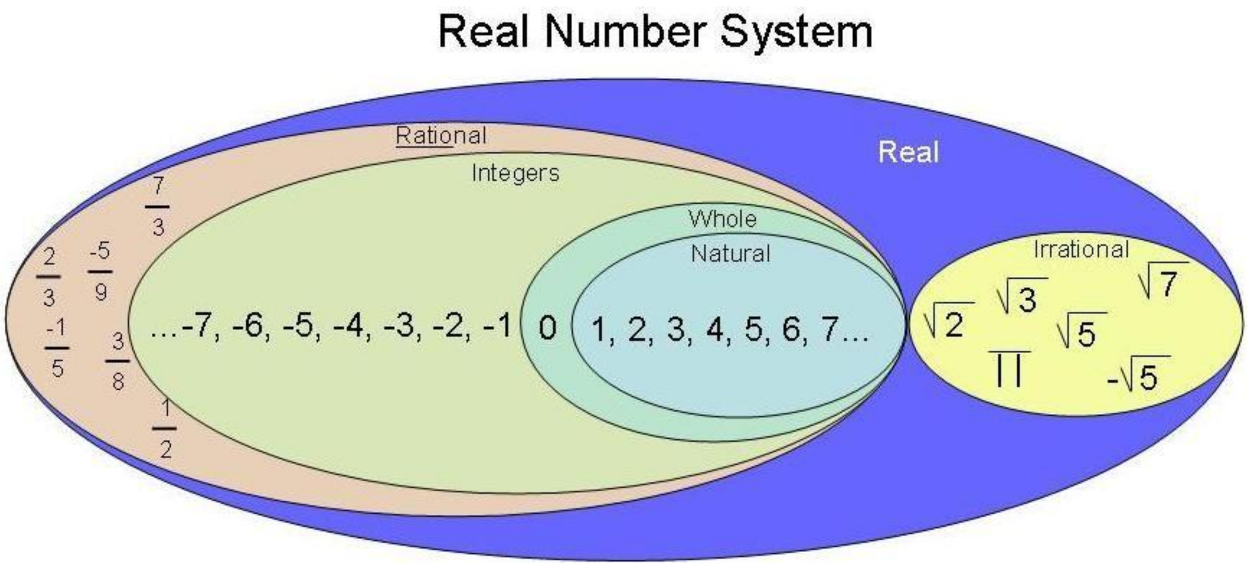
Number System

Number system can be understood by this diagram easily.



➢➢ **Natural Numbers (N):**

Natural Numbers are counting numbers from 1, 2, 3, 4, 5, ...

N = {1, 2, 3, 4, 5, ………..}

➢➢ **Whole Numbers (W):**

Whole numbers are natural numbers including zero. They are 0, 1, 2, 3, 4, 5,...............

W = {0, 1, 2, 3, 4, 5, ...}, In other way we can say that W = 0 + N

➢➢ **Positive Numbers:**

Positive numbers are, 1, 2, 3, 4, 5.................

➢➢ **Negative Numbers:**

Negative numbers are -3, -2, -1)

➢➢ **Integers (Z):**

❖❖ Whole Numbers together with negative numbers.

❖❖ Integers are set containing the positive numbers, 1, 2, 3, 4, ..., and negative numbers,............-3, -2, -1, together with zero.

❖❖ Zero is neither positive nor negative but is both.

- In other words, Integers are defined as a set of whole numbers and their opposites.
- Z = {......., -3, -2, -1, 0, 1, 2, 3, ...}

- **Rational Numbers (Q):**
  - Numbers which can be express in the form of P/Q, Where Q does not equal to zero are called rational number.
  - Proper Fraction: Value of Numbers smaller than 1 e.g. ½ or ¾.
  - Improper Fraction: Value of Numbers greater than 1 e.g. 5/2
  - Mixed Fraction: $2\frac{1}{2}$ = 5/2
  - Powers and square roots may be rational numbers if their standard form is a rational number.
  - In rational numbers, the denominator cannot be zero

    Ex 1:- 2 can be expressed in the form of p/q as 2/1,
    Ex 2:- 13/9 = 1.444.......,

    Ex 3:- 4/3 = 1.333...,

    Ex 4:- 2/3 = 0.6666666.......

