$$

$$\beta = P(Z \geq 0.95) = 1 - P(Z < 0.95) = 1 - 0.8289 = 0.1711$$

3) a)  $p = 0.6, n = 50$

$$x \leq 24.5$$

$$\mu_x = np = 50 \times 0.6 = 30$$

$$\sigma_x = \sqrt{np(1-p)} = \sqrt{50 \times 0.6 \times 0.4} = 3.464$$

$$z = \frac{x - \mu_x}{\sigma_x} = \frac{24.5 - 30}{\sqrt{12}} = -1.5877$$

$$P(Z \leq -1.5877) \approx 0.056$$

b)  $p = 0.3, x \geq 24.5$

$$\mu_x = np = 50 \times 0.3 = 15$$

$$\sigma_x = \sqrt{np(1-p)} = \sqrt{50 \times 0.3 \times 0.7} = 3.24$$

$$z = \frac{x - \mu_x}{\sigma_x} = \frac{24.5 - 15}{\sqrt{12}} = -1.5877$$

4) a)  $|\bar{x} - 1000| > 1.96 \frac{5}{\sqrt{25}}$

$$\frac{|\bar{x} - \mu_0|}{\sigma/\sqrt{n}} > 1.96$$

$$\mu_0 = 1000, \sigma = 5, n = 25$$

$$|z| > 1.96$$

$$\alpha = P(|z| > 1.96)$$

$$P(|z| > 1.96) = 2 \cdot P(Z < -1.96)$$

$$= 2 \cdot P(Z > 1.96) = 2 \cdot 0.025 = 0.05 = 5\%$$

$$= 2 \times 0.025 = 0.05 = 5\%$$



b)  $\mu_1 = 998.5$  AAYUSHI  
PRIYA

$$|\bar{x} - 1000| \leq 1.96 \cdot \frac{5}{\sqrt{25}}$$

$$1.96 \cdot \frac{5}{\sqrt{25}} = 1.96$$

$$\bar{x} > 1000 + 1.96 \text{ or } \bar{x} < 1000 - 1.96$$

$$\bar{x} > 1001.96 \text{ or } \bar{x} < 998.04$$

$$998.04 \leq \bar{x} \leq 1001.96$$

$\mu = 998.5$

$$\beta = P(998.04 \leq \bar{x} \leq 1001.96 | \mu_1 = 998.5)$$

$$\frac{\sigma}{\sqrt{n}} = 1 : z_1 = \frac{998.04 - 998.5}{1} = -0.46$$

$$z_2 = \frac{1001.96 - 998.5}{1} = 3.46$$

$$\beta = P(z_1 \leq Z \leq z_2) = P(-0.46 \leq Z \leq 3.46)$$

$$\beta = P(Z \leq 3.46) - P(Z \leq -0.46)$$

$$= 0.9997 - 0.3228$$

$$= 0.6769 = 67.69\%$$

### 3. Z-TEST

1)  $H_0: \mu = 150, H_1: \mu \neq 150$

$\sigma = 5, n = 36, \bar{x} = 148.5, \alpha = 0.05$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{148.5 - 150}{5/\sqrt{36}} = -1.8$$

$\alpha = 0.05, Z_{\alpha/2} = \pm 1.96$

$Z = -1.8$  is b/w  $-1.96$  &  $1.96$

Fail to reject null hypothesis

$H_0: \mu = 150$  at 5% level of significance

b)  $P = 2 \times P(Z < |z|)$

$$= 2 \times P(Z < -1.8) = 2 \times 0.0359$$

$$= 0.0718$$

2.  $H_0: \mu = 25, H_1: \mu < 25$

$\sigma = 2, n = 49, \bar{x} = 24.4, \alpha = 0.01$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{24.4 - 25}{2/\sqrt{49}} = -2.1$$

$\alpha = 0.01, Z_{\alpha} = -2.33$

$$Z = -2.1$$

We fail to reject null hypothesis

$H_0: \mu = 25$  at 1% level of significance.

