Optimization Models in Finance (26:711:564)
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HOMEWORK 3 *(due Tuesday, September 26, 2023)*

Problem 1

You have to determine an investment strategy for the next three years. At present (time 0) the amount of $100,000 is available for investment. Five investments are available. The cash flows associated with investing $1 in each of them are given in the table below. Returns from investments can be reinvested immediately. For example, if we decide to invest 10,000 in A and 10,000 in C, then the resulting cash flows will be -10,000 in year 1, -5,000 in Year 2, etc.

At most $75,000 can be placed in any single investment. Cash earns 8% per year.

The cash flows are given in the table below.

Time	0	1	2	3
Investment A	-$1.00	$0.50	$1.00	$0.00
Investment B	-$1.00	$1.20	$0.00	$0.00
Investment C	$0.00	-$1.00	$0.50	$1.00
Investment D	-$1.00	$0.00	$0.00	$1.90
Investment E	$0.00	$0.00	-$1.00	$1.50

This table should be read as follows: for every dollar committed to Investment A, there is an outflow of $1 at time 0 (now), inflow of $0.50 at time 1, and inflow of $1 at time 2. Each dollar committed to Investment C results in an outflow of $1 at time 1, inflow of $0.50 at time 2, and inflow of $1 at time 3, etc. Money obtained from Investments A or B at time 1 can be used to invest in C.

Problem 2

You are holding hold a bond portfolio of four bonds, 100 units of each of them. You are considering rebalancing it. The bid and ask prices of the bonds are given in the table below.

	Bid Price	Ask Price
Bond 1	$980	$990
Bond 2	$960	$972
Bond 3	$970	$985
Bond 4	$940	$954

The cash payments of the bonds in the next three years are as follows.

	Bond 1	Bond 2	Bond 3	Bond 4
Year 1	$100	$70	$80	$60
Year 2	$110	$80	$90	$50
Year 3	$1100	$1090	$1020	$1110

Cash on hand earns 5% interest. You want to re-balance the portfolio in such a way that at any time in the future your cash position will be at least as good as the position that would result from your current portfolio. How much money you can take out today under this condition?

Problem 3

The pension fund manager of the Association of Business Professors identified three reliable mutual funds, which have a long time successful record of operation: the Growth Fund, the Global Fund and the Income Fund. All these funds invest in four asset categories: U.S. large capitalization stocks, U.S. small capitalization stocks, foreign stocks and U.S. bonds, in proportions given in the following table.

Asset Category	Growth	Global	Income
U.S. Large Stocks	45%	20%	25%
U.S. Small Stocks	40%	10%	5%
Foreign Stocks	10%	60%	5%
U.S. Bonds	5%	10%	65%
Average return rate	17%	14%	10%

The table also shows an average annual return of these funds observed over a long time period.

The pension fund offers to its participants a Balanced Package. The minimum and maximum investments of the package in each asset category package is given in the table below.

Asset Category	Minimum	Maximum
U.S. Large Stocks	25%	35%
U.S. Small Stocks	15%	25%
Foreign Stocks	20%	30%
U.S. Bonds	20%	30%

The pension fund has 5 million dollars. How much should the manager invest in the Growth Fund, Global Fund and Income Fund to construct the package and to maximize the annual return?

1) Can we put objective function as one of our constraint

2) if constraint has this form c1 <= f(x) <= c2 can we bifurcate the constraints

3) In the first problem we can not short anything

4) In the second problem can we short the bond