Algebraic Equations for Sums

1. Mean Returns Calculation

The mean return for each asset across all scenarios is calculated as:

$$\mu_i = \frac{1}{n} \sum_{j=1}^{n} R_{ij}$$

Where:

- μ_i: Mean return of the i-th asset.
- R_{ij}: Return of the i-th asset in the j-th scenario.
- n: Number of scenarios.

2. Weighted Returns Calculation

The weighted portfolio return for each scenario is computed as:

$$Z_k = \sum_{i=1}^{m} w_i R_{ik}$$

Where:

- Z_k: Portfolio return in the k-th scenario.
- w_i: Weight of the i-th asset.
- R_{ik}: Return of the i-th asset in the k-th scenario.
- m: Number of assets.

3. Probability Calculation

The probability of an event less than or equal to η is:

$$P(\eta) = \frac{\text{Number of events } (Z_k \leq \eta)}{n}$$

Where:

- $P(\eta)$: Cumulative probability at threshold η.
- Z_k: Portfolio returns across scenarios.

4. Expected Portfolio Return for Event η

Conditional expectation of portfolio returns for events $Z_k \leq \eta$:

$$E[Z_k | Z_k \leq \eta] = \frac{1}{\text{Number of events } (Z_k \leq \eta)} \sum_{Z_k \leq \eta} Z_k$$

5. $F^{-2}(p)$ Calculation

The $F^{-2}(p)$ value at probability p is:

$$F^{-2}(p) = p \cdot \frac{1}{\text{Number of events } (Z_k \leq \eta)} \sum_{Z_k \leq \eta} Z_k$$

6. Optimization Problem Objective

Maximize expected portfolio return:

$$\text{Objective: } \max_{w} \mu^\top w$$

Where:

- μ: Vector of mean returns (μ_i).
- w: Vector of weights (w_i).

7. Optimization Problem Constraints

Constraint 1: Conditional Return Constraint for Each Probability p

For each probability p:

$$\frac{1}{\text{Number of events } (Z_k \leq \eta)} \sum_{Z_k \leq \eta} \sum_{i=1}^{m} w_i R_{ik} \geq \frac{1}{p} F^{-2}(p)$$

Constraint 2: Weight Normalization

$$\sum_{i=1}^{m} w_i = 1$$

Constraint 3: Non-Negative Weights

$$w_i \geq 0 \quad \forall i$$

8. Delta Calculation for Constraint Violations

For each threshold t:

$$\delta_t = \frac{1}{P(t)} F^{-2}(P(t)) - \frac{1}{\text{Number of events } (Z_k \leq t)} \sum_{Z_k \leq t} Z_k$$

Where δ_t is the violation of the constraint at threshold t.

9. Stopping Criterion

Stop the iteration when:

$$\max_{t} \delta_t \leq 0$$