Performance of a Heat Exchanger

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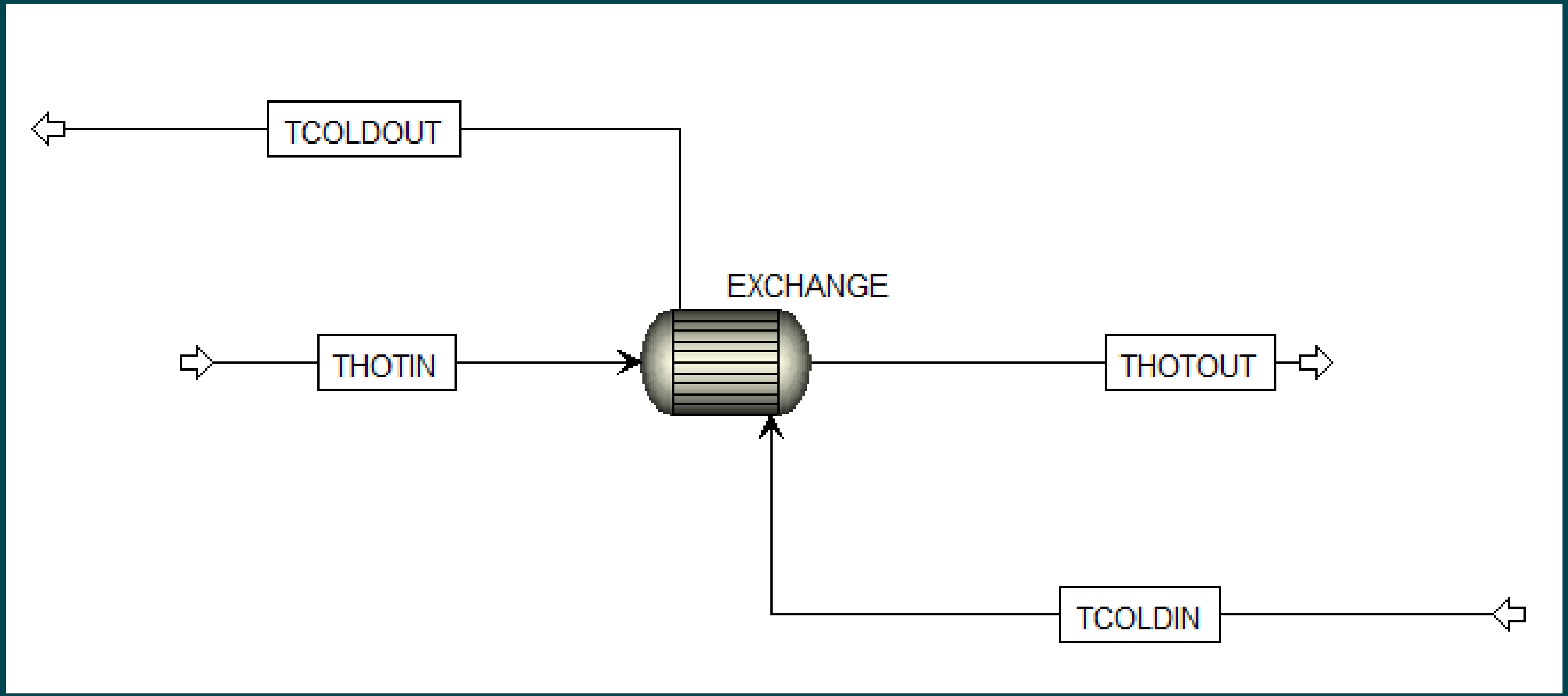
Problem Statement



Problem Statement

Predicting the performance and Designing of a heat exchanger: Heat exchangers are widely used in chemical processing industries for heat transfer between fluids. Modelling the heat transfer within the exchanger requires solving the heat balance equation at the shell and tube side as well as finding optimum area and number of tubes.

Prerequisites

Partial Differential Equations

What are partial differential equations?

A partial differential equation is an equation containing an unknown function of two or more variables and its partial derivatives with respect to these variables.

