

Day 2 - Linear Machine Learning Algorithm

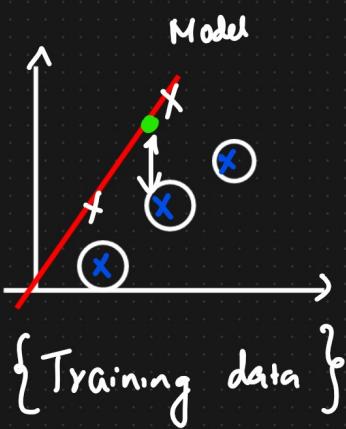
Agenda

- ① Ridge and Lasso Regression
- ② Assumption of Linear Regression
- ③ Logistic Regression
- ④ Confusion Matrix
- ⑤ Practical Implementation

① Ridge And Lasso Regression

$$\text{Cost function} = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

$$\theta_0 = 0$$



$$J(\theta_0, \theta_1) = 0 \quad \downarrow \downarrow \downarrow$$

Underfitting { High Bias
High Variance }

- { ① Model Accuracy is bad with Training data
② Model Accuracy is also bad with Test data }

Model performs well → Training data

Fails to perform well → Test Data ✓

(High Variance)

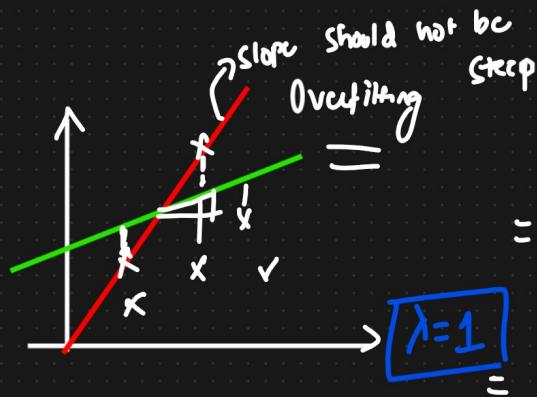
Model 1

Training Acc = 90%.

Test Acc = 80%.



{ Overfitting }
Low Bias, High Variance



Model 2

Training Acc = 92%.

Test Acc = 91%.



{ Generalized Model }

{ Low Bias
High Variance }

$$J(\theta_1) = 0$$

$$= \frac{1}{2m} \sum_{i=1}^m (\hat{y}_i - y^{(i)})^2$$

$$= (\hat{y}_i - y^{(i)})^2 + \lambda (\text{slope})^2 \quad \checkmark$$

Model 3

Training Acc = 70%.

Test Acc = 65%.



Underfitting

High Bias, High Variance

$$h_\theta(x) = \hat{y} \quad \theta_1 = 2 \quad \theta_0 = 0$$

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$h_\theta(x) = \theta_1 x \quad \text{slope}$$

Ridge (L2 Regularization)

$$= 0 + 1(2)^2$$

iterations { Hyperparameter }

$$= 4/4 \downarrow \downarrow \text{bb}$$

R², adjusted R²

$$= (\hat{y}^{(i)} - y^{(i)})^2 + \lambda (\text{slope})^2 \quad \lambda \rightarrow \text{Hyperparameter} \checkmark$$



