Simulation and Modeling

UNIT I : Introduction to Simulation.

The process of designing a mathematical or logical model of a real-system and then conducting computer based experiments with the model to describe, explain and predict the behaviour of the real system is called simulation.

Components of a System.

- Discrete
- Continuous

1. Entity
2. Attribute
3. Activity
4. State
5. Event
   - (a) endogeneous.
   - (b) Exogeneous.

General Simulation Language.

1. GPSS
2. SIMSCRIPT II.5
3. MODSIM III Modern Object Oriented Language
4. SIMULA
5. SLAM Block-Structured Language.
6. CSIM.

Special Simulation packages.

1. Network II.5
   - Simulation for computer system.

(2) The Network simulator ns-2
- http://www.isi.edu/nsnam/ns/

(3) OPNET
- simulator for communication networks including wireless network.

(4) COMNET III
- simulator for communication network.

(5) SIMFACTORY
→ simulators for manufacturing operations

system and system Environment.

Components of a system.

(1) Entity :→ an object of interest in the system.

(2) Attribute :→ a property of an entity.

(3) Activity :→ a time period of specified length.

(4) State :→ the collection of variables necessary to describe the system at any time, relative to the objectives of the study.

(5) Event :→ an momentary occurence that may change the state of the system.

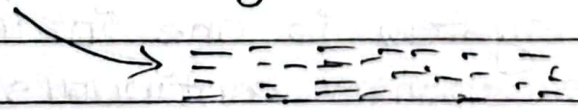
Examples of system and components.

System	Entities	Attribute	Activities	Events
Banking →	Customers →	Checking account balance →	Making deposits →	Amount increases, Arrival, Department

State variables
→ Number of busy tellers,
number of customer waiting.

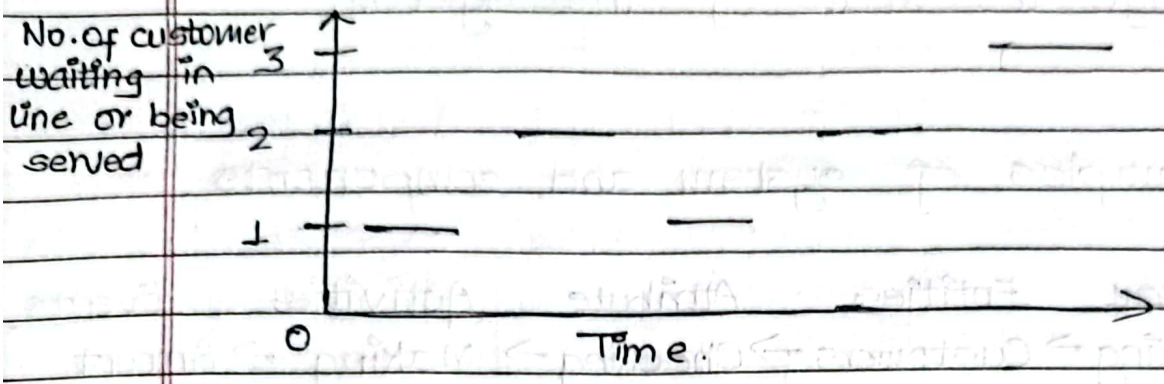
Types of system → 2 types
(1) Discrete system
(2) Continuous system.

System can be categorised as discrete or continuous.
- Bank : a discrete system.
- The head of water behind a dam : a continuous system.



Discrete System.

Discrete system.



A discrete system is one in which the state variables change only at a discrete set of points in time. Ex – Bank – the number of customer changes only when a customer arrives or when the service provided a customer is completed.

Continuous system.

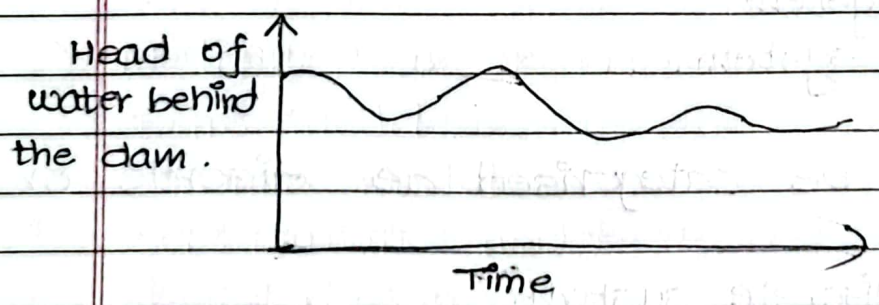


fig – Constinous system state variable.

A continuous system is one in which the state variable change continuously over time. Head of water behind the dam.

Model of a system.

Model.

→ A representation of a system for the purpose of studying the system.

→ A simplication of the system.

→ Sufficiently detailed to permit valid conclusions to be drawn about the real system.

Types of Models

① Static or Dynamic simulation Models.

② Deterministic or Stochastic Simulation Models.

① Static or Dynamic Simulation Models:

→ Static simulation model (called Monte Carlo simulation) represents a system at a particular point in time.

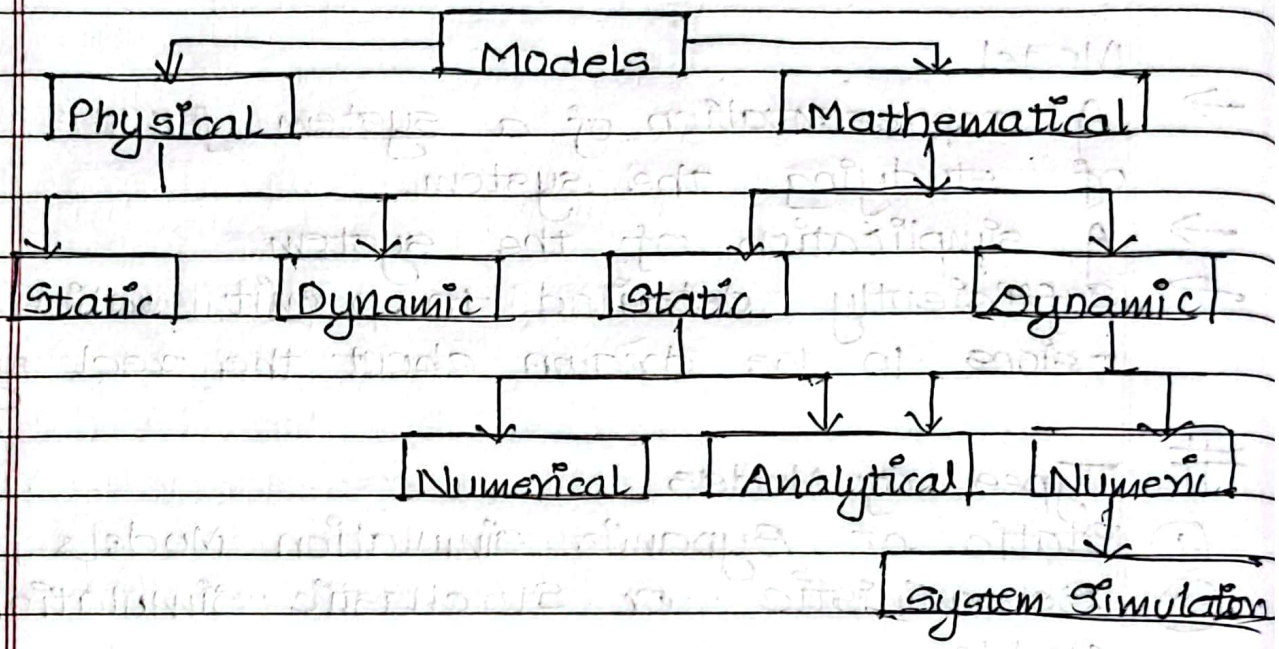
→ Dynamic simulation model represents system as they change over time.

② Deterministic or stochastic Simulation Models.

→ Deterministic simulation models contains no random variables & have a known set of inputs which will result in a unique set of outputs.

→ Stochastic simulation models has one or more random variables as input. Random inputs leads to random output.

Types of Models



→ A model is defined as a representation of a system for the purpose of studying the system.

→ The various types of models are :

* Mathematical or physical model.
* Static model.
* Dynamic model
* Deterministic model.
* Stochastic model
* Discrete model.

* Mathematical model : $E=mc^2$

User symbolic notation & the mathematical equations to represent a system.

* Static Model:
Represents a system at a particular point of time and also known as Monte-Carlo simulation.

* Dynamic Model:
Represents system as they change over time eg. Simulation of a bank.

* Deterministic Model
Contains no random variables. They have a known set of inputs which will result in a unique set of outputs. Ex: Arrival of patients to the Dentist at the scheduled appointment time.

* Stochastic Model:
Has one or more random variables as input. Random inputs leads to random output. eg simulation of a bank involves random interarrival and service time.

* Discrete - Event System Simulation:
Modeling of systems in which the state variables changes only at a discrete set of points in time.
The simulation models are analysed by numerical rather than by analytical methods.

Advantages of simulation.

① Simulation is best suited to analyze complex and large practical problems when it is not possible to solve them through a mathematical model.

② Simulation is flexible and changes in the system variables can be made to select the best solution among the various alternatives.

③ In simulation, the experiments are carried out with the model without disturbing the system.

④ Policy decisions can be made much faster by knowing the option well in advance & by reducing the risk of experimenting in the real system.

Limitations / Disadvantages of simulation.

① A good simulation model may be very expensive. Simulation often required significant amount of computer time and is therefore expensive.

② Simulation generates a way of evaluating solutions but does generate solutions ~~to~~ themselves.

③ Simulation is not precise. It does not yield an answer but merely provides a set of the system responses to different operation conditions.

