AI1103-Assignment 6

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Latex codes :

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Let X and Y be two random variables satisfying $X \geq 0, Y \geq 0, E(X) = 3, Var(X) = 9, E(Y) = 2$ and $Var(Y) = 4$. Which of the following statements are correct?

A) $0 \leq Cov(X, Y) \leq 4$
B) $E(XY) \leq 3$
C) $Var(X + Y) \leq 25$
D) $E(X + Y)^2 \geq 25$

Solution

$$E(X^2) = Var(X) + (E(X))^2 = 18 \qquad (0.0.1)$$

Similarly,

$$E(Y^2) = Var(Y) + (E(Y))^2 = 8 \qquad (0.0.2)$$

We can use the Covariance inequality for this question,

$$(Cov(X, Y))^2 \leq Var(X)Var(Y) \qquad (0.0.3)$$

The proof of this inequality is as shown,

$$Var\left(\frac{X}{\sigma_X} \pm \frac{Y}{\sigma_Y}\right) = Var\left(\frac{X}{\sigma_X}\right) + Var\left(\frac{\pm Y}{\sigma_Y}\right)$$

$$+ 2Cov\left(\frac{X}{\sigma_X}, \frac{\pm Y}{\sigma_Y}\right) \qquad (0.0.4)$$

$$= \frac{1}{\sigma_X{}^2}Var(X) + \frac{1}{\sigma_Y{}^2}Var(Y)$$

$$+ 2Cov\left(\frac{X}{\sigma_X}, \frac{\pm Y}{\sigma_Y}\right) \qquad (0.0.5)$$

$$= 2 \pm 2\frac{Cov(X, Y)}{\sigma_X \sigma_Y} \qquad (0.0.6)$$

Since Variance is always positive,

$$Var\left(\frac{X}{\sigma_X} \pm \frac{Y}{\sigma_Y}\right) \geq 0 \qquad (0.0.7)$$

$$2 \pm 2\frac{Cov(X, Y)}{\sigma_X \sigma_Y} \geq 0 \qquad (0.0.8)$$

$$1 \pm 1\frac{Cov(X, Y)}{\sigma_X \sigma_Y} \geq 0 \qquad (0.0.9)$$

$$|(Cov(X, Y))| \leq (\sigma_X)(\sigma_Y) \qquad (0.0.10)$$

$$(Cov(X, Y))^2 \leq Var(X)Var(Y) \qquad (0.0.11)$$

1) Substituting values of variance we get,

$$-6 \leq Cov(X, Y) \leq 6 \qquad (0.0.12)$$

Therefore, option A is incorrect.

2) From equation 0.0.12,

$$Cov(X, Y) = E(XY) - E(X)E(Y) \qquad (0.0.13)$$

$$-6 \leq E(XY) - E(X)E(Y) \leq 6 \qquad (0.0.14)$$

$$0 \leq E(XY) \leq 12 \qquad (0.0.15)$$

Therefore, Option B is correct.

3) Now,

$$E(X + Y)^2 = E(X^2) + E(Y^2) + 2E(XY)$$
$$\qquad (0.0.16)$$

$$E(X + Y)^2 = 26 + 2E(XY) \qquad (0.0.17)$$

From equation 0.0.16,

$$26 \leq E(X + Y)^2 \leq 50 \qquad (0.0.18)$$

Therefore, Option D is correct.

4) Now,

$$Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y)$$
$$\qquad (0.0.19)$$

$$= 13 + 2Cov(X, Y) \qquad (0.0.20)$$

From equation 0.0.12,

$$1 \leq Var(X + Y) \leq 25 \qquad (0.0.21)$$

Therefore, Option C is correct.