Course Title: BIT 152 Discrete Structures (Second Semester)

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Our recommended books:

1.Discrete Mathematics & its applications, Seventh Edition,Mc Graw Hill Publication, 2022

Discrete Mathematics

Introduction

Discrete Mathematics is a part of mathematics. It is devoted to the study of discrete or discontinuous elements or objects or unconnected elements.

Different kinds of problems can be solved using discrete mathematics.

1. What is the shortest path between two cities using a transportation system?
2. What is the probability of winning a lottery?
3. How many steps to do such a sorting?
4. Is there a link between two computers in a network?

Why study Discrete Mathematics?

There are several important reasons for studying discrete mathematics.

1. To develop your mathematical maturity that is your ability to understand & create mathematical arguments.
2. Discrete Mathematics is the gateway to more advanced courses in all parts of mathematical sciences.
3. Discrete Mathematics provides mathematical foundations for many Computer Science courses including Data Structures, algorithms, Database, Automata Theory, Compiler Theory, Computer Security & Operating Systems.
4. Discrete Mathematics contains the necessary mathematical background for solving problems in operations research, chemistry, biology, engineering & so on.

Unit-1

Logic and Proof Methods

Logic

Logic is a rule. The rules of logic give precise meaning to mathematical statements. These rules are used to distinguish between valid and invalid mathematical arguments.

Logic has numerous applications in computer science. These rules are used in the design of computer circuits, the construction of computer programs and the verification of the correctness of programs.

Propositions

A proposition is a statement that is either true or false, but not both. As for example,

Example 1

1. Washington, D.C., is the capital of USA. (True)

2. KTM is the capital of Nepal. (True)

3. 1+1=2(True)

4. 2+2=3(False)

Propositions 1, 2, 3 are true and 4 are false.

Example 2

5. What time is it?

6. Read this carefully.

7. x+1=2

8. x+y=z

Sentences 5 & 6 are not propositions because they are not declarative sentences. Sentences 7 & 8 are not propositions because they are neither true nor false.

Truth Table

Since we need to know the truth value of a proposition in all possible scenarios, we consider all the possible combinations of the propositions which are joined together by Logical Connectives to form the given compound proposition. This compilation of all possible scenarios in a tabular format is called a **truth table**.

Negation (¬)

Let p be a proposition. The statement "The sun rises in the west", called the negation of negation of p.The negation of p is denoted by ¬p (not p).
Example1
Today is friday.
Today is not friday.
Or, It is not Friday today.
The truth table for the negation of a proposition.

P	**¬ p**
F	T
T	F

Conjunction

Let p and q be propositions. The proposition “p & q” denoted by $p \wedge q$. It is the proposition that is true when both p & q are true & is false otherwise.

The proposition $p \wedge q$ is called the conjunction of p & q.

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

As for example,

Find the conjunction of the propositions p & q where p is the proposition “Today is Friday” & q is the proposition “It is raining today”.

Solution:

The conjunction of these propositions, $p \wedge q$ is the proposition, Today is Friday & it is raining today.

Disjunction

Let p and q be propositions. The proposition “p or q” denoted by $p \vee q$. It is the proposition that is false when both p & q are false & is true otherwise.

The proposition $p \vee q$ is called the disjunction of p & q. The truth table of disjunction is

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

As for example,

Find the disjunction of the propositions p & q where p is the proposition “Today is Friday” & q is the proposition “It is raining today”.

Solution:

The disjunction of these propositions, $p \lor q$ is the proposition,

-Today is friday or it is raining today.

- Students who have taken Math or Computer Science can take this class.

Conditional or implication / if-then (→)
An implication $p \rightarrow q$ is the proposition “if p, then q”. It is false if p is true and q is false. The rest cases are true. The truth table $p \rightarrow q$ is sometimes called conditional statements.

The truth table is as follows −

p	q	p →q
T	T	T
T	F	F
F	T	T
F	F	T

Implication is denoted by if p then q →p implies q.
If p, q → p only if q
q if p → q whenever p
q when p → q is necessary for p.
As for example,
If I am elected, then I will take lower taxes.
If you got 100% on the final, then you will get an A

Biconditional or If and only if (⇔ or ↔)

Let p & q be propositions. The biconditonal **p ↔ q** is the proposition that is true when p &q have the same truth values & is false otherwise.
p ↔ q (p→q & q→p)
p ↔ q (p→q ∧ q→p)
p ↔ q (p if & only if q)

The truth table is as follows −

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Exclusive OR ( )

Let p & q be propositions. The exclusive of p & q, denoted by $p \oplus q$ (or p XOR q), the proposition that is true when exactly one of p & q is true & is false otherwise.

