AI1103-Assignment 2

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Python codes :

https://github.com/Aayush−2492/Assignment−2/tree/main/code

Latex codes :

https://github.com/Aayush−2492/Assignment−2

Question 29

A discrete random variable X takes values from 1 to 5 with probabilities as shown in the table. A student calculates the mean of X as 3.5 and her teacher calculates the variance of X as 1.5. Which of the following statements is true?

k	1	2	3	4	5
$P(X = k)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{9}$

A) Both the student and the teacher are right
B) Both the student and the teacher are wrong
C) The student is wrong but the teacher is right
D) The student is right but the teacher is wrong

Solution

Let n represent the length of the interval.

$$P_n(x = k) = \frac{1}{3(2n-1)} - \frac{|\frac{k}{n} - 3|}{9n(2n-1)} \tag{0.0.1}$$

$\forall k \in [0, 6n]$ and $n =$ odd and positive number

For the given question, $n = 1$. Verify with the values in the table.

Note that $\sum_{k=0}^{6n} P_n(x = k) = 1$

Let x_k represent the values in sample space $\forall k \in [0, 6n]$ and k ϵ $\mathbb{N}$

$$x_k = k \tag{0.0.2}$$

$$Mean = E(X) = \sum_{k=0}^{6n} x_k P_n(x = k) \tag{0.0.3}$$

Substituting values from 0.0.1 and 0.0.2, we get,

$$E(X) = \sum_{k=0}^{6n} k \left(\frac{1}{3(2n-1)} - \frac{|\frac{k}{n} - 3|}{9n(2n-1)} \right) \tag{0.0.4}$$

$$= \frac{1}{3(2n-1)} \sum_{k=0}^{6n} k - \frac{1}{9n^2(2n-1)} \sum_{k=0}^{6n} k|k - 3n| \tag{0.0.5}$$

$$= \frac{1}{3(2n-1)} \left(\frac{6n(6n+1)}{2} \right) - \frac{1}{9n^2(2n-1)} \left(\sum_{k=0}^{3n} k(3n-k) + \sum_{k=3n+1}^{6n} k(k-3n) \right) \tag{0.0.6}$$

$$= \frac{6n^2 - 2n - 1}{(2n-1)} \tag{0.0.7}$$

Substituting n=1, we get,

$$E(X) = 3 \tag{0.0.8}$$

$$\neq 3.5 \tag{0.0.9}$$

Therefore, the student is wrong.

We know that

$$E(g(X)) = \sum_{k=1}^{n} g(x_k) P_k \tag{0.0.10}$$

For $g(X) = X^2$,

$$E(X^2) = \sum_{k=1}^{n} (x_k)^2 P_k \tag{0.0.11}$$

Substituting values from 0.0.1 and 0.0.2, we get,

$$E(X^2) = \sum_{k=0}^{6n} k^2 \left(\frac{1}{3(2n-1)} - \frac{|\frac{k}{n} - 3|}{9n(2n-1)} \right) \quad (0.0.12)$$

$$= \frac{1}{3(2n-1)} \sum_{k=0}^{6n} k^2 - \frac{1}{9n^2(2n-1)} \sum_{k=0}^{6n} k^2 |k - 3n| \quad (0.0.13)$$

$$= \frac{1}{3(2n-1)} \left(\frac{6n(6n+1)(12n+1)}{6} \right) - \frac{1}{9n^2(2n-1)} \left(\frac{9n^2(3n+1)(9n+1)}{2} \right) \quad (0.0.14)$$

$$= \frac{144n^3 - 45n^2 - 34n - 3}{6(2n-1)} \quad (0.0.15)$$

Substituting n=1, we get,

$$E(X^2) = 10.333 \quad (0.0.16)$$

Now, for variance,

$$Variance = E(X^2) - E(X)^2 \quad (0.0.17)$$

Substituting mean and value from equation 0.0.16 in equation 0.0.17, we get,

$$Variance = 10.333 - 3^2 \quad (0.0.18)$$
$$= 10.333 - 9 \quad (0.0.19)$$
$$= 1.333 \quad (0.0.20)$$
$$(0.0.21)$$

Therefore, the teacher is wrong.

Option (B) is correct