Convex Optimization Lab session 4

1. Consider the polynomial $f(x) = x^n$. We wish to approximate this polynomial with a degree $n-1$ polynomial $a_0 + a_1 x + a_2 x^2 + \ldots + a_{n-1} x^{n-1}$ on the interval $[-1, 1]$. Consider partitioning the interval $[-1, 1]$ into $2N+1$ equispaced points $-1 = x_{-N}, x_{-N+1}, \ldots, x_{-1}, x_0 = 0, x_1, \ldots, x_{N-1}, x_N = 1$.

   Suppose we wish to pick the coefficients a_i such that the following cost function is minimized:

$$\sum_{k=-N}^{N} \left(x_k^n - \sum_{i=0}^{n-1} a_i x_k^i \right)^2$$

   (a) Formulate the problem of finding the coefficients a_i, and write a `CVX/CVXPY` code to compute the coefficients.

   (b) Find the coefficients for $n = 5$ $n = 10$, $n = 20$. Pick N to be large enough.

   (c) Plot the optimal cost as a function of n.

   (d) Repeat the above excerises with the cost function

$$\sum_{k=-N}^{N} \left| x_k^n - \sum_{i=0}^{n-1} a_i x_k^i \right|.$$