C++
Bitwise Operations

Lecture- 37

Sanket Singh

Converting a decimal number to binary and vice versa

Decimal → binary

lsb → least signyficant bit

msb → most signyficant bit

\# Binary & decimal → 101

1101

msb

lsb

0 0 1 1 0 1 0 1

Converting a decimal number to binary and vice versa

PW SKILLS

Decimal

0 1 2 3 4 5 6 7 8 9

$(11)_{10} \rightarrow (\_\ \_\ \_\ \_\ \_)_2$

$\begin{array}{c} 0 \\ 1 \end{array}$

most significant digit

1 2 3 4

least significant place

$(183)_{10}$

$1 \times 10^2 + 8 \times 10^1 + 3 \times 10^0$

$\begin{array}{r} 100 \\ 80 \\ 3 \\ \hline 183 \end{array}$

$(1234)_{10} \rightarrow 1000 + 200 + 30 + 4$

$\rightarrow 1 \times 10^3 + 2 \times 10^2 + 3 \times 10^1 + 4 \times 10^0$

$6 \rightarrow 1\,1\,\cancel{0}\,1$

$\cancel{1}^{0}\;1\;0$

$7 \leftarrow 1\,1\,1$

$0\;1\;0 \rightarrow 2$

2

$(0\,0\,1\,1\,0\,1)$

2

$0\times2^5 + 0\times2^4 + 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$

$1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$

$8 + 4 + 0 + 1$

$8+4+1 \Rightarrow 13$

$1011 \rightarrow 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0$

$8 + 0 + 2 + 1$

$\rightarrow 11$

("0 0 1 1 1 0 1 1 0")

$$\begin{array}{ccccccccc} 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array}$$

9

index → bit pos

(n - index - 1)

decimal ?

ans → 0 + 0 + 2 + 4 + 0 + 16 + 32 + 64 + 0 + 0

$$\sum \text{char} \times (1 << (n - idx - 1))$$

$$\sum \left(\text{char} \times 2^{(n-idx-1)}\right)$$

$2^0 \to 1 \longrightarrow 1$

$2^1 \to 2 \longrightarrow 10$

$2^2 \to 4 \longrightarrow 100$

$2^3 \to 8 \longrightarrow 1000$

$2^4 \to 16 \longrightarrow 10000$

$2^5 \to 32 \longrightarrow 100000$

$\vdots$

$2^x \to \underbrace{10000\ldots\ldots}_{x \text{ times}}$

$7 \to 00000000111$

$7<<3$

00111000

$<< \to$ left shift

One ------ none or some zeroes

101

$5<<2$

$000 10100$

SKILLS

$y << x$

Giving $\underbrace{0\,0\,0\,0\,----}_{x \text{ zeros}}$

$1 << x \rightarrow 2^x$

$1\underbrace{0000.....}_{x \text{ zeros}}$

$1 << x \rightarrow 2^x$

Decimal → Binary

18 → $1\times10^1 + 8\times10^0$

1834 → $1\times10^3 + 8\times10^2 + 3\times10^1 + 4\times10^0$

$10 \,\overline{)\,1834\,}\,(183$

10

83

80

34

30

4

$10 \,\overline{)\,183\,}\,(18$

10

83

80

3

2	18	→	0
2	9	→	1
2	4	→	0
2	2	→	0
	1	→	1

$10010 \rightarrow 18$

PW SKILLS

0 → 0000 ✓
1 → 0001 ✓
2 → 0010 ✓
3 → 0011 ✓
4 → 0100 ✓
5 → 0101 ✓
6 → 0110 ✓
7 → 0111 ✓
8 → 1000 ✓

even → lsb → 0

odd → lsb → 1

4

18 → 9

(18 % 2 == 0)

_ _ _ 0 1 0

(9 % 2 == 0);

