26:711:564 **OPTIMIZATION MODELS IN FINANCE**
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Currency Conversion

The Problem

We have currently cash reserves in 10 different currencies in the amounts given in the second column of the following table (in millions).

	Initial Position	Desired Position
EUR	70	62
USD	20	20
AUD	8	6
GBP	3	8
NZD	15	10
CAD	7	15
CHF	2	3
JPY	1,500	1,800
HKD	35	40
SGD	18	13

Our objective is to convert these amounts in order to have reserves at least equal to the amounts given in the last column of the table. The second table (included in the Excel spreadsheet currency_2023f_data) contains the conversion rates (cross rates) as of August 30, 9 AM.

The numbers in this table should be read as follows: to buy 1 U.S. dollar for Euro requires € 0.9159 (the element in the second row and the first column), 1 U.S. dollar costs 0.59566 Australian dollars (AUD), etc. If we want to buy 1 Euro for U.S. dollars we have to pay $ 1.09168 (the element in the EUR row and USD column), etc.

Design the conversion plan to meet the desired goals and to maximize the U.S.$ value of the obtained positions.

The Model

Let us denote by n the number of currencies considered ($n = 10$ in our case). Let a_j, $j = 1, \dots, n$ denote the initial positions and let b_i, $i = 1, \dots, n$, denote the desired positions. Finally, we denote by r_{ij} the cost of currency i in currency j (the element in row i and column j of the rates table).

We introduce the **decision variables** x_{ij} representing the amounts of currency i purchased for currency j, $i, j = 1, \dots, n$. For $i = j$ the variable x_{ii} represents the amount kept in currency i.

The decision variables have to satisfy the following equations and inequalities, called **constraints**:

$$\sum_{i=1}^{n} r_{ij} x_{ij} = a_j, \quad j = 1, \dots, n \quad \text{(amount of currency } j \text{ spent)}$$

$$\sum_{j=1}^{n} x_{ij} \geq b_i, \quad i = 1, \dots, n, \quad \text{(amount of currency } i \text{ obtained)}$$

$$x_{ij} \geq 0, \quad i = 1, \dots, n, \quad j = 1, \dots, n.$$

In the first equation we assume that only the original amounts can be converted, not the amounts obtained from other conversions.

The set X of conversion plans x satisfying the constraints is called the **feasible set**.

The value of the obtained positions can be expressed as the dollar value the amounts obtained, that is

$$f(x) = \sum_{i=1}^{n} \frac{1}{r_{2i}} \sum_{j=1}^{n} x_{ij}.$$

It is called the **objective function**. The problem is to maximize the objective function over x in the feasible set X, that is to find a point $\hat{x} \in X$ (the **optimal solution**) such that

$$f(\hat{x}) \geq f(x) \quad \text{for all} \quad x \in X.$$

The problem above is an example of an **optimization problem**, in particular, a **linear programming problem**.