A	B	A **XOR** B
0	0	0
0	1	1
1	0	1
1	1	0

Example

Let p & q be propositions that state "A student can have a salad with dinner" & "A student can have soup with dinner" respectively. What is $p \oplus q$?

Solution: The exclusive OR of p &q is the statement that is true when exactly one of p & q is true. That is, $p \oplus q$ is the statement "A student can have soup or salad, but not both, with dinner.

Converse, Contrapositive and Inverse

-The proposition $q \rightarrow p$ is called the converse of $p \rightarrow q$.

Table for the Bit operation OR,AND & XOR				
X	Y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

-The contrapositive of $p \to q$ is the proposition $\neg q \to \neg p$.

-The proposition $\neg p \to \neg q$ is called the inverse of $p \to q$.

p	**q**	**¬ q**	**p ∨ ¬ q**	**p ∧ q**	**(p ∨ ¬q) → (p ∧ q)**
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Precedence of Logical Operators

Precedence of Logical Operators	
Operators	Precedence
¬	1
∧	2
∨	3
→	4
↔	5

Logic & Bitwise operators

True value	Bit
T	1
F	0

Definition

A Bit string is a sequence of zero or more bits. The length of this string is the number of bits in the string.

Example-15

101010011 is a bit string of length nine.

Express bit operations to bit strings.

-Bitwise OR

-Bitwise AND

-Bitwise XOR

01	1011	0110	
11	0001	1101	
11	1011	1111	Bitwise OR
01	0001	0100	Bitwise AND
10	1010	1011	Bitwise XOR

Applications of Propositional Logic

Translating English sentences to Propositional Logic

Example 1

How can this English sentence be translated into a logical expressions?

“You can access the Internet from campus only if you are a Computer Science major or you are not a freshman.”

[Solution:

$a \rightarrow (c \vee \neg f)$

Example 2

How can this English sentence be translated into a logical expressions?

“You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old”.

Solution: Let q, r & s represent "You cannot ride the roller coaster". You are under 4 feet tall" & "You are older than 16 years old" respectively. Then, the sentence can be translated to

$(r \wedge \neg s) \rightarrow \neg q$.

Propositional Equivalences

Definition

A compound proposition that is always true is called tautology.

A compound proposition that is always false is called a contradiction.

A compound proposition that is neither a tautology nor a contradiction is called a contingency.

True value =Tautology=$p \vee \neg p$=true

False value=Contradiction=$p \wedge \neg$ p=false

Example of Tautology & a Contradiction			
P	P	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

Example of Contingency

-Example − Prove $(A \vee B) \wedge (\neg A)$ a contingency

The truth table is as follows −

A	B	A ∨ B	¬ A	(A ∨ B) ∧ (¬ A)
True	True	True	False	False
True	False	True	False	False
False	True	True	True	True
False	False	False	True	False

As we can see every value of $(A \vee B) \wedge (\neg A)$ has both "True" and "False", it is a contingency.

Propositional Equivalences

Two statements X and Y are logically equivalent if any of the following two conditions hold

- The truth tables of each statement have the same truth values.

- The bi-conditional statement X⇔Y is a tautology.

Example − Prove ¬(A∨B)and[(¬A)∧(¬B)] are equivalent

Testing by 1st method (Matching truth table)

A	B	A ∨ B	¬ (A ∨ B)	¬ A	¬ B	[(¬ A) ∧ (¬ B)]
True	True	True	False	False	False	False
True	False	True	False	False	True	False
False	True	True	False	True	False	False
False	False	False	True	True	True	True

Here, we can see the truth values of ¬(A∨B)and[(¬A)∧(¬B)] are same, hence the statements are equivalent.

Testing by 2nd method (Bi-conditionality)

A	B	¬ (A ∨ B)	[(¬ A) ∧ (¬ B)]	[¬ (A ∨ B)] ⇔ [(¬ A) ∧ (¬ B)]
True	True	False	False	True
True	False	False	False	True
False	True	False	False	True
False	False	True	True	True

As [¬(A∨B)]⇔[(¬A)∧(¬B)] is a tautology, the statements are equivalent.

Logical Equivalences

The propositions p & q are called logically equivalent if p↔ q is tautology. p≡ q are logically equivalent. De Morgan's Laws
¬(p∧q) ≡¬p∨¬q
¬(p∨q) ≡¬p∧¬q

Example 2

Show that ¬(p∨q) & ¬p∧¬q are logically equivalent.