    Ex 5:-5/11 = 0.454545......

- **Irrational Numbers ($Q^{a)}$:**
  - Cannot be expressed as a ratio of integers.
  - As decimals, they never repeat or terminate.
  - They go on forever or infinity.
    - Example: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{7}$, $\sqrt{8}$

      $\sqrt{2}$ = 1. 41421356......Irrational

  π = 22/7 = 3.14159265....... Irrational

- **Real Numbers R:**
  - Real Numbers are every number, irrational or rational.
  - Any number that you can find on the number line.
  - It is a number required to label any point on the number line, or it is a number that names the distance of any point from 0.
  - R = Q + Q1

❖❖ Natural Numbers are Whole Numbers, which are Integers, which are Rational Numbers, which are Real Number also.

❖❖ Irrational Numbers are Real Numbers, but not all Real Numbers are Irrational Numbers.

Examples:

0.45	rational real
3.1415926535..................	irrational, real
3.14159	rational, real
0	whole, integer, rational, real
= 5/3	rational, real
√ 2= 1.41421356......	irrational, real

Remainder and Quotient

If 'a' &'d' are natural numbers, with ≠ 0, it can be proved that there exist unique integers 'q' and 'r', such that = + and $0 \le r < d$. The number **'q'** is called the quotient, while **r** is called the remainder. Following can be summarized as remainder theorem.

For instance, "The remainder is 1 when 7 is divided by3" means 7 = (3×2) + 1

Even, Odd Numbers

- A number n is even if the remainder is zero when n is divided by 2: n = 2a+0 or n=2a.
- A number n is odd if the remainder is one when n is divided by 2:n=2a+1)

Prime Numbers

Number which have exactly 2 distinct factor are called prime numbers.

A prime number is a positive integer that is divisible by itself and 1 only.

Examples: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, etc.

Please note the following facts---

❖❖ Zero is not a prime number because zero is divisible by more than two numbers. Zero can be divided by 1, 2, 3 etc.

(0 ÷ 1 = 0, 0÷ 2 = 0 ...)

❖❖ One is not a prime number because it does not have two factors. It is divisible by only 1.

Prime number can also be written in the form of 6n+1 or 6n-1 but every number written in this form is not a prime number.

Co-Prime Numbers:-

A pair of numbers are said to be co-prime if they don't have any number in common except 1.

Ex1:- 2,3

Ex:-(2,3),(4,5),(9,11)

Composite Numbers

Composite numbers are numbers that have more than two factors. A composite number is divisible by at least one number other than 1 and itself.

Examples: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, etc.

Please note that zero and 1 are neither prime numbers nor composite numbers.

Every whole number is either prime or composite, with two exceptions 0 and 1 which are neither prime nor composite.

Tests of Divisibility

Divisibility By 2: A number is divisible by 2 if its unit's digit is any of 0, 2, 4, 6, and 8.

Divisibility By 3: A number is divisible by 3 if the sum of its digits is divisible by 3.

Divisibility By 4: A number is divisible by 4 if the number formed by the last two digits is divisible by 4.

Divisibility By 5: A number is divisible by 5 if its unit's digit is either 0 or 5. Thus, 20820 and 50345 are divisible by 5, while 30934 and 40946 are not.

Divisibility By 6: A number is divisible by 6 if it is divisible by both 2 and 3.

Divisibility By 7: There are following steps :

1. Double the digit at the ones place.
2. Subtract the number from the number without ones digit
3. If the difference is divisible by 7 then the number is divisible by 7.
   Note:-If there is very large number than you can repeat the process till number comes in 3 digits.

Divisibility By 8: A number is divisible by 8 if the number formed by the last three digits of the given number is divisible by 8.

Divisibility By 9: A number is divisible by 9 if the sum of its digits is divisible by 9.

Divisibility By 10: A number is divisible by 10 if it ends with 0.

Divisibility By 11: A number is divisible by 11, if the difference of the sum of its digits at odd places and the sum of its digits at even places, is either 0 or a number divisible by 1e)

<u>Note:-</u> If a number is divisible by **p** as well as **q**, where **p** and **q** are co---prime pairs, then the given number is divisible by **p*q**. If **p** and **q** are not co---primes, then the given number need not be divisible by **p*q**, even when it is divisible by both **p** and **q**.

