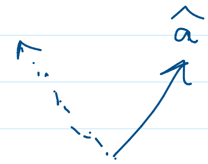
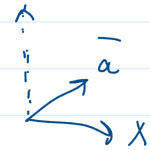
$$\hat{i} + 2\hat{j} - \hat{k} \equiv \begin{bmatrix}1\\2\\-1\end{bmatrix}$$

Q1. $\mathbb{R}^3 \equiv \mathbb{R}^{3\times 1}$ $\quad \begin{bmatrix}\vdots\end{bmatrix}$



$\hat{a}, \hat{b} \in \mathbb{R}^3 \qquad X \in \mathbb{R}^3 \Rightarrow \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$



Eq^n : $\hat{a} \times X = \hat{b}$

suppose sol^n exists, $\quad \hat{a}\cdot\underbrace{(\hat{a}\times X)} = \hat{a}\cdot\hat{b}$

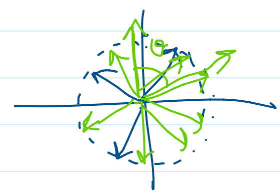
$$\underbrace{0 = \hat{a}\cdot\hat{b}}$$

Cond^n for eq^n to have a sol^n: $\underline{\hat{a}\cdot\hat{b} = 0}$

$\bar{a}\cdot\bar{b}$, $\quad a = \begin{bmatrix}\vdots\end{bmatrix}$, $\quad b = \begin{bmatrix}\vdots\end{bmatrix}$

$\bar{a}\cdot\bar{b} \equiv a^T b$

Q2. $p = \begin{bmatrix}x_1\\x_2\end{bmatrix} \equiv \hat{p} = x_1\hat{i} + x_2\hat{j}$



$A = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$; $p = \begin{bmatrix}\cos\phi\\ \sin\phi\end{bmatrix}$

$\{p, Ap, A^2p \dots\}$

Claim: $A^n = \begin{bmatrix}\cos n\theta & -\sin n\theta\\ \sin n\theta & \cos n\theta\end{bmatrix}$

for $n = 1$, $\quad A^1 = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ ✓

Let it be true for m

$A^{m+1} = A^m \cdot A = \begin{bmatrix}\cos m\theta & -\sin m\theta\\ \sin m\theta & \cos m\theta\end{bmatrix}\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

first term $= \cos m\theta\cos\theta - \sin m\theta\sin\theta$ ✓

$= \cos(m\theta + \theta) = \cos((m+1)\theta)$

Claim is valid

$$A^n p = \begin{bmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{bmatrix} \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix} = \begin{bmatrix} \cos(n\theta+\phi) \\ \sin(n\theta+\phi) \end{bmatrix}$$

If $\theta = \frac{a}{b} \cdot 2\pi$ $\quad A^b p = \begin{bmatrix} \cos(2a\pi + \phi) \\ \sin(2a\pi + \phi) \end{bmatrix} = p$

Set is finite

If not, set is infinite (countably infinte)

Q3. $\underbrace{x^2 + y^2 - z^2 + 7xy - 3yz + 6xz}$ $= 3$

$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow u A u^T$$

$1 \times 3 \quad 3 \times 3 \quad 3 \times 1 \rightarrow 1 \times 1 \rightarrow$ scalar

$$\underset{1}{a_{11}} x^2 + \underset{1}{a_{22}} y^2 + \underset{-1}{a_{33}} z^2 + (a_{12} + a_{21}) xy + (a_{13} + a_{31}) xz + (a_{23} + a_{32}) yz$$

$a_{12} = a_{21} = \frac{7}{2}$; $a_{13} = a_{31} = \frac{6}{2} = 3$; $a_{23} = a_{32} = -\frac{3}{2}$

$$A = \begin{bmatrix} 1 & 7/2 & 3 \\ 7/2 & 1 & -3/2 \\ 3 & -3/2 & -1 \end{bmatrix} \rightarrow \text{sym}$$

But if A is not sym $\rightarrow$ inf sol^n

$[x\, y\, z]\, A \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow$ quadratic form

Q4. a) $|A| \neq 0 \iff$ invertible
iff

$$Au = 0$$
$3\times3 \quad 3\times1$ $\quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$|A| \neq 0 \leftarrow$ is satisfied by only $u = 0$
$|A| = 0 \leftarrow$ is satisfied by $u \neq 0$

