Speed Math

Multiplication:

1. Multiplication using multiples
Assume that we should find out the result of 12 x 15.
12 x 15   (Here we can write this 15 as 5 x 3)
= 12 x 5 x 3 (now 12 x 5 becomes 60)
= 60 x 3 (For this you just calculate 3 x 6, that is 18 and add one Zero to it. that is 180)
= 180

2. Multiplication by distribution
Assume that we should find out the result of 12 x 17
12 x 17
(Here we can divide this 17 as 10+7. Here, multiplying 12 with 17 is same as multiplying 12 with 10 and 7 separately and then adding the results)
So, we can write it as
= (12 x 10) + (12 x 7)
= 120 + 84
= 204

3. Multiplication by "giving and taking"
12 x 47 (Here its little difficult for us to calculate the multiplication of 12 and 47 mentally. so just check for the ROUNDED number nearer to 47. Yes it is 50. so.....
= 12 x (50 - 3)
= (12 x 50) - (12 x 3) = 600 - 36
= 564

4. Multiplication by 5
If we have to multiply a number with 5, just divide the number with 2 and then multiply the result with 10. Confused? Its very simple step actually....
428 x 5 (Now just divide the number with 2)
= 428 x 1/2 = 214 (Now multiply it with 10)
= 214 x 10
= 2140 (This is our result)
What's the logic behind this step?
Very simple.
Lets say the number is X.
Now we are dividing the number with 2. so here X becomes X/2.
And then we are multiplying it with 10. So it will become 10X / 2
Now cancel it with 2. so it becomes 10X / 2 = 5X = 5 multiplied by X. That's it ;)

5. Multiplication by 10 ------------ just move the decimal point one place to the right
16 x 10 = 160
5.9 x 10 = 59
169.93 x 10 = 1699.3

6. Multiplication by 50 ------ divide with 2 and then multiply by 100
Well, this is also same process as we did for 5. Here we should add an extra zero. That's it
18 x 50
= (18/2) = 9
= 9 x 100
= 900

7. Multiplication by 100 -------- move the decimal point two places to the right
45 x 100
= 4500

8. Multiplication by 500-------- divide with two and multiply with 1000
21 x 500
= 21/2 x 1000
= 10.5 x 1000
= 10500

9. Multiplication by 25 ---------- use the analogy Rs 1 = 4 x 25 Paise
25 x 14 (just divide the 14 as 10+4)
= (25 x 10) + (25 x 4)
= 250 + 100 ---> Rs2.50 + Rs1
= 350

10. Here you can use another technique too. Which we have used for multiplication with 5.
Multiplication by 25 ----------- divide by 4 and multiply by 100
36 x 25
= (36/4) x 100
= 9 x 100
= 900

11. Multiplication by 11 (if sum of digits is less than 10)
72 x 11
= 7+2 =9, it is less than 10.
So,
= place this term 9 between 7 &2
= 792 (That's the answer)

12. Multiplication by 11 (if sum of digits is greater than 10)
87 x 11
=> 8 + 7 = 15
Because here 15 is greater than 10, first use 5 and then add 1 to the first term 8,
Which give you the answer
= 957

13. Multiplication of numbers ending in 5 with the same first terms (square of a number)
25 x 25
First term = (2 + 1) x 2 = 6
Last term = 25
Answer = 625 => square of 25
75 x 75
First term = (7 + 1) x 7 = 56
Last term = 25
Answer = 5625 => 75 square

Squares:

1. Easiest way to find the squares of numbers which have 5 as a unit digit.
5^2= 25
15^2 =225
25^2= 625
35^2 =1225

45^2=2025 and so……on
It is Clear that at last 25 appears in every case and the others numbers are calculate on the basis of N*(N+1) i.e.
In 15^2, write 25 at tens place and for further hundred and thousand place 1*(1+1)= 2 so the complete answer is 225.
To find square of any number in 2 digits use this trick:
We all know this formula
$(a + b)^2 = a^2 + 2ab + b^2$
Just apply this to find the squares in seconds.
$(43)^2 = 4^2 + 2*4*3 + 3^2$ =16 + 24+9
Copy single digit from last and take the remaining number as carry forward: First write 9 at unit place then 4 at tens place carry 2 and 16 + 2 = 18 write 18 at hundred place so the number form is:1849
Take the different examples and solve with the trick

Square Root:
By knowing the shortcut to find the square root of a number, you will be able to find out the square root of any number within seconds.
Now let's go through the steps…
Step 1: First of all group the number in pairs of 2 starting from the right.