④ It is a trial & errors methods that may produce different solutions in repeated runs.

⑤ The difficulty is finding the optimal values increases due to an increase in the number of parameters.

Application Areas → when to simulate

① In science and engineering research.
The experiments that are simulated on computer being much faster, cheaper, easier frequently provide better insight into the system than laboratory experiments do physically.

② Simulation in soft science:
Simulation can be expected to play even a more vital role in biology, sociology economical medicine, psychology etc where experiments could be very expensive, dangerous or even impossible.

③ Simulation for business execution:
There are many problems faced by management that can be solved by standard operations research tools like dynamic programming. Here, computer simulation can be used to make better decisions.

Simulation study steps:
1. Problem formulation
2. Simulation model building.
3. Experimental design & analysis
4. Evaluate and iterate
5. Implementation.

① Problem formulation
- ⓐ Define the problem
- ⓑ Define the system.
- ⓒ Establish performance metrics.
- ⓓ Build conceptual model.
- ⓔ Document model assumptions.

② Simulation model building:
- ⓐ Model translation
- ⓑ Input data modeling
- ⓒ Verification
- ⓓ Validation

Date
Page

③ Experimental design & analysis
ⓐ Preliminary runs
ⓑ Final experiments
ⓒ Analysis of results

④ Evaluate and iterate
ⓐ Documentation
ⓑ Model manual
ⓒ User manual

⑤ Implementation
Finally use it for purpose.

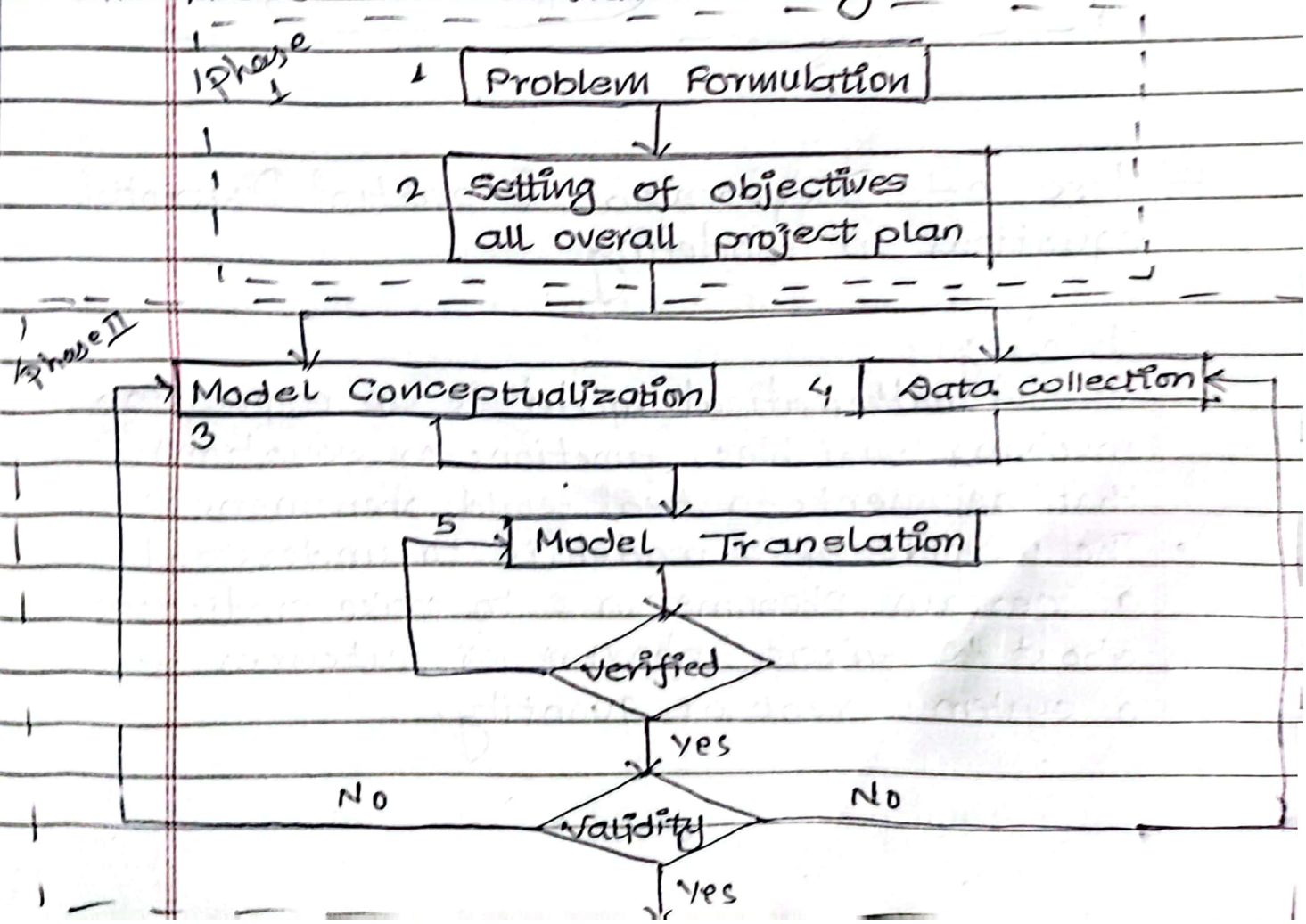
CamScanner

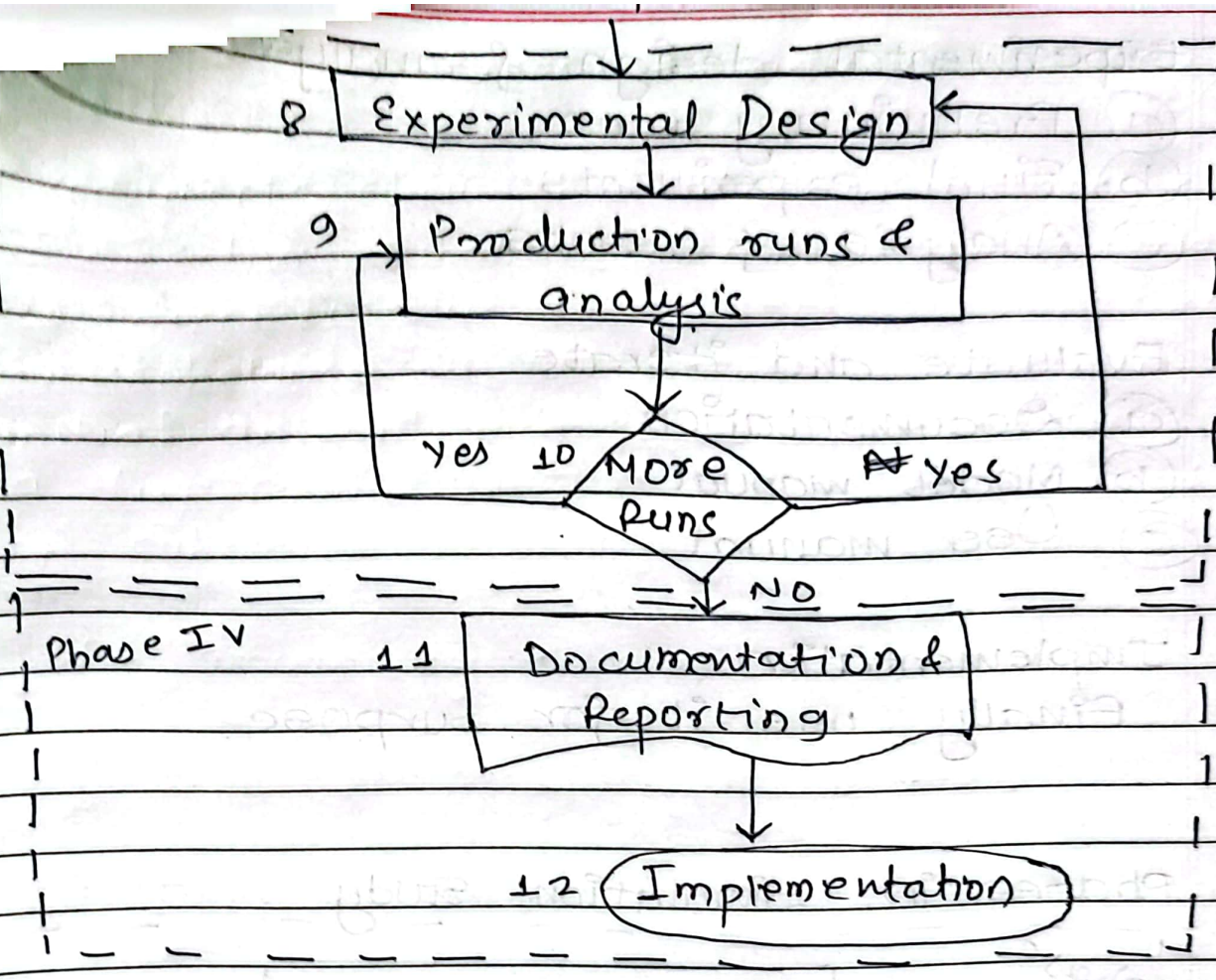
③ Experimental design & analysis
ⓐ Preliminary runs.
ⓑ Final experiments
ⓒ Analysis of results.

④ Evaluate and iterate
ⓐ Documentation
ⓑ Model manual
ⓒ User manual

⑤ Implementation
Finally use it for purpose.

Phases in Simulation Study





Uses of Differential & partial Differential equations in modeling

Modeling

A mathematical model is an expression involving variables, functions or equations that represents a real world phenomenon.

