AI1103-Challenging problem

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Three points are chosen on the line of unit length.Find the probability that each the 3 line segments have length greater than $\frac{1}{4}$.

SOLUTION

Let the line be situated between the points $(0, 0)$ and $(1, 0)$

Consider two points $(x, 0)$ and $(y, 0)$ which divide the line in three parts. Conditions on values of x and y are :

A) $\frac{1}{4} \leq x \leq 1$

B) $(y - x) \geq \frac{1}{4}$

C) $(1 - y) \geq \frac{1}{4}$ i.e $0 \leq y \leq \frac{3}{4}$
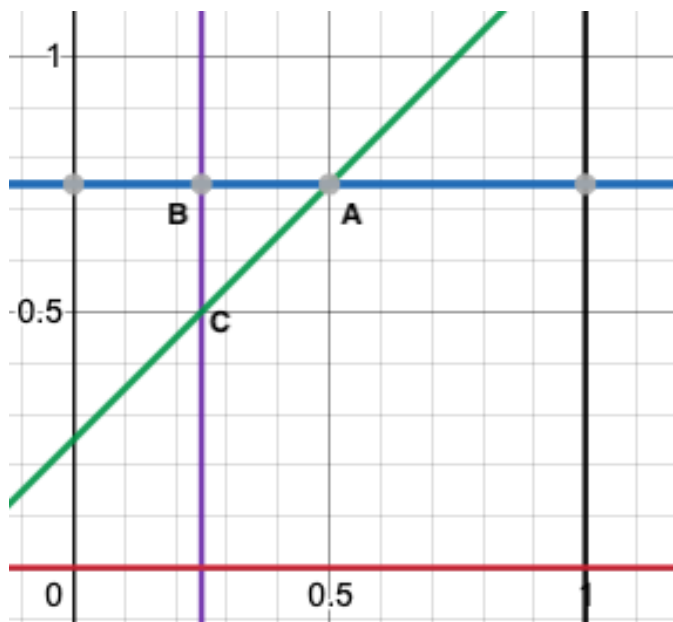
If we consider (x,y) as coordinates of a 2D Cartesian plane, we get,

Fig. 3: Graph

Sample space : $0 \leq x \leq y \leq 1$

Using the above three conditions,

$$\text{Probability} = \frac{\text{Area of ABC}}{\text{Area of Sample Space}} = \frac{1}{16}$$
$$(0.0.1)$$