3.  $H_0: \mu = 5000, H_1: \mu \neq 5000$

$n = 100, \bar{x} = 4950, s = 120,$

$\alpha = 0.05, n \geq 30.$

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{4950 - 5000}{120/\sqrt{100}} = -4.167$$

$Z_{\alpha/2} = \pm 1.96$

$Z = -4.167$

$H_0: \mu = 5000$  at 5% level of significance

4.  $H_0: \mu = 2, H_1: \mu < 2$

$n = 64, \bar{x} = 1.98, s = 0.08, \alpha = 0.01$

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{1.98 - 2}{0.08/\sqrt{64}} = -2$$

$\alpha = 0.01, Z_{\alpha} = -2.33$

$Z = -2$

We fail to reject null hypothesis

$H_0: \mu = 2$  at 1% level of significance

5.  $H_0: p = 0.6, H_1: p \neq 0.6$

$\alpha = 0.05$

$n = 200, x = 108, \hat{p} = \frac{x}{n} = \frac{108}{200} = 0.54$

Standard error:  $SE = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.6(1-0.6)}{200}} = 0.03464$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.54 - 0.6}{\sqrt{\frac{0.6(1-0.6)}{200}}} = -1.732$$

$Z_{\alpha/2} = \pm 1.96$

$Z = -1.732$

$2 \times P(Z < -|z|)$

p-value =  $2 \times P(Z < -1.732)$

$$= 2 \times 0.0416 = 0.0832$$



$$b) H_0: p = 0.05, H_1: p > 0.05$$

$$n = 400, x = 28$$

$$\hat{p} = \frac{28}{400} = 0.07 \quad \alpha = 0.01$$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.07 - 0.05}{\sqrt{\frac{0.05(1-0.05)}{400}}} = 1.835$$

Z-value is approx 2.33

$$Z \approx 1.835$$

$$P(Z > 1.835) \approx 0.0332$$

#### 4. F-TEST

$$1) \mu_0 = 500 \text{ mg}, n = 9, \bar{x} = 485 \text{ mg}$$

$$s = 20 \text{ mg}, \alpha = 0.05$$

$$H_0: \mu = 500 \text{ \& } H_1: \mu \neq 500$$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{485 - 500}{20/\sqrt{9}} = -2.25$$

$$df = n - 1 = 9 - 1 = 8$$

$$\alpha = 0.05$$

$$t \text{ values are } \pm t_{\alpha/2, df} = \pm t_{0.025, 8} = \pm 2.306$$

$$t = -2.25$$

$$(-2.306 < -2.25 < 2.306)$$

we do not reject null hypothesis

$$2. H_0: \mu = 100; H_1: \mu < 100$$

$$\alpha = 0.01, n = 12, \bar{x} = 96.5$$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{96.5 - 100}{8/\sqrt{12}} = -1.516$$

$$df = n - 1 = 12 - 1 = 11$$

$$\alpha = 0.01, -t_{\alpha, df} = -t_{0.01, 11} = -2.718$$

$$t = -1.516$$

$$(-1.516 > -2.718)$$

we do not reject  $H_0$ . Avg battery life is less than 100 hrs at 1% level of significance

$$3. H_0: \mu = 70, H_1: \mu > 70$$

$$\alpha = 0.05, n = 10, \bar{x} = 74, s = 5$$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{74 - 70}{5/\sqrt{10}} = 2.53$$

$$df = n - 1 = 10 - 1 = 9$$

$$t_{\alpha, df} = t_{0.05, 9} = 1.833$$

$$t = 2.53 > 1.833$$

We reject  $H_0$ .

#### 5. $\chi^2$ TEST

$$1) H_0: \sigma^2 = 0.04, H_1: \sigma^2 \neq 0.04$$

$$\alpha = 0.05 \quad s^2 = 0.06, n = 10$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(10-1)0.06}{0.04} = 13.5$$

$$df = n - 1 = 10 - 1 = 9$$

$$\chi^2_{1-\alpha/2} = \chi^2_{0.975} \quad \chi^2_{\alpha/2} = \chi^2_{0.025}$$

$$\chi^2_{0.975} = 2.70 \quad \chi^2_{0.025} = 19.02$$

$$\chi^2 < 2.70 \text{ or } \chi^2 > 19.02$$

$$\chi^2 = 13.5 \quad (2.7 < 13.5 < 19.02)$$

do not reject  $H_0$ .