5 → 010~~1~~

5 >> 1

right shift

010~~1~~ → 2

5/2 → 2

$$\begin{array}{cccccc} 5 & 4 & 3 & 2 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{array}$$

$$2^5 + 2^4 + 2^2 + 2^1 \rightarrow 32 + 16 + 4 + 2$$

$$2 \,|\, 54$$

PW SKILLS

x

$5 \to 101$

$x << 1$

$5 << 1 \to 5 \times 2$

$0\,101\cancel{0} \to 10$

$10 >> 1 \to 5$

$(10/2)$

$1 << y$

2^y

$x << 1 \rightarrow x \times 2$

$x << 2 \rightarrow x \times 2^2$

$x << 3 \rightarrow x \times 2^3$

$x >> 1 \rightarrow x/2$

$x >> 2 \rightarrow x/2^2$

$x >> 3 \rightarrow x/2^3$

$1 << 3$

1×2^3

2^3

Practice

__builtin_popcount(n) →

Find the number of set bits for any integer n.

set bits

5 → 0000 0101

8 bit

13 → 00001101 → 3

19 → 00010011 → 3

$13 \rightarrow 1101$

$12 \rightarrow 1100$

$16 \rightarrow 10000$

$15 \rightarrow 01111$

x

$x-1$

$10 \rightarrow 1010$

$9 \rightarrow 1001$

$3 \rightarrow 011$

$2 \rightarrow 010$

$6 \rightarrow 110$

$5 \rightarrow 101$

$23 \rightarrow 10111$

$22 \rightarrow 10110$

$22 \rightarrow 10110$

$21 \rightarrow 10101$

last set bit

$x \rightarrow$ ______ ______

$x-1 \rightarrow$ ______ ______

Same flipped

$13 \rightarrow 1101$

$12 \rightarrow 1100$

$6 \rightarrow 110$

$5 \rightarrow 101$

x

$22 \rightarrow 10110$

$21 \rightarrow 10101$

$16 \rightarrow 10000$

$15 \rightarrow 01111$

$9 \rightarrow 1001$

$8 \rightarrow 1000$

$23 \rightarrow 10111$

$22 \rightarrow 10110$

7 6 5 4 3 2 1 0

204 11001100

128 + 64 + 8 + 4

192 200 204

203 11001011

128 + 64 + 8 + 2 + 1

192 200 203

Same   flipped

22

&

21

$x \rightarrow$ 10110

$x-1 \rightarrow$ 10101

10100

$x \,\&\, x-1$

bitwise and

22 & 21

$22 \rightarrow \quad 22 \,\&\, 21 \rightarrow 10110 \,\&\, 10101$

$20 \rightarrow 10100$

$20 \,\&\, 19 \rightarrow 10100 \,\&\, 10011$

$16 \rightarrow \underline{10000}$

$16 \,\&\, 15 \rightarrow 10000 \,\&\, 01111$

$\rightarrow 0$

$$\boxed{x = x \,\&\, x-1}$$

Brian Kernighan's

22 → _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

64 bit

PW SKILLS

Practice

Given an integer n, find the maximum power of two that is smaller than n.

$n = 32 \rightarrow 32$

$y < x$

$n = 24 \rightarrow 16$

$n = 100 \rightarrow 64$

2^x

$1\underbrace{0000}_{x \text{ no. of zeros}}$

6 5 4 3 2 1 0

x 0 0 1 0 1 1 0 1 0

y 1 0 0 0 0 0 0

64 + 16 + 8 + 2

64 + 26

90

4 3 2 1 0

1 1 0 0 0 ← 24

16 + 8 → 24

1 0 0 0 0 ← 2^4 = 16

Practice

Given an integer n, find the maximum power of two that is smaller than n.

$\times 2$

$x \rightarrow 1\,0\,1\,1\,0\,1\,0$

$\&\quad x-1 \quad 1\,0\,1\,1\,0\,0\,1$

$x \quad 1\,0\,1\,1\,0\,0\,0$

$\&\quad x-1 \quad 1\,0\,1\,0\,1\,1\,1$

$x \quad 1\,0\,1\,0\,0\,0\,0$

$\&\quad x-1 \quad 1\,0\,0\,1\,1\,1\,1$

$x \quad 1\,0\,0\,0\,0\,0\,0$

$\&\quad x-1$

$0 \quad 0$

Practice

Given an integer n, find the maximum power of two that is smaller than n.

