In [1]:
```
import numpy as np
from prettytable import PrettyTable
import matplotlib.pyplot as plt
```

In [2]:
```
#A. Finding Roots of the Equations:
```

In [3]:
```
#q1
```

In [4]:
```
def f(x):
    return x*x - np.sin(x)
```

In [5]:
```
tab = PrettyTable()
a = 0.5
b = 1
eps = 0.0001
tab.field_names = ["Step", "a","b","x","f(x)"]
i =0
```

In [6]:
```
while (f(a)*f(b)<0):
    x = (a+b)/2
    tab.add_row([i,"%.4f"%a,"%.4f"%b,"%.4f"%x,"%.4f"%f(x)])
    i = i+1
    if (f(x) == 0):
        break
    elif (f(x)*f(a)<0):
        b = x
        if(abs(a-x)<eps):
            break
    else:
        a = x
        if(abs(b-x)<eps):
            break
```

In [7]:
```
print(tab)
```

```
+------+--------+--------+--------+---------+
| Step |   a    |   b    |   x    |   f(x)  |
+------+--------+--------+--------+---------+
|  0   | 0.5000 | 1.0000 | 0.7500 | -0.1191 |
|  1   | 0.7500 | 1.0000 | 0.8750 | -0.0019 |
|  2   | 0.8750 | 1.0000 | 0.9375 |  0.0728 |
|  3   | 0.8750 | 0.9375 | 0.9062 |  0.0341 |
|  4   | 0.8750 | 0.9062 | 0.8906 |  0.0157 |
|  5   | 0.8750 | 0.8906 | 0.8828 |  0.0068 |
|  6   | 0.8750 | 0.8828 | 0.8789 |  0.0024 |
|  7   | 0.8750 | 0.8789 | 0.8770 |  0.0003 |
|  8   | 0.8750 | 0.8770 | 0.8760 | -0.0008 |
|  9   | 0.8760 | 0.8770 | 0.8765 | -0.0003 |
|  10  | 0.8765 | 0.8770 | 0.8767 | -0.0000 |
|  11  | 0.8767 | 0.8770 | 0.8768 |  0.0001 |
|  12  | 0.8767 | 0.8768 | 0.8768 |  0.0000 |
+------+--------+--------+--------+---------+
```

In [8]:
```
#q2
```

In [9]:
```
def f(x):
    return (4*x - np.exp(x))
```

In [10]:
```
def f1(x):
    return (4 - np.exp(x))
```

In [11]:
```
x0 = 1
eps = 0.0001
a= eps+1
t = PrettyTable()
```

In [12]:
```
t.field_names = ["Step", "Xi","f(Xi)"]
i = 0
while abs(a)>eps:
    t.add_row([i, "%.4f"%x0, "%.4f"%f(x0)])
    i = i+1
    a = f(x0) / f1(x0)
    x0 = x0 - a
```

In [13]:
```python
print(t)
```

```
+------+--------+---------+
| Step |   Xi   |  f(Xi)  |
+------+--------+---------+
|  0   | 1.0000 |  1.2817 |
|  1   | 0.0000 | -1.0000 |
|  2   | 0.3333 | -0.0623 |
|  3   | 0.3572 | -0.0004 |
|  4   | 0.3574 | -0.0000 |
+------+--------+---------+
```

