RV College of Engineering®

DEPARTMENT OF MATHEMATICS

CIE-II Scheme and Solution

Academic year 2025-2026 (Third Semester BE)

LINEAR ALGEBRA AND PROBABILITY THEORY (MA231TC)

(Common to CS, CD, CY, IS)

<table>
<tr><th>Q. No's</th><th>Question</th><th>M</th></tr>
<tr><td colspan="3">PART A</td></tr>
<tr><td>1</td><td>$k=-2$</td><td>1</td></tr>
<tr><td>2</td><td>6, 2</td><td>2</td></tr>
<tr><td>3</td><td>$\begin{bmatrix}3 & 1\\ 1 & 3\end{bmatrix}$.</td><td>2</td></tr>
<tr><td>4</td><td>$\| f \| = \sqrt{\int_0^2 f(x)f(x)dx} = \sqrt{\frac{32}{5}}$</td><td>2</td></tr>
<tr><td>5</td><td>$\frac{3+3k}{10} = 1 \Rightarrow k = \frac{7}{3}$.</td><td>1</td></tr>
<tr><td>6</td><td>$\int_0^{\frac{1}{2}} kx dx = \frac{1}{4} \Rightarrow k = 2$.</td><td>2</td></tr>
<tr><td colspan="3">PART B</td></tr>
<tr><td>1a</td><td>Given vectors are $x_1 = (1,1,1,1)$, $x_2 = (1,-1,2,2)$, $x_3 = (1,2,-3,-4)$
Steps of Gram-Schmidt method:
$v_1 = x_1 = (1,1,1,1)$
$v_2 = x_2 - \left(\frac{x_2 . v_1}{v_1 . v_1}\right) v_1$
$v_2 = (1,-1,2,2) - \frac{(1,-1,2,2).(1,1,1,1)}{(1,1,1,1).(1,1,1,1)}(1,1,1,1) = (0,-2,1,1)$
$v_3 = x_3 - \left(\frac{x_3 . v_1}{v_1 . v_1}\right) v_1 - \left(\frac{x_3 . v_2}{v_2 . v_2}\right) v_2$
$v_3 = (1,2,-3,-4) - \frac{(1,2,-3,-4).(1,1,1,1)}{(1,1,1,1).(1,1,1,1)}(1,1,1,1) - \frac{(1,2,-3,-4).(0,-2,1,1)}{(0,-2,1,1).(0,-2,1,1)}(0,-2,1,1)$
$= (2, -\frac{2}{3}, -\frac{1}{6}, -\frac{7}{6})$
$Proj(v,S) = \left(\frac{v.v_1}{v_1.v_1}\right) v_1 + \left(\frac{v.v_2}{v_2.v_2}\right) v_2 + \left(\frac{v.v_3}{v_3.v_3}\right) v_3 \Rightarrow \left(-\frac{1}{5}, \frac{12}{5}, \frac{3}{5}, \frac{6}{5}\right)$.</td><td>1
1
2
2</td></tr>
<tr><td>1b</td><td>Probability distribution
<table><tr><td>X</td><td>20</td><td>4</td><td>-12</td></tr><tr><td>$f(x_i)$</td><td>$\frac{2}{15}$</td><td>$\frac{8}{15}$</td><td>$\frac{1}{3}$</td></tr></table>Mean $\mu = \sum x\, f(x) = \frac{12}{15} = 0.8$.</td><td>3
1</td></tr>
<tr><td>2</td><td>$A = \begin{bmatrix}1 & 2 & 2\\ -1 & 1 & 2\\ -1 & 0 & 1\\ 1 & 1 & 2\end{bmatrix}$ $Let\ x_1 = (1,-1,-1,1), x_2 = (2,1,0,1), x_3 = (2,2,1,2)$
$v_1 = x_1 = (1,-1,-1,1) \Rightarrow \| v_1 \| = 2$,
$v_2 = x_2 - \frac{x_2 \cdot v_1}{v_1 \cdot v_1} v_1 = (2,1,0,1) - \frac{1}{2}(1,-1,-1,1) = \left(\frac{3}{2}, \frac{3}{2}, \frac{1}{2}, \frac{1}{2}\right) \Rightarrow \| v_2 \| = \sqrt{5}$
$v_3 = x_3 - \frac{x_3 \cdot v_1}{v_1 \cdot v_1} v_1 - \frac{x_3 \cdot v_2}{v_2 \cdot v_2} v_2 = (2,2,1,2) - \frac{1}{4}(1,-1,-1,1) - \frac{\frac{15}{2}}{5}\left(\frac{3}{2}, \frac{3}{2}, \frac{1}{2}, \frac{1}{2}\right) = \left(-\frac{1}{2}, 0, \frac{1}{2}, 1\right) \Rightarrow \| v_3 \| = \sqrt{\frac{3}{2}}$.
Normalizing the vectors $Q = [\, v_1 \ v_2 \ v_3 \,]$</td><td>2
2
3</td></tr>