Step 2: To get the ten's place digit, Find the nearest square (equivalent or greater than or less than) to the first grouped pair from left and put the square root of the square.

Step 3: To get the unit's place digit of the square root
Remember the following

If number ends in	Unit's place digit of the square root
1	1 or 9(10-1)
4	2 or 8(10-2)
9	3 or 7(10-3)
6	4or 6(10-4)
5	5
0	0

Lets see the logic behind this for a better understanding
We know,
$1^2=\underline{1}$
$2^2=\underline{4}$
$3^2=\underline{9}$
$4^2=1\underline{6}$
$5^2=2\underline{5}$
$6^2=3\underline{6}$
$7^2=4\underline{9}$
$8^2=6\underline{4}$
$9^2=8\underline{1}$
$10^2=10\underline{0}$

Now, observe the unit's place digit of all the squares.
Do you find anything common?
We notice that,
Unit's place digit of both 1^2 and 9^2 is 1.
Unit's place digit of both 2^2 and 8^2 is 4

Unit's place digit of both 3^2 and 7^2 is 9
Unit's place digit of both 4^2 and 6^2 is 6.

Step 4: Multiply the ten's place digit (found in step 1) with its consecutive number and compare the result obtained with the first pair of the original number from left.

Remember,
If first pair of the original number > Result obtained on multiplication then select the greater number out of the two numbers as the unit's place digit of the square root.

If first pair of the original number < the result obtained on multiplication, then select the lesser number out of the two numbers as the unit's place digit of the square root.

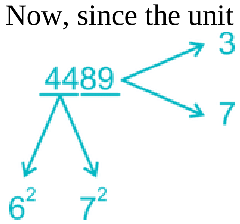
Let us consider an example to get a better understanding of the method

Example 1: $\sqrt{784}$=?
Step 1: We start by grouping the numbers in pairs of two from right as follows
7 84
Step 2: To get the ten's place digit,
We find that nearest square to first group (7) is 4 and $\sqrt{4}=2$
Therefore ten's place digit=2

Step 3: To get the unit's place digit,
We notice that the number ends with 4, So the unit's place digit of the square root should be either 2 or 8(Refer table).

Step 4: Multiplying the ten's place digit of the square root that we arrived at in step 1(2) and its consecutive number(3) we get,
$2\times3=6$

ten's place digit of original number > Multiplication result
$7>6$
So we need to select the greater number (8) as the unit's place digit of the square root.
Unit's place digit =8
Ans:$\sqrt{784}=28$
Example: Find the square root of 4489.
We group the last pair of digits and the rest of the digits together.
Now, since the unit digit of 4489 is 9. So we can say that unit digit of its square root will be either 3 or 7.

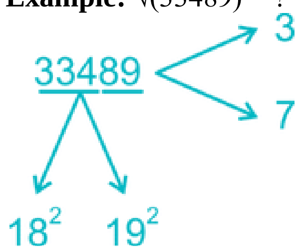


Now consider first two digits i.e. 44. Since 44 comes in between the squares of 6 and 7 (i.e. $6^2 < 44 < 7^2$), so we can definitely say that the ten's digit of the square root of 4489 will be 6. So far, we can say that the square root will be either 63 or 67.
Now we will find the exact unit digit.
To find the exact unit digit, we consider the ten's digit i.e. 6 and the next term i.e. 7.
Multiply these two terms

$$6 \times 7 = 42 < 44$$

Since, 44 is greater than 42. So square root of 4489 will be the bigger of the two options i.e. 67.
Let us take another example.

Example: $\sqrt{(33489)} = ?$



Unit digit will be 3 or 7.
Since, $18^2 < 334 < 19^2$
So, square root will be 183 or 187.
Now consider 18 and 19.

$$18 \times 19 = 342 > 334$$

Now, 334 is less than 342. So, the square root will be lesser of the two numbers i.e. 183.

Cube Root:

Step 1:Find the cube root of the last digit.
Points to be remembered while using this method.
(1)If the last digit is 8 then cube root will be 2.
(2)If the last digit is 2 then cube root will be 8.
(3)If the last digit is 7 then cube root will be 3.
(4)If the last digit is 3 then cube root will be 7.
(5)If the last digit is any other digit other than 2,8,3,7 then put the same number.
From this step you will get the unit's or one's place digit.

To find the tenth place digit you need to follow the below steps.
Step 2: Strike out the last 3 digits of the given number.
Step 3: find the nearest cube of the remaining number.
Step 4: find the cube root of the nearest cube which will give you ten's place digit.