- The purpose of a model is to understand a certain phenomenon & to make predictions about the future behavior or outcomes of a system, event or quantity.

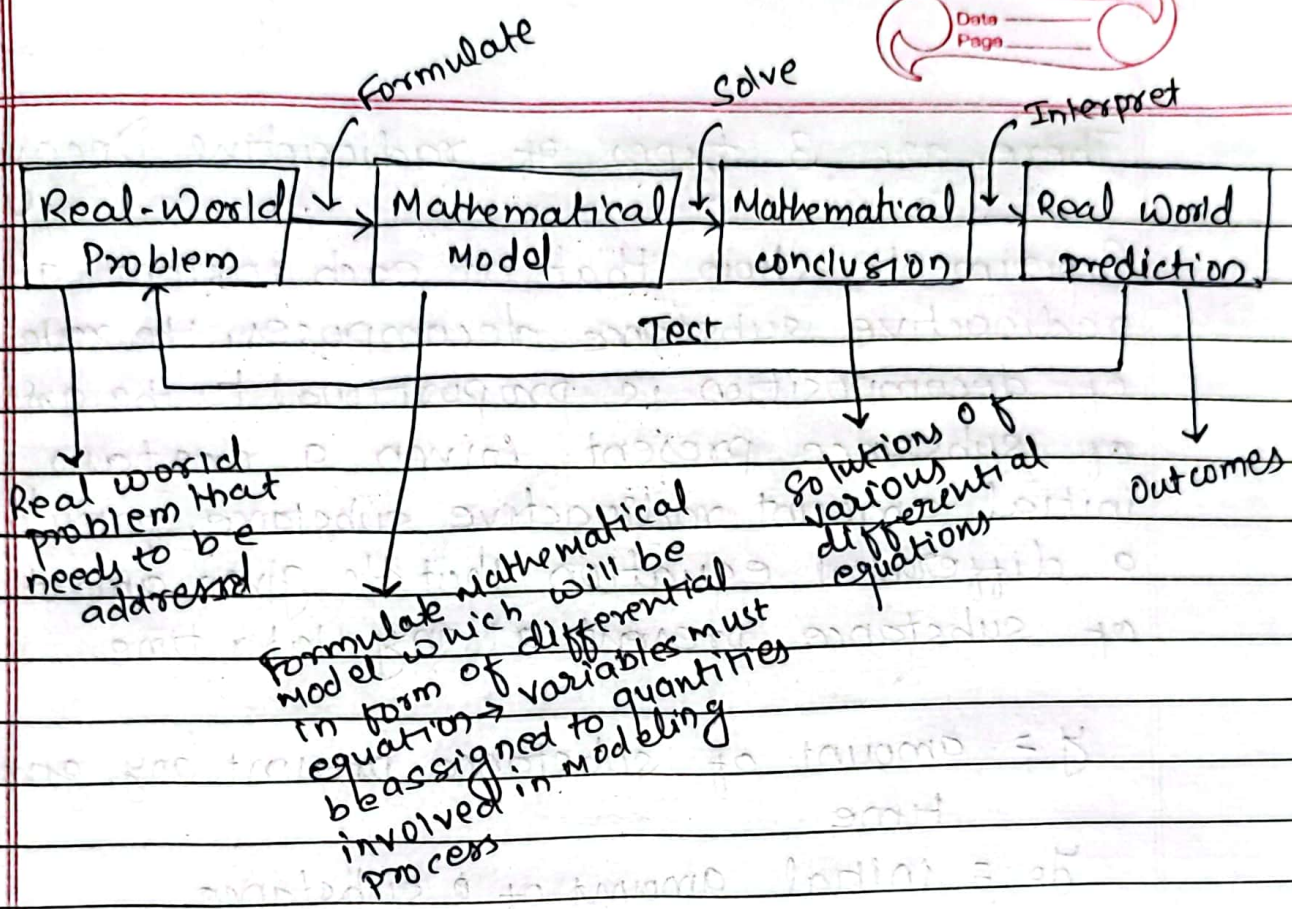
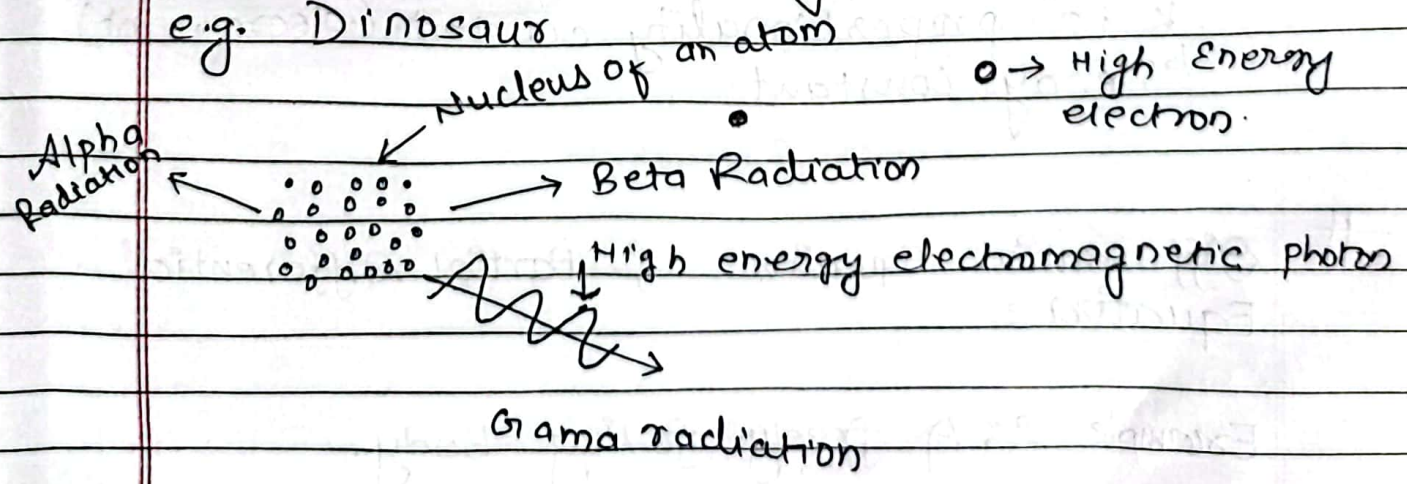


Fig: Illustration of the modelling process

Example 1: Radioactive Decay

→ Radioactive Carbon Dating Technique
e.g. Dinosaur



There are 3 types of radioactive Decay.

Experiments show that at each instant a radioactive substance decomposes, the rate of decomposition is proportional to the amount of substance present. Given a certain initial amount radioactive substance, formulate a differential equation that the given amount of substance present at any later time.

y = amount of substance present any given time

y_0 = initial amount of substance

$$\frac{dy}{dt} \propto y$$

working eqn.

$$\frac{dy}{dt} = -ky$$

k is propertionality constant (decrement)
↳ Decay constant.

Differential Equation & Partial Differential Equation:

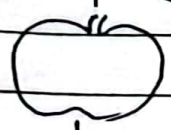
Example 2: A freely falling body.

A object is falling under the influence of gravity. As it falls downward, it is

subject to air resistance that creates an upward force proportional to the velocity of the object. Obtain the differential equation that describes the velocity and the position of the object at any given time. Assume that the mass & acceleration due to gravity are constant.

$kv \rightarrow$ velocity

$\rightarrow$ proportionality constant



mg

Apply relevant physical law Newton law of acceleration (2nd)

$$* \boxed{F_{net} = ma}$$

$\hookrightarrow$ Force.

$$\Rightarrow mg - kv = ma$$

where, $a = \frac{dv}{dt}$

i.e $mg - kv = m \cdot \frac{dv}{dt}$

or, $\frac{dv}{dt} + \frac{k}{m} v = g \rightarrow$ First order differential equation.

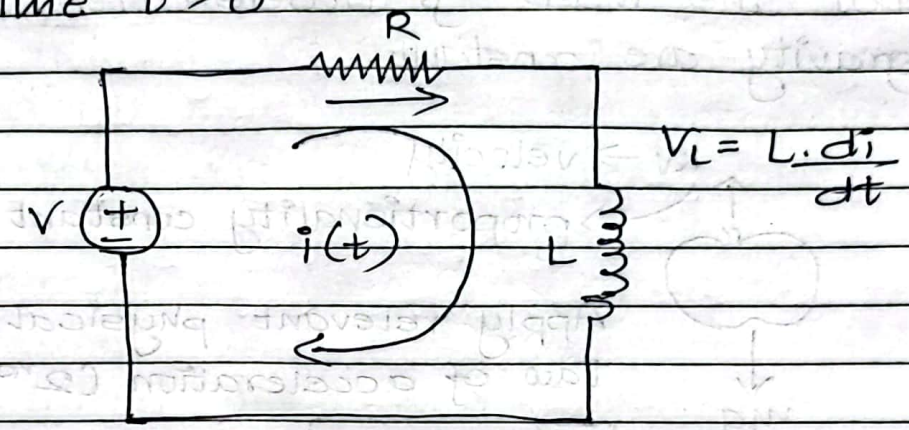
y = position of the object at any given time.

$v = \frac{dy}{dt}$

$$\boxed{\frac{d^2y}{dt^2} + \frac{k}{m} \cdot \frac{dy}{dt} = g} \rightarrow \text{second order differential equation.}$$

Example 3: RL Circuit

A series RL circuit is connected to a voltage source V. Develop a model of the circuit describing the current at any time $t > 0$



L = Inductance

$$V - iR - L.\frac{di}{dt} = 0$$

KKL → Kirchoff's voltage law.

or, $L.\frac{di}{dt} + iR = V$

Or, $\boxed{\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L}}$ → Describe current at any time t

↳ First order differential Equation.