$$(\text{Small value}) + 1(1.5)^2$$

Convergence

$$= (\text{Small value}) + 2.25$$



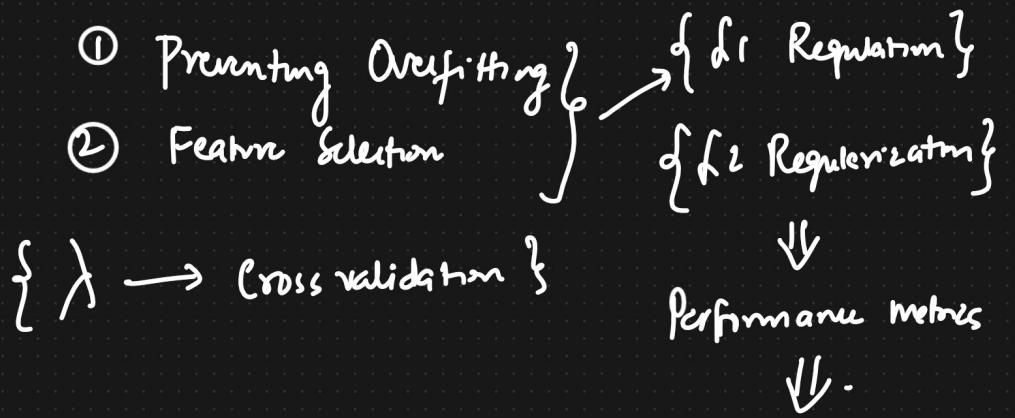
$$\approx 3 \downarrow \downarrow$$

feature selection

Lasso (L1 Regularization)

$$= (\hat{y} - y)^2 + \lambda |\text{slope}| \quad |\theta_0 + \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \dots + \theta_n|$$

$$h_\theta(x) = \hat{y} = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4 + \dots + \theta_n x_n$$



Ridge Regression (λ_2 Norm)

$$\text{Cost function} = (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda (\text{slope})^2$$

④

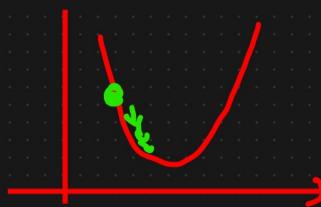
Purpose : Preventing Overfitting

$$|\theta_0 + \theta_1 + \theta_2 + \theta_3 + \dots + \theta_n|$$

Lasso Regression (λ_1 Reg)

$$\text{Cost function} = (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda |\text{slope}|$$

Purpose : 1) Prevent Overfitting
2) Feature Selection



Assumption of Linear Regression

① Normal / Gaussian Distribution \rightarrow Model will get trained well

✓ ② {Standardization {Scaling data} \rightarrow Z-score $\mu=0, \sigma=1$ }

③ Linearity

$$X_3 \quad \boxed{X_1 \quad \cancel{X_2}} \quad \boxed{Y}$$

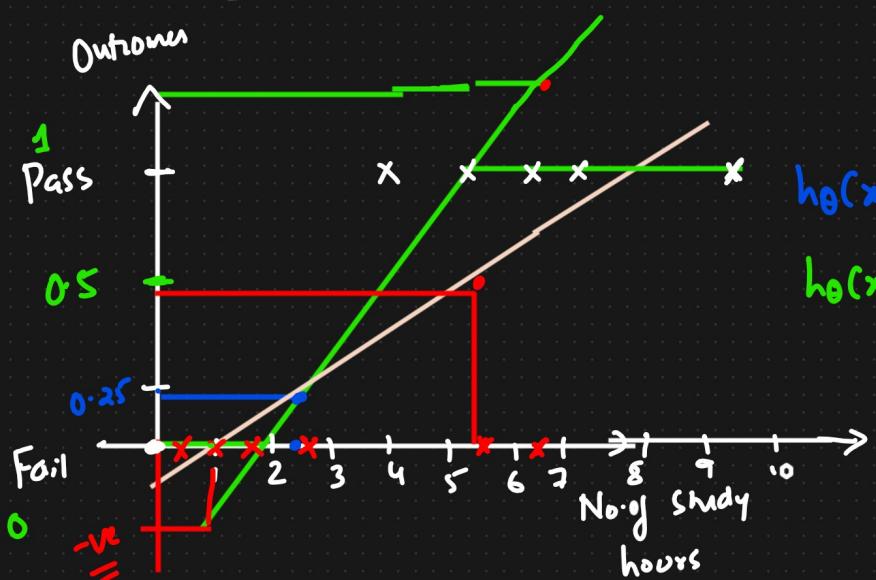
Variation Inflation factor?