In [14]:
```python
#B. Interpolation
```

In [15]:
```python
class Values:
    def __init__(self, x_values, y_values):
        self.n = len(x_values)
        self.table = [[0 for i in range(self.n+1)] for j in range(self.n)]
        for i in range(self.n):
            self.table[i][0] = x_values[i]
            self.table[i][1] = y_values[i]

    def forward(self):
        for j in range(2, self.n+1):
            for i in range(self.n-j+1):
                self.table[i][j] = self.table[i+1][j-1] - self.table[i][j-1]
        Table = PrettyTable()
        field_names = ["x", "y"]
        for i in range(2, self.n+1):
            field_names.append("Del"+str(i-1)+"y")
        Table.field_names = field_names
        for i in range(self.n):
            row = [round(self.table[i][0],4), round(self.table[i][1],4)]
            for j in range(2, self.n+1):
                if i > self.n-j:
                    row.append(" ")
                else:
                    row.append(round(self.table[i][j],4))
```

```
            Table.add_row(row)
        print(Table)

    def backward(self):
        for j in range(2, self.n+1):
            for i in range(j-1, self.n):
                self.table[i][j] = self.table[i][j-1] - self.table[i-1][j-1]
        Table = PrettyTable()
        field_names = ["x", "y"]
        for i in range(2, self.n+1):
            field_names.append("nabla"+str(i-1)+"y")
        Table.field_names = field_names
        for i in range(self.n):
            row = [round(self.table[i][0],4), round(self.table[i][1],4)]
            for j in range(2, self.n+1):
                if i < j-1:
                    row.append(" ")
                else:
                    row.append(round(self.table[i][j],4))
            Table.add_row(row)
        print(Table)

    def central(self):
        forward(self)
```

In [16]:

```
#q3
```

In [17]:

```
x= np.linspace(-1,1,21)
y = np.exp(x)
tab = Values(x, y)
tab.forward()
```

x	y	Del1y	Del2y	Del3y	Del4y	Del5y	Del6y	Del7y	Del8y	Del9y	Del10y	Del11y	Del12y	Del13y	Del14y	Del15y	Del16y	Del17y	Del18y	Del19y	Del20y
-1.0	0.3679	0.0387	0.0041	0.0004	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.0	0.0	-0.0	0.0	-0.0	0.0	-0.0	0.0
-0.9	0.4066	0.0428	0.0045	0.0005	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.0	0.0	-0.0	0.0	-0.0	0.0	
-0.8	0.4493	0.0473	0.005	0.0005	0.0001	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.0	0.0	-0.0	0.0		
-0.7	0.4966	0.0522	0.0055	0.0006	0.0001	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.0	0.0	-0.0	0.0			
-0.6	0.5488	0.0577	0.0061	0.0006	0.0001	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.0	0.0	-0.0	0.0				
-0.5	0.6065	0.0638	0.0067	0.0007	0.0001	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.0	0.0					
-0.4	0.6703	0.0705	0.0074	0.0008	0.0001	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.0	0.0						
-0.3	0.7408	0.0779	0.0082	0.0009	0.0001	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0							
-0.2	0.8187	0.0861	0.0091	0.001	0.0001	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0								
-0.1	0.9048	0.0952	0.01	0.0011	0.0001	0.0	0.0	0.0	0.0	0.0	0.0	0.0									
0.0	1.0	0.1052	0.0111	0.0012	0.0001	0.0	0.0	0.0	0.0	0.0	0.0										
0.1	1.1052	0.1162	0.0122	0.0013	0.0001	0.0	0.0	0.0	0.0	0.0											
0.2	1.2214	0.1285	0.0135	0.0014	0.0001	0.0	0.0	0.0	0.0												
0.3	1.3499	0.142	0.0149	0.0016	0.0002	0.0	0.0	0.0													
0.4	1.4918	0.1569	0.0165	0.0017	0.0002	0.0	0.0														
0.5	1.6487	0.1734	0.0182	0.0019	0.0002	0.0															
0.6	1.8221	0.1916	0.0202	0.0021	0.0002																
0.7	2.0138	0.2118	0.0223	0.0023																	

```
| 0.8  | 2.2255 | 0.2341 | 0.0246 |        |        |       |       |       |       |       |        |        |
|        |        |        |        |        |        |        |        |
| 0.9  | 2.4596 | 0.2587 |        |        |        |       |       |       |       |       |        |        |
|        |        |        |        |        |        |        |        |
| 1.0  | 2.7183 |        |        |        |        |       |       |       |       |       |        |        |
|        |        |        |        |        |        |        |        |
+------+--------+--------+--------+--------+--------+-------+-------+-------+-------+-------+--------+--------+-----
---+--------+--------+--------+--------+--------+--------+--------+--------+
```