Simulation software:

① Arena

www.arenasimulation.com

② AutoMod
www.automod.com.

③ Delmin / Quest
www.delmia.com
www.3ds.com.

④ Extend
www.flexsim.com

⑤ Flexsim
~~www.flexsim.com~~ www.imaginethatinc.com.

⑥ MicroSaint
www.maad.com

⑦ ProModel
www.promodel.com.

⑧ SIMULA 8
www.simula.com.

⑨ WITNESS
www.witness-for-simulation.com.

UNIT-2

Simulation of Continuous and Discrete system.

Continuous system Models.

→ A continuous system is one in which the predominant activities of the system cause smooth changes in the attributes of the system ~~pa~~ entities.

→ When such a system is modeled mathematically the variables of the model representing the attributes are controlled by continuous function.

→ In continuous system, the relationship describes the rate at which the variables representing the attributes changes, so that the model consist of differential equation.

→ The simplest differential equation models have one or more linear differential equation with constant coefficients.

→ Non-linear ties are introduced either if any of the derivatives of dependent variables are raised to power greater than one or all combined in any other way.

Differential and Partial Differential Eqn:

Differential Equation:

A differential equation is an equation, involving an unknown function & its

derivatives.

→ The order of the differential equation is the order of the highest derivative of the unknown function involved in the equation:

$$\frac{dy}{dt} = f(y,t)$$

CamScanner

derivatives.

→ The order of the differential equation is the order of the highest derivative of the unknown function involved in the equation:

$$\frac{dy}{dt} = f(y,t)$$

Linear Differential Equation:

A linear differential equation of order n is a differential equation written in the following form:

$$a_n(x)\frac{d^ny}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \dots + a_1(x)\frac{dy}{dx} + a_0(x)y = f(x)$$

Example 1: Balance Equation.

Most ordinary differential equation are balance equations. A balance equation basically says.

Change = Increase − Decrease

For example, an ordinary differential equation for the amount of water x in a tank.

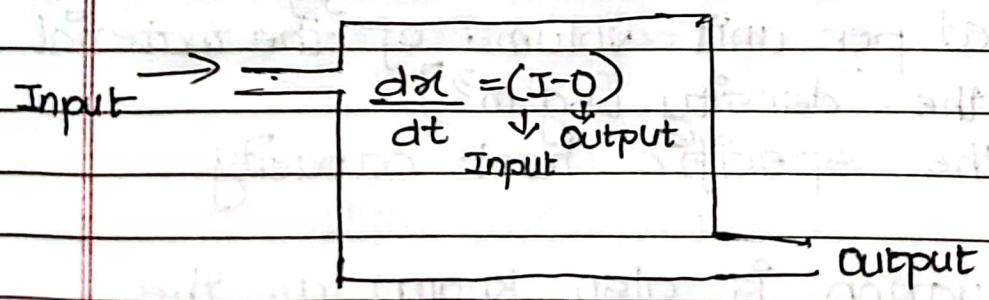


Fig: Water tank Problem Model by Balance Differential Equation.

Partial Differential Equation

→ When more than one independent variable occurs in differential equation, the equation is said to be partial differential equation.

→ It can involve the derivation of the same dependent variable with respect to each of the independent variable.

Example :

An equation describing the flow of heat in a three-dimentional body. There are four independent variables representing the three dimensions & time & one dependent variable representing temperature.

The general heat conduction equation.

$$\frac{\partial}{\partial x}\left[k.\frac{\partial T}{dx}\right] + \frac{\partial}{\partial y}\left(k.\frac{\partial T}{\partial y}\right) + \frac{\partial}{dz}\left(k.\frac{\partial T}{\partial z}\right) + q_v$$

$$= \rho C_p.\frac{\partial T}{\partial t} \Rightarrow \text{Partial Differentiator}$$

where k is the materials conductivity $[w m^{-1} k^{-1}]$

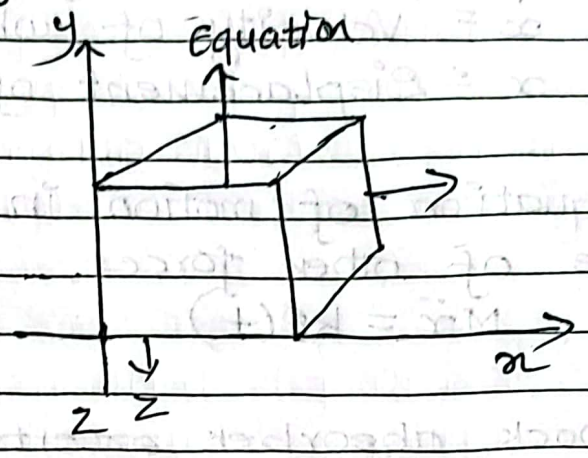
→ q_v is the rate at which energy in generated per unit volume of the material

→ ρ is the density $[kg.m^3]$

→ C_p is the specific heat capacity

This equation is also known as the Fourier - Biot equation & provides the

base tool for heat conduction analysis.



→ Differential equation occur repeatedly in scientific & engineering studies.

→ The most physical & chemical processes involve rate of change which requires differential equation for their mathematical description.

Heat conduction equation

$$\frac{\partial}{\partial x}\left(k.\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k.\frac{\partial T}{\partial y}\right)+\frac{d}{dz}\left(k.\frac{\partial T}{\partial z}\right)$$

$$+q_v = \rho C_p.\frac{\partial T}{\partial t}$$ → Partial Differential Equation

↳ Fourier - Biot Equation.

Automobile wheel suspension system.

The differential equation describing auto-mobile wheel suspension system.

$$M\ddot{x} + \underbrace{\theta\dot{x}}_{\text{shock absorber}} + \underbrace{kx}_{\text{compression}} = kf(t)$$

where, $\ddot{x}$ = Acceleration of wheel
$\dot{x}$ = Velocity of wheel.
x = Displacement of point

The equation of motion in the ~~absense~~ absence of other forces,

$$M\ddot{x} = kf(t)$$

→ The shock absorber exerts a resisting force that depends on the velocity.
$D\dot{x}$ → Directly proportional to velocity

→ If the force is directly proportional to the compression, it can be represented by kx.

→ Since, both these force oppose the motion

$$M\ddot{x} = kf(t) - D\dot{x} - kx$$

Differential & partial differential equation

Automobile Suspension Problem.

$$M\ddot{x} + D\dot{x} + kx = kF(t)$$

Derivative

$\ddot{x}$ → x''

Solving the equation for the highest order derivative gives

$$M\ddot{x} = kF(t) - D\dot{x} - kx$$

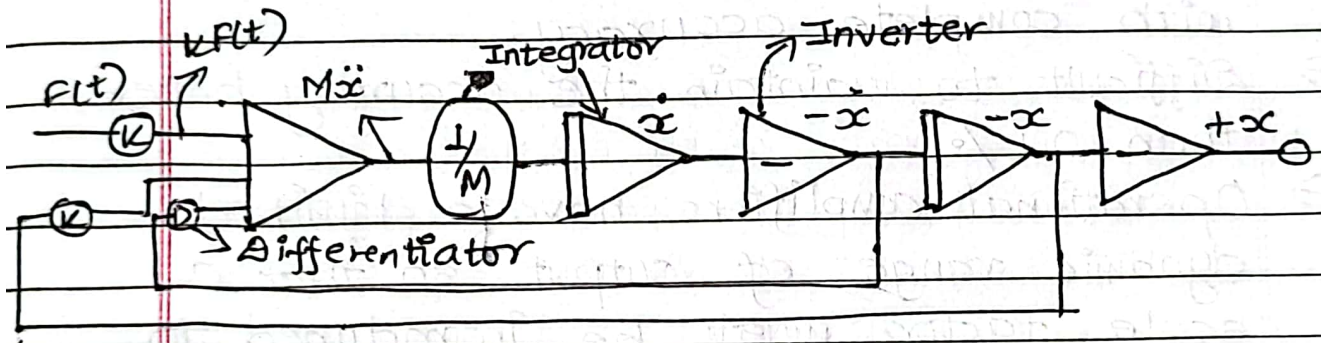


Fig: Automobile Suspension Problem

Suppose a variable representing the input F(t) is supplied, assume there exist variable representing $-x$ & $-x'$. These three variable can be scaled and added to produce $M\ddot{x}$. Integrating it with a scale factor $1/M$ produces $\dot{x}$. Changing sign produce $-x'$, further integrating produces $-x$, a further sign inverter is included to produce $+x$ as output.

Analog computer.

Limitations of Analog Computer

→ Difficulty to carry the accuracy of measuring a voltage beyond of certain point.

→ A number of assumptions are made.

→ Does not solve the mathematical model with complete accuracy.

→ Difficult to maintain the accuracy better than 0.1%.

→ Operational amplifiers have a limited dynamic range of output so that a scale factor must be introduced to keep it within the range (↑↓)

Digital Computers:

→ Virtually any degree of accuracy can be programmed

→ With the use of floating-point representation of numbers an extremely wide range of variations can be tolerated.

→ Easy to use for many different problem

→ A digital computer has the advantage of being easily used for many different problems.

Hybrid Computer

Hybrid computer describe combinations of traditional analog computer elements, giving smooth continuous outputs and elements, carrying out such non linear digital operatio as storing values, switching & performing logical operations.

Hybrid computer may be used to simulate system that are mainly continuous but do have some digital elements.

eg: Artificial Satellite for which both the continuous equation of motion & digital control signals must be simulated.