Solution:

Truth tables for ¬(p∨q) & ¬p∧¬q						
p	Q	p∨q	¬(p∨q)	¬p	¬q	¬p∧¬q
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Example 3

Show that $p \rightarrow q$ & $\neg (p \vee q)$ are logically equivalent.

Truth tables for p → q & ¬(p∨q)				
p	q	¬p	¬p∨q	p→ q
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Example 4 Show that $p \vee (q \wedge r)$ & $(p \vee q) \wedge (p \vee r)$ are logically equivalent.(Distributive law)

Solution:

(Do yourself)

Laws of Logically Equivalences

1. Idempotent Laws

(i) $p \vee p \equiv p$

(ii) $p \wedge p \equiv p$.

Proof

p	p	$p \vee p$	$p \wedge p$
T	T	T	T
F	F	F	F

Table 12.14

In the above truth table for both p, $p \vee p$ and $p \wedge p$ have the same truth values. Hence $p \vee p \equiv p$ and $p \wedge p \equiv p$.

2. Commutative Laws

(i) $p \vee q \equiv q \vee p$

(ii) $p \wedge q \equiv q \wedge p$.

Proof (i)

p	q	$p \vee q$	$q \vee p$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

Table 12.15

The columns corresponding to $p \vee q$ and $q \vee p$ are identical. Hence $p \vee q \equiv q \vee p$.

Similarly (ii) $p \wedge q \equiv q \wedge p$ can be proved.

3. Associative Laws

(i) $p \vee (q \vee r) \equiv (p \vee q) \vee r$ (ii) $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$.

Proof

The truth table required for proving the associative law is given below.

p	q	r	$p\vee q$	$q\vee r$	$(p\vee q)\vee r$	$p\vee(q\vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

Table 12.16

The columns corresponding to $(p \vee q) \vee r$ and $p \vee (q \vee r)$ are identical.

Hence $p \vee (q \vee r) \equiv (p \vee q) \vee r$.

Similarly, (ii) $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$ can be proved.

4. Distributive Laws

(i) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

(ii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Proof (i)

p	q	r	$q\wedge r$	$p\vee(q\wedge r)$	$p\vee q$	$p\vee r$	$(p\vee q)\wedge(p\vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

Table 12.17

The columns corresponding to $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are identical. Hence $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.

Similarly (ii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ can be proved.

5. Identity Laws

(i) $p \vee \mathbf{T} \equiv \mathbf{T}$ and $p \vee \mathbf{F} \equiv p$

(ii) $p \wedge \mathbf{T} \equiv p$ and $p \wedge \mathbf{F} \equiv \mathbf{F}$

p	$\mathbb{T}$	$\mathbb{F}$	$p \vee \mathbb{T}$	$p \vee \mathbb{F}$
T	T	F	T	T
F	T	F	T	F

Table12.18

(i) The entries in the columns corresponding to $p \vee \mathbf{T}$ and $\mathbf{T}$ are identical and hence they are equivalent. The entries in the columns corresponding to $p \vee \mathbf{F}$ and p are identical and hence they are equivalent.

Dually

(ii) $p \wedge \mathbf{T} \equiv p$ and $p \wedge \mathbf{F} \equiv \mathbf{F}$ can be proved.

6. Complement Laws

(i) $p \vee \neg p \equiv \mathbf{T}$ and $p \wedge \neg p \equiv \mathbf{F}$ (ii) $\neg\mathbf{T} \equiv \mathbf{F}$ and $\neg\mathbf{F} \equiv \mathbf{T}$

Proof

p	$\neg p$	$\mathbb{T}$	$\neg\mathbb{T}$	$\mathbb{F}$	$\neg\mathbb{F}$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F	F	T	T	F
F	T	T	F	F	T	T	F

Table 12.19

(i) The entries in the columns corresponding to $p \vee \neg p$ and $\mathbf{T}$ are identical and hence they are equivalent. The entries in the columns corresponding to $p \wedge \neg p$ and $\mathbf{F}$ are identical and hence they are equivalent.

(ii) The entries in the columns corresponding to $\neg\mathbf{T}$ and $\mathbf{F}$ are identical and hence they are equivalent. The entries in the columns corresponding to $\neg\mathbf{F}$ and $\mathbf{T}$ are identical and hence they are equivalent.

7. Involution Law or Double Negation Law

$\neg(\neg p) \equiv p$

Proof

p	$\neg p$	$\neg(\neg p)$
T	F	T
F	T	F

Table 12.20

The entries in the columns corresponding to $\neg(\neg p)$ and p are identical and hence they are equivalent.