In [18]:

```
#q4
```

In [19]:

```
x = [0.20 ,0.22 ,0.24 ,0.26 ,0.28 ,0.30]
y = [1.6596 ,1.6698, 1.6804, 1.6912, 1.7024, 1.7139]
tab = Values(x,y)
tab.forward()
```

```
+------+--------+--------+--------+---------+---------+---------+
|  x   |   y    | Del1y  | Del2y  |  Del3y  |  Del4y  |  Del5y  |
+------+--------+--------+--------+---------+---------+---------+
| 0.2  | 1.6596 | 0.0102 | 0.0004 | -0.0002 |  0.0004 | -0.0007 |
| 0.22 | 1.6698 | 0.0106 | 0.0002 |  0.0002 | -0.0003 |         |
| 0.24 | 1.6804 | 0.0108 | 0.0004 | -0.0001 |         |         |
| 0.26 | 1.6912 | 0.0112 | 0.0003 |         |         |         |
| 0.28 | 1.7024 | 0.0115 |        |         |         |         |
| 0.3  | 1.7139 |        |        |         |         |         |
+------+--------+--------+--------+---------+---------+---------+
```

In [20]:

```
X = 0.21
h = x[1]-x[0]
p = (X-x[0])/h
Y = y[0] + tab.table[2][0]*p+(p*p-p)*0.5*tab.table[3][0]+p*(p-1)*(p-2)*tab.table[4][0]/6
print(Y)
```

```
1.7646
```

In [21]:

```
x = [0.20 ,0.22 ,0.24 ,0.26 ,0.28 ,0.30]
y = [1.6596 ,1.6698, 1.6804, 1.6912, 1.7024, 1.7139]
tab = Values(x,y)
tab.backward()
```

```
+------+--------+---------+---------+---------+---------+---------+
|  x   |   y    | nabla1y | nabla2y | nabla3y | nabla4y | nabla5y |
+------+--------+---------+---------+---------+---------+---------+
| 0.2  | 1.6596 |         |         |         |         |         |
| 0.22 | 1.6698 |  0.0102 |         |         |         |         |
| 0.24 | 1.6804 |  0.0106 |  0.0004 |         |         |         |
| 0.26 | 1.6912 |  0.0108 |  0.0002 | -0.0002 |         |         |
| 0.28 | 1.7024 |  0.0112 |  0.0004 |  0.0002 |  0.0004 |         |
| 0.3  | 1.7139 |  0.0115 |  0.0003 | -0.0001 | -0.0003 | -0.0007 |
+------+--------+---------+---------+---------+---------+---------+
```

In [22]:
```
X = 0.29
h = x[1]-x[0]
p = (X-x[5])/h
Y = y[5] + tab.table[2][5]*p+(p*p+p)*0.5*tab.table[3][5]+p*(p+1)*(p+2)*tab.table[4][5]/6
print(Y)
```

```
1.713875
```

In [23]:
```
#q5
```

In [24]:
```
x = [0, 1, 3, 4, 5]
y = [0, 1, 81, 256, 625]
```

In [25]:
```
def lagrange(Xval,Yval,x):
    y = 0
    for i in range(len(Xval)):
        n = d = 1
        for j in range(len(Xval)):
            if i != j:
                n = n*(x-Xval[j])
                d = d*(Xval[i]-Xval[j])
        y = y + n*Yval[i]/d
    return(round(y,4))
```

In [26]:
```
print(lagrange(x,y,2))
```

```
16.0
```

In [27]:
```
#C. Numerical Integration
```

In [28]:
```
#q6
```

In [29]:
```
def function(x):
    y = np.sin(x)/np.exp(x)
    return y
```

In [30]:
```
def trapezoid(f,l,s):#first last step
    x = np.linspace(f,l,s+1)
    I = 0
    for i in range(len(x)):
        if x[i]>f or x[i]<l:
            I += 2*function(x[i])
        else:
            I += function(x[i])

    return I * 0.5* (x[1]-x[0])
```

In [31]:
```
trapezoid(0,np.pi,20)
```

Out[31]:
```
0.5194637027711455
```

In [32]:
```
#q7
```

In [33]:
```
def f1(x):
    y = np.exp(-x*x/2)
    return y
```

In [34]:
```
def sim3(f,l,s):#first last step
    x = np.linspace(f,l,s+1)
    I = 0
    for i in range(len(x)):
        if x[i]>f or x[i]<l:
            if i%2 == 0:
                I += 2*f1(x[i])
```

```python
            else:
                I += 4*f1(x[i])
        else:
            I += f1(x[i])
    I = I*np.sqrt(1/(2*np.pi))
    I = I * (x[1]-x[0])/3
    return I
```