$$2. H_0: \sigma^2 \leq 0.0025, H_1: \sigma^2 > 0.0025$$

$$\alpha = 0.05, n = 20, s^2 = 0.004,$$

$$\sigma^2 = 0.0025$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(20-1)0.004}{0.0025} = 30.4$$

$$df = n - 1 = 20 - 1 = 19$$

$$\chi^2_{\alpha} = \chi^2_{0.05} \approx 30.14$$

$$\chi^2 > 30.14$$

$$30.4 > 30.14$$

Reject  $H_0$

$$3. H_0: \sigma^2 \leq 0.01, H_1: \sigma^2 > 0.01$$

$$\alpha = 0.05$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \quad n = 12, s^2 = 0.015, \sigma^2 = 0.01$$

$$\chi^2 = \frac{(12-1) \times 0.015}{0.01} = 16.5$$

(5)

$$df = n - 1 = 12 - 1 = 11$$

$$\chi^2_{\alpha} = \chi^2_{0.05} = 19.68$$

$$(16.5 < 19.68)$$

We do not reject null hypothesis

## 6. F-TEST

$$1) (\sigma^2_A = \sigma^2_B)$$

$$F = S^2_B / S^2_A \quad S^2_B \text{ is larger}$$

$$n_A = 10, S^2_A = 0.002 \text{ cm}^2, n_B = 11,$$

$$S^2_B = 0.005 \text{ cm}^2, \alpha = 0.10$$

$$df_B = n_B - 1 = 10$$

$$df_A = n_A - 1 = 9$$

$$F_{\alpha/2} \text{ is } F_{0.05, 10, 9} = 3.259$$

$$F = \frac{S^2_B}{S^2_A} = \frac{0.005}{0.002} = 2.5$$

$2.5 < 3.259$  we don't reject null hypothesis.

$$2) F = S^2_Y / S^2_X$$

$$n_X = 8, S^2_X = 1.5, n_Y = 10, S^2_Y = 2.8$$

$$\alpha = 0.01$$

$$df_Y = n_Y - 1 = 9 \quad df_X = n_X - 1 = 7$$

$$F_{0.01, 9, 7} \approx 6.71$$

$$F = \frac{S^2_Y}{S^2_X} = \frac{2.8}{1.5} \approx 1.867$$

$$1.867 < 6.71$$

Do not reject null hypothesis

$$3) n_A = 8, S^2_A = 20, n_B = 7, S^2_B = 12$$

$$\alpha = 0.05$$

$$df_A = n_A - 1 = 7, df_B = n_B - 1 = 6$$

$$F_{0.05, 7, 6} \approx 4.21$$

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$$F = \frac{S^2_A}{S^2_B} = \frac{20}{12} \approx 1.667$$

$$1.667 < 4.21$$

$\therefore$  At 5% significant level, not enough evidence to conclude method A has greater variance than method B.

$$4) n_A = 10$$

$$\bar{x}_A = \frac{10.2 + 10.4 + 10.1 + 10.5 + 10.3 + 10.2 + 10.4 + 10.3 + 10.1 + 10.2}{10}$$

$$= 10.27 \text{ mm}$$

$$S^2_A \approx 0.019 \text{ mm}^2$$

$$n_B = 12$$

$$\bar{x}_B = \frac{10.7 + 10.9 + 10.8 + 10.6 + 11.0 + 10.9 + 10.7 + 10.8 + 10.9 + 11.1 + 10.8 + 10.7}{12}$$

$$= 10.825 \text{ mm}$$

$$S^2_B = 0.017 \text{ mm}^2$$

$$\alpha = 0.10$$

$$df_A = n_A - 1 = 9 \quad df_B = n_B - 1 = 11$$

$$F_{0.05, 9, 11} \approx 2.90$$

$$F = \frac{S^2_A}{S^2_B} = \frac{0.019}{0.017} \approx 1.118$$

$$F_{0.95, 9, 11} = \frac{1}{F_{0.05, 11, 9}} \approx \frac{1}{3.10} = 0.32$$

$$0.32 < 1.5 < 2.90$$

At 10% significant level, we fail to reject null hypothesis

$$5) H_0: \sigma^2_A \leq \sigma^2_B$$

$$H_1: \sigma^2_A > \sigma^2_B \quad \alpha = 0.01$$

$$n_A = 15, n_B = 15$$

$$\bar{x}_A = 83.87 \quad \bar{x}_B = 77.2$$

$$S^2_A = 14.98 \quad S^2_B = 4.4$$

$$F = \frac{S^2_A}{S^2_B} = \frac{14.98}{4.4} = 3.405$$



$$df_A = n_A - 1 = 14; df_B = n_B - 1 = 14$$

$$F_{0.01, 14, 14} = 3.70$$

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$3.405 < 3.70 \therefore$  we fail to reject null hypothesis