8. De Morgan's Laws

(i) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

(ii) $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Proof of (i)

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Table 12.21

The entries in the columns corresponding to $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are identical and hence they are equivalent. Therefore $\neg(p \wedge q) \equiv \neg p \vee \neg q$. Dually (ii) $\neg(p \vee q) \equiv \neg p \wedge \neg q$ can be proved.

9. Absorption Laws

(i) $p \vee (p \wedge q) \equiv p$

(ii) $p \wedge (p \vee q) \equiv p$

p	q	$p \wedge q$	$p \vee q$	$p \vee (p \wedge q)$	$p \wedge (p \vee q)$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	F	T	F	F
F	F	F	F	F	F

Table 12.22

(i) The entries in the columns corresponding to $p \vee (p \wedge q)$ and p are identical and hence they are equivalent.

(ii) The entries in the columns corresponding to $p \wedge (p \vee q)$ and p are identical and hence they are equivalent.

Example 12.17

Establish the equivalence property: $p \rightarrow q \equiv \neg p \vee q$

Solution

p	q	$\neg p$	$p \rightarrow q$	$\neg p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Table 12.23

The entries in the columns corresponding to $p \rightarrow q$ and $\neg p \vee q$ are identical and hence they are equivalent.

Example 12.18

Establish the equivalence property connecting the bi-conditional with conditional:

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Solution

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Table 12.24

Compound Proposition

Can be replaced by one that is logically equivalent to it without changing the truth value of the compound proposition.

Example 6

Show that ¬(p→q) & p∧¬q are logically equivalent.

$\neg(p \rightarrow q) \equiv \neg(\neg p \vee q)$, Conditional disjunction(Example 3)

$\equiv \neg(\neg p) \wedge \neg q$, Second De Morgan's law

$\equiv p \wedge \neg q$, Double negation law

Example 7 Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

Solution: $(p \wedge q) \rightarrow (p \vee q) \equiv \neg(p \wedge q)\ \neg\ (p \vee q)$ by Example-2

$p \rightarrow q \equiv \neg(p \vee q)$ (by example-2)

$\equiv(\neg p \vee \neg q) \vee (p \vee q)$ First De Morgan's law

$\equiv(\neg p \vee p) \vee (\neg q \vee q)$ Associative & Commutative

$\equiv T \vee T$ (Example-1)

$\equiv T \vee T$ (domination law)

$\equiv T$

proved

Example - 8 Show that $\neg(p \vee (\neg p \wedge q))$ & $\neg p \wedge \neg q$ are logically equivalent.

Solution: (do itself)

Dual compound propositions

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Predicate Logics

Predicates and quantifiers

Predicate
Predicate is a property of statements .As for example, $x>3$

Subject of statement   predicate

$x=y+3$ & $x+y=z$

#Let $p(x)$ denote the statement $x>3$. What are the true values of $p(4)$ &$p(2)$.

Hint: $4>3$ is true

$2>3$ is false

Quantifiers

Quantifiers are to create a proposition from a propositional function. This process is called quantification. Quantifiers are two ways.

1. Universal quantification
2. Existential quantification

1. Universal quantification

The universal quantification of p(x) is the proposition "p(x) is true for all values of x in the universe of discourse."

Where ∀ x p(x) denotes the universal quantification of p(x).

Here, ∀ is called universal quantifier.

∀ x p(x) is read as "for all x p(x)".

Example 1

Let P(x) be the statement "x+1>x". What is the truth value of the quantification?

∀ xP(x),where the universe of discourse consists of all real numbers?

Solution:

Since P(x) is true for all real numbers x, the quantification ∀ xP(x) is true.

Example 2

Let Q(x) be the statement "x<2".What is the truth value of the quantification ∀ xQ(x),where the universe of discourse consists of all real numbers?

Solution:

Q(x) is not true for every real number x. Since Q(3) is false.Thus, ∀ xQ(x) is false.

When all the elements in the universe of discourse can be x_1, x_2,……x_n follows that ∀x P(x) is the same as conjunction $P(x_1) \wedge P(x_2) \wedge \ldots\ldots \wedge P(x_n)$. Since this conjunction is true.

2. Existential quantifier

The existential quantification of p(x) is the proposition "There exists an element x in the universe of discourse such that P(x) is true".It is expressed as ∃ xp(x).

∃ is called existential quantifier. ∃ xp(x) is read as" There is an x such that P(x)".

