Tutorial - 3

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Q1. $A, B \in \mathbb{R}^{n\times n}$

$A + iB \to$ invertible

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 2 & 4 & -1 & 2 \\ 3 & 1 & 9 & -1 \\ 5 & 2 & 5 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ -B & A \end{bmatrix} \in \mathbb{R}^{2n\times 2n}$$

$R_1 \to R_1 + R_3$

$R_2 \to R_2 + R_3$

$$\xrightarrow{R_1 \to R_1 - iR_2} \begin{vmatrix} A+iB & B-iA \\ -B & A \end{vmatrix}$$

$$\begin{bmatrix} 4 & 2 & 11 & 2 \\ 7 & 6 & 4 & 3 \\ 3 & 1 & 9 & -1 \\ 5 & 2 & 5 & 1 \end{bmatrix}$$

$$\xrightarrow{C_2 \to C_2 + iC_1} \begin{vmatrix} A+iB & 0 \\ -B & A-iB \end{vmatrix}$$

$$= |A+iB|\,|A-iB|$$

Q2.
$$\begin{bmatrix} 2 & 0 & 6 & 0 & 4 \\ 5 & 3 & 2 & 2 & 7 \\ 2 & 5 & 7 & 5 & 5 \\ 2 & 0 & 9 & 2 & 7 \\ 7 & 8 & 4 & 2 & 1 \end{bmatrix}$$

$C_5 \to C_5 + 10C_4 + 100C_3 + 1000C_4 + 10000C_5$

$$\to \begin{vmatrix} 2 & 0 & 6 & 0 & 20604 \\ 5 & 3 & 2 & 2 & 53227 \\ 2 & 5 & 7 & 5 & 25755 \\ 2 & 0 & 9 & 2 & 20927 \\ 7 & 8 & 4 & 2 & 78421 \end{vmatrix}$$

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$17 \times x \quad ; x \in \mathbb{Z}$

Q3. $x(x^2 + ax + b) = 0 \;\; \begin{matrix}\nearrow \alpha \\ \to 0 \\ \searrow \end{matrix} \quad \to \quad x^3 + ax^2 + bx = 0$

Q3. $x(x^2+ax+b)=0 \to \alpha, 0, \beta \quad \longrightarrow \quad x^3+ax^2+bx=0$

$x(x^2+px+q)=0 \to \alpha, 0, \gamma \quad \longrightarrow \quad x^3+px^2+qx=0$

$$\begin{vmatrix} 1 & a & b & 0 \\ 0 & 1 & a & b \\ 1 & p & q & 0 \\ 0 & 1 & p & q \end{vmatrix} \longrightarrow \begin{vmatrix} 1 & a & b & \alpha^3+\alpha^2 a+\alpha b \\ 0 & 1 & a & \alpha^2+a\alpha+b \\ 1 & p & q & \alpha^3+\alpha^2 p+\alpha q \\ 0 & 1 & p & \alpha^2+p\alpha+q \end{vmatrix}$$

$C_4 \to C_1\alpha^3+C_2\alpha^2+C_3\alpha+C_4$

$$\downarrow$$

$$\begin{vmatrix} 1 & a & b & 0 \\ 0 & 1 & a & 0 \\ 1 & p & q & 0 \\ 0 & 1 & p & 0 \end{vmatrix} \to \det = 0$$

Tut Sheet 3

2.
$$x+2y+3z=20$$
$$x+3y+z=13$$
$$x+6y+\beta z=\beta$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 1 \\ 1 & 6 & \beta \end{vmatrix} = 1(3\beta-6)-2(\beta-1)+3(3)$$
$$= \beta+5$$

For Cramer's Rule, $\det \neq 0 \Rightarrow \beta \neq -5 \to$ unique sol^n

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 20 \\ 1 & 3 & 1 & 13 \\ 1 & 6 & -5 & -5 \end{array}\right] \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 20 \\ 0 & 1 & -2 & -7 \\ 0 & 4 & -8 & -25 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 4R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 20 \\ 0 & 1 & -2 & -7 \\ 0 & 0 & 0 & 3 \end{array}\right]$$

$\beta = -5 \to$ no sol^n

3. $a\hat{i} + b\hat{j} + c\hat{k}$
$c\hat{i} + a\hat{j} + b\hat{k}$
$b\hat{i} + c\hat{j} + a\hat{k}$ $\to \begin{bmatrix} a & c & b \\ b & a & c \\ c & b & a \end{bmatrix}$

vectors are lin dep $\to$ det $= 0$

$$\det = (a^3 + b^3 + c^3) - 3abc = 0$$

$$= \frac{(a+b+c)}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right] = 0$$

$a = b = c$ or $a + b + c = 0$ $\to$ cond^n for lin dep

9. $M = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix}$

$1/15 - 1/16 = 1/240$

$1/10 - 1/12 = 1/60 \, (-1)^3$

$$\text{Cof} = \begin{bmatrix} 1/240 & -1/60 & 1/72 \\ 1/60 & 4/45 & 1/12 \\ 1/72 & 1/12 & 1/12 \end{bmatrix} = \text{adj}$$

$\text{adj} = \text{cof}^T = \text{cof}$

$$\qquad \tfrac{1}{72} \qquad \tfrac{1}{12} \qquad \tfrac{1}{12} \;\rfloor$$

$$\text{adj} = \text{cof}^T = \text{cof}$$

$$|\text{adj}| = \frac{1}{2160}$$

$$H^{-1} = \frac{\text{adj}(H)}{|H|} = \begin{bmatrix} 9 & -36 & 30 \\ -36 & 192 & -180 \\ 30 & -180 & 180 \end{bmatrix}$$

Q 11.

$$W_{f_1 f_2 \cdots f_n}(x) := \begin{array}{c} n \\ \begin{vmatrix} f_1 & f_2 & f_3 & \cdots & f_n \\ f_1' & f_2' & f_3' & \cdots & f_n' \\ \vdots & \vdots & \vdots & & \vdots \\ f_1^{(n-1)} & f_2^{(n-1)} & \cdots & \cdots & f_n^{(n-1)} \end{vmatrix} \end{array} \; n$$

$c_1, c_2 \cdots c_n$:

$$c_1 f_1 + c_2 f_2 + \cdots \; c_n f_n = 0 \quad ; \quad x \in (a,b)$$

$$c_1 f_1' + c_2 f_2' + \cdots , \; c_n f_n' = 0$$

$$\vdots$$

$$c_1 f_1^{(n-1)} + c_2 f_2^{(n-1)} + \cdots \; c_n f_n^{(n-1)} = 0$$

$$\begin{bmatrix} f_1 & f_2 & \cdots & f_n \\ f_1' & f_2' & \cdots & f_n' \\ \vdots & \vdots & & \vdots \\ f_1^{(n-1)} & f_2^{(n-1)} & & f_n^{(n-1)} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = 0 \qquad x_0 \in (a,b)$$

$\det \neq 0$

$$\Rightarrow \; c_1 = c_2 = c_3 = \cdots = c_n = 0$$