Detection Probability Calculations for Fluctuating Targets under Clutter by RADAR Systems.

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Quick overview of working of a RADAR

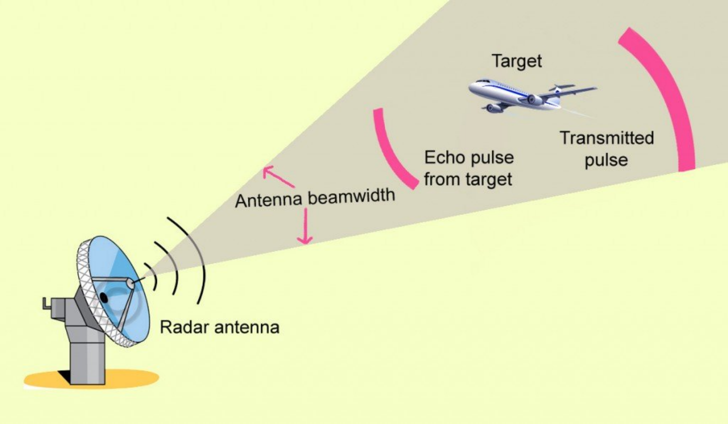

Figure: Working of a RADAR

Some RADAR Terminologies

1. RCS(Radar Cross Section) - Effective area of cross section of the target which reflects the radio/micro waves. It is a common measure of how detectable an object is by radar.
2. Noise - Random signals that get mixed with the reflected signals and not originated from any physical objects. In this presentation we will only take thermal noise into consideration.
3. Clutter - Unwanted reflected waves from ground, sea, rain, birds, etc.
4. Detection Threshold - Threshold level such that if signal to noise power ratio is above it, it is taken as target signal and if below that, it is taken as noise signal. This demands the need to include statistics in RADAR systems and gives rise to terms like probability of detection.

Diagram illustrating detection threshold

Figure: Statistics of target detection

Abstract

1. Classical RCS models(like Swerling model) treated the target RCS value as a random variable irrespective of its aspect angle with respect to radar. This leads to inaccuracy in detection probability calculations and the RCS model used in this presentation takes aspect angle into account.
2. This paper describes a method for generating single-pulse detection probabilities by using a newly proposed aspect-dependent RCS model including the clutter component as well.
3. We will compare various Weibull clutter models to emphasize the importance of clutter parameters selection when analyzing radar performance.
4. We will compare the probability detection for different values of Weibull parameter β and specify the importance of choosing the right β for the specific terrain type.
5. Real life applications include flight path optimizations, improving radar systems for better detection,etc.

Symbols used

Symbols	Meaning
θ	Azimuth angle
ψ	Elevation angle
P_d	Probability of detection
A_t	Target amplitude
A_c	Clutter amplitude
A_n	Noise amplitude
T_d	Detection Threshold
S_i	Mean Radar Cross Section at θ_i

Table: Symbols used in presentation

Some Probability Distributions Used

1. Gaussian Distribution

$$f(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$$

where μ is the mean and σ^2 is the variance.

Some Probability Distributions Used

2. Weibull Distribution

$$f(x;\alpha,\beta) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} exp\left(\frac{-x}{\alpha}\right)^{\beta}, x \geq 0$$

This distribution will be required to generate the values of clutter amplitude for a particular azimuth angle(θ_i).

Some Probability Distributions Used

3. Rayleigh Distribution

$$f(x;\sigma) = \frac{x^2}{\sigma^2} exp\left(\frac{-x}{2\sigma^2}\right), x \geq 0$$

This distribution will be required to generate the values of noise amplitude for a particular azimuth angle(θ_i).

Generating RCS model of Target

We take a sample of values of Azimuth angles(θ_i).
For each sampled azimuth angle θ_i, we generate N random values of azimuth angles $\Theta_{i,j}$ ($j = 1$ to N) from the normal distribution function taking θ_i as the mean of the Gaussian distribution and setting the standard deviation at $\Delta\theta = 1\ ^\circ$ such that $\theta_i - 3^\circ \leq \Theta_{i,j} \leq \theta_i + 3^\circ$.

$$p(\Theta_{i,j}|\theta_i, \Delta\theta_i) = \frac{1}{\sqrt{2\pi\Delta\theta_i}} exp\left(\frac{-(\Theta_{i,j} - \theta_i)^2}{2\Delta\theta_i}\right)$$

We do the same for elevation angles and generate N random values of elevation angles($\Phi_{i,j}$) with mean elevation angle$= 0$ for simplification purposes.

$$p(\Phi i, j|\phi_i, \Delta\phi_i) = \frac{1}{\sqrt{2\pi\Delta\phi_i}} exp\left(\frac{-(\Phi_{i,j} - 0)^2}{2\Delta\phi_i}\right)$$

Generating RCS model of Target

$N = 30000$ is sufficient here. Using the new RCS model in reference paper (2), we generate N power values for each θ_i,

$$\sigma_t^{i,j} = RCS(\Theta_{i,j}, \Phi_{i,j})$$

Target amplitude is,

$$A_t^{i,j} = \sqrt{2\sigma_t^{i,j}}$$

$$S_i = \frac{1}{N}\sum_{j=1}^{N}(\sigma_t^{i,j})$$

Therefore for each azimuth angle θ_i we will take RCS as S_i.

