

## Optimization Models in Finance

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ASSIGNMENT 6 (*due Thursday, March 23, 2023*)

### Problem 1

Use duality to solve the following nonlinear programming problem:

$$\begin{aligned} \min \quad & \sum_{j=1}^n c_j (x_j)^2 \\ & \sum_{j=1}^n a_j x_j = 1, \\ & 0 \leq x_j \leq u_j, \quad j = 1, \dots, n, \end{aligned}$$

where  $c_j > 0$ ,  $a_j > 0$  and  $u_j > 0$ , for all  $j = 1, \dots, n$ .

### Problem 2

You have collected data on monthly return rates of 10 securities, as shown in the attached spreadsheet.

- (a) Estimate the expected returns and the covariance matrix of the returns.
- (b) Determine the mean—variance efficient frontier, with shorting allowed. Describe this frontier employing the Two-Fund Theorem. Use the minimum variance portfolio as one of the funds involved.
- (c) Analyze the efficient frontier (by calculating 10 points on each of them) for the cases when shorting is not allowed. Compare with case b).