1011010 → 1000000

24 → 16

2^4

10000

11000

11111

$2^5 - 1$

$31 = 2^5 - 1$

$31 + 1 = 2^5$

2

1 1 1 0 0 1 0 1

?

N | N>>1

$$
\begin{array}{r}
11100101 \\
01110010 \\
\hline
11110111
\end{array}
$$

$0 \rightarrow 1$

$1 \rightarrow 1$

$0 \, 0$

PW SKILLS

$$N \quad (1\,0\,0\,0\,0\,0\,0\,0)$$

$$N = N \mid (N >> 1) \quad \rightarrow \quad N = 1\,1\,0\,0\,0\,0\,0\,0$$

$$N = N \mid N >> 2 \qquad N \quad 0\,0\,1\,1\,0\,0\,0\,0$$

$$N = N \mid N >> 4 \qquad 1\,1\,1\,1\,0\,0\,0\,0$$

$$0\,0\,0\,0\,1\,1\,1\,1$$

$$N = N \mid N >> 8 \qquad 1\,1\,1\,1\;\;1\,1\,1\,4\;\;0\,0\,0\,0\,0\,0\,0\,0\,0$$

$N = 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0$

2^x

$N = N \mid N >> 1$

$N = N \mid N >> 2$

$N = N \mid N >> 4$

$N = N \mid N >> 8$

$N = N \mid N >> 16$

1 0 0 0 0 0 0 0 0 | 0 1 0 0 0 0 0 0 0

7

1 1 0 0 0 0 0 0

0 0 1 1 0 0 0 0

1 1 1 1 0 0 0 0

0 0 0 0 1 1 1 1

1 1 1 1 1 1 1 1

SKILLS

1 0 0 0 1 1 0 1

0 1 0 0 0 1 1 0

1 1 0 0 1 1 1 1

0 0 1 1 0 0 1 1

1 1 1 1 1 1 1 1

0 0 0 0 1 1 1 1

$$N = N \mid N >> 1$$

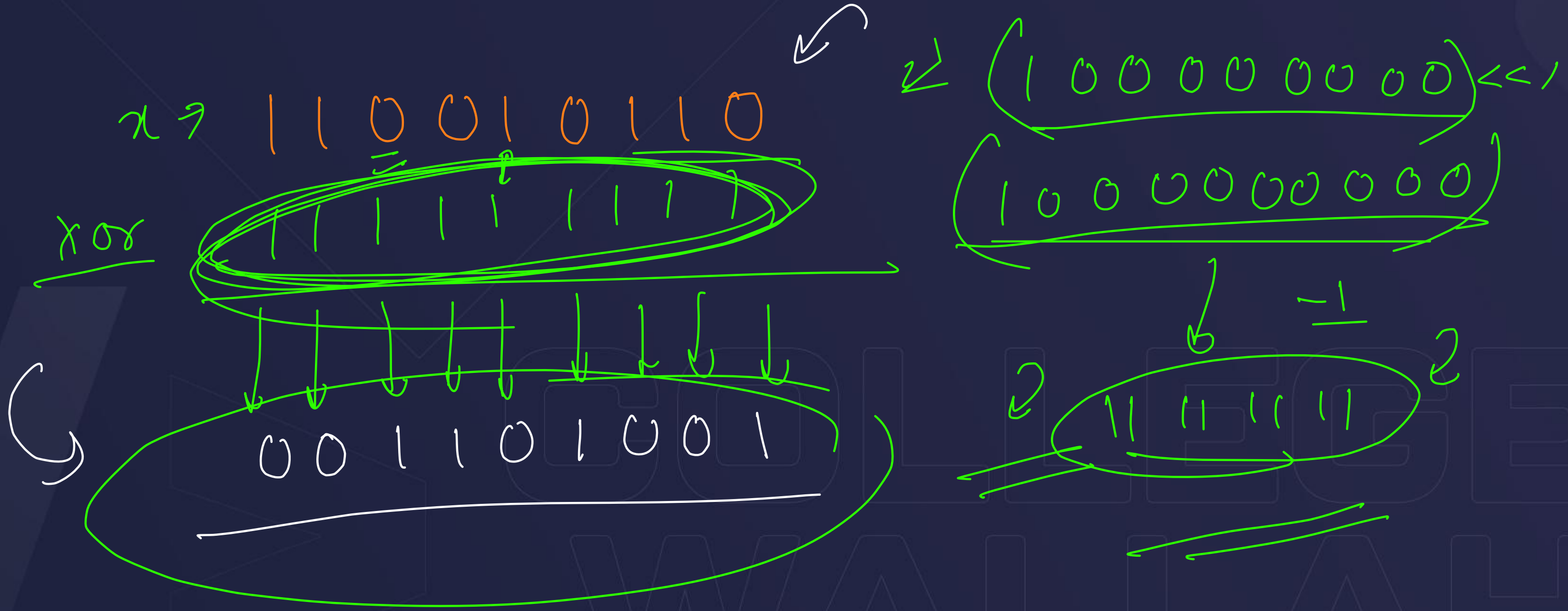
$$N = N \mid N >> 2$$

$$N = N \mid N >> 4$$

SKILLS

Practice

Given an integer n, flip all its bits i.e. if the given bit is 1, change it to 0 and vice versa.



Practice

coding

Given an integer n, flip all its bits i.e. if the given bit is 1, change it to 0 and vice versa.

23 → 10111

11111

1 0

0 1

(1 ~ ((1 << 2) - 1)) ^ 2

100000 << 1

1000000 → 32

31 → 11111

Practice

Calculate the minimum number of bit flips to convert from one number to another.

Example :

5 can be written as 0101 and
