



UNIT - V
Inferential Statistics

1 Confidence Interval

1. A machine fills bottles with soda. A quality control inspector takes a random sample of 100 bottles and finds that the sample mean volume is 498 ml with a population standard deviation of 10 ml. Construct a 95% confidence interval for the true mean volume filled by the machine.
2. A company wants to estimate the average weight of its packaged flour. A random sample of 150 bags shows a sample mean weight of 2.02 kg. The population standard deviation is known to be 0.1 kg. Construct a 99% confidence interval for the true mean weight of the flour bags.
3. A researcher wants to estimate the average sleep duration of university students. A random sample of 9 students yields the following sleep hours per night: 6.5, 7.0, 6.8, 5.9, 7.1, 6.3, 7.4, 6.0, 6.7. Assuming the sleep durations are normally distributed, construct a 95% confidence interval for the population mean sleep duration.
4. A health clinic is researching the average recovery time (in days) for patients after a minor surgery. A random sample of 7 patients gives the following recovery times: 12, 15, 14, 10, 13, 11, 16. Assuming recovery times are approximately normally distributed, construct a 90% confidence interval for the mean recovery time.
5. A survey of 500 voters in a city finds that 285 support a particular candidate. Construct a 95% confidence interval for the true proportion of voters in the city who support this candidate.
6. In a quality control check, a manufacturer finds that 18 out of 120 items produced in a day are defective. Find a 99% confidence interval for the true proportion of defective items produced by the manufacturer.
7. A random sample of 10 metal rods has a sample variance of 4.5 mm^2 . Assume the population of rod diameters is normally distributed. Construct a 95% confidence interval for the population variance.
8. The breaking strength of a certain type of wire is tested for a sample of 16 wires, yielding a sample standard deviation of 1.2 N. Assume the data are normally distributed. Find a 99% confidence interval for the population variance.
9. A researcher is comparing the variability in lifespans of two brands of batteries. A sample of 12 batteries from Brand A has a variance of 18.2 hours^2 , and a sample of 10 batteries from Brand B has a variance of 10.5 hours^2 . Assume both populations are normally distributed. Construct a 90% confidence interval for the ratio of the variances of Brand A to Brand B.



10. Two machines are used to manufacture bolts. A sample of 14 bolts from Machine 1 has a sample variance of 0.022 mm^2 , and a sample of 16 bolts from Machine 2 has a sample variance of 0.015 mm^2 . Assume normality in both populations. Find a 98% confidence interval for the ratio of the variances of Machine 1 to Machine 2.

2 Type-I and Type-II error

1. A quality control manager believes that the defect rate in a production line is $p = 0.4$. To test this belief, a random sample of 12 items is inspected. If 2 or fewer defective items are found, the null hypothesis that $p = 0.4$ is rejected in favor of the alternative $p < 0.4$. Use the binomial distribution to answer the following:
- Find the probability of committing a Type-I error if the true defect rate is $p = 0.4$.
 - Find the probability of committing a Type-II error if the true defect rate is $p = 0.2$.
2. The lifetime of a certain type of battery is normally distributed with standard deviation $\sigma = 10$ hours. A battery manufacturer claims that the mean lifetime is $\mu = 100$ hours. To test the null hypothesis:

$$H_0 : \mu = 100 \quad \text{vs} \quad H_1 : \mu < 100,$$

a sample of size $n = 25$ is taken, and H_0 is rejected if the sample mean $\bar{x} < 96.9$.

- Calculate the probability of Type-I error.
 - Calculate the probability of Type-II error if the true mean is $\mu = 95$.
3. A fabric manufacturer believes that the proportion of orders for raw material arriving late is $p = 0.6$. If a random sample of 50 orders shows that 24 or fewer arrived late, the hypothesis that $p = 0.6$ should be rejected in favor of the alternative $p < 0.6$. Assume the normal approximation to the binomial distribution is appropriate.
- Find the probability of committing Type-I error if the true proportion is $p = 0.6$.
 - Find the probability of committing Type-II error if the true proportion is $p = 0.3$.
4. A packaging machine is set to fill bags with 1000 grams of flour. The amount dispensed per bag follows a normal distribution with known standard deviation $\sigma = 5$ grams. To test:

$$H_0 : \mu = 1000 \quad \text{vs} \quad H_1 : \mu \neq 1000,$$

a sample of size $n = 25$ is taken, and the rejection region is defined as

$$|\bar{x} - 1000| > 1.96 \cdot \frac{5}{\sqrt{25}}.$$



- (a) What is the probability of Type-I error?
- (b) What is the probability of Type-II error if the true mean is $\mu = 998.5$?