Example 1: Find the cube root of 157464 in 5 seconds.
$\sqrt[3]{157464}=?$
Step 1: First we need to find the cube root of the last digit of the given number.
Here the last digit is 4. 4 is a number other than 2, 8, 3, 7
hence we put the number as it is.
We get our one's place digit as 4.

Now to get tenth place digit
Step 2: We need to strike the last 3 digits of the given number.
In this example the last 3 digits are 464 which we will strike off as shown below
157~~464~~

Step 3: We need to find the nearest cube to the remaining number(157)
We find that 125 is the nearest cube to 157.

Step 4: We need to find the cube root of the nearest cube(125)
$\sqrt[3]{125}=5$
From this step we get our ten's place digit as 5.
From step 1 and step 4 we get the
$\sqrt[3]{157464}=54$

Ans: 54

Fraction:

The following points are found useful while comparing two or more fractions.

(a) If the denominators of the fractions are same, the largest is one whose numerator is the largest.

Example: Which is the largest fraction among the following?

$\frac{3}{8}, \frac{7}{8}$ and $\frac{5}{8}$

Solution: $\frac{7}{8}$

(b) If the numerators of the fractions are same, the largest is one whose denominator is the smallest.

Example: Which is the largest fraction among the following?

$\frac{5}{2}, \frac{5}{7}$ and $\frac{5}{9}$

Solution: $\frac{5}{2}$

(c) If neither the numerators nor the denominators of the fractions are same, then they are converted into equivalent fractions of the same denominator by taking the L.C.M. of the denominators of the given fractions. Then the fractions are compared accordingly.

Example: Which is the largest fraction among the following?

$\frac{1}{2}, \frac{2}{3}, \frac{4}{5}$ and $\frac{5}{8}$

Solution: L.C.M. of 2, 3, 5 and 8 = 120.

$\frac{1}{2} = \frac{1 \times 60}{2 \times 60} = \frac{60}{120}$

Then,

$\frac{2}{3} = \frac{2 \times 40}{3 \times 40} = \frac{80}{120}$

$\frac{4}{5} = \frac{4 \times 24}{5 \times 24} = \frac{96}{120}$

And, $\frac{5}{8} = \frac{5 \times 15}{8 \times 15} = \frac{75}{120}$

Now, the denominator of these fractions are same and the largest numerator is 96. Hence, the largest fraction is $\frac{96}{120}$, that is, $\frac{4}{5}$.

(d) Two fractions can also be compared by cross-multiplication method.

Example: Which is grater $\frac{6}{13}$ or $\frac{7}{5}$?

Solution: Step I: By cross-multiplying the two given fractions

$\frac{6}{13} \times \frac{5}{7}$

We get 6 × 7 = 42 and 13 × 5 = 65.

Step 2: Since 65 is greater than 42 and in 65, the numerator of $\frac{5}{7}$ is included,

$\therefore \frac{5}{7}$ is greater than $\frac{6}{13}$

(e) If the difference of the numerator and denominator of each of the given fractions be same, then the fraction of the largest numerator is the largest.

(f) In the given fraction, $\frac{x}{y}, \frac{x+a}{y+b}, \frac{x+2a}{y+2b}, \ldots,$
$\frac{x+na}{y+nb}$, where a < b

(a) If $\frac{\text{Increase in Numerator}}{\text{Increase in Denominator}}$ > first fraction, the last value is the greatest.

(b) If $\frac{\text{Increase in Numerator}}{\text{Increase in Denominator}}$ < first fraction, the last value is the least.

(c) If $\frac{\text{Increase in Numerator}}{\text{Increase in Denominator}}$ = first fraction all values are equal.

Example: Which of the following fraction is the largest?
$\frac{2}{3}, \frac{3}{4}, \frac{5}{6}$ and $\frac{9}{10}$
Solution: In each of the given fractions, the difference between the numerator and denominator is same and the largest numerator is 9. The largest fraction is $\frac{9}{10}$.

Example: Which one of the following fractions is the greatest?
$\frac{3}{8}, \frac{4}{11}, \frac{5}{14}, \frac{6}{17}, \frac{7}{20}$
Solution: Since, $\frac{\text{increase in numerator}}{\text{increase in Denominator}} = \frac{1}{3}$ is less than the first fraction $\frac{3}{8}$, therefore, the first fraction $\frac{3}{8}$ the greatest.