④ Multi Collinearity

Logistic Regression (classification) → Binary classification

No. of study No. of play P/F

— — P
— — F



Linear Regression ??

$$h_{\theta}(x) < 0.5 \Rightarrow 0 \rightarrow \text{Fail}$$

$$h_{\theta}(x) \geq 0.5 \Rightarrow 1 \rightarrow \text{PASS}$$

{ Sigmoid function }

0 to 1

Decision Boundary Logistic Regression

$$h_{\theta}(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \dots + \theta_n x_n$$

$$\boxed{h_{\theta}(x) = \theta^T x}$$

$$h_{\theta}(x) = \boxed{\theta_0 + \theta_1 x_1}$$

Squash

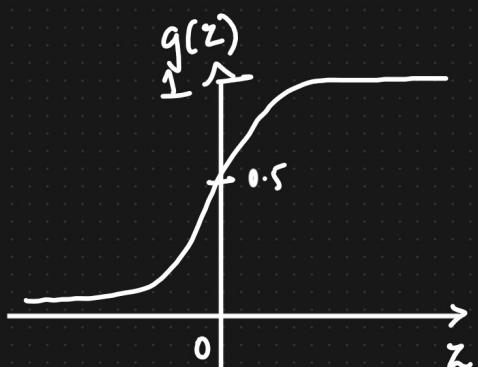


$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1)$$

$$\text{let } z = \theta_0 + \theta_1 x$$

$h_{\theta}(x) = g(z)$ Sigmoid or Logistic function

$$h_{\theta}(x) = \frac{1}{1+e^{-z}}$$



$$g(z) \geq 0.5 \quad \left\{ \begin{array}{l} \checkmark \\ \cdot \end{array} \right.$$

When $z \geq 0$

$$\boxed{h_{\theta}(x) = \frac{1}{1 + e^{-(\theta_0 + \theta_1 x)}}}$$

Training Set

$$\{(x^1, y^1), (x^2, y^2), (x^3, y^3), \dots, (x^n, y^n)\}$$

$$y \in \{0, 1\} \rightarrow 2 \text{ o/p}$$

$$h_{\theta}(z) = \frac{1}{1 + e^{-z}} \quad \boxed{z = \theta_0 + \theta_1 z}$$

(change parameter θ_1 ?)

Cost function

Linear Regression $J(\theta_0) = \frac{1}{m} \sum_{i=1}^m \frac{1}{2} (h_{\theta}(x^i) - y^i)^2$

$$h_{\theta}(x) = \theta_0 + \theta_1 x \quad \boxed{}$$

Logistic Regression

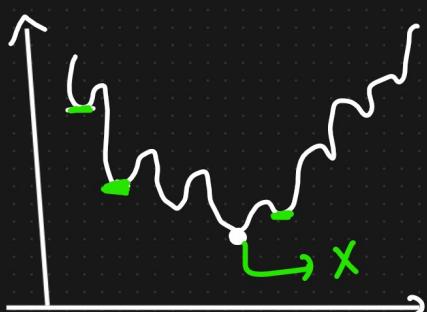
$$h_{\theta}(x) = \frac{1}{1 + e^{-(\theta_0 + \theta_1 x)}}$$

Logistic Regr
Cost function $= \frac{1}{2} (h_{\theta}(x^{(n)}) - y^{(n)})^2 \quad \left. \begin{array}{l} \text{We cannot use this} \\ \text{cost function for logistic} \end{array} \right\}$