Classification of PDE's

PDE's are classified in two ways:

- As linear and non-linear
- As homogeneous and non-homogeneous

In a differential equation, when the variables and their derivatives are only multiplied by constants, then the equation is linear. Whereas when they are multiplied by themselves, it becomes non-linear.

A linear partial differential equation with constant coefficients in which all the partial derivatives are of the same order is called as homogeneous linear partial differential equation, otherwise it is called a non-homogeneous linear partial differential equation.

Solving PDE's in Matlab

How to solve PDEs in Matlab?

We use the pdepe function to solve partial differential equations in Matlab.

The syntax of the PDE is as follows:

$$c\left(x,t,u,\frac{\partial u}{\partial x}\right)\frac{\partial u}{\partial t} = x^{-m}\frac{\partial}{\partial x}\left[x^m f\left(x,t,u,\frac{\partial u}{\partial x}\right)\right] + s\left(x,t,u,\frac{\partial u}{\partial x}\right)$$

Here,

c is a diagonal matrix, f is defined as the flux term, and s is defined as the source term.

The boundary conditions are defined as:

$$p(x,t,u) + q(x,t)g(x,t,u,u_x) = 0 \ \text{at} \ x = a,b$$

How to code pdepe in Matlab?

In order to solve a partial differential equation, we must program the PDE function, boundary conditions and initial conditions as separate Matlab function codes.

Then we must call these three subroutines using pdepe function.

eg. u=pdepe(m,'pdefn'.'pdeic','pdebc',x,t)

where m defines the coordinate system, 'pdefn' is the PDE function, 'pdeic' is the Initial condition function, 'pdebc' is the boundary condition function and x and t are the independent variables.

