**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans. Finding Z-Scores at X=50;

Z = (X - µ) / σ

Z = (50 - 45) **/** 8

Z = 0.625

Finding probability P(X>50) = 1-stats.norm.cdf(abs(z\_score))

1. stats**.**norm**.**cdf(abs(0.625))

Is 0.265

OR Find probability P(X<=50); p\_value=stats.norm.cdf(abs(z\_score))

p\_value=stats.norm.cdf(abs(0.625))

p\_value

Is 0.734

P(X>50) = 1 - P(X<=50)

1 - 0.734

Is 0.266

So the answer is B. 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans. a) probability of employees >44 = Pr(x>44)=1-Pr(x<=44)

Z = (X-μ)/σ *= (X - 38)/6*

Pr(X<=44) = Pr(z<=(44 - 38)/6)

Pr(z<=1) = .84134 = 84.134%

Probability that employee will be greater then 44 = 100 - 84.134 = 15.866

Probability of employees between 38 & 44 Pr(x<=44) - Pr(x>=38)

Here, Pr(x<=44) = 0.84134

Pr(x>=38) = Pr(z>=(38-38)/6) = Pr(z>=0) = 0.5

Therefore, Pr(x<=44) - Pr(x>=38) = 0.84134 - 0.5 = 0.34134 = 34.134%

So, There are more employees at the processing center are orlder then 44 in between

38 and 44 so it is TRUE.

b) Probability of employees less then 30 = Pr(x<30)

Z = (X - μ)/σ *= (30 - 38)/6*

Pr(x<30) = Pr(z<(30 - 38)/6)

= Pr(z< -1.3333)

= .09176

= 9.17%

So, number of employees with probability 0.0917 of are under 30 = 400\*0.0917 = 36.68 = 36

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans. The distribution of 2*X*1 is normal with a mean of 2μ and a variance of 4σ^2

The distribution of is normal with a mean of 2μ and a variance of 2σ^2.

Both random variables have the same mean (2μ), but the variance of *X*1 is larger (4σ^2) than the variance of *X*1 + *X*2 (2σ^2). This means that *X*1 + *X*2 is less variable than *X*1.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans. The probability of getting value b/w a and b is 0.99

So the probability of getting value outside a and b is 1 - 0.99 = 0.01

The probability towards left of a = -0.01/2 = 0.05

The probability towards right of b = 0.01/2 = 0.05

So let calculate the probability of X - the random variable at a& b which has these probability

By finding standard normal variable(z), need to calculate X;

Z = (X - μ)/σ

For a probability of 0.005, z value is -2.57

Z\*μ + σ = x

-(-2.57)\*20+100 = 151.4

(2.57)\*20+100 = 48.6

So the answer is d) 48.5,151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans.

*#*Mean profits from two different divisions of a company = Mean1 + Mean2

Mean **=** 5**+**7

print('Mean Profit is Rs', Mean**\***45,'Million')

Mean Profit is Rs 540 Million

# Variance of profits from two different divisions of a company = SD^2 = SD1^2 + SD2^2

SD **=** np**.**sqrt((9)**+**(16))

print('Standard Deviation is Rs', SD**\***45, 'Million')

Standard Deviation is Rs 225.0 Million

# A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

print('Range is Rs',(stats**.**norm**.**interval(0.95,540,225)),'in Millions')

Range is Rs (99.00810347848784, 980.9918965215122) in Millions

# B. Specify the 5th percentile of profit (in Rupees) for the company

# To compute 5th Percentile, we use the formula X=μ + Zσ; wherein from z table, 5 percentile = -1.645

X**=** 540**+**(**-**1.645)**\***(225)print('5th percentile of profit (in Million Rupees) is',np**.**round(X,))

5th percentile of profit (in Million Rupees) is 170.0

#Probability of Division 1 making a loss P(X<0)

stats**.**norm**.**cdf(0,5,3)

0.0477903522728147

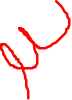
#*Probability of Division 2 making a loss P(X<0)*

stats**.**norm**.**cdf(0,7,4)

0.040059156863817086

#C. To compare the probability of making a loss for each division, we need to calculate the probability that each division's profit is negative.

For a normally distributed random variable *X* with mean *μ* and standard deviation *σ*, the probability of *X* being less than zero can be calculated using the cumulative distribution function (CDF) of the normal distribution:



P(X<0)=CDF(0,μ,σ)*P*(*X*<0)=CDF(0,*μ*,*σ*)



Let's calculate the probabilities for both divisions:

For Profit1:

* Mean (*μ*1​) = $5 million
* Standard deviation (*σ*1​) = √32 million

For Profit2:

* Mean (*μ*2​) = $7 million
* Standard deviation (*σ*2​) = √42 million

Using the cumulative distribution function (CDF) of the normal distribution, we can find the probabilities of making a loss for each division. Then we compare these probabilities to determine which division has a larger probability of making a loss.

Let's calculate:

First, let's calculate the standard deviations for both Profit1 and Profit2:

For Profit1: *σ*1​=√32​

For Profit2: *σ*2​=√42​

Next, let's calculate the z-scores for a profit of $0 million for both divisions using the formula:

Z

Z = 0-μ/σ

For Profit1: Z1 = 0-5/√32

For Profit2: Z2 = 0-7/√42​

Now, we'll use the cumulative distribution function (CDF) of the normal distribution to find the probabilities of making a loss for each division.

Let's calculate the z-scores and probabilities of making a loss for both divisions:

For Profit1: *σ*1​=√32​ , *μ*1​=5

For Profit2: *σ*2​=√42​ , *μ*2​=7

Using the standard normal distribution table or a statistical calculator, we can find the cumulative probabilities for each z-score.

Let's calculate:

For Profit1: *σ*1​=√32​=5.657 , *μ*1​=5

For Profit2: *σ*2​=√42​=6.4807 , *μ*2​=7

Now, let's calculate the z-scores:

For Profit1: Z1 = 0-5/5.567 ≈ −0.884

For Profit2: Z2 = 0-7/6.4807 ​≈ −1.079

Using a standard normal distribution table or a statistical calculator, we find the cumulative probabilities for Z1 and Z2.

For P(X1<0) = 0.1889

For P(X2<0) = 0.1406

Hence, the division associated with Profit1 has a larger probability of making a loss in a given year.

So the answer is Division by 2