AI1103-Assignment 7

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Python codes :

https://github.com/Aayush−2492/Assignments/tree/main/Assignment7/code

Latex codes :

https://github.com/Aayush−2492/Assignments/tree/main/Assignment7

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Consider a parallel system with two components. The lifetimes of the two components are independent and identically distributed random variables each following an exponential distribution with mean 1. The expected lifetime of the system is:

A) 1

B) $\frac{1}{2}$

C) $\frac{3}{2}$

D) 2

Solution

Consider two random variables X and Y which represent the lifetime of the two components namely A and B.

$$X \sim Exp(\lambda_X) \tag{0.0.1}$$

$$Y \sim Exp(\lambda_Y) \tag{0.0.2}$$

Let $f_X(x)$ denote the probability distribution function for random variable X.

$$f_X(x) = \begin{cases} \lambda_X \cdot e^{-\lambda_X \cdot x} & x \geq 0 \\ 0 & otherwise \end{cases} \tag{0.0.3}$$

Let $f_Y(y)$ denote the probability distribution function for random variable Y.

$$f_Y(y) = \begin{cases} \lambda_Y \cdot e^{-\lambda_Y \cdot y} & y \geq 0 \\ 0 & otherwise \end{cases} \tag{0.0.4}$$

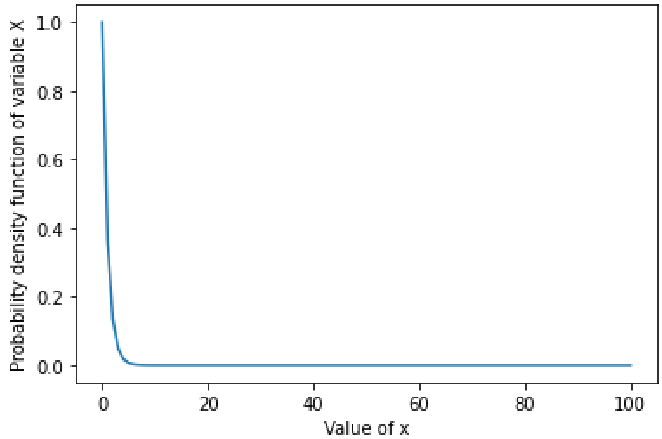

Fig. 4: P.D.F. of X

Let $F_X(x)$ denote the cumulative distribution function for random variable X.

$$F_X(x) = \begin{cases} 1 - e^{-\lambda_X \cdot x} & x \geq 0 \\ 0 & otherwise \end{cases} \tag{0.0.5}$$

Let $F_Y(y)$ denote the cumulative distribution function for random variable Y.

$$F_Y(y) = \begin{cases} 1 - e^{-\lambda_Y \cdot y} & y \geq 0 \\ 0 & otherwise \end{cases} \tag{0.0.6}$$

$$E(X) = \frac{1}{\lambda_X} \tag{0.0.7}$$

$$E(Y) = \frac{1}{\lambda_Y} \tag{0.0.8}$$

From 0.0.7 and 0.0.8,

$$\lambda_X = \lambda_Y = 1 \tag{0.0.9}$$

Let Z be a random variable such that $Z = max(X, Y)$

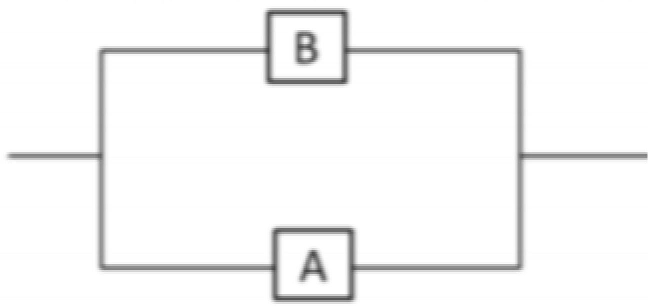

Fig. 4: Parallel system

$$P(Z \leq z) = P(max(X, Y) \leq z) \tag{0.0.10}$$
$$= P(X \leq z, Y \leq z) \tag{0.0.11}$$
$$= P(X \leq z) \cdot P(Y \leq z) \tag{0.0.12}$$
$$= (F_X(z) - F_X(0)) \cdot (F_Y(z) - F_Y(0)) \tag{0.0.13}$$
$$= 1 + e^{-(\lambda_X)\cdot z} + e^{-(\lambda_X)\cdot z} - e^{-(\lambda_X+\lambda_Y)\cdot z} \tag{0.0.14}$$

$P(Z \geq z)$ denotes the probability that the lifetime of the system is z.

$$P(Z \geq z) = e^{-(\lambda_X+\lambda_Y)\cdot z} - e^{-(\lambda_X)\cdot z} - e^{-(\lambda_X)\cdot z} \tag{0.0.15}$$

Let V be a random variable such that V represents the lifetime of the system.

$$P(V = v) = e^{-(\lambda_X+\lambda_Y)\cdot v} - e^{-(\lambda_X)\cdot v} - e^{-(\lambda_X)\cdot v} \tag{0.0.16}$$

$$E(V) = \int_0^\infty P(V = v)\, dv \tag{0.0.17}$$
$$= \int_0^\infty (e^{-(\lambda_X+\lambda_Y)\cdot v} - e^{-(\lambda_X)\cdot v} - e^{-(\lambda_X)\cdot v})\, dv \tag{0.0.18}$$
$$= \frac{1}{\lambda_X} + \frac{1}{\lambda_Y} - \frac{1}{\lambda_X + \lambda_Y} \tag{0.0.19}$$

From 0.0.9,

$$E(V) = \frac{3}{2} \tag{0.0.20}$$

Therefore, option (C) is correct.