3 z-test

1. A snack company claims that the average weight of a bag of chips is 150 grams. The weights are normally distributed with a known standard deviation of $\sigma = 5$ grams. A quality control officer randomly selects a sample of 36 bags and finds that the average weight is $\bar{x} = 148.5$ grams.
 - (a) Test the hypothesis $H_0 : \mu = 150$ against $H_1 : \mu \neq 150$ at the 5% level of significance.
 - (b) What is the p -value of the test?
2. An engine manufacturer claims that the average fuel consumption of its new model is 25 km/litre. Assume fuel consumption follows a normal distribution with a known standard deviation $\sigma = 2$ km/litre. A consumer organization tests 49 cars and finds the sample mean fuel consumption to be $\bar{x} = 24.4$ km/litre. Test the hypothesis $H_0 : \mu = 25$ against $H_1 : \mu < 25$ at the 1% level of significance.
3. A manufacturer claims that the average lifetime of a type of LED bulb is 5000 hours. A consumer group tests a random sample of 100 bulbs and finds a sample mean lifetime of $\bar{x} = 4950$ hours and sample standard deviation $s = 120$ hours. Test the hypothesis $H_0 : \mu = 5000$ against $H_1 : \mu \neq 5000$ at the 5% level of significance using the z -test.
4. A soft drink company claims that its bottles contain 2 liters of soda on average. A random sample of 64 bottles is taken and the sample mean is found to be $\bar{x} = 1.98$ liters with a sample standard deviation of $s = 0.08$ liters. Test the null hypothesis $H_0 : \mu = 2$ against the alternative $H_1 : \mu < 2$ at the 1% level of significance using the z -test.
5. A political analyst claims that 60% of voters support a new education policy. To test this claim, a random sample of 200 voters is surveyed, and 108 indicate their support. Test the hypothesis $H_0 : p = 0.60$ against the alternative $H_1 : p \neq 0.60$ at the 5% level of significance using a z -test. Also calculate the P -value.
6. A factory claims that no more than 5% of its products are defective. In a recent quality inspection of 400 items, 28 defective items were found. Test the null hypothesis $H_0 : p = 0.05$ against the alternative $H_1 : p > 0.05$ at the 1% level of significance using a z -test. Find the p -value and state your conclusion.

4 t-test

1. A dietitian claims that the average sodium content in a certain brand of soup is 500 mg. A random sample of 9 cans has a mean sodium content of 485 mg and a sample standard deviation of 20 mg. Test the hypothesis $H_0 : \mu = 500$ against $H_1 : \mu \neq 500$ at the 5% level of significance.



2. A battery manufacturer claims that its batteries last at least 100 hours on average. A consumer organization tests 12 batteries and finds the sample mean to be 96.5 hours with a sample standard deviation of 8 hours. Test the hypothesis $H_0 : \mu = 100$ against $H_1 : \mu < 100$ at the 1% level of significance.
3. An energy drink company claims that their product increases alertness scores. The average baseline alertness score is 70. A sample of 10 individuals who consumed the drink has a mean score of 74 with a sample standard deviation of 5. Test the hypothesis $H_0 : \mu = 70$ against $H_1 : \mu > 70$ at the 5% level of significance.

5 χ^2 -test

1. A machine is supposed to produce metal rods with a variance of 0.04 cm^2 . A sample of 10 rods shows a variance of 0.06 cm^2 . Test at the 5% significance level whether the variance differs from 0.04 cm^2 .
2. A manufacturer claims that the variance of the diameter of ball bearings is at most 0.0025 cm^2 . To verify this claim, a random sample of 20 ball bearings is taken, and the sample variance is found to be $s^2 = 0.004 \text{ cm}^2$. Using a significance level of 5%, test the null hypothesis $H_0 : \sigma^2 \leq 0.0025$ against $H_1 : \sigma^2 > 0.0025$. Conduct a right-tailed Chi-square test.
3. The variance of thickness in paper sheets is claimed to be no more than 0.01 mm^2 . A sample of 12 sheets shows a variance of 0.015 mm^2 . Test at 5% significance level if the variance exceeds 0.01 mm^2 .

6 F-test

1. Two different machines produce bolts. From 10 bolts from machine A, variance is 0.002 cm^2 , and from 11 bolts from machine B, variance is 0.005 cm^2 . Test at 10% significance level whether the two machines have equal variances.
2. A company claims that the variance of strength from supplier X is less than that from supplier Y. From 8 samples of X, variance is 1.5, and from 10 samples of Y, variance is 2.8. Test the claim at 1% significance level.
3. A researcher wants to check if the variance of two teaching methods differs. Sample variances from method A (8 samples) and method B (7 samples) are 20 and 12 respectively. Test at the 5% significance level if method A has greater variance than method B.
4. Machine A and Machine B produce bolts. The diameters (in mm) of bolts from each machine are measured as follows:
 - Machine A: 10.2, 10.4, 10.1, 10.3, 10.5, 10.2, 10.4, 10.3, 10.1, 10.2
 - Machine B: 10.7, 10.9, 10.8, 10.6, 11.0, 10.9, 10.7, 10.8, 10.9, 11.1, 10.8, 10.7



At the 10% significance level, test whether the variances of diameters from the two machines are equal.

5. Two teaching methods are evaluated by measuring the test scores of students:

- Method A: 78, 85, 80, 90, 88, 82, 84, 79, 81, 87, 83, 86, 80, 85, 88
- Method B: 75, 80, 79, 77, 76, 78, 75, 79, 74, 80, 77, 78, 76, 75, 79

At the 1% significance level, test if the variance of scores for Method A is greater than that of Method B.

7 Goodness-of-fit

1. A set of four identical coins were tossed 240 times and the results are recorded below:

Number of Heads	0	1	2	3	4
Frequency	9	32	72	85	42

Test the hypothesis that the data follows a binomial distribution with $p = 0.5$ (i.e., fair coins). Use a 5% level of significance.

2. Define X as the number of defective bulbs found in a box of 20 bulbs selected from a production line. Eighty boxes are inspected, and the number of defective bulbs per box is recorded as follows:

Number of Defectives (X)	0	1	2	3
Frequency	30	28	16	6

Assume that the number of defective bulbs in a box follows a Poisson distribution. Test the goodness of fit of the data with a binomial distribution at the 5% level of significance.

3. A spinner with six equal sectors is spun 150 times, and the following outcomes are recorded:

Sector (x)	1	2	3	4	5	6
Frequency (f)	20	21	27	29	26	27

Test at the 0.05 level of significance whether the spinner is fair using the chi-square goodness-of-fit test.