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$$T(n) = a\,T\left(\frac{n}{b}\right) + \theta(n^k \log^p n)$$

$a \geq 1$, $b > 1$, $k \geq 0$, p Real Number

Case I if $a > b^k$ then $T(n) = \theta(n^{\log_b a})$

Case 2. If $a = b^k$.

(a) If $p > -1$ then $T(n) = \theta(n^{\log_b a} \log^{p+1} n)$

(b) $p = -1$ $T(n) = \theta(n^{\log_b a} \log \log n)$

(c) $p < -1$ $T_n = \theta(n^{\log_b a})$

Case-3 If $a < b^k$.

(a) if $p \geq 0$ then $T(n) = \theta(n^k \log^p n)$

(b) If $p < 0$ then $T(n) = \theta(n^k)$

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Q.1

$T(n) = 4\,T(\frac{n}{2}) + n.$

$a = 4, \quad b = 2, \quad k = 1, \quad P = 0.$

$b^k = 2^1 = 2$

$a > b^k.$ $\boxed{\text{Case:- I.}}$

$T(n) = \Theta(n^{\log_b a}) = \Theta(n^{\log_2 4}) = \Theta(n^2)$ Ans.

Q.2

$T(n) = 16^n\,T(\frac{n}{3}) + \frac{1}{n}.$

$a = 16^n \quad b = 3, \quad k = -1$

Not solve using master Theorem.

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Case-2.

(i) $T(n) = 2T(\frac{n}{2}) + n \log n.$

$a=2, \quad b=2, \quad K=1, \quad P=1$

$a = 2 \quad b^K = 2^1 = 2$

Case ② $a = b^K.$

Case (a) $\boxed{P > -1}$

$T(n) = \theta(n^{\log_b a} \log^{P+1} n)$

$= \theta(n^{\log_2 2} \log^{1+1} n)$

$$\boxed{T(n) = \theta(n \log^2 n)}$$

(ii) $T(n) = 2T(\frac{n}{2}) + n \log^{-1} n.$

$a=2, \quad b=2, \quad K=1, \quad P=-1$

Case 2/b

$T(n) = \theta(n \log \log n)$ Ans.

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(iii) $T(n) = 2T(\frac{n}{2}) + \frac{n}{\log^2 n}$

$$T(n) = 2T(\frac{n}{2}) + n \log^{-2} n.$$

$a=2$, $b=2$, $k=1$, $P=-2$.

Case II/c.

$$T(n) = \Theta(n^{\log_b a}) = \Theta(n).$$

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Case - III:

$$T(n) = 2T\left(\frac{n}{2}\right) + n^2$$

$a = 2,\ b = 2,\ k = 2,\ P = 0$

$a < b^k,\ 2 < 2^2,\ 2 < 4.$

Case-3/(a)

$$T(n) = \Theta(n^2 \log^0 n) = \Theta(n^2)$$ Ans.

Q.2 $T(n) = 3T\left(\frac{n}{2}\right) + \frac{n^2}{\log n}.$

$a = 3,\ b = 2,\ k = 2,\ P = -1$

$a < b^k,\ 2 < 4$

Case-III/(b)

$$\boxed{T(n) = \Theta(n^2)}$$

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$T(n) = T(\sqrt{n}) + \log n.$

Let $n = 2^m$

$T(2^m) = T(2^{m/2}) + \log 2^m = T(2^{m/2}) + m$

Let $T(2^m) = S(m)$

$S(m) = S(\frac{m}{2}) + m.$

$a=1, \; b=2, \; k=1, \; p=0.$

$a < b^k \; , \; 1 < 2^1$

Case-III.1a

$T(n) = \theta(m^k \log^p n) = \theta(m^1)$

$\Rightarrow$ ~~n = log~~ $\quad n = 2^m$

$\log n = m \log_2 2 = m$

$$\boxed{T(n) = \theta(\log n)}$$