1. *Funding an expense stream.* Your task is to fund an expense stream over n time periods. We consider an expense stream $e \in \mathbf{R}^n$, so that e_t is our expenditure at time t.

   One possibility for funding the expense stream is through our bank account. At time period t, the account has balance b_t and we withdraw an amount w_t. The value of our bank account accumulates with an interest rate ρ per time period, less withdrawals:

   $$b_{t+1} = (1+\rho)b_t - w_t.$$

   We assume the account value must be nonnegative, so that $b_t \geq 0$ for all t.

   We can also use other investments to fund our expense stream, which we purchase at the initial time period $t = 1$, and which pay out over the n time periods. The amount each investment type pays out over the n time periods is given by the *payout matrix* P, defined so that P_{tj} is the amount investment type j pays out at time period t per dollar invested. There are m investment types, and we purchase $x_j \geq 0$ dollars of investment type j. In time period t, the total payout of all investments purchased is therefore given by $(Px)_t$.

   In each time period, the sum of the withdrawals and the investment payouts must cover the expense stream, so that

   $$w_t + (Px)_t \geq e_t$$

   for all $t = 1, \ldots, n$.

   The total amount we invest to fund the expense stream is the sum of the initial account balance, and the sum total of the investments purchased: $b_1 + \mathbf{1}^T x$.

   (a) Show that the minimum initial investment that funds the expense stream can be found by solving a convex optimization problem.

   (b) Using the data in `expense_stream_data.*`, carry out your method in part (a). On three graphs, plot the expense stream, the payouts from the m investment types (so m different curves), and the bank account balance, all as a function of the time period t. Report the minimum initial investment, and the initial investment required when no investments are purchased (so $x = 0$).