Design of shell and tube heat exchanger

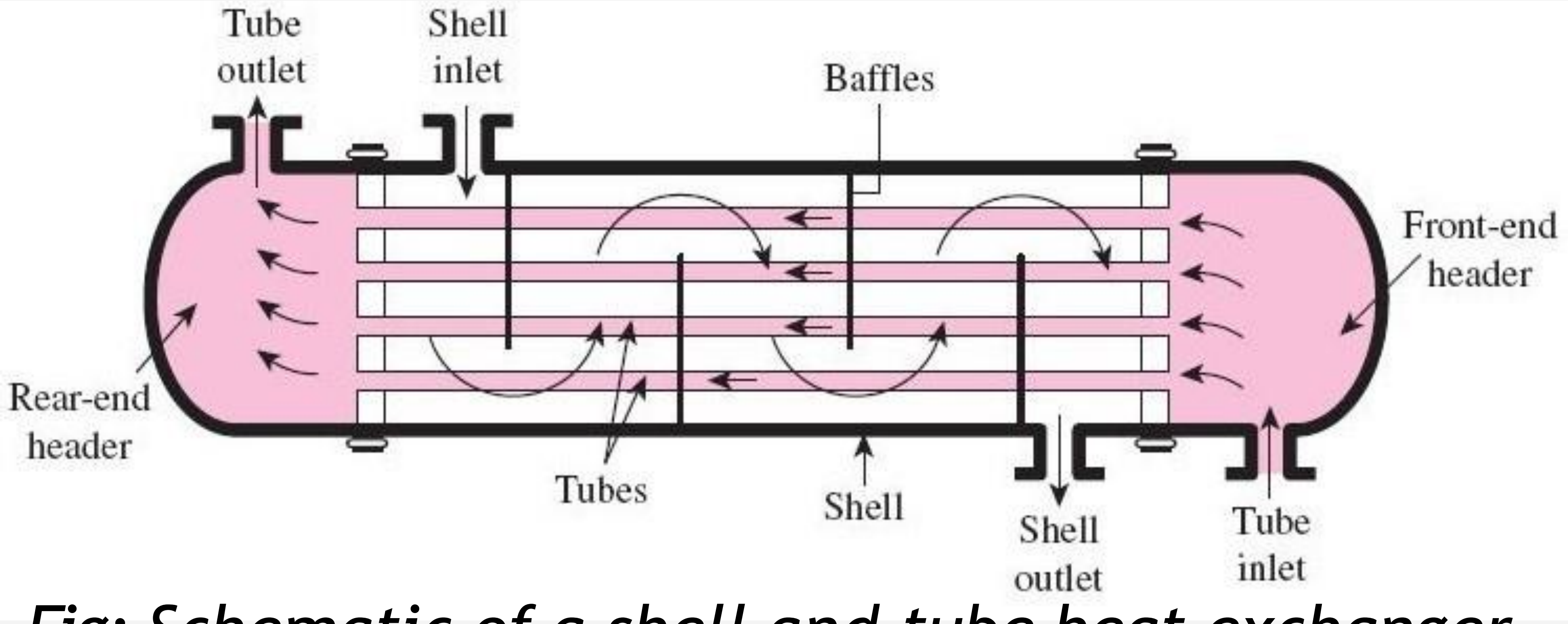


Fig: Schematic of a shell and tube heat exchanger

- 0.0666 < (Shell diameter/Tube length) < 0.2
- 1.25 < (Pitch/Outer Diameter) < 1.5

- Two methods for design calculation of shell and tube heat exchanger

  -Kern and Bell-Delaware methods

Design of shell and tube heat exchanger

- The overall heat transfer rate can be calculated using Equation -

$$Q = U * A * LMTD$$

where, Q = overall heat transfer rate

U = Overall heat transfer coefficient

A = Heat transfer area

LMTD = Logarithmic mean temperature difference

- The LMTD can be calculated using Equation -

$$LMTD = (\Delta T1 - \Delta T2) / \ln(\Delta T1 / \Delta T2)$$

where, $\Delta T1$ = Hot fluid inlet temperature - Cold fluid outlet temperature

$\Delta T2$ = Hot fluid outlet temperature - Cold fluid inlet temperature

Design of shell and tube heat exchanger

- The correction factor can be calculated using Equation-

$$C = (\Delta T1 - \Delta T2) / (\ln(\Delta T1 / \Delta T2) - F)$$

where,

F = Correction factor

- The number of tubes based on the heat transfer area required can be calculated using Equation - :

$$Nt = A / (\pi * D * L)$$

where, Nt = Number of tubes

A = Heat transfer area

D = Tube inside diameter

L = Tube length

Understanding the Problem Statement

Understanding the Problem

- We are designing a shell and tube heat exchanger by solving energy balance equations on both shell as well as tube side
- Firstly we will see the governing equations as well as boundary conditions given in the problem.
- Then we will find steady state solutions using "dsolve" operator.
- We will use these values as initial conditions for "pdepe" solver
- In the end, we will see the LMTD temperature optimum area and number of tubes obtained.

Governing Equations:

$$\frac{\delta T^t}{\delta t} = \frac{K_1}{\rho^t {c_p}^t}\frac{\delta^2 T_t}{\delta z^2} + \frac{4U}{\rho^t {c_p}^t d}(T^s - T^t)$$

$$\frac{\delta T^s}{\delta t} = \frac{K_2}{\rho^s {c_p}^s}\frac{\delta^2 T_s}{\delta z^2} - \frac{U\pi D}{\rho^s {c_p}^s A^s}\left(T^s - T^t\right)$$

Boundary Conditions:

$T^t(0,t) = 30^{\circ}C$

$T^t(L,t) = 50^{\circ}C$

$T^s(0,t) = 200^{\circ}C$

$T^s(L,t) = 40^{\circ}C$

Given Data:

$U = 800\ W/m^{2\circ}C$ $\quad (Overall\ heat\ coefficient)$

$d = 0.02\ m$ $\quad (Diameter\ of\ tubes)$

$D = 0.667m$ $\quad (Diameter\ of\ shell)$

$K_1 = 0.631\ W/m^{\circ}C$ $\quad (Thermal\ conductivity\ of\ cooling\ water)$

$K_2 = 0.625\ W/m^{\circ}C$ $\quad (Thermal\ conductivity\ of\ gas\ oil)$

Mass flow rate of gas oil $= \dot{m}_h = 6.25$ kg/s

Tube side	Shell side
cooling water	Gas Oil
$T_{c,in} = 30^{\circ}C$	$T_{h,in} = 200^{\circ}C$
$T_{c,out} = 30^{\circ}C$	$T_{h,out} = 40^{\circ}C$