<u>Divisibility By 12:</u> if a number is divisible by 3 & 4 both than the number is divisible by 12.

Like Wise you can find the divisibility of 14,15,16,18 and others numbers which have factors in the form of co-Prime pairs.

<u>Divisibility By 13:</u> There are following steps :

1. Four times the digit at the ones place.
2. Add the number to the number without ones digit
3. If the result is divisible by 13 then the number is divisible by 13.
   Note:-If there is very large number than you can repet the process till number comes in 3 digit.

<u>Divisibility By 17:</u>

There are following steps:

1. Five times the digit at the ones place.
2. Subtract the number from the number without ones digit
3. If the difference is divisible by 17 then the number is divisible by 17.
   Note:-If there is very large number than you can repet the process till number comes in 3 digit.

<u>Divisibility By 19:</u> There are following steps :

1. Double the digit at the ones place.
2. Add the number to the number without ones digit
3. If the result is divisible by 19 then the number is divisible by 19.

Note:-If there is very large number than you can repet the process till number comes in 3 digit.

<u>Highest power of a number in a factorial:</u>

We will look at the three different variations of questions based on this concept.
So, let us get started.
The first variety of question on this concept is –

1. How to find the highest power of a prime number in a factorial.
Let us take an example to understand this. Say, we need to find the highest power of 3 in 20!
In the exam, they can ask you this question in two ways:

Question 1. A
If 20! contains 3^k, where k is a positive integer, what is the highest value of k?
Or they can ask the question as show below:

Question 1.B
What is the highest power of 3 in 20!?
The solution is the same for either of the above questions and there are two ways to solve it. We will first solve it using method 1 which is **Brute Force method**, where we simply count the number

of 3’s. We’ll then analyse the advantages and disadvantages of this method and then move to a better method (method 2).

Solution
Method 1

- We need to find the highest power of 3 in 20!

Step 1

- **Firstly, we will jot down all the multiples of 3 which are less or equal to 20.**
  - Multiples of 3 which are less than or equal to 20 are 3, 6, 9, 12, 15, and 18.

Step 2

- **We will prime factorize the multiples of 3 to get the greatest power of 3 in each of them.**

So,

  - $3 = 3^1$
  - $6 = 2^1*3^1$
  - $9 = 3^2$
  - $12 = 2^2*3^1$
  - $15 = 3^1*5^1$
  - $18 = 2^1*3^2$

Step 3

- **We will add up all the highest powers of 3 obtained from each of its multiple.**
  So, the highest power of 3 in 20! = 1 + 1+ 2+ 1+ 1+ 2 = 8
    And thus, k = 8

Disadvantage of Method 1:
In this question, it was easy to find the highest power of 3, using method 1, because the factorial value was small. However, this method becomes tedious when the factorial numbers are high. For example, instead of 20! If we had 200! then solving the question using the above method would have taken considerable time.
So, **we’ll use another method to solve this type of questions and we would recommend you use the same**.

Method 2

- In this method, we **take number whose factorial is given.** And we keep on dividing it by the powers of the prime number whose highest power we are looking for. On each division, we are simply looking for the quotient.
- The highest power of 3 in 20! = $(\frac{20}{3^1})_Q + (\frac{20}{3^2})_Q$ = 6 + 2 = 8
  - Here, **( )_Q denotes the quotient of the division operation**.
    - We know that 20 = 6 * 3 + 2, where 6 is the quotient and 2 is the remainder. So, we’ll just take the quotient 6.
      - Similarly, 20 = 2 * 9 + 2. In this case the quotient is 2, so we’ll just take that.
  - We’ll continue dividing the factorial number until the quotient, becomes 0.
    - Like in the above case the quotient of $20/3^3$ is 0. So, we are not finding it and stopping before that only.
  - Here’s another example, let’s say we need to find the highest power of 3 in 60!
    - Then, the highest power of 3 in 60! = $(\frac{60}{3^1}Q) + (\frac{60}{3^2})_Q + (\frac{60}{3^3}Q)$= 20 + 6 + 2 = 28
      - Notice, we have not considered $(\frac{60}{3^4}Q)$ and terms involving other higher powers of 3 in the denominator, because in all those cases the quotient is 0.
    - Once you get all the quotients, as you can see, we just need to add up all the quotients and we’ll have the highest power of that prime number in that factorial.