Clutter RCS model generation

Using the Weibull density function, we generate N random values of clutter amplitude($A_c^{i,j}$) for each azimuth angle θ_i and from uniform distribution, we generate N random values of phase between 0 and 2π ($\psi_c^{i,j}$).

$$p(A_c|\alpha, \beta) = \frac{\beta}{\alpha}\left(\frac{A_c}{\alpha}\right)^{\beta-1} exp\left(\frac{-A_c}{\alpha}\right)^{\beta} \tag{1}$$

where, α is the scale parameter and β is the shape parameter.
The parameter α and β are varied as per the domain/surroundings.

Noise RCS model generation

Using the Rayleigh density function, we generate N random values of noise amplitude($A_n^{i,j}$) and from uniform distribution, we generate N random values of phase between 0 and 2π ($\psi_n^{i,j}$).

$$p(A_n) = \frac{A_n}{\sigma_n^2} exp\left(\frac{-A_n^2}{2\sigma_n^2}\right)$$

where σ_n^2 is characteristic noise power in each channel.

Calculating detection probability

Let $Q_{i,j}$ and $I_{i,j}$ represent the components of the input signal $A_s^{i,j}$ for each azimuth angle θ_i,

$$Q_{i,j} = A_t^{i,j} cos(\psi_t^{i,j}) + A_c^{i,j} cos(\psi_c^{i,j}) + A_n^{i,j} cos(\psi_n^{i,j})$$

$$I_{i,j} = A_t^{i,j} sin(\psi_t^{i,j}) + A_c^{i,j} sin(\psi_c^{i,j}) + A_n^{i,j} sin(\psi_n^{i,j})$$

$$A_s^{i,j} = \sqrt{Q_{i,j}^2 + I_{i,j}^2}$$

$$p_d^i = \frac{1}{N} \sum_{j=1}^{N} (A_s^{i,j} \geq T_d)$$

Therefore for each azimuth angle(θ_i) in sample we have generated the detection probability of the aircraft (including the clutter).
T_d value calculation is predetermined.

Conclusion-Simulations

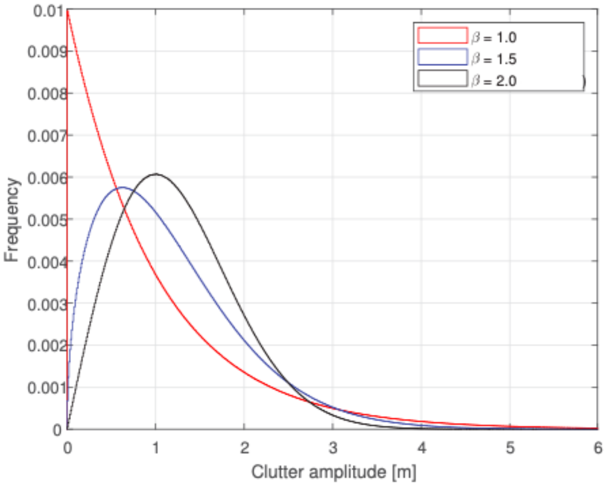

Figure: PDF of Weibull distribution for different values of parameter β

Conclusion-Simulations

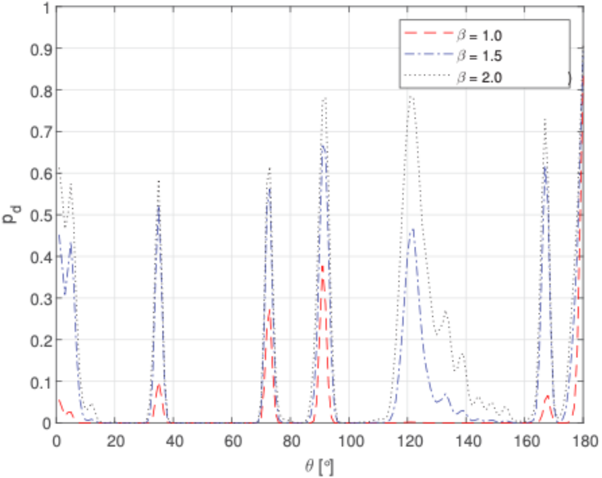

Figure: p_d v/s θ for different values of parameter β

Observations from the above simulations

Taking as a reference the classic Swerling I model together with the Gaussian background (β =2.0), the detection probability is expected to be low with $p_d = 0.28$. However, closely examining above figure shows that the target is more likely to be detected ($p_d \geq$0.5) when $\theta_i = 0^\circ, 90^\circ$ and 180°.

Conclusion

1. We presented a method to calculate the detection probability of an aerial target by combining a newly proposed approach for modeling aspect-dependent RCS fluctuation together with classic statistical clutter models.
2. By comparing the probability detection for different values of Weibull parameter β we specify the importance of choosing the right β for the specific terrain type.