Properties obtained from Literature

Physical Properties

Water, from steam tables:

Temperature, °C	30	40	50
C_p, kJ kg^{-1}°C^{-1}	4.18	4.18	4.18
k, kWm^{-1}°C^{-1}	618×10^{-6}	631×10^{-6}	643×10^{-6}
μ, mNm^{-2}s	797×10^{-3}	671×10^{-3}	544×10^{-3}
ρ, kg m^{-3}	995.2	992.8	990.1

Gas oil, from Kern (1950).

Temperature, °C	200	120	40
C_p, kJ kg^{-1}°C^{-1}	2.59	2.28	1.97
k, Wm^{-1}°C^{-1}	0.13	0.125	0.12
μ, mNm^{-2}s	0.06	0.17	0.28
ρ, kg m^{-3}	830	850	870

Matlab Code for the Problem

Steady State Equations

```
syms T_tubes(z)
syms T_shell(z)
rho1 = 992.8;                   % density of hot water (kg/m^3)
Cp1 = 4180;                     % specific heat capacity of hot water (J/kg*K)
d = 0.02;                       % diameter of tube (m)
u1 = 0.02747/(pi*(d^2)/4);      % velocity of hot water (m/s)
rho2 = 850;                     % density of oil (kg/m^3)
Cp2 = 2280;                     % specific heat capacity of oil (J/kg*K)
D=0.667;                        % diameter of shell (m)
u2 = 0.00735/(pi*(D^2)/4);      % velocity of cooling water (m/s)

k1 = 0.631;
L = 4;                          % length of tube (m)
As = pi*0.667^2/4;
U=800;                          % Overall Heat Transfer coefficient (W/(m^-2 C^-1))

DT_tubes = diff(T_tubes);
ode_1=diff(T_tubes,z,2)== ((U*4)/(rho1*Cp1*d*k1))*(150);
cond_Adash = T_tubes(0) == 50;
cond_Bdash = DT_tubes(0) == 0;
conds_1 =[cond_Adash cond_Bdash];
tubessolve(z)= dsolve(ode_1,conds_1)


DT_shell = diff(T_shell);
ode_2=diff(T_shell,z,2)== ((U*pi*D)/(rho2*Cp2*As*k1))*(150);
cond_A = T_shell(0) == 200;
cond_B = DT_shell(0) == 0;
conds_2 =[cond_A cond_B];
shellsolve(z)= dsolve(ode_2,conds_2)
```

Steady State Results

$$\mathrm{tubessolve}(z) = 4.5826 * z^2 + 50$$

$$\mathrm{shellsolve}(z) = 0.29424 * z^2 + 200$$

PDE function

```
function [c,f,s] = pde_he(x,t,u,DuDx)
rho = [992.8 , 850];
cp = [4180 , 2280];
k1 = 0.631;
k2 = 0.125;
U = 800;
d=0.02;
D=0.667;
As = pi*D^2/4;

c = [1 ; 1];
f = [-(k1/(rho(1)*cp(1))); -k2/(rho(2)*cp(2))].*DuDx;
s = [(4*U)*(u(2)-u(1))/(rho(1)*cp(1)*d) ; - U*pi*D*(u(2)-
u(1))/(rho(2)*cp(2)*As)];

end
```

Initial Condition

```
function u = pde_ic(x)
u = [4.5826*x^2+50; 0.29424*x^2+200];
end
```

