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Course Code: MA231TC

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III Semester B.E. Regular / Supplementary Examination February/March 2026

Common to CD / CS / CY / IS

Linear Algebra And Probability Theory

Time : 3 Hours **Maximum Marks : 100**

Instructions to the students

1. Answer all questions from Part A . Part A questions should be answered in first three pages of the answer book only.
2. Answer Five full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Use of scientific calculator & handbook for mathematics is permitted.

Part A

Question No	Question	M	CO	BT
1.1	The value of k such that the polynomials $1-8t+5t^2, k-3t+2t^2, 2-t+t^2$ are linearly dependent is ______.	02	1	1
1.2	Let $T: R^3 \to R^3$ be a linear transformation defined by $T(x,y,z) = (2x-y+z,\ x+y-z,\ 3x-z)$. Find $T(1,2,-1)$.	02	2	1
1.3	If $\begin{pmatrix}2\\3\end{pmatrix}$ and $\begin{pmatrix}3\\-2\end{pmatrix}$ are the eigenvectors of a symmetric matrix $A_{2\times2}$ corresponding to the eigenvalues 1 and 2 respectively, then $A =$ ______.	02	1	1
1.4	The singular values of the matrix $A = \begin{pmatrix}1 & 0\\0 & 2\\0 & 0\end{pmatrix}$ are ___.	02	2	1
1.5	If a continuous random variable X has the probability density function $f(x) = \frac{1}{4}$ for $0 < x < 4$, and 0 elsewhere with mean 2 then the variance $V(X)$ is __________.	02	1	1
1.6	If the joint probability mass function of discrete random variables X and Y is $P(X=x, Y=y) = \frac{(x+y)}{8}$ for $x=0,1$ and $y=1,2$ then $P(X+Y\le 2)$ is ________.	02	1	1
1.7	A radioactive source emits 5 particles on an average during a six-second period. If it follows Poisson distribution, the probability that it emits exactly one particle during a six-second period is _____ .	02	1	1
1.8	A population has mean 48.4 and standard deviation 6.3. The probability that the mean of a sample of size 64 will be less than 46.70 is ____.	02	2	2

1.9	A sample of 16 resistors has a sample standard deviation of 2.5 ohms. The manufacturer claims the standard deviation should be 2 ohms. Formulate the null and alternative hypotheses to test whether the population standard deviation has increased.	02	1	1
1.10	The average zinc concentration recovered from a sample of measurements taken from 36 different locations in a river is found to be 2.6 grams per milliliter. Find the 95% confidence interval for the mean zinc concentration in the river. Assume that the population standard deviation is 0.3 grams per milliliter.	02	2	2

Part B

Question No	**Question**	**M**	**CO**	**BT**
2a	Determine whether the vectors $v_1 = (1,2,1), v_2 = (2,3,1), v_3 = (3,5,2)$ are linearly dependent. If they are linearly dependent, find the linear dependence relation.	05	2	3
2b	Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 3 & 8 & 13 \end{bmatrix}$. Obtain the bases for the column space and null space of A.	06	3	3
2c	Determine the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $T(1,1) = (0,1)$ and $T(-1,1) = (3,2)$. Hence write matrix of the linear transformation T.	05	2	3
3a	Factorize the matrix $A = \begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ as $A = U\Sigma V^T$ using singular value decomposition process.	10	3	3
3b	Construct an orthonormal basis for the column space of the matrix $\begin{pmatrix} 1 & 5 & 2 \\ -1 & 1 & 3 \\ 1 & 1 & 4 \\ -1 & 1 & -1 \end{pmatrix}$, using the Gram–Schmidt orthogonalization process.	06	2	2
	OR			
4a	Find the QR factorization of the matrix A using the Gram–Schmidt process, where $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$.	06	2	3
4b	Obtain a diagonal matrix D and an orthogonal matrix P such that $A = PDP^T$, where $A = \begin{pmatrix} 4 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 5 \end{pmatrix}$.	10	2	3
5a	A Discrete random variable has the PMF as follows: $\begin{array}{ccccccccc} x & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ P(X=x) & 0 & k & 2k & 2k & 3k & k^2 & 2k^2 & 7k^2+k \end{array}$ Find (i) k, (ii) $P(X \geq 5)$, (iii) $P(X < 3)$, (iv) $P(2 < X \leq 5)$, (v) mean and (vi) variance.	08	2	2
5b	The joint probability density function of two random variables X and Y is given by $h(x,y) = \begin{cases} 2, & 0 < x < y < 1 \\ 0, & \text{elsewhere.} \end{cases}$. Find the marginal distributions of	08	3	3

X and Y and find the covariance between X and Y.

OR

6a A continuous random variable has the density function

$$f(x) = \begin{cases} kx^2 & -3 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

Find k and hence find $P(x < 2), P(x < 3)\, and\, P(x > 1)$. 08 2 2

6b The joint probability distribution of two random variables X and Y is given by the following table.

Y X	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

(i) Find the marginal distribution of X and Y (ii) Also determine μ_X and μ_Y and hence evaluate cov(X,Y). 08 3 3

7a If the mean and standard deviation of the number of correctly answered questions in a test given to 4096 students are 2.5 and $\sqrt{1.875}$. Find an estimate of the number of candidates answering correctly (i) 8 or more questions (ii) 2 or less and (iii) 5 questions. 08 3 3

7b The line width for semiconductor manufacturing is assumed to be normally distributed with a mean of 0.5 micrometer and a standard deviation of 0.05 micrometer. (i) What is the probability that a line width is greater than 0.62 micrometer? (ii) What is the probability that a line width is between 0.47 and 0.63 micrometer? (iii) The line width of 90% of samples is below what value? 08 4 4

OR

8a The time between the arrival of electronic messages at your computer is exponentially distributed with a mean of two hours. (i) What is the probability that you do not receive a message during a two-hour period? (ii) If you have not had a message in the last four hours, what is the probability that you do not receive a message in the next two hours? (iii) What is the expected time between your fifth and sixth messages? 08 3 3

8b A rowing team consists of four rowers whose weights are 69, 71, 73, 75kgs. Find all possible random samples of size 2 (a) with replacement and (b) without replacement. In each case, compute mean and standard deviation of the sampling distribution of means. Also find the mean and standard deviation of the population. Verify the formulas for $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$ in each case. 08 4 4

9a A factory claims that the average weight of a product it manufactures is 500 grams. The weights of the products are known to be normally distributed. A random sample of 36 products has a mean weight of 490 grams, and the population standard deviation is known to be 24 grams. Using z- test check whether the factory's claim about the average weight is valid at the 5% level of significance. 08 4 4

i. State the null and alternative hypotheses.

ii. Calculate the test statistic.

iii. Find the critical value and p-value.

iv. State your conclusion with proper reasoning.

9b Define X as the number of underfilled bottles from a filling operation in a carton of 24 bottles. Seventy-five cartons are inspected and the following observations on X are recorded: 08 4 4

Values:	0	1	2	3
Frequency:	39	23	12	1

Based on these 75 observations, test the goodness of fit with a binomial distribution for the above data at a 0.05 level of significance.

OR

10a A nutritionist claims that the average calorie content of a certain brand of snack bar is 200 calories. A random sample of 10 snack bars is analyzed, and the calorie content is found to be: 205, 198, 202, 195, 201, 203, 199, 197, 200, 204. Assuming the calorie content is normally distributed; test the nutritionist's claim at a significance level of 0.10 08 4 4

i. State the null and alternative hypotheses.
ii. Calculate the sample mean and sample standard deviation.
iii. Calculate the appropriate test statistic.
iv. Determine the degrees of freedom and the critical value(s).
v. Make a decision and interpret your findings.

10b Two different machines are used to manufacture steel rods in a factory. To test whether the variability in rod lengths differs between the two machines, the following data was collected from random samples:

Machine A: Rod lengths (in cm): 102, 98, 100, 99, 101, 97, 100, 103
Machine B: Rod lengths (in cm): 96, 95, 97, 98, 94, 93, 97, 99

Assuming the data is drawn from normal populations, test at the 10% level of significance whether there is a significant difference in the variances of rod lengths produced by the two machines 08 4 4