ACA - Mastering Algorithms