In [35]:
```python
sim3(-4,4,50)
```

Out[35]: 0.9999508833741578

In [36]:
```python
#D. Numerical Differential Equations:
```

In [37]:
```python
def f1(x):
    return x*x+x
```

In [38]:
```python
#q8
```

In [39]:
```python
x = np.linspace(0,2,21)
y = np.zeros(len(x))
h = x[1]-x[0]
y[0]= 1
MT = PrettyTable()
MT.field_names = ["X", "Y"]
```

In [40]:
```python
MT.add_row([round(x[0],4),round(y[0],4)])
for i in range(1,len(y)):
    y[i] = y[i-1] +h*f1(x[i-1])
    MT.add_row([round(x[i],4),round(y[i],4)])
```

In [41]:
```python
print(MT)
```

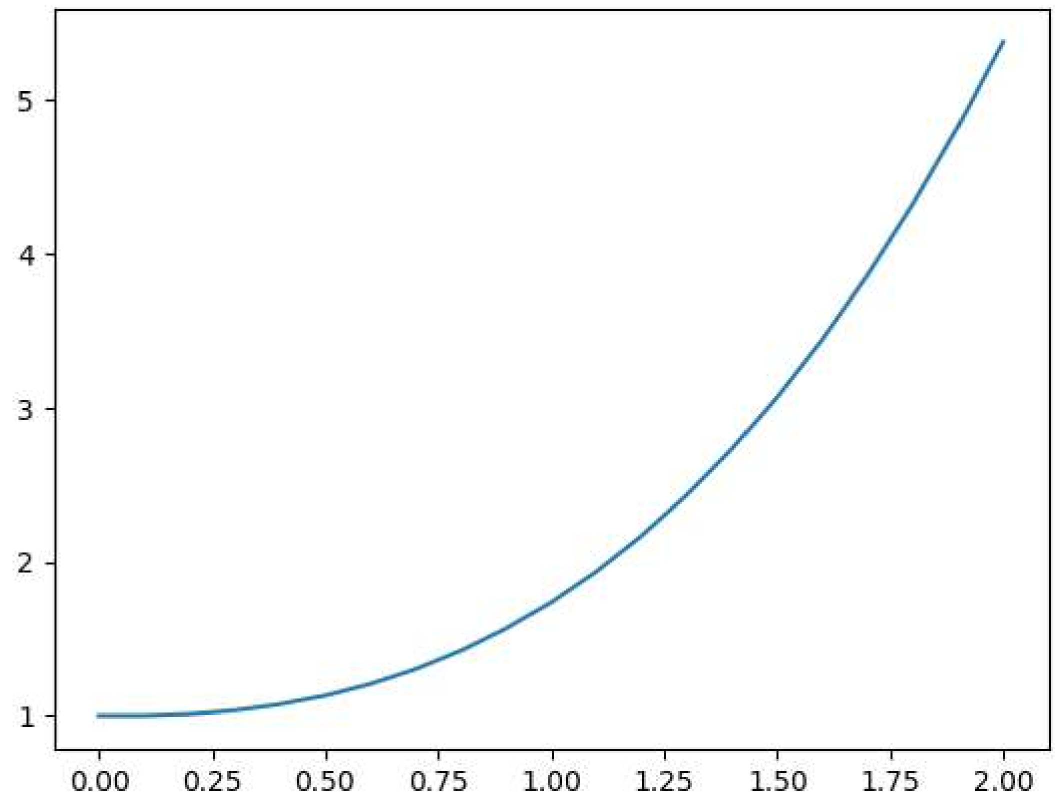
X	Y
0.0	1.0
0.1	1.0
0.2	1.011
0.3	1.035
0.4	1.074
0.5	1.13
0.6	1.205
0.7	1.301
0.8	1.42
0.9	1.564
1.0	1.735
1.1	1.935
1.2	2.166
1.3	2.43
1.4	2.729
1.5	3.065
1.6	3.44
1.7	3.856
1.8	4.315
1.9	4.819
2.0	5.37