Feedback System: → Stable

The system takes feedback from the outpu i.e input is coupled with output. A significant factor in the performance of many systems is that coupling occurs between the input and output of the system. The term feedback is used to describe the phenomenon.

One Example of feedback system in whic there is continuous control in the aircr system.

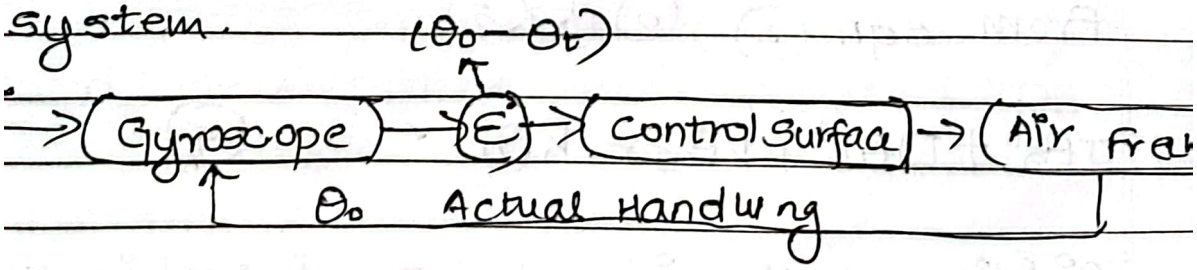


Fig: An aircraft under autopilot control (continuous system)

Here the input is a desired aircraft heading and the output is the actual heading. The gyroscope of the autopilot is able to detect the difference between the two headings. A feedback is established by using the difference to operate control surface. Since change of heading will then effect, the signal being used to control the heading. The difference between the desired signal θ_i and actual θ_o is called the error signal, since it is a measure of the extent to which the system the desired ~~cont~~ conditions. It is denoted by e.

$$e = \theta_i - \theta_o \quad ---- (1)$$

$$\text{Torque} = Ke - D\theta_o' \quad ---- (2)$$

We also know that in terms of angular acceleration

$$\text{Torque} = I\theta_o'' \quad ---- (3)$$

$\theta_o'' \rightarrow \frac{d\theta^2}{dt^2}$

$I \rightarrow$ moment of Inertia.

From eqn (1), (2) & (3)

$$I\theta_o'' + D\theta_o' + k\theta_o = k\theta_i \quad ---- (4)$$

Dividing both sides by I & making the following substitution in eqn (4)

$$\theta_o'' + \frac{D}{I}\theta_o' + \frac{k}{I}\theta_o = \frac{k}{I}\theta_i$$

$$\therefore 2\varepsilon\omega = \frac{D}{I}, \quad \omega^2 = \frac{k}{I}$$

$$\theta_o'' + 2\varepsilon\omega\theta_o' + \omega^2\theta_o = \omega^2\theta_i \quad \text{-----} (5)$$

where ε is damping factor. This is a second order differential equation.

Interactive System:

Interactive systems are computer systems characterized by significant amount of interaction between humans & the computer.

Most user have grown up using Macintosh or Windows computer operating system which are prime example of graphical interactive system.

Editors CAD - CAM (Computer Aided Design, Computer Aided Manufactures) system and data entry systems are all computer system involving a high degree of human-computer interaction. Games & simulations are interactive system. Web browsers & integrated development environments (IDE's) are also examples of very complex interactive systems.

Clock time in System Simulation

Clock time is updated based on the following two models.

1. Fixed time step Model
2. Event-to-Event model

① Fixed time step model:

In this the timer simulated by the computer is updated at a fixed time interval. The system is checked to see if any event has taken place during that interval. All the events which take place during the time time interval are considered to have occured simultaneous at the end of the interval.

This is very Imp
Upasana

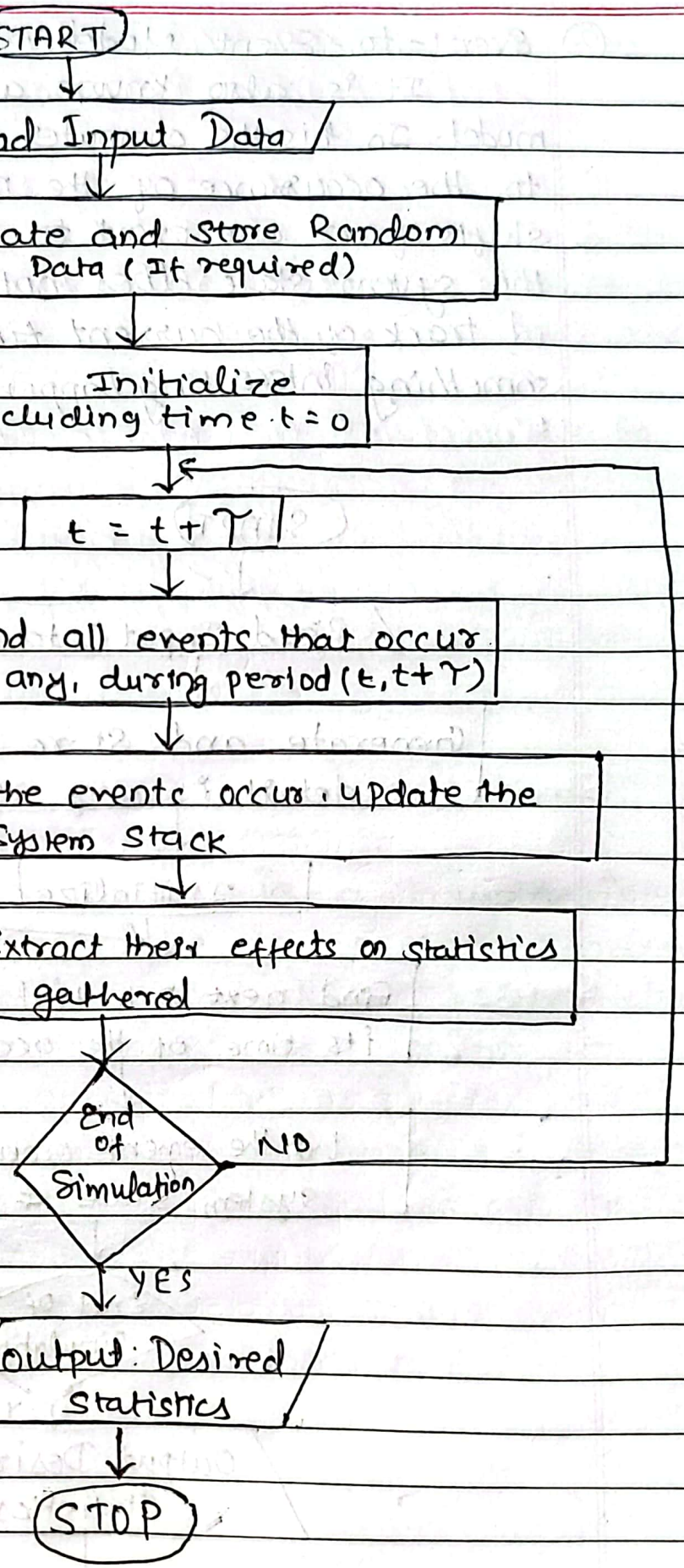


Fig: Fixed time step simulation

② Event-to-Event Model :→

It is also known as the next-event model. In this the computer advances the time to the occurrence of the next event. So it shifts from one event to the next event & the system state does not change in between. A track of the current time is kept when something interesting happens to the system.

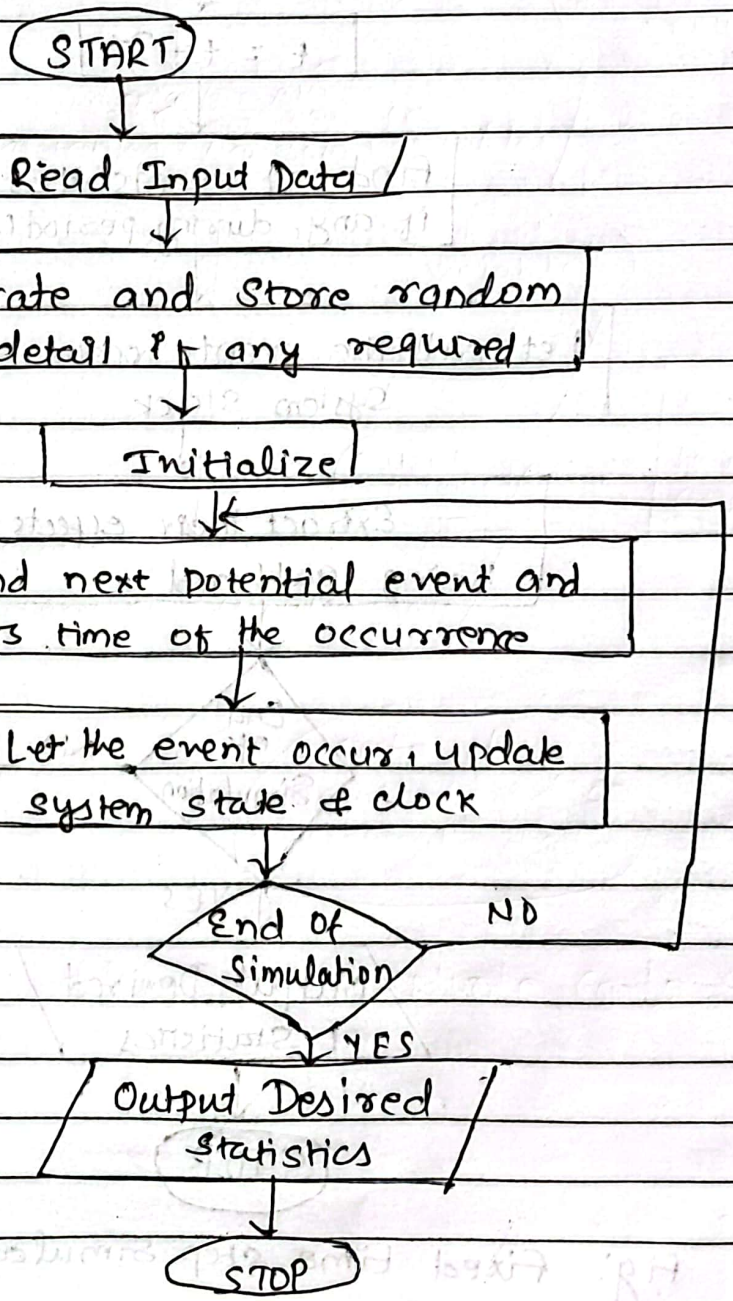


fig: Next-Event Simulation.

