

AI1103-Assignment 4

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Python codes :

```
https://github.com/Aayush-2492/Assignments/tree/
main/Assignment4/code
```

Latex codes :

```
https://github.com/Aayush-2492/Assignments/tree/
main/Assignment4
```

$$P(X \leq b | X \geq a) = \frac{P((X \leq b) \cdot (X \geq a))}{P(X \geq a)} \quad (0.0.9)$$

$$= \frac{P(a \leq X \leq b)}{P(X \geq a)} \quad (0.0.10)$$

$$= \frac{e^{-a} - e^{-b}}{e^{-a}} \quad (0.0.11)$$

$$= 1 - e^{-(b-a)} \quad (0.0.12)$$

Therefore the required probability depends on $b - a$
Option (A) is correct.

QUESTION 29

Let X be a random variable with probability density function:

$$f(t) = \begin{cases} e^{-t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Let $b > a > 0$. Then the probability $P(X \leq b | X \geq a)$

- A) $b-a$
- B) a
- C) b
- D) $a+b$

SOLUTION

Let $F_X(t)$ denote the Cumulative Distribution Function for random variable X .

$$F_X(t) = \int_{-\infty}^t f(t) dt \quad (0.0.1)$$

$$= \int_{-\infty}^0 0 dt + \int_0^t e^{-t} dt \quad (0.0.2)$$

$$= -e^{-t} \Big|_0^t \quad (0.0.3)$$

$$= 1 - e^{-t} \quad (0.0.4)$$

$$P(a \leq X \leq b) = F_X(b) - F_X(a) \quad (0.0.5)$$

$$= e^{-a} - e^{-b} \quad (0.0.6)$$

$$P(X \geq a) = \lim_{k \rightarrow \infty} F_X(k) - F_X(a) \quad (0.0.7)$$

$$= e^{-a} \quad (0.0.8)$$

Plot for probability density function.