In [42]:

```
plt.plot(x,y)
```

Out[42]:

```
[<matplotlib.lines.Line2D at 0x1ce82a9dac0>]
```



In [43]:
```
#q9
```

In [44]:
```
x = np.linspace(0,2,11)
y = np.zeros(len(x))
y[0] = 1
MT = PrettyTable()
MT.field_names = ["X", "Y"]
```

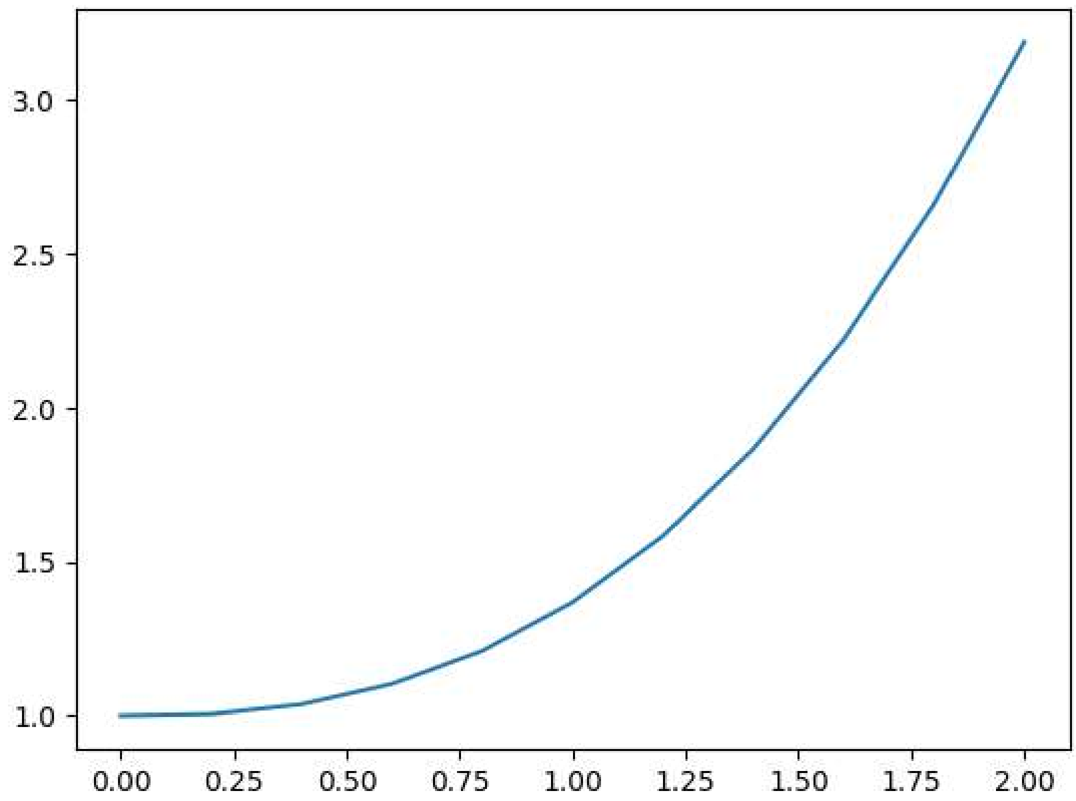
In [45]:
```
MT.add_row([round(x[0],4),round(y[0],4)])
for i in range(1,len(y)):
    y[i] = y[i-1] + h *  0.5 * (f1(x[i-1]) + f1(x[i-1] + h) )
    MT.add_row([round(x[i],4),round(y[i],4)])
```

In [46]:
```
print(MT)
```

```
+-----+--------+
|  X  |   Y    |
+-----+--------+
| 0.0 |  1.0   |
| 0.2 | 1.0055 |
| 0.4 | 1.037  |
| 0.6 | 1.1025 |
| 0.8 |  1.21  |
| 1.0 | 1.3675 |
| 1.2 | 1.583  |
| 1.4 | 1.8645 |
| 1.6 |  2.22  |
| 1.8 | 2.6575 |
| 2.0 | 3.185  |
+-----+--------+
```