Differentiate between clocktime & simulation time used in system simulation

Clock Time	Simulation Time.
① It is the total time for which the CPU remains time.	It is the total amt of tim that CPU spends for simulat
② Clock time is measured continuously through all the operation that a CPU undergoes.	It only deals with the amount of time elevated for simulation.
③ It is usually more comparatively.	It is usually less comparatively.

Monte Carlo simulation:

Monte Carlo simulation is a computerised mathematical technique to generate random sample data based on some known distributi for numerical experiments. This method is applied to risk quantitative analysis and decision making problems. This method is us by professionals of various profiles such as finance, project management, energy, manufacturing, research and development, insurance, oil & gas transportation, etc.

Following are the three important characteristics of monte-Carlo method:

→ It's output must generate random samples.
→ It's input distribution must be known.
→ Its result must be known while performing an experiment.

Flowchart for Monte Carlo Simulation

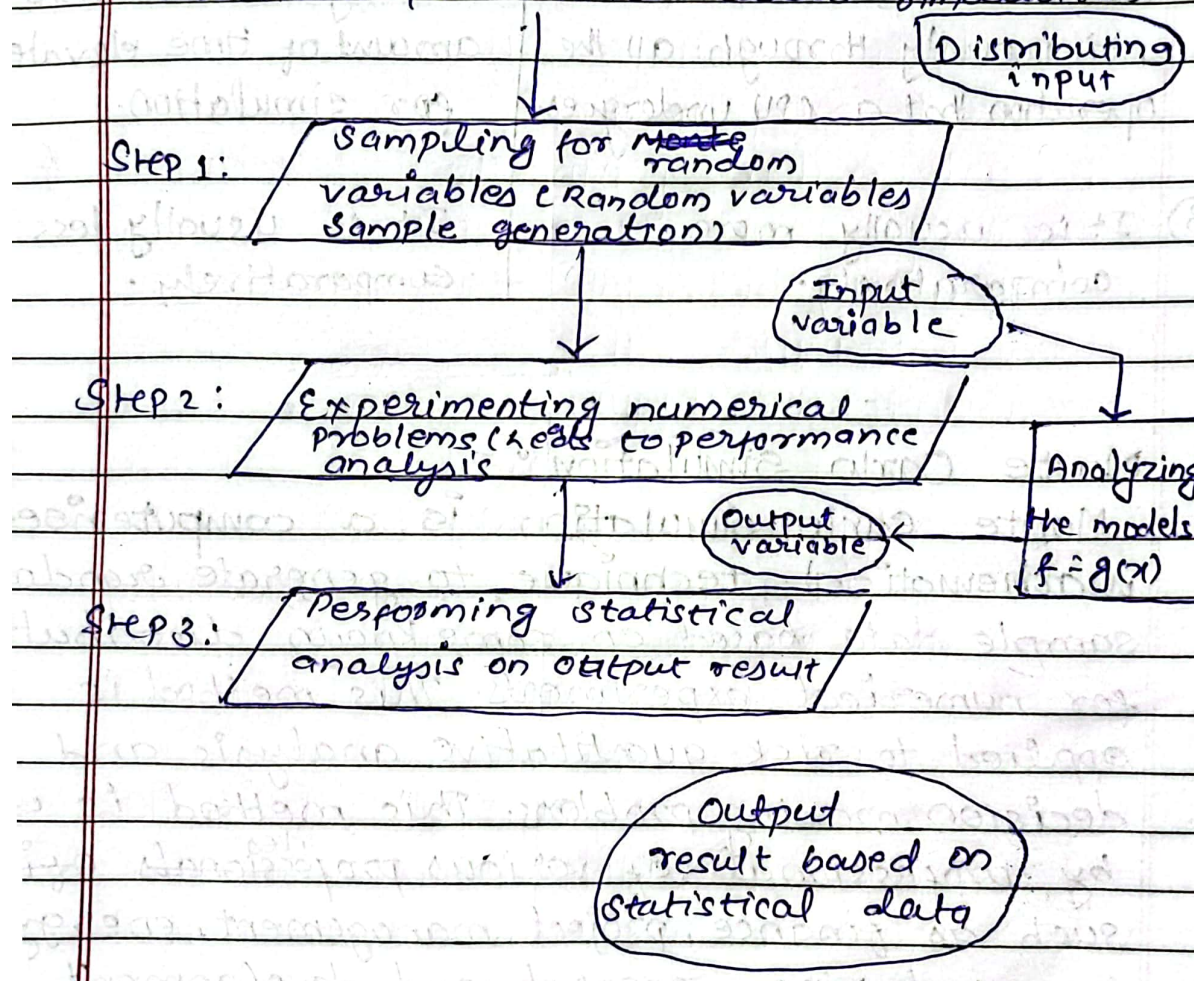
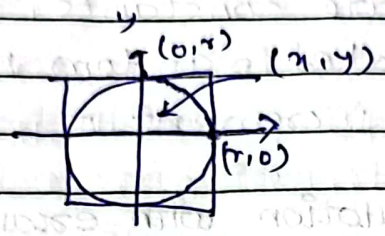


fig: Flow chart of Monte Carlo Simulation

Example:

Determine the value of PI using Monte carlo Method.



~~Area of quadrant of circle~~

~~Area of Rectangle~~

$$\frac{\text{A. of quadrant of circle}}{\text{A. of rectangle}} = \frac{\text{No. of points inside the curve}}{\text{No. of point inside the rectangle.}}$$

or, $$\frac{1/4\, \pi r^2}{r^2} = \frac{n}{N}$$

$$\boxed{\therefore \pi = \frac{4n}{N}}$$

We use random number generation method to determine the sample points that lie inside or outside the curve. Let (x_0, y_0) be an initial guess for the sample point then from a linear congruential method of random number generation.

$x_{i+1} = (ax_i + c) \bmod m$
$y_{i+1} = (ay_i + c) \bmod m$

where a & c are constants, m is the upper limit of generated random number. If $y <= y_i$, then increment n.

Q. Explain hybrid simulation with example.

Hybrid simulation :

In reality, the system is neither a pure continuous nor a pure discrete nature. For simulating, such system, the combination of analog and digital computers are used. Such setup is known as hybrid computers. The simulation provided by the hybrid computers is known as hybrid simulation.

The form taken by ~~hr~~ hybrid simulation depends upon the application. One computer may be simulating the system being studied while other is providing a simulation of the environment into which the system is to operate.

It is also possible that the system being simulated is an interconnection of continuous & discrete system, which can best be modeled by an analog & digital computer being linked together.

The major difficulty in use of the hybrid simulation is that it requires high speed converters to transform

signals from analog to digital form & vice-vers

Q. What is non-stationary poisson process? How can we convert it into stationary poisson process.

Non-Stationary Poisson process & stationary Poisson Process:

The non-stationary Poisson process is a poisson process for which the arrival rate varies with time.

The counting process $N(t)$ is a non-stationary Poisson process if:

(a) The process has independent increments.

(b) $Pr[N(t+dt) - N(t)] = 0 = 1 - \lambda(t)dt$
$= 1 = \lambda(t)dt$
$> 1 = 0$

where,
$\lambda(t)$ = the arrival rate at time t.
dt = differential sized interval

The definition is identical to the stationary Poisson process, with the exception that the arrival rate, $\lambda(t)$ is now a function of time.

Stationary Poisson Process: A counting process $N(t)$ is a stationary Poisson process with rate λ if,

(a) The process has independent increments.
(b) The process has stationary increments.
and

$$Pr\{[N(t+dt) - N(t)] \begin{cases} =0 \\ =1 \\ >1 \end{cases} \begin{cases} = 1 - \lambda dt \\ = \lambda dt \\ = 0 \end{cases}$$

A non-stationary Poisson process can be transformed into a stationary Poisson process with arrival rate 1.

Define arrival Pattern. Explain non-stationary Poisson process.

Arrival Pattern

Arrival defines the way customers enter into the system. Mostly the arrivals are random with random intervals between two adjacent arrivals. Typically the arrival is described by a random distribution of intervals also called Arrival Pattern. Arrivals may occur at scheduled times or at random times. When at random times, the inter arrival times are usually characterized by a probability distribution and most important model for random arrival is the Poisson process.

In schedule arrival interarrival time of customers are constant.

The non-stationary Poisson process is a Poisson process for which the arrival

rate varies with time.

More specifically, it can be defined as follows:

The counting process $N(t)$ is a non-stationary Poisson process if:

(a) The process has independent increments.

(b) $Pr[N(t+dt) - N(t)] = \begin{cases} = 0 & = 1 - \lambda(t)\,dt \\ = 1 & = \lambda(t)\,dt \\ > 1 & = 0 \end{cases}$

where $\lambda(t)$ = the arrival rate at time t.

dt = differential sized interval.

