AI1103-Assignment 3

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Python codes :

https://github.com/Aayush−2492/Assignments/tree/
    main/Assignment3/code

Latex codes :

https://github.com/Aayush−2492/Assignments/tree/
    main/Assignment3

QUESTION 29

Raju has four fair coins and one fair dice. At first Raju tosses a coin. If the coin shows head then he rolls the dice and the number that dice shows is taken as his score. If the coin shows tail then he tosses three more coins and the total number of tails shown (including the first one) is taken as his score. If Raju tells that his score is 2 then the probability that he rolled the dice is (up to two decimal places):

SOLUTION

Let X_i denote the random variable function for the ith coin $i \in \{1, 2, 3, 4\}$.
$X_i \in (0, 1)$ where 0 represents head and 1 represents tail $i \in \{1, 2, 3, 4\}$.

	Head	Tail
X_i	0	1

Let Y denote the random variable function for the dice.
$Y \in (1, 2, 3, 4, 5, 6)$ where 1 represents dice showing 1 and so on.

$$P(X_i = q) = \frac{1}{2} \qquad (0.0.1)$$

$q \in \{0, 1\}$ and $i \in \{1, 2, 3, 4\}$.

$$P(Y = q) = \frac{1}{6} \qquad (0.0.2)$$

$q \in \{1, 2, 3, 4, 5, 6\}$.

Let B denote the event that out of last three coins, only one shows tail.

$$P(B) = \frac{3}{8} \qquad (0.0.3)$$

Since tossing a coin and rolling a dice are independent events,

$$P((X_i = p) \cdot (Y = q)) = P(X_i = p) \cdot P(Y = q) \quad (0.0.4)$$

$p \in \{0, 1\}$ and $q \in \{1, 2, 3, 4, 5, 6\}$.

Let A denote the event that the score is 2.

Clearly,

$$
\begin{aligned}
P(A) &= P(Y = 2|X_1 = 0) \cdot P(X_1 = 0) \\
&\quad + P(B|X_1 = 1) \cdot P(X_1 = 1) \qquad (0.0.5) \\
&= P((X_1 = 0) \cdot (Y = 2)) + P((X_1 = 1) \cdot B) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad (0.0.6) \\
&= P(X_1 = 0) \cdot P(Y = 2) + P(X_1 = 1) \cdot P(B) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad (0.0.7) \\
&= \frac{1}{2} \cdot \frac{1}{6} + \frac{1}{2} \cdot \frac{3}{8} \qquad (0.0.8) \\
&= \frac{13}{48} \qquad (0.0.9)
\end{aligned}
$$

We have to find $P(X_1 = 0|A)$

$$
\begin{aligned}
P(X_1 = 0|A) &= \frac{P(A \cdot (X_1 = 0))}{P(A)} \qquad (0.0.10) \\
&= \frac{P(X_1 = 0) \cdot P(Y = 2)}{P(A)} \qquad (0.0.11) \\
&= \frac{\frac{1}{2} \cdot \frac{1}{6}}{\frac{13}{48}} \qquad (0.0.12) \\
&= \frac{4}{13} \qquad (0.0.13) \\
&= 0.31 \qquad (0.0.14)
\end{aligned}
$$

Therefore, required probability = 0.31