In [47]:
```
plt.plot(x,y)
```

Out[47]:
```
[<matplotlib.lines.Line2D at 0x1ce82c3d580>]
```



In [48]:
```
#q10
```

In [49]:
```
x = np.linspace(0, 1, 11)
y = np.zeros(len(x))
y[0] = y[-1] = 0
MT = PrettyTable()
MT.field_names = ["X", "Y"]
```

In [50]:
```
A = np.zeros((9, 9))
B = np.zeros(9)
h = x[1] - x[0]
p = -64
q = 0
r = -10
```

In [51]:
```
a = 1 - 0.5 * h * p
b = -2 + h * h * q
c = 1 + 0.5 * h * p
d = h*h*r*r
```

In [52]:
```
np.fill_diagonal(A, b)
np.fill_diagonal(A[1:], a)
np.fill_diagonal(A[:, 1:], c)
B = np.full(9, d)


B[0] -= a * y[0]
B[-1] -= c * y[-1]
```

In [53]:
```
y[1:-1] = np.linalg.solve(A, B)
```

In [54]:
```
for i in range(len(x)):
    MT.add_row([round(x[i],1), round(y[i],3)])
```

In [55]:
```
print(MT)
```

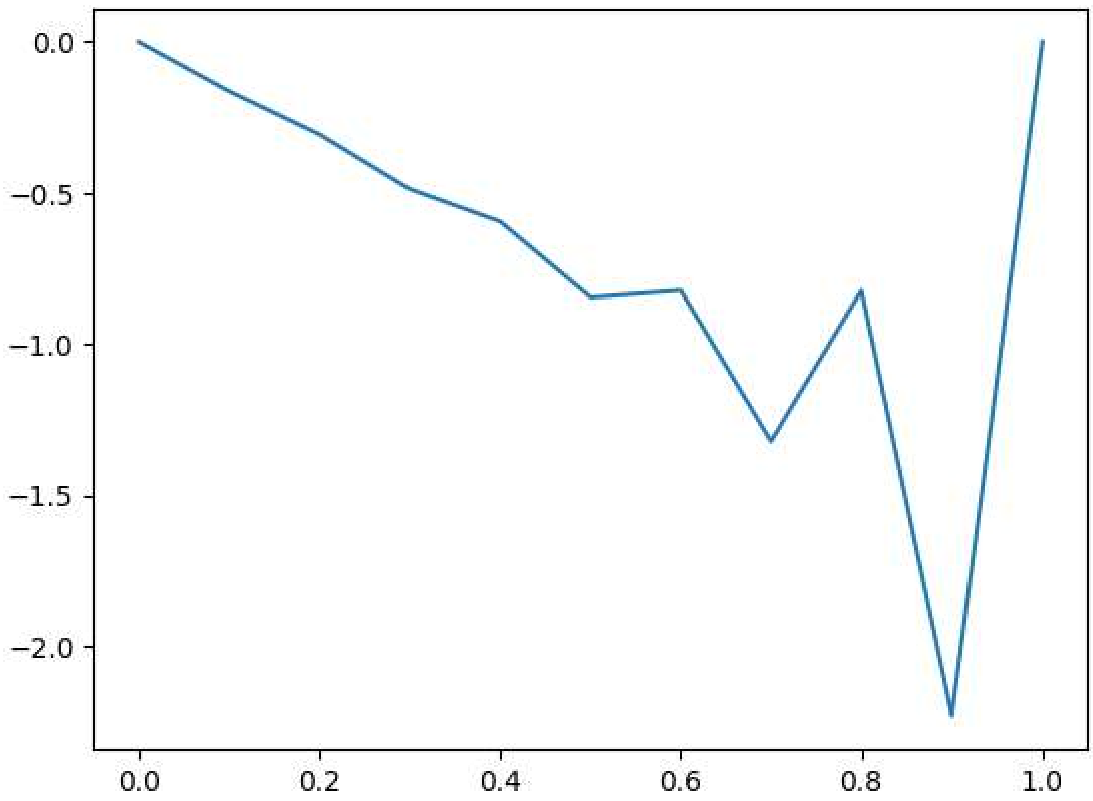
```
+-----+--------+
|  X  |   Y    |
+-----+--------+
| 0.0 |  0.0   |
| 0.1 | -0.163 |
| 0.2 | -0.306 |
| 0.3 | -0.488 |
| 0.4 | -0.595 |
| 0.5 | -0.845 |
| 0.6 | -0.822 |
| 0.7 | -1.321 |
| 0.8 | -0.823 |
| 0.9 | -2.228 |
| 1.0 |  0.0   |
+-----+--------+
```