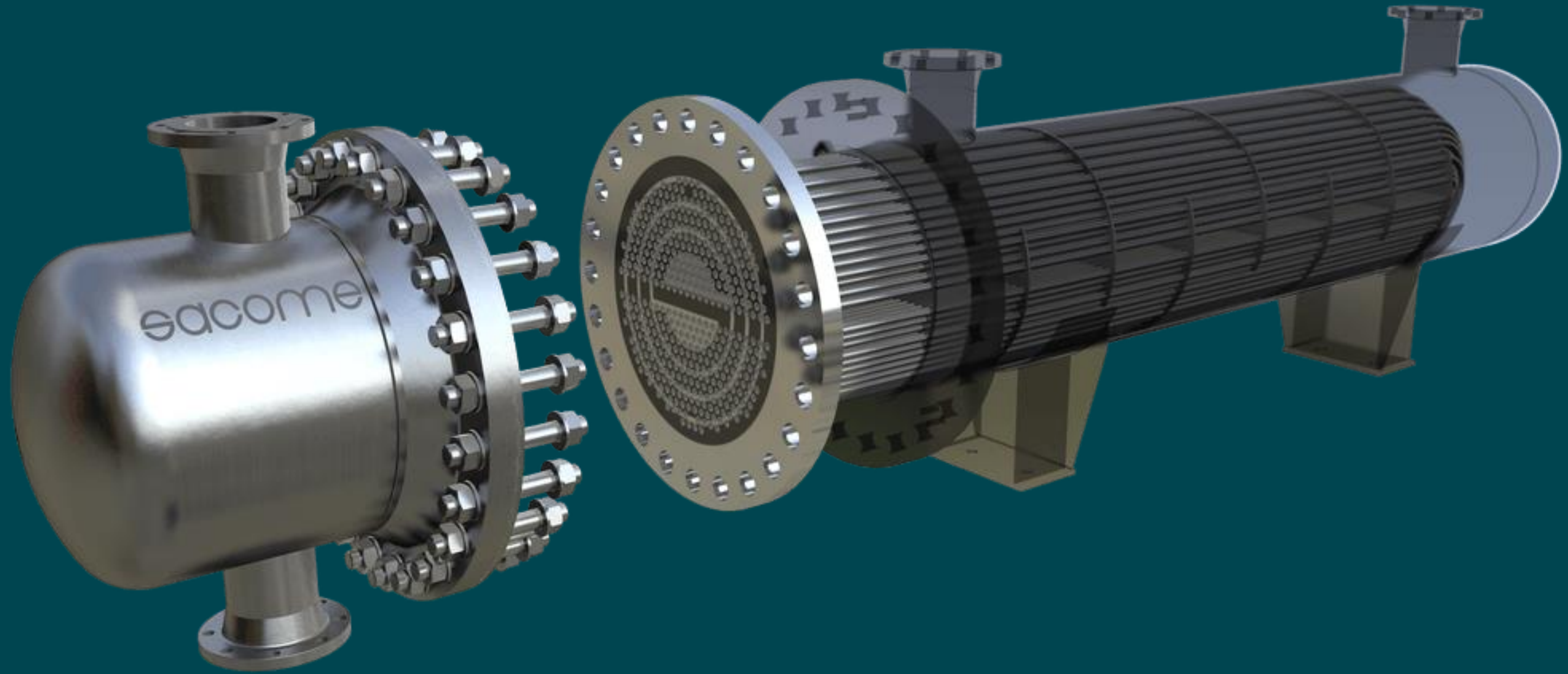
Boundary Condition

```
function [pl,ql,pr,qr] = pde_bc(xl,ul,xr,ur,t)
pl=[ul(1)-50;ul(2)-200];
ql=[0;0];
pr=[ur(1)-30;ur(2)-40];
qr=[0;0];
end
```