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	$Q=\begin{bmatrix} \frac{1}{2} & \frac{3\sqrt{5}}{10} & -\frac{\sqrt{6}}{6} \\ -\frac{1}{2} & \frac{3\sqrt{5}}{10} & 0 \\ -\frac{1}{2} & \frac{\sqrt{5}}{10} & \frac{\sqrt{6}}{6} \\ \frac{1}{2} & \frac{\sqrt{5}}{10} & \frac{\sqrt{6}}{3} \end{bmatrix}$ $R=Q^TA=\begin{bmatrix} 2 & 1 & \frac{1}{2} \\ 0 & \sqrt{5} & \frac{3\sqrt{5}}{2} \\ 0 & 0 & \frac{\sqrt{6}}{2} \end{bmatrix}.$	1 2
3	$\begin{vmatrix} 2-\lambda & 2 & 0 \\ 2 & 5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix}=0 \Longrightarrow \lambda^3-10\lambda^2+27\lambda-18=0.$ Eigen values are 6, 3, 1 Eigen vector corresponding to $\lambda=6 \Rightarrow X_1=\begin{bmatrix}1\\2\\0\end{bmatrix}$ Eigen vector corresponding to $\lambda=3 \Rightarrow X_2=\begin{bmatrix}0\\0\\1\end{bmatrix}$ Eigen vector corresponding to $\lambda=1 \Rightarrow X_1=\begin{bmatrix}-2\\1\\0\end{bmatrix}$ $\therefore$ Orthogonal matrix $P=\begin{bmatrix} \frac{1}{\sqrt{5}} & 0 & \frac{-2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & 0 & \frac{1}{\sqrt{5}} \\ 0 & 1 & 0 \end{bmatrix}$ and $D=\begin{bmatrix}6&0&0\\0&3&0\\0&0&1\end{bmatrix}.$	2 2 2 2 1 1
4	$AA^T=\begin{bmatrix}3&2&2\\2&3&-2\end{bmatrix}\begin{bmatrix}3&2\\2&3\\2&-2\end{bmatrix}=\begin{bmatrix}17&8\\8&17\end{bmatrix}$ Eigen values of $AA^T=\begin{bmatrix}17&8\\8&17\end{bmatrix}$ are 25 and 9. $A^TA=\begin{bmatrix}3&2\\2&3\\2&-2\end{bmatrix}\begin{bmatrix}3&2&2\\2&3&-2\end{bmatrix}=\begin{bmatrix}13&12&2\\12&13&-2\\2&-2&8\end{bmatrix}$ Eigen values of $A^TA=\begin{bmatrix}13&12&2\\12&13&-2\\2&-2&8\end{bmatrix}$ are 25, 9 and 0. Eigen vectors of AA^T *are* $\begin{bmatrix}1\\1\end{bmatrix}$ *and* $\begin{bmatrix}-1\\1\end{bmatrix}$ Eigen vectors of A^TA are $\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}-1\\1\\-4\end{bmatrix}$ and $\begin{bmatrix}-2\\2\\1\end{bmatrix}$ Hence singular value decomposition is. $\begin{bmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}5&0&0\\0&3&0\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{18}} & \frac{1}{\sqrt{18}} & \frac{-4}{\sqrt{18}} \\ -\frac{2}{3} & \frac{2}{3} & \frac{1}{3}\end{bmatrix}.$	2 1 ~~3~~ 2 3 2 1
5a	$P(x)\geq 0$, for all x and $\sum p(x)=\frac{1}{2}\left\{\frac{2}{3}+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^3+\cdots\right\}=\frac{1}{2}\left\{\frac{\frac{2}{3}}{1-\frac{2}{3}}\right\}=1.$ $P(X=x)$ is a probability distribution function. $P(X=even)=\frac{1}{2}\left\{\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^4+\cdots\right\}=\frac{2}{5}.$	2 2
5b	The cumulative density function is given by $F(x)=\begin{cases} 0, & x\leq 0 \\ \int_0^x(1-x)dx, & 0<x<1 \\ \int_0^1(1-x)dx+\int_1^x x-1dx, & 1<x<2 \\ 1, & x>2. \end{cases} \Rightarrow F(x)=\begin{cases} 0, & x\leq 0 \\ x-\frac{x^2}{2}, & 0<x<1 \\ \frac{x^2}{2}-x+1, & 1<x<2 \\ 1, & x>2. \end{cases}$ $P(X>1.4)=1-F(X\leq 1.4)=1-0.58=0.42.$	1 2 1 2 2