Method 1

Is $I - uu^T$ inv / not

$u = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $x^2 + y^2 + z^2 = 1$

$$uu^T = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} x & y & z \end{bmatrix} = \begin{bmatrix} x^2 & xy & xz \\ xy & y^2 & yz \\ xz & yz & z^2 \end{bmatrix}$$

$$|I - uu^T| = (-1)^3 \begin{vmatrix} -1+x^2 & +xy & +xz \\ +xy & -1+y^2 & +yz \\ +xz & +yz & -1+z^2 \end{vmatrix}$$

$R_1 \to xR_1$
$R_2 \to yR_2$
$R_3 \to zR_3$

x common from C_1 . . . . .

$$\begin{vmatrix} x^2-1 & x^2 & x^2 \\ y^2 & y^2-1 & y^2 \\ z^2 & z^2 & z^2-1 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} x^2+y^2+z^2-1 & x^2+y^2+z^2-1 & x^2+y^2+z^2-1 \\ . & . & . \\ . & . & . \end{vmatrix} = 0$$

$I - uu^T$ is not inv

Method 2

$$(I - uu^T)u = I \cdot u - u\underbrace{u^T u}_{\bar{u}\cdot\bar{u} = 1}$$
$$= u - u$$
$$= 0$$

$$\Rightarrow |I - uu^T| = 0$$
$\hookrightarrow$ not inv

b) $f(x) = (I - 2uu^T)x \quad ; \quad x \in \mathbb{R}^3$

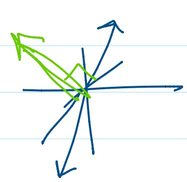
$a \in \mathbb{R} \longrightarrow b \in \mathbb{R}$
$f(x) = x - 4$
$5 \to 1$

$$\begin{bmatrix} & \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} \to \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

For, $X = \lambda u \quad ; \quad \lambda \in \mathbb{R}$

$$(I - 2uu^T)\lambda u$$
$$\lambda u - 2\lambda u = -\lambda u$$

$$\lambda u \longmapsto -\lambda u$$



For $X = a \quad ; \quad a \perp u$ ✓

$$(I - 2uu^T)a$$
$$a - 2u\underbrace{u^T a}_{\bar{u}\cdot\bar{a} = 0} = a$$

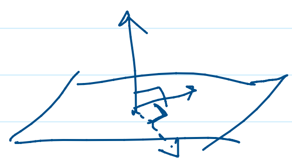
$$a - 2u \underbrace{u^T a}_{\bar{u}\cdot\bar{a} = 0} = a$$

$$a \longmapsto a$$

mirror normal $= \bar{u}$

$k \cdot a + l \cdot u$

Q5. $x + y + z = 0 \rightarrow$ normal $= \frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$

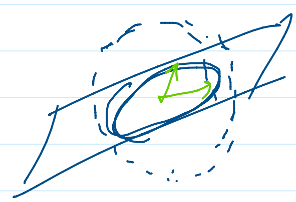


Find $\hat{u}$ that lies on plane

$\hat{v} = \hat{u} \times \hat{n}$

$(0,0,0) - (1,-1,0) = \frac{\hat{i}}{\sqrt{2}} - \frac{\hat{j}}{\sqrt{2}}$

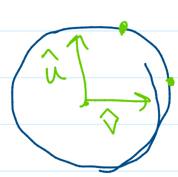
$\hat{v} = \hat{u} \times \hat{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ & & \end{vmatrix}$



$x^2 + y^2 + z^2 = 1 \longrightarrow$ sphere

$\hat{u}, \hat{v} \longrightarrow$ lie on the plane
$\longrightarrow$ unit vectors $\rightarrow$ lie on sphere $\Big\}$ crossection circle

Top view



$\hat{a}$ lies on circle

$\hookrightarrow \underbrace{\hat{u}\cos\theta}_{\text{like } \hat{i}} + \underbrace{\hat{v}\sin\theta}_{\text{like } \hat{j}}$

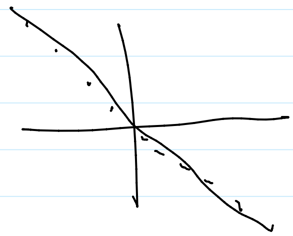
$f : \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad : \quad \mathbb{R} \to \mathbb{R}^2 \to$ line through origin

$f : \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad : \quad \mathbb{R} \to \mathbb{R}^2 \to$ line through origin

λ

$\lambda \longrightarrow \begin{bmatrix} \lambda \\ -\lambda \end{bmatrix}$

$(\lambda, -\lambda)$



$x + y = 0$