Algorithm 1 FABRIK

Input:

Joint positions $p_i, i \in 1, 2, ...n$

p_1 is for the base joint

p_n is for the end-effector joint

Desired position p_d

Number of iterations k

Error tolerance $tol = 10^{-4}$

Output:

New joint positions $p_i, i \in 1, 2, ...n$

$dists_i = ||p_{i+1} - p_i||, i \in 1, 2, ...n - 1$

if $||p_d - p_1|| > \sum_{i=1}^{n-1} dists_i$ **then** ▷ Check if p_d is unreachable

  Return *null*

end if

$basePos = p_1$ ▷ Needed for backward-reaching steps

for $x = 1$ to k **do**

  if $||p_n - p_d|| \leq tol$ **then**

    break

  end if

  $p_n = p_d$

  for $i = n - 1$ to 1 **do**

    $r = ||p_{i+1} - p_i||$

    $\lambda = dists_i / r$

    $p_i = (1 - \lambda)p_{i+1} + \lambda p_i$

  end for

  $p_1 = basePos$

  for $i = 1$ to $n - 1$ **do**

    $r = ||p_{i+1} - p_i||$

    $\lambda = dists_i / r$

    $p_{i+1} = (1 - \lambda)p_i + \lambda p_{i+1}$

  end for

end for

Return p_i

Algorithm 2 Calculate constrained angle

Input:
Desired axis $axis \in \{x, y\}$
Joint position p_i
Joint orientation o_i
Joint constraint $constr_i$
Calculated position p

Output:
Angle θ

$altAxis \leftarrow$ y or x, different from $axis$
$p_{z,altAxis} \leftarrow$ position on Z-$altAxis$ plane
$vec \leftarrow p_{z,altAxis} - p_i$
$\theta \leftarrow$ vector angle between $o_{i,z}$ and vec
if $||p_{z,altAxis} - (p_i + o_{i,axis})|| > ||p_{z,altAxis} - (p_i - o_{i,axis})||$ **then**
    $\theta \leftarrow -\theta$ ▷ Signed angle
end if
if θ not in $constr_{i,axis}$ **then**
    $\theta \leftarrow$ closest value in $constr_{i,axis}$
end if
Return θ

Algorithm 3 3D FABRIK with constraints

Input:

Joint positions $p_i, i \in 1, 2, ...n$

p_1 is for the base joint

p_n is for the end-effector joint

Joint orientations $o_i, i \in 1, 2, ...n$

o_i's z-axis points to p_{i+1}

Desired position p_d

Joint motion constraints $constr_i, i \in 1, 2, ...n-1$

Each $constr_i$ is w.r.t. a ZX or ZY plane of o_i

Number of iterations k

Error tolerance $tol = 10^{-4}$

Output:

New joint positions $p_i, i \in 1, 2, ...n$

New joint angles $\theta_i, i \in 1, 2, ...n$

$dists_i = ||p_{i+1} - p_i||, i \in 1, 2, ...n-1$

if $||p_d - p_1|| > \sum_{i=1}^{n-1} dists_i$ **then** ▷ Check if p_d is unreachable

Return *null*

end if

$basePos = p_1$ ▷ Needed for backward-reaching steps

for $x = 1$ to k **do**

if $||p_n - p_d|| \leq tol$ **then**

break

end if

$p_n = p_d$

for $i = n-1$ to 1 **do**

$r = ||p_{i+1} - p_i||$

$\lambda = dists_i / r$

$p_i = (1 - \lambda)p_{i+1} + \lambda p_i$

$\theta_{i,x} \leftarrow constrainedAngle(x\text{-axis}, p_i, o_i, p_{i+1})$

$\theta_{i,y} \leftarrow constrainedAngle(y\text{-axis}, p_i, o_i, p_{i+1})$

$rotMat_x \leftarrow RotationMatrix(\theta_{i,x}, \{o_{i,z}, o_{i,x}\})$

$rotMat_y \leftarrow RotationMatrix(\theta_{i,y}, \{o_{i,z}, o_{i,y}\})$

$p_i = p_{i+1} - dists_i \cdot (rotMat_x \; rotMat_y) o_{i,z}$

end for

$p_1 = basePos$

for $i = 1$ to $n-1$ **do**

$r = ||p_{i+1} - p_i||$

$\lambda = dists_i / r$

$p_{i+1} = (1 - \lambda)p_i + \lambda p_{i+1}$

$\theta_{i,x} \leftarrow constrainedAngle(x\text{-axis}, p_i, o_i, p_{i+1})$

$\theta_{i,y} \leftarrow constrainedAngle(y\text{-axis}, p_i, o_i, p_{i+1})$

$rotMat_x \leftarrow RotationMatrix(\theta_{i,x}, \{o_{i,z}, o_{i,x}\})$

$rotMat_y \leftarrow RotationMatrix(\theta_{i,y}, \{o_{i,z}, o_{i,y}\})$

$p_{i+1} = p_i + dists_i \cdot (rotMat_x \; rotMat_y) o_{i,z}$

end for

end for

Return $\{p_i, \theta_i\}$