11 can be written as 1011, so number of bit flips required will be 3.

Practice

Calculate the minimum number of bit flips to convert from one number to another.

Example :

5 can be written as 0101 and
11 can be written as 1011, so number of bit flips required will be 3.

$x \to 23 \longrightarrow 10111$

$x \to 31 \longrightarrow 11111$

01000

10111
11111

01000

32 → 100000

23 ∩ 010111

110111 → 5

0001000

Basic bitwise operations

- AND
- OR
- NOT
- XOR

these are applied on the corresponding bits of the data.

logical
and &&
or ||
not !

Boolean input

(x<y) and (z<x)

boolean

boole

Basic bitwise operations

- AND → &
- OR → |
- NOT → ~
- XOR → ^

PW SKILLS

And

X	Y	Result
0	0	0
0	1	0
1	0	0
1	1	1

OR

X	Y	Result
0	0	0
0	1	1
1	0	1
1	1	1

XOR

X	Y	Result
0	0	0
0	1	1
1	0	1
1	1	0

NOT

X	Result
0	1
1	0

$7 \,\&\, 4 \rightarrow \underline{4}$

$7 \rightarrow 111$ $\quad 4 \rightarrow 100$

$3 \,|\, 6 \rightarrow \underline{7}$

$3 \rightarrow 011$ $\quad 6 \rightarrow 110$

$$
\begin{array}{rr}
 & 111 \\
\& & 100 \\
\hline
 & 100
\end{array}
\qquad
\begin{array}{rr}
 & 011 \\
| & 110 \\
\hline
 & 111
\end{array}
\rightarrow
$$

$8 \,\wedge\, 6$

$8 \rightarrow 1000$ $\quad 6 \rightarrow 0110$

$$
\begin{array}{rr}
 & 1000 \\
\wedge & 0110 \\
\hline
 & 1110
\end{array}
$$

$\underline{14}$

AND vs OR

Practice

Given an integer array. Find the length of longest subarray which has maximum possible bitwise AND value.

$x > y$

$x \,\&\, y \le y$

$[2, 3, 1, 2, 3, 3, 6, 4]$

i

j

Practice

Given an integer array. Find the length of longest subarray which has maximum possible bitwise AND value.