Now, let us look at the second variety of questions.

2. How to find the highest power of a power of prime number in a factorial

In the last section, we learned how to find the highest power of a prime number in a factorial. In this section, we will extend the same concept to find the highest power of a power of prime number(i.e. a number in the form of p^q, where p is a prime number and q is a positive integer greater than 1) in a factorial.
Let us understand with an example.

Question 2
What is the highest power of 8 in 70!?

Common Mistake:
On seeing this question, a lot of students follow the approach shown below:

- The highest power of 8 in 70! = $(\frac{70}{8^1})_Q + (\frac{70}{8^2})_Q$ = 8 + 1 = 9

However, this is INCORRECT, because 8 is not a prime number and we cannot directly divide by a non-prime number to find the highest power of it in a factorial.
The following section explains the correct step-by-step procedure of solving such type of questions.

Solution
Step 1

- **Prime factorize the given number to find the prime factor and its highest power.**
  - Now, prime factorization of 8 = 2^3
    - Prime factor of 8 = 2
    - And the highest power of its prime factor (i.e. 2) in 8 = 3

Step 2

- Find the highest power of the prime factor of the given number in the given factorial
- So, we'll first find the highest power of 2 in case of 70!

  So, the highest power of 2 in 70! = = 35 +17+8 + 4 +2 +1= 67

Step 3

- Now that we know the highest power of 2, can be written as 2^{67}. But we need the highest power of 8 and we know that we need three twos to make an 8 (since 8 = 2^3)
- **So, we'll simply divide 67 by 3 in this case and see how many 8's we can make. In this division, all we need to do is to take the quotient.**

  So, the highest power of 8 in 70! = $(\frac{67}{3})_Q = 22$

So, one major learning here is that don't divide the factorial by a non-prime number. First break the number down into its prime factorized form and then find the highest power of that number. And after that figure out what will be the highest power of that non-prime number.
Till now, we have seen how to find the highest power of a prime number or a power of a prime number, in the next section, we will see how to find the highest power of a number that has two distinct prime number.

Recommended: 3 Important Properties of Prime Numbers

3. How to find the highest power of a number that has two distinct prime numbers
Finding the highest power in this case is only a little bit different from the last section as instead of one prime factor there will be more than one prime factor i.e. the number will be in the form of $p_1 * p_2$, where p_1 and p_2 are prime numbers. So, with the help of an example, let us understand how to solve this type of question.

Question 3
What is the highest power of 10 in 100!?

Solution

Step 1

- **Prime factorize the given number.**
  - So prime factorization of 10 = 2*5

So, now we know that to make one 10 we need one 2 and one 5. So, in our last step, let's see how many 10s we can make in 100!

Step 2

- The highest power of 2 is 100! = = 50 + 25 + 12 + 6 + 3 +1= 97
- And, the highest power of 5 in 100! = $(\frac{100}{5^1})_Q + (\frac{100}{5^2})_Q$ = 20 + 4 = 24
- Hence, the highest power of 2 in 100! is 97 i.e. 100! contains 97 twos or 2^{97} and the highest power of 5 in 100! is 24 i.e. 100! Contains 24 fives or 5^{24}.

  We know that we need one 2 and one 5 to make one 10.

  So, the highest number of 10s that we can make from 97 twos (2^{97}) and 24 fives (5^{24}) is 24.
  That is, the highest power of 10 in 100! = 24

  If you have noticed the highest power of 10 in 100! is equal to the highest power of 5. This is because the factorial of any number is always going to have more 2s than 5s as 5 > 2 and we need equal number of 2 and 5 to make one 10. So, if you are able to make this inference beforehand, you can save time by finding the highest power of only one number instead of two numbers.

  Therefore, only find the highest power of the greatest prime factor in the factorial.

    - So, the highest power of 10 in 100! = the highest power of 5 in 100! = $(\frac{100}{5^1})_Q + (\frac{100}{5^2})_Q$ = 20 + 4 = 24