In [56]:

```
plt.plot(x, y)
```

Out[56]:

```
[<matplotlib.lines.Line2D at 0x1ce82b686a0>]
```



In [57]:
```
#E. Curve Fitting
```

In [58]:
```
#q11
```

In [59]:
```
x = np.array([1, 2, 3, 4, 5, 6])
y = np.array([2.4, 3.1, 3.5, 4.2, 5.0, 6.0])
m = len(x)
```

In [60]:
```
m = len(x)
xy_sum = np.sum(x * y)
x_sum = np.sum(x)
```

```
y_sum = np.sum(y)
x_squared_sum = np.sum(x ** 2)
```

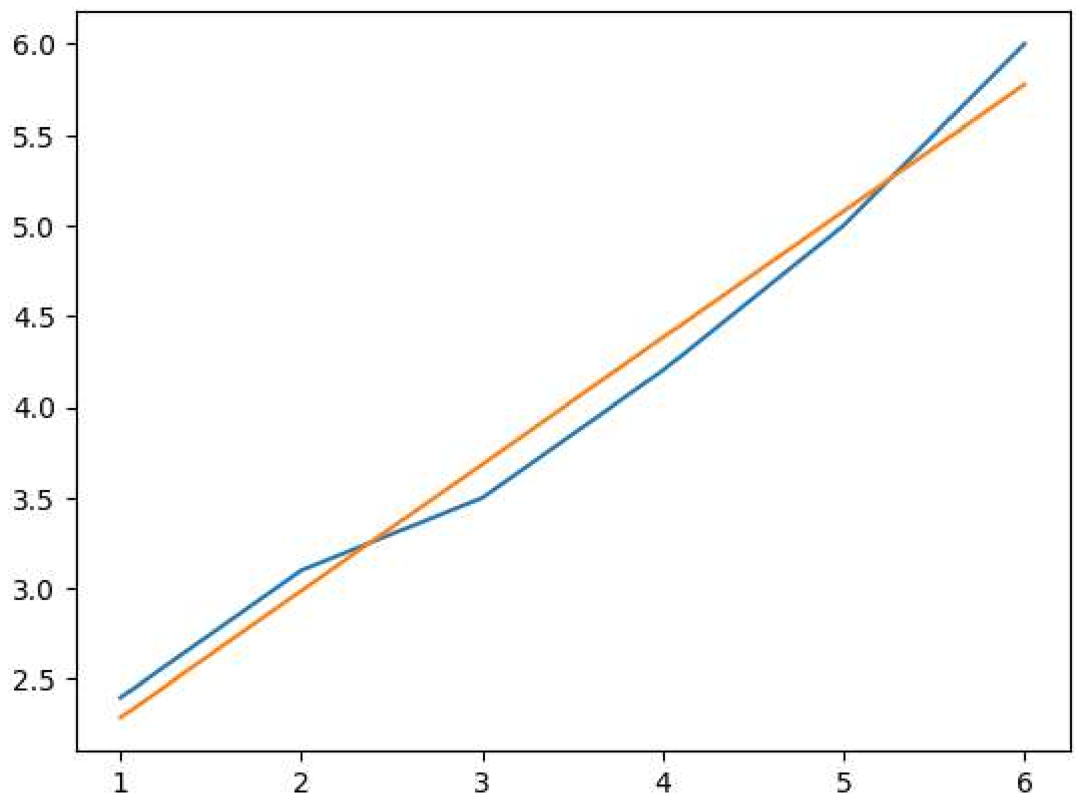
In [61]:
```
a1 = (m * xy_sum - x_sum * y_sum) / (m * x_squared_sum - x_sum ** 2)
a0 = (y_sum - a1 * x_sum) / m
```

