AI1103-Assignment 4

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Python codes :

https://github.com/Aayush−2492/Assignments/tree/main/Assignment4/code

Latex codes :

https://github.com/Aayush−2492/Assignments/tree/main/Assignment4

Question 29

Let X be a random variable with probability density function:

$$f(t) = \begin{cases} e^{-t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Let $b > a > 0$. Then the probability $P(X \leq b | X \geq a)$

A) b-a
B) a
C) b
D) a+b

Solution

Let $F_X(t)$ denote the Cumulative Distribution Function for random variable X.

$$F_X(t) = \int_{-\infty}^{t} f(t)\,dt \tag{0.0.1}$$
$$= \int_{-\infty}^{0} 0\,dt + \int_{0}^{t} e^{-t}\,dt \tag{0.0.2}$$
$$= -e^{-t}\Big|_0^t \tag{0.0.3}$$
$$= 1 - e^{-t} \tag{0.0.4}$$

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$$P(X \leq b | X \geq a) = \frac{P((X \leq b), (X \geq a))}{P(X \geq a)} \tag{0.0.5}$$
$$= \frac{P(a \leq X \leq b)}{P(X \geq a)} \tag{0.0.6}$$
$$= \frac{F_X(b) - F_X(a)}{\lim_{k\to\infty} F_X(k) - F_X(a)} \tag{0.0.7}$$
$$= \frac{e^{-a} - e^{-b}}{e^{-a}} \tag{0.0.8}$$
$$= 1 - e^{-(b-a)} \tag{0.0.9}$$

Therefore the required probability depends on $b - a$ Option (A) is correct.

Plot for probability density function.