$$h_{\theta}(x) = \frac{1}{1 + e^{-(\theta_0 + \theta_1 x)}}$$

Gradient Descent

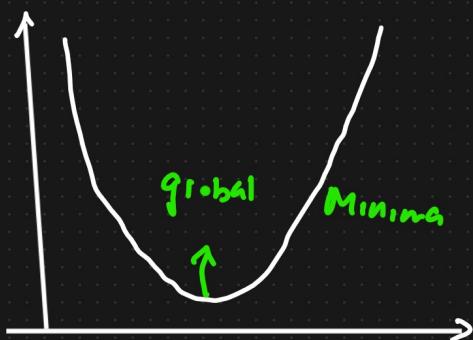
Non convex function



Local Minima Problem

Gradient Descent

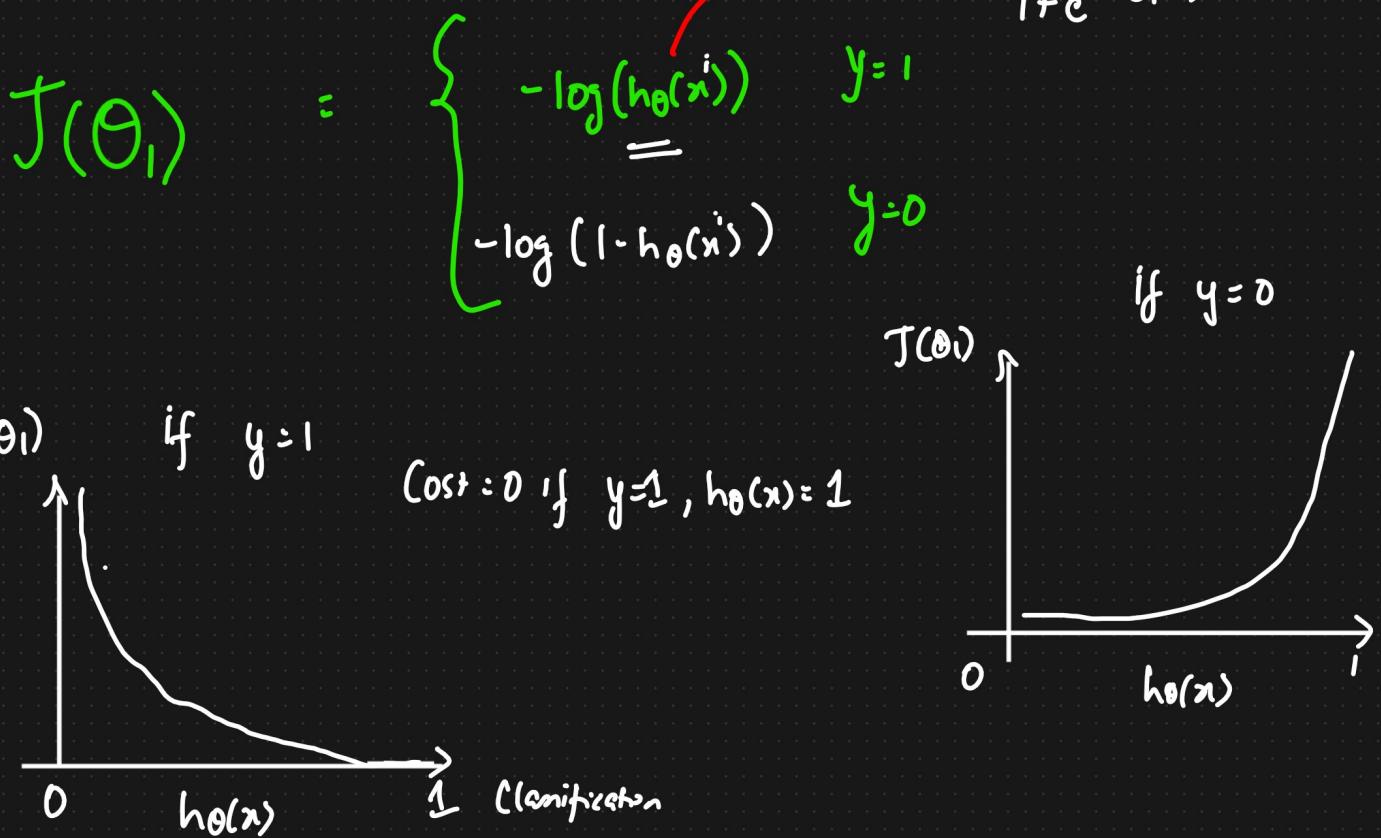
Convex function



Global Minima

Logistic Regression Cost function

$$h_{\theta}(x) = \frac{1}{1 + e^{-(\theta^T x)}}$$



$$\text{Cost}(h_{\theta}(x^i), y) = \begin{cases} -\log(h_{\theta}(x^i)) & \text{if } y=1 \\ -\log(1-h_{\theta}(x^i)) & \text{if } y=0 \end{cases}$$