$x \,\&\, x = x$

ans = ~~1~~ 3

[12, 3, 1, 6, 1, 6, 6, 6, 6, 4, 3, 8, 13, 13, 13, 8]

max_el = ~~$-\infty$~~ ~~12~~ 13

count = ~~1~~ ~~2~~ 3

subarray that has all max

Basic Properties of XOR

- A ^ 0 = A
- A ^ A = 0
- If A ^ B = C, then A ^ C = B

PW SKILLS

Practice

XOR → Same ^ Same → 0

0 ^ x → x

Given an integer array where every element occurs twice except one occurs only once. Find that unique element.

Sort

[2, 1, 3, 2, 1, 5, 5, 6, 3]

2 ^ 3 ^ 1

2 ^ 1 ^ 3 →

2 ^ 1 ^ 3 ^ 2 ^ 1 ^ 5 ^ 5 ^ 6 ^ 3

2 ^ 2 ^ 3 ^ 3 ^ 1 ^ 1 ^ 5 ^ 5 ^ 6

0 ^ 0 ^ 0 ^ 0 ^ 6 → 6

Practice

Given an integer array nums, in which exactly two elements appear only once and all the other elements appear exactly twice. Find the two elements that appear only once

[1, 2, 1, 3, 2, 5] → 1 ^ 2 ^ 1 ^ 3 ^ 2 ^ 5

6

3 ^ 5

any set bit pos

k^{th} → 1

$k=2$

num 10110

0010 & 1

num >> (k)

$k^{th}=2$

1010

$(10110) >> 2 \rightarrow 00101$

& 1

00000

Modulo operator (%)

remainder

print ans modulo 10^9+7

remainder

$13 \% 4 = 1$

$$\begin{array}{r|l} 4 & 13 \; (3 \\ & \underline{12} \\ & 1 \end{array}$$

int → 32bit

$(10^8 \times 10^8) \% 5$

$C = (x \times y)$

c do 5

$$(10^8 \times 10^8) \% 5$$

$$\downarrow$$

$$\underline{\underline{10^{16}}}$$

10^9+7

$$(a+b)\%c \Rightarrow (a\%c + b\%c)\%c$$

$$(\underbrace{[0,c-1] + [0,c-1]}) \%c$$

$$\hookrightarrow [0,c-1]$$

$$(a\times b)\%c \Rightarrow (a\%c \times b\%c)\%c$$

$(a-b) \% c \rightarrow (a \% c - b \% c) \% c$

$\downarrow$

$(22-14) \% 5 \rightarrow (22 \% 5 - 14 \% 5) \% 5$

$8 \% 5 \rightarrow 3$

$(2-4) \% 5$

$(-2) \% 5$

$$(a-b)\%c = (a\%c - b\%c + c)\%c$$

$$(22-14)\%5 = (22\%5 - 14\%5 + 5)\%5$$

$$(2 - 4 + 5)\%5$$

$$3\%5 \to 3$$

$A \equiv B \bmod C$

- represents $A \% C = B \% C$

Basic operations in modulo arithmetic

(+ , - , *)

Modulo the result by 10^9+7

Why do we use $10^9 + 7$?

$[0, 10^9+7-1]$

int

Practice

Print factorial of first 25 natural numbers and modulo the result by 10^9+7.

$7! = 7 \times 6!$ $\quad 8! = 8 \times 7!$

$0! = 1$

$1! = 1$

$x! = (x)\times(x-1)\times(x-2)\ldots\ldots\times 3\times 2\times 1$

0	1	2	3	4	5	
1	1	2	6	24		....

$8!$

$\text{fact}[i] = i \times \text{fact}[i-1]$

Practice

Print factorial of first 25 natural numbers and modulo the result by 10^9+7.

$$(a \times b) \% c = (a \% c \times b \% c) \% c$$

$$fact[i] = (i \% c \times fact[i] \% c) \% c$$

Calculation of Inverse Modulo

Practice

Calculate the inverse modulo of first 10 natural numbers and modulo the result by $10^9 + 7$.

SKILLS

Thank you!