7. GOODNESS-OF-FIT

$$1 \gamma p = 0.5, N = 240, n = 4$$

$$P(X=k) = {}^n C_k p^k (1-p)^{n-k}$$

$$E_k = N \times P(X=k)$$

$$k=0 \\ E_0 = 240 \cdot {}^4 C_0 (0.5)^0 (0.5)^4 = 15$$

$$k=1 \\ E_1 = 240 \cdot {}^4 C_1 (0.5)^1 (0.5)^3 = 60$$

$$k=2 \\ E_2 = 240 \cdot {}^4 C_2 (0.5)^2 (0.5)^2 = 90$$

$$k=3 \\ E_3 = 240 \cdot {}^4 C_3 (0.5)^3 (0.5)^1 = 60$$

$$k=4 \\ E_4 = 240 \cdot {}^4 C_4 (0.5)^4 (0.5)^0 = 15$$

no. of heads	observed ( $O_i$ )	Expected ( $E_i$ )	$\frac{(O_i - E_i)^2}{E_i}$
0	9	15	$\frac{(9-15)^2}{15} = 2.4$
1	32	60	$\frac{(32-60)^2}{60} = 13.07$
2	72	90	$\frac{(72-90)^2}{90} = 3.6$
3	85	60	$\frac{(85-60)^2}{60} = 10.42$
4	42	15	$\frac{(42-15)^2}{15} = 48.6$

$$\chi^2_{\text{calc}} = 2.4 + 13.07 + 3.6 + 10.42 + 48.6 = 78.09$$

$$df = k - p - 1 = 5 - 0 - 1 = 4$$

At 5% level of significance

$$\alpha = 0.05 \quad df = 4$$

$$\chi^2_{\text{crit}} = 9.488$$

$$\chi^2_{\text{calc}} = 78.09 > \chi^2_{\text{crit}} = 9.488$$

We reject null hypothesis

$$2 \gamma N = 80$$

$$\bar{x} = \frac{30.0 + 28.1 + 16.2 + 6.3}{80} = 0.975$$

$$P(X=k) = \frac{e^{-\lambda} \lambda^k}{k!}$$

$$E_k = N \times P(X=k)$$

$$k=0 \\ E_0 = 80 \times \frac{e^{-0.975} (0.975)^0}{0!} = 30.17$$

$$k=1 \\ E_1 = 80 \times \frac{e^{-0.975} (0.975)^1}{1!} = 29.38$$

$$k=2 \\ E_2 = 80 \times \frac{e^{-0.975} (0.975)^2}{2!} = 14.34$$

$$k=3 \\ E_3 = 80 \times \frac{e^{-0.975} (0.975)^3}{3!} = 4.66$$

$$\text{Observed } (O_i): 16 + 6 = 22$$

$$\text{Expected } (E_i): 14.34 + 4.66 = 19$$

No. of defectives	observed ( $O_i$ )	Expected ( $E_i$ )	$\frac{(O_i - E_i)^2}{E_i}$
0	30	30.17	0.0009
1	28	29.38	0.0657
$\geq 2$	22	19	0.4734

$$\chi^2_{\text{calc}} = 0.0009 + 0.0657 + 0.4734 = 0.5403$$

$$df = k - p - 1 = 3 - 1 - 1 = 1$$

$$\chi^2_{\text{crit}} = 3.841$$

$$3.841 > 0.5403$$

We do not reject null hypothesis

⑦

$$3) N=150, k=6$$

$$E_i = \frac{150}{6} = 25$$

Sector (x)	Observed ( $O_i$ )	Expected ( $E_i$ )	$(O_i - E_i)^2 / E_i$
1	20	25	$(20-25)^2/25 = 1$
2	21	25	$(21-25)^2/25 = 0.64$
3	27	25	$(27-25)^2/25 = 0.16$
4	29	25	$(29-25)^2/25 = 0.64$
5	26	25	$(26-25)^2/25 = 0.04$
6	27	25	$(27-25)^2/25 = 0.16$

$$\chi^2_{\text{calc}} = 1 + 0.64 + 0.16 + 0.64 + 0.04 + 0.16 = 2.64$$

$$df = 6 - 1 = 5$$

$$\alpha = 0.05 \quad \chi^2_{\text{crit}} = 11.07$$

$$2.64 < 11.07$$

we do not reject null hypothesis

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