Example 1

Let P(x) denote the statement "x>3".What is the truth value of the quantification

∃ xP(x),where the universe of discourse consists of all real numbers?

Solution:

Since "x>3" is true, when x=4.The existential quantification of P(x) which is ∃ xP(x) is true.

Precedence of Quantifiers

The quantifier's ∀ & ∃ have higher precedence than all logical operators from propositional calculus. As for example, ∀ x P(x) ∨ Q(x) is the disjunction of ∀ x P(x) & Q(x).

Binding Variables

The quantifiers are said to bind the variable x in these expressions. Variables in the scope of some quantifier are called bound variables. All other variables in the expression are called free variables. A propositional function that does not contain any free variables is a proposition and has a truth value. This can be done using a combination of universal quantifiers, existential quantifiers & value assignments.

Negating Quantified Statements

Earlier we have explain example in which the statement **$\forall x : x^2 > 2$** is false and **$\forall x : x^2 +1 > 0$** is true for x = 1. The first statement is false because x =1 is unable to satisfy the predicate. In this case, we find a solution that says we can negate a ∀ statement by flipping ∀ into ∃. After that, we will negate the predicate inside.

For example: The negation of $\forall x : P(x)$ is $\exists x : \boxed{P(x)}$.

If the statement predicate **$\forall x : P(x)$** is true, then $\exists x : \boxed{P(x)}$. Here, the x that satisfies $\boxed{P(x)}$ is known as the counterexample that claims **$\forall x : P(x)$.** Similarly, if we want to negate $\exists x : P(x)$, we have to claim that P(x) fails to hold for any value of x. So we again flip the quantifier and then negate the predicate like this:

the negation of $\exists \boxed{x : P}(x)$ is $\forall x : P(x)$

$\exists x \neg P(x)$

$\neg \exists x\ P(x) \longrightarrow$ negation

$\forall x\ \neg P(x)$

$\neg \forall x\ P(x) \longrightarrow$ negation

Nested Quantifiers

The nested quantifier is used by a lot of serious mathematical statements. For example:

Let us assume a statement that says, "For every real number, we have a real number which is greater than it". We are going to write this statement like this:

$\forall x\ \exists y: y > x$

Or assume a statement that says, "We have a Boolean formula such that every truth assignment to its variables satisfies it". We are going to write this statement like this:

∃ formula F ∀ assignments A: A satisfies F.

Rule of inferences (Induction & Reasoning)

Mathematical logic is often used for logical proofs. Proofs are valid arguments that determine the truth values of mathematical statements.

An argument is a sequence of statements. The last statement is the conclusion and all its preceding statements are called premises (or hypothesis). The symbol "∴", (read

therefore) is placed before the conclusion. A valid argument is one where the conclusion follows from the truth values of the premises.
Modus ponens states that if both an implication & its hypothesis are known to be true then the conclusion of this implication is true.

Rules of Inference provide the templates or guidelines for constructing valid arguments from the statements.

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \wedge (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \wedge (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \to q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p q $\therefore p \wedge q$	$((p) \wedge (q)) \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$	Resolution

Example 3

State which rule of inference is the basis of the following argument: “It is below freezing now. Therefore, it is below freezing or raining now.”

Solution: Let p be proposition "It is below freezing now" & q be proposition "It is raining now". Then, this argument is of the form

$$\frac{p}{\therefore p \vee q.}$$

.This is an argument that uses the addition rule.

Example 4

It is below freezing & raining now.

This argument of the form $\dfrac{p \wedge q}{\therefore p}$

Valid arguments

An argument form is called valid if whenever all the hypothesis are true, the conclusion is also true.

$(p_1 \wedge p_2 \wedge \ldots\ldots\ldots \wedge p_n) \rightarrow q$ is true.

Resolution

Many of these programs make use of a rule of inference known as resolution. This rule of inference is based on the tautology.

$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$

Final disjunction in the resolution rule, q r is called resolvent.

Fallacies

Incorrect argument is called fallacies. The proposition $[[p \rightarrow q) \wedge q] \rightarrow p$ is not a tautology. Since it is false when p is false & q is true. This type of incorrect reasoning is called the fallacy of affirming the conclusion.

Example 1

If you do every problem in this book, then you will learn discrete mathematics. You learned discrete mathematics.

Solution:

Let p be proposition "You did every problem in this book. Let q be the proposition "you learned discrete mathematics. The, this argument is of the form: if $p \rightarrow q$ & q, then p. This is an example of incorrect arguments.

Methods of proving Theorems

Direct proofs

$p \to q$