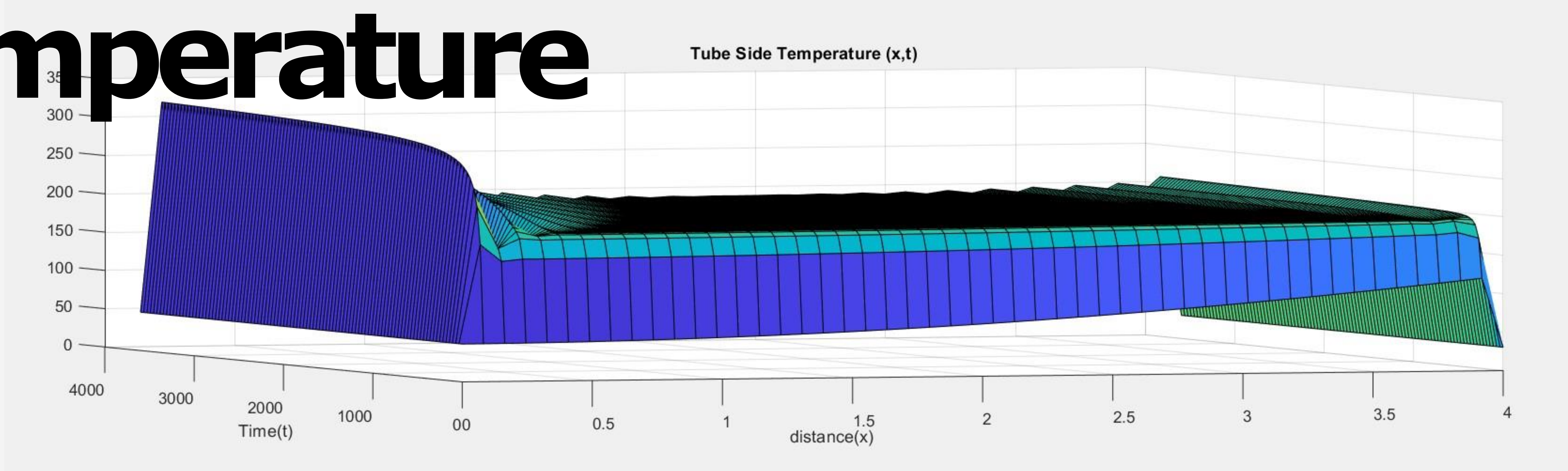
Driver Code

```
clc;
clear;
m = 0;
x = linspace(0, 4, 50);
t = linspace(0, 3600, 100);
sol = pdepe(m,'pde_he','pde_ic','pde_bc',x,t);


T_tube = sol(:,:,1);                        %temperature tube side
T_shell = sol(:,:,2);                        %temperature shell side
subplot(2,1,1);
surf(x,t,T_tube);
title('Tube Side Temperature (x,t)');
xlabel('distance(x)');
ylabel('Time(t)');
subplot(2,1,2);
surf(x,t,T_shell);

title('Shell Side Temperature(x,t)');
xlabel('distance (x)');
ylabel('Time(t)');
```