In [62]:
```
def y1(x):
    return a0 + a1 *x
print(y1(2.5))
```

```
3.336190476190475
```

In [63]:
```
plt.plot(x,y)
plt.plot(x,y1(x))
```

Out[63]:
```
[<matplotlib.lines.Line2D at 0x1ce82bdb820>]
```



In [64]:
```
#q12
```

In [65]:
```
x = np.array([2,4,6,8,10])
y =  np.array([4.077, 11.084, 30.128, 81.897, 222.62])
```

In [66]:
```
Y = np.log(y)
m = len(x)
xY_sum = np.sum(x * Y)
x_sum = np.sum(x)
Y_sum = np.sum(Y)
x_squared_sum = np.sum(x ** 2)
```

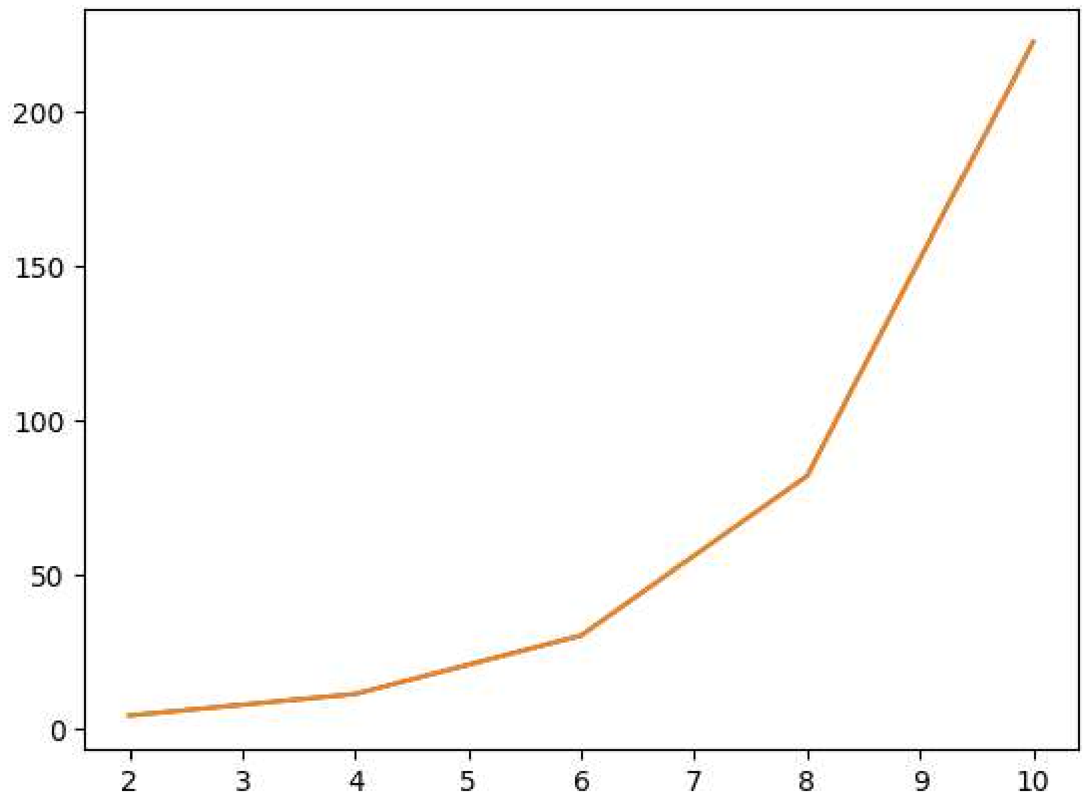
In [67]:
```
a1 = (m * xY_sum - x_sum * Y_sum) / (m * x_squared_sum - x_sum ** 2)
a0 = (Y_sum - a1 * x_sum) / m
```

In [68]:
```
def y2(x):
    Y2 = a0 + a1*x
    return np.exp(Y2)
print(y2(9))
```

```
135.027025924343
```

In [71]:
```
plt.plot(x,y)
plt.plot(x,y2(x))
```

Out[71]:
```
[<matplotlib.lines.Line2D at 0x1ce82ba3520>]
```


0
50
100
150
200
2
3
4
5
6
7
8
9
10