Discrete Event Simulation

→ Discrete event simulation concerns the model of a system as it evolves over time by a representation in which the state variables change instantaneously at separate points in time.

→ In more mathematical terms, we might say that the system can change at only a countable number of points in time.

→ These points in time are the ones at which an event occurs, where an event is defined as an instantaneous occurance that may change the state of the system.

Example:

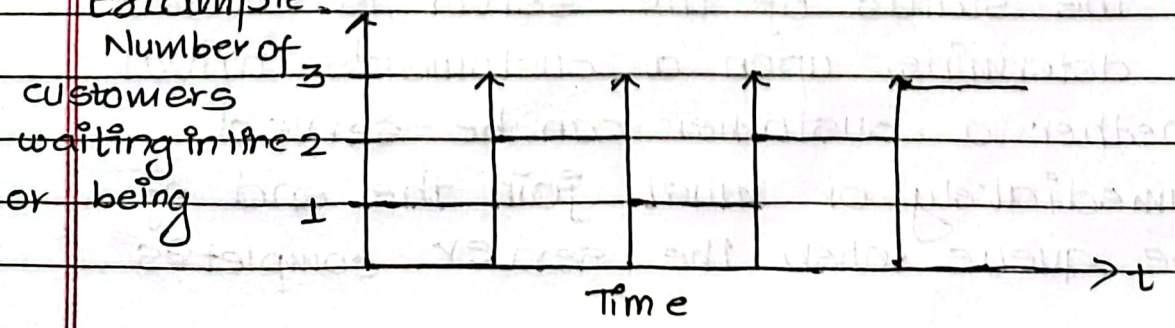


Fig: Example of discrete system simulation

Q. What is non-stationary poisson process? How can we convert it into a stationary poisson process?

Note: arival time lai ⊥ parne bittikai it becomes stationary poisson process.

Example:

Consider a service facility with a single-server - eg a one-operator barbershop or an information desk at an airport - for which we would like to estimate the (expected) average delay in queue (line) of arriving customers, where the delay in queue of a customer is the length of the time interval from the instant of his arrival at the facility to the instant he begins being served.

For the objective of estimating the average delay of a customer, the state variables for a discrete-event simulation ~~of the serve~~ model of the facility would be the status of the server i.e either idle or busy, the number of customer waiting in queue to be served (if any) and the time of arrival of each person waiting in queue.

The status of the server is needed to determine, upon a customer's arrival, whether a customer can be served immediately or must join the end of the queue when the server completes

serving a customer, the number of customers in the queue is used to determine whether the server will become idle or begin serving the first customer in the queue. The time of arrival of a customer is needed to compute his delay in queue, which is the time he begins being served minus his time of arrival.

There are two types of events for this system
→ The arrival of a customer and the completion of service for a customer, which results in the customer's departure.
→ An arrival is a event since it causes the (state variable) server status to change from idle to busy or the (state variable) number of customer in the queue to increase by 1.
→ Correspondingly, a departure is an event because it causes the server status to change from busy to idle or the number of customers in the queue to decrease by 1.

Queue: The line where the entities or customer wait is generally known as Queue. The combination of all entities in system being served and being waiting for service will be called as queuing system.

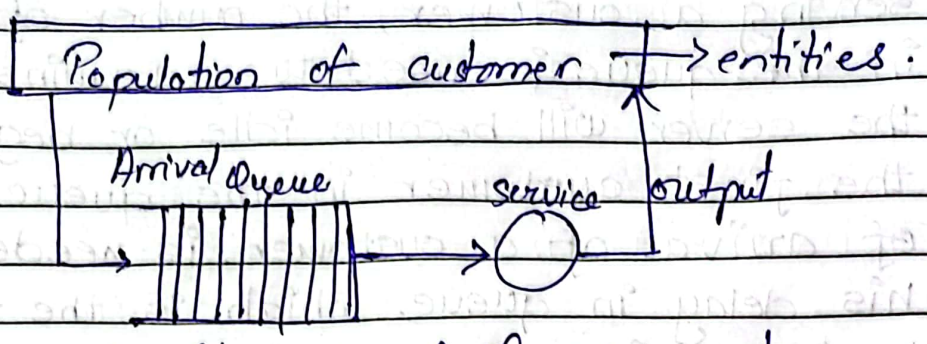


Fig: Elements of Queuing system

The key elements of Queuing system are customer and service customer refers to anything that arrives that arrives at a facility and requires service. i.e people, machine, trucks, motorbike, etc.

Server refers to any resources that provides the requested services. Eg. receptionist, teller CPU, washing Machine etc.

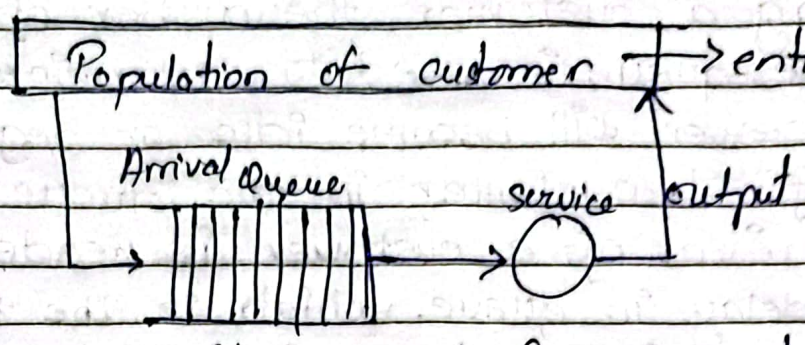


Fig: Elements of Queuing system

The key elements of Queuing system are customer and service customer refers to anything that arrives that arrives at a facility and requires service. i.e people, machine, trucks, motorbike, etc.

Server refers to any resources that provides the requested services. Eg. receptionist, teller CPU, washing Machine etc.

What do you mean by Queuing System? Explain the characteristics of Queueing system with example.

* Characteristics of Queuing System.

1. Calling Population
2. Arrival Process
3. Service Process
4. Queueing Discipline & Queuing Behaviour.
5. Number of Servers.

1. Calling Population :→ The population of potential customers those requires service from system is called calling population. It may be finite or infinite.

System having large calling population in is usually considered as infinite. For e.g. customers at bank, restraunt.

Any system having countable population is usually considered as finite e.g. a certain number of machines to be repaired by a service men.

In finite model, arrival rate depends on the number of customers being serviced and waiting. But in infinite population model, arrival rate is not affected by the number of customers being served & waiting.

(2) Arrival Process:

The arrival process for infinite-population models is usually characterized in terms of inter-arrival times of successive customers.

Arrival's may occur at scheduled tim or at random time. When at random times, the inter arrival times are usually characterized by a probability distribution & most important model for random arrival is the Poisson process. In schedule arrival interarrival times of customers are constant.

③ Service Process:

Service process can be measured by the number of customers served per some unit of time or time taken to complete the service. Once entities have ~~enta~~ entered to the system they must be served. The service can be provided in single or batch. If it is batch, as in the case of arrival the batch size can fixed or random.

Service time may be constant duration or of random duration

↳ Markov service process

④ Queuing Discipline and Queuing Behavior:

~~Queing~~ Queue Discipline refers to the rule that a server uses to choose the next customer ~~from~~ from the the queue when the server completes the service of the current customer.

Common Queue discipline include:

① FIFO
② LIFO
③ Service in Random order (SIRO)
④ Shortest Processing Time First (SPTF) &
⑤ ~~Sorting~~ Service According to Priority (PR)

FIFO : It states that customers are served one at a time & that the customer has been waiting the longest is served first.

LIFO : This also serves customer one at a time, however the customer with the shortest waiting time will be served first.

Characteristics of Queuing system.

1. Calling Population
2.

Queue Behaviour :
Queue behaviour refers to the action of customers while in a queue waiting for servers to begin. Different queue behaviour are :

- Balk / Balking :→
  It means leaving the queue while the customer see the line is too long.

- Renege / Renegeing :
  Leave after being in the line when they see that the line is moving too slowly.

- Jackey / Jackeying :
  Move from one to a shortest line.

(5) Number of servers:

Servers represent the entity that provides services to the customer. A system may consist of single server or multiple servers.

→ A system with multiple servers is able to provide parallel services to the customer.

Markov Service Process:

A Markov service process is a special service process in which entities are processed one at a time in FCFS (First come first served) order and service times are independent and exponential. As with the case of Markov arrivals, a Markov service process is memoryless, which means that the expected time until an entity is finished remains constant regardless of how long it has been in service.

Queuing Notation (Kendall notation of Queuing system)

Kendall's notation is used for parallel server systems. The basic format of the notation is of form: A/B/C/N/K, where A, B, C, N, K respectively indicate arrival pattern, service pattern, number of servers

B C

system capacity (N) and calling population (K)

The symbols used for the probability distribution for inter arrival time, and service time are D for deterministic, M for exponential (or Markov) and Ek for Erlang. → Telecom

e.g.:

→ Kendall's notation

ⓐ M/D/2/5/∞ → Stands for a queuing system having exponential arrival time, deterministic service time, 2 servers, capacity of 5 customers and infinite population.

ⓑ M/D/2 means exponential arrival time, deterministic service time, 2 servers, infinite service capacity and infinite population.