$$\boxed{\text{Cost}(h_{\theta}(x^i), y) = -y \log(h_{\theta}(x^i)) - (1-y) \log(1-h_{\theta}(x^i))}$$

if $y=1$ \Downarrow cost function.

$$\text{Cost}(h_{\theta}(x^i), y) = \begin{cases} -\log(h_{\theta}(x^i)) & \text{if } y=1 \\ -\log(1-h_{\theta}(x^i)) & \text{if } y=0 \end{cases}$$

$$J(\theta_0) = -\frac{1}{2m} \sum_{i=1}^m \left[(y^i \log(h_\theta(x^i)) + (1-y^i) \log(1-h_\theta(x^i))) \right]$$

↓
cost $h_\theta(x^i) = \frac{1}{1+e^{-\theta_0 x^i}}$

Repet until convergence

→ {
 $\theta_j := \theta_j - \alpha \frac{\partial J(\theta)}{\partial \theta_j}$
} {

Performance Metrics {Classification Problem}

x_1	x_2	y	\hat{y}	Pred	Actual
-	-	0	1	1	3
-	-	1	1	0	2
-	-	0	0	0	1
-	-	1	1	1	1
-	-	1	1	0	1
-	-	0	1	1	0
-	-	1	0	0	1

		1	0	Actual
		TP	FP ↓	Confusion matrix
		FN ↓	TN	
Predicted	1	3	2	
Actual	0	1	1	

$$\text{Accuracy} = \frac{TP + TN}{TP + FP + FN + TN}$$

$$\textcircled{1} \quad \begin{aligned} & 0 \rightarrow 900 \\ & 1 \rightarrow 100 \end{aligned} \quad \begin{aligned} \text{Imbalance} &= \frac{3+1}{3+2+1+1} = \frac{4}{7} \\ \text{DATAMIN} &= 0.57 = 57\% \end{aligned}$$

$$\begin{aligned} & 0 \rightarrow 600 \\ & 1 \rightarrow 400 \end{aligned} \quad \text{Balanced} \quad \begin{aligned} 0 : 900 \\ 1 : 100 \end{aligned} \quad \text{Balanced} =$$

$$\{ \text{Model} \rightarrow 0 = \frac{900}{1000} = 90\% \}$$

• TPR, Sensitivity

① Precision

$$\frac{TP}{TP + FP}$$

② Recall

$$\left\{ \frac{TP}{TP + FN} \right\}$$

③ F-Score.

		Actual	
		1	0
pred	1	TP	FP
	0	FN	TN

↓↓↓

{ Tom Stock market is going to crash } → Precision
 { Spam classification } → Recall
 { Has CANCER OR NOT } → Recall

$$\underline{\underline{F - Beta}} = (1 + \beta^2) \frac{\text{Precision} \times \text{Recall}}{\beta^2 \times \text{Precision} + \text{Recall}}$$

$$\beta = 1 \approx (1+1) \frac{\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}$$

$$\text{F1-Score} = \frac{2 (\text{Precision} \times \text{Recall})}{\text{Precision} + \text{Recall}}$$

$$\frac{\text{Harmonic Mean}}{=} \frac{2xy}{x+y}$$

$$\beta = 0.5 \quad (1 + (0.5)^2) \frac{P \times R}{(0.25) P + R}$$

$$\beta = 2 \quad FN \gg FP$$

F2 Score