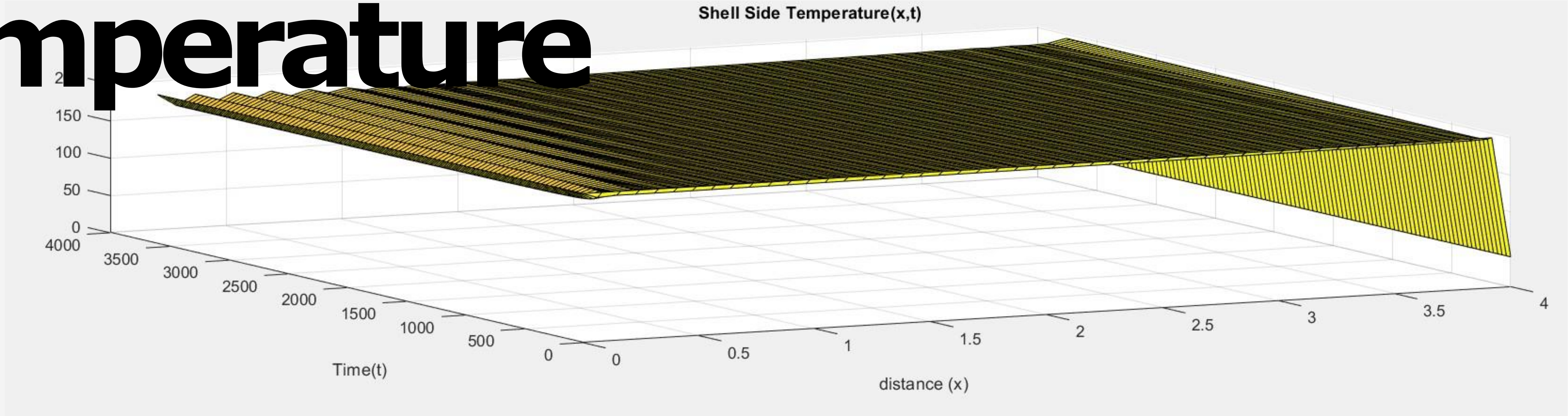
Results



Plot for Tube Side Temperature



Plot for Shell Side Temperature



Heat Exchanger final Design Calculations

```
clc
clear
% GIVEN
T_hotin = 200;
T_hotout = 40;

T_coldin = 30;
T_coldout = 50;
U = 800;
D = 0.02;
L = 4;
Mhot = 6.25;         % mass of gas oil in kg/s
cphot = 2280;        % J/(kgC)

Q = Mhot*cphot*(T_hotin - T_hotout)

T_LMTD = ((T_hotin - T_coldout) - (T_hotout - T_coldin))/(log((T_hotin- T_coldout)/(T_hotout -
T_coldin)))

% for correction factor

R = (T_hotin - T_coldout)/(T_coldout - T_coldin)
S = (T_coldout - T_coldin)/(T_hotin - T_hotout)

% from curve
Ft = 0.94;

T_LMTDcorrected = T_LMTD * Ft

Required_Area = Q/(U*T_LMTDcorrected)
NO_of_tubes = Required_Area/(pi*D*L)
```

Final Results

$$Q = 2280\ kW$$

$$T_{LMTD} = 51.6977\ °C$$

$$T_{LMTD\ corrected} = 49.5958\ °C$$

$$Required\ Area = 58.6470\ m^2$$

$$\text{No.}\ of\ \text{tubes} = 233$$

Advantages of Shell and Tube Heat Exchangers

The Benefits of Shell and Tube Heat Exchangers

High Heat Transfer Efficiency: Shell and tube heat exchangers have a large surface area and can handle high-pressure and high-temperature differences between fluids, resulting in a high heat transfer rate.

Versatility: Shell and tube heat exchangers are suitable for a wide range of fluids, including liquids, gases, and vapors, making them a versatile choice for many industrial applications.

Low Maintenance: Shell and tube heat exchangers require minimal maintenance and are easy to clean due to their simple design.

The Benefits of Shell and Tube Heat Exchangers

Longevity: Shell and tube heat exchangers are durable and can last for many years if properly maintained, making them a cost-effective solution.

Scalability: Shell and tube heat exchangers can be designed and built in a variety of sizes and configurations to meet the specific needs of different applications.

Low Fouling: The design of shell and tube heat exchangers minimizes fouling, which can lead to reduced efficiency and higher maintenance costs in other types of exchangers.

Applications of Shell and Tube Heat Exchangers in Chemical Processes

Heat Recovery: Shell and tube heat exchangers are used to recover heat from exhaust gases, process streams, or cooling water, which can then be reused in the production process, leading to energy savings.

Condensation: Shell and tube heat exchangers are used for condensing vapors into liquids. This process is often used to recover valuable products or to improve the efficiency of chemical reactions.

Cooling: Shell and tube heat exchangers are used to cool process streams and equipment, such as reactors, distillation columns, or storage tanks. They can also be used to cool heat exchanger duty streams, which can help to reduce energy costs.

Applications of Shell and Tube Heat Exchangers in Chemical Processes

Heating: Shell and tube heat exchangers can also be used to heat process streams, such as in the case of preheating feed streams to reactors or evaporators.

Chemical Reactions: Shell and tube heat exchangers are used to control the temperature of chemical reactions by either heating or cooling the reactants. This can help to increase reaction rates, improve product quality, or prevent unwanted side reactions.

Thank You !

Now we are ready for questions