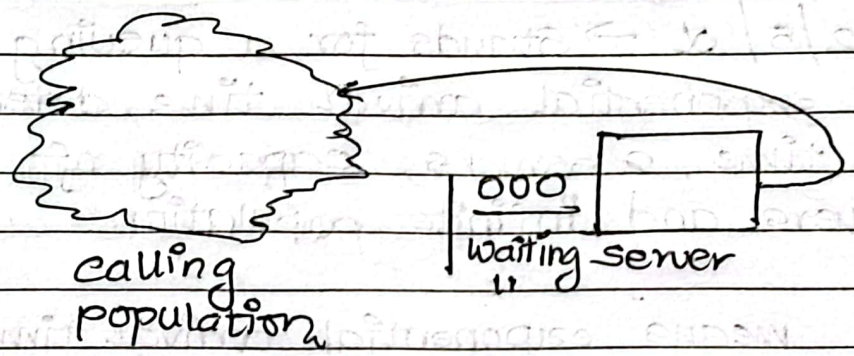
Application of queuing system

1) Telecommunication
2) Traffic control → Automobile → Air traffic
3) Computer process
4) Manufacturing process

Queuing Model
- Single server
- Multi-server
- Multiple single server queue

1) Single Server Queue:

A queuing system is described by its calling population, the nature of arrivals, the service mechanism, system capacity and the queuing discipline. A single channel queuing system is portrayed in figure below:



So, in a single server queue.

- Calling population is infinite.
  → Arrival rate does not change.
- Units are served according to FIFO.
- Arrivals are defined by the distribution of time between arrival.
  → Inter-arrival time.
- Service time are according
- Arrival rate must be less than service rate.
  → Stable system

- Otherwise waiting line will grow unbounded.

→ Unstable system.

Arrival Event

If server idle units get service, otherwise units enter queue.

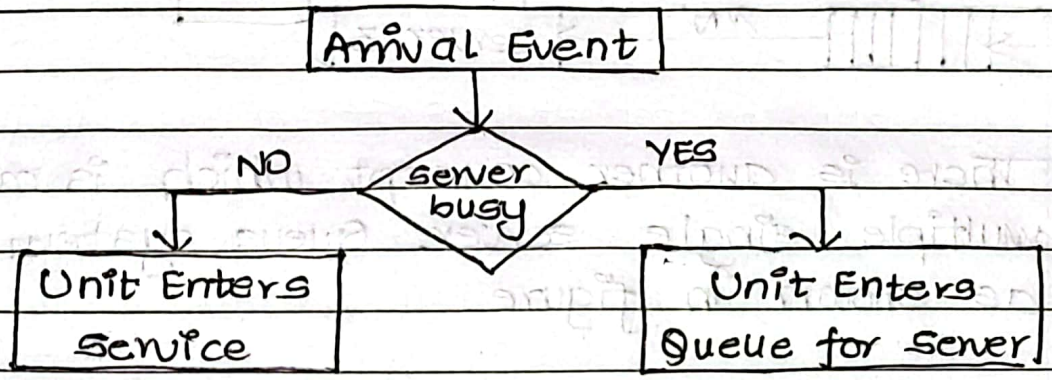
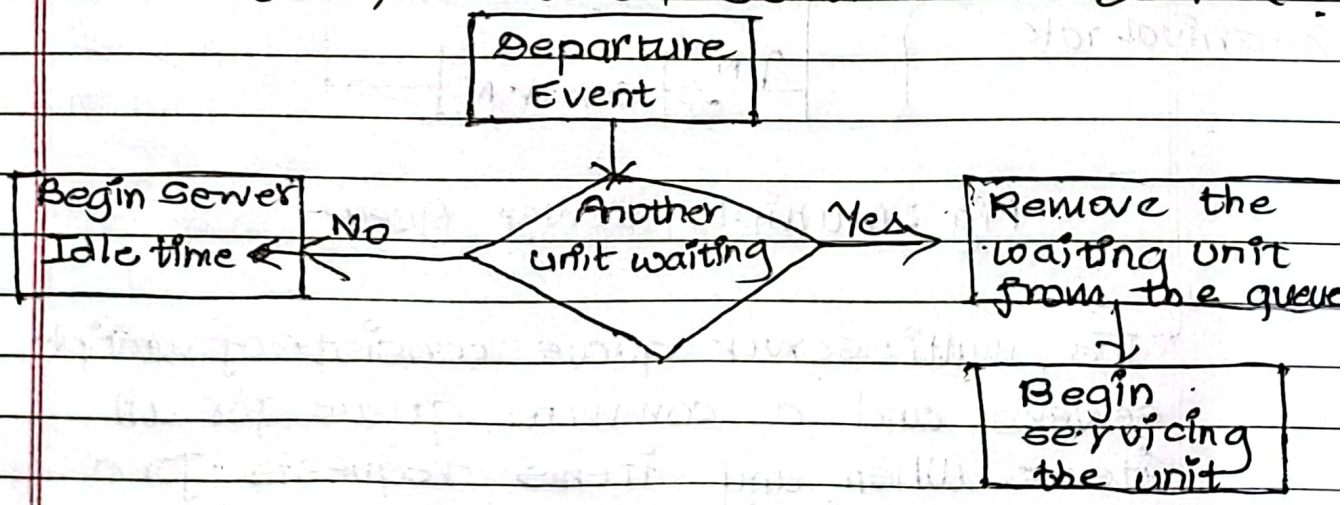


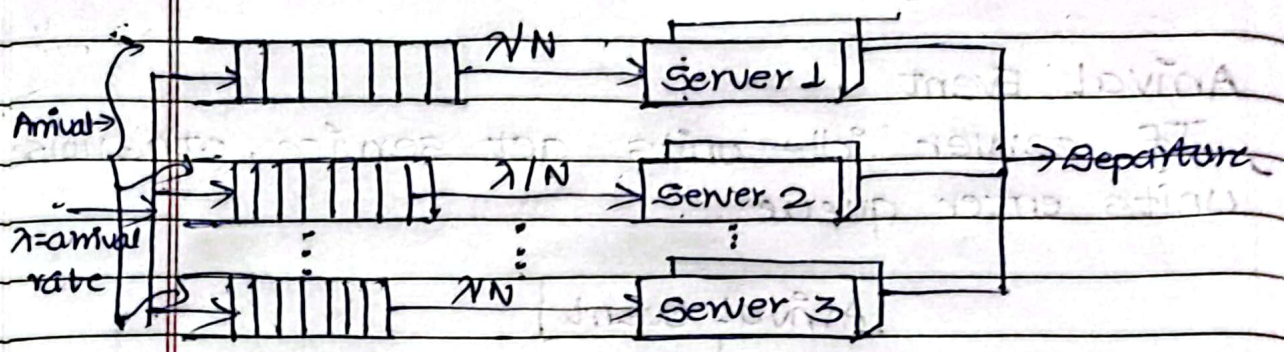
Fig: Unit entering system flow diagram.

Departure Event:

If queue is not empty begin servicing next unit, otherwise service will be idle.



③ Multiple single server Queue :



There is another concept which is called multiple single server Queue system as shown in figure.

④ Multiple server Queue

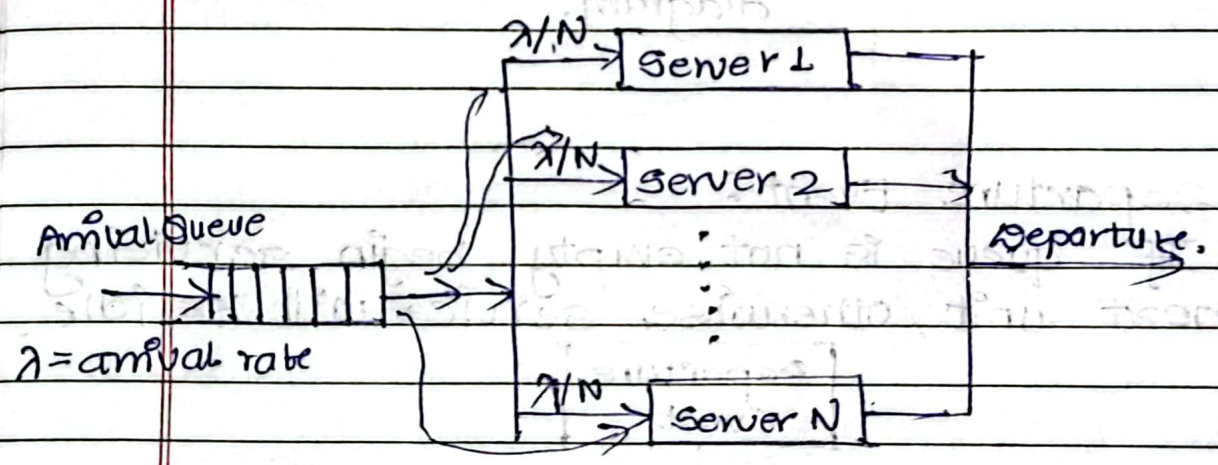


Fig : Multiple Server Queue.

The multi-server queue consist of multiple servers and a common queue for all items. When any items requests for a server, it is allocated if at least one server is available. Else the

queue begins to start until the server is free. In this system, we assume that all servers are identical i.e there is no difference which server is choosen for which item.

∴ The total server utilization for all server system is

$$\rho = \frac{\lambda}{\mu N}$$

where μ = service rate

λ = arrival ~~time~~ rate

Description → Fig shows a generalization of the simple model we have been discussing for multiple servers, all sharing a common queue.

If an item arrives and at least one server is available, the item is immediately dispatched to the server.

It is assumed that all servers are identical, thus if more than one server is available, it makes no difference which server is choosen for item.

If all server are busy, a queue begins to form. As soon as one server becomes free, an item is dispatched from the queue using the dispatching discipline as force.

Q. What do you mean by Multi-server Queue? Explain.