DEPARTMENT OF MATHEMATICS

LINEAR ALGEBRA AND PROBABILITY THEORY (MA231TC)

UNIT 2: LINEAR ALGEBRA – II

1. If $y = (3, 4)$ and $u = (1, 2)$, obtain the orthogonal projection of y onto u.

2. Without finding the characteristic equation, verify whether $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ is an eigenvector of the matrix $A = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}$. If yes, then find the corresponding eigenvalue.

3. Given $A = \begin{bmatrix} 2 & 1 & 5 \\ -2 & -3 & -2 \\ 3 & 3 & 1 \end{bmatrix}$. Decompose the matrix A as $A = QR$, using the Gram-Schmidt process.

4. Factorize the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix}$ as $A = PDP^{-1}$.

5. Using the Gram-Schmidt process, orthonormalize the columns of the matrix

$$A = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix}$$

6. Obtain the Singular Value Decomposition of $A = \begin{bmatrix} 5 & 7 & 0 \\ 5 & 1 & 0 \end{bmatrix}$.

7. Obtain the third row of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ - & - & - \end{bmatrix}$, such that the rows are orthogonal.

8. Choose the second row of $A = \begin{bmatrix} 0 & 1 \\ a & b \end{bmatrix}$ so that A has the eigenvalues 4 and 7.

9. Convert the basis vectors $(-3, 1, 0, 2, -1), (1, 2, -3, -1, 2), (3, 2, -1, -1, 3)$ to an orthonormal basis of a subspace of $\mathbb{R}^5$, using Gram-Schmidt orthogonalization.

10. Obtain the matrix P which diagonalizes the matrix $A = \begin{bmatrix} 7 & -4 & -2 \\ -4 & 1 & -4 \\ -2 & -4 & 7 \end{bmatrix}$. Also find the matrices P^{-1} and D.

11. Obtain the QR factorisation of the matrix A, by applying Gram-Schmidt process, where $A = \begin{bmatrix} 1 & -2 & 2 \\ 2 & -4 & 1 \\ -2 & 0 & 4 \\ 1 & 0 & 2 \\ 2 & -2 & -1 \end{bmatrix}$.

12. A matrix can be resolved as $U\Sigma V^T$, by singular value decomposition. Find the matrices U and Σ for the matrix $A = \begin{bmatrix} 4 & 2 \\ 4 & 2 \\ -2 & -1 \end{bmatrix}$.

13. If $y = (3, 4)$ and $u = (2, 2)$, obtain the orthogonal projection of y onto u

14. Without finding the characteristic equation, verify whether $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is an eigenvector of the matrix $A = \begin{bmatrix} 5 & 2 \\ 3 & 6 \end{bmatrix}$. If yes, then find the corresponding eigenvalue.

15. Given $A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & -3 & 1 \\ 0 & 2 & 3 \end{bmatrix}$, decompose the matrix A as $A = QR$, using the Gram-Schmidt process.

16. Factorize the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ as $A = PDP^{-1}$.

17. Using the Gram-Schmidt process, orthonormalize the columns of the matrix

$$A = \begin{bmatrix} 1 & -2 & 2 \\ 2 & -4 & 1 \\ -2 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

18. Obtain the Singular Value Decomposition of $A = \begin{bmatrix} 6 & -3 & 0 \\ -3 & 6 & 0 \end{bmatrix}$.

19. A matrix can be resolved as $U\Sigma V^T$, by singular value decomposition. Find the matrices U and Σ for the matrix $A = \begin{bmatrix} 6 & -2 \\ -3 & 1 \\ 6 & -2 \end{bmatrix}$

20. Obtain the QR factorisation of the matrix A, by applying Gram-Schmidt process, where $A = \begin{bmatrix} -6 & 1 & 0 \\ 1 & -3 & 2 \\ 4 & 2 & -2 \\ 0 & 1 & -5 \\ 5 & 2 & -1 \end{bmatrix}$.

21. Convert the basis vectors $(3, 2, -2, 1, 3), (6, 0, 4, -1, 4), (6, -4, 4, 2, -1)$ to an orthonormal basis of a subspace of $\mathbb{R}^5$, using Gram-Schmidt orthogonalization.

22. Obtain the matrix P which diagonalizes the matrix $A = \begin{bmatrix} 5 & -4 & -2 \\ -4 & 5 & 2 \\ -2 & 2 & 2 \end{bmatrix}$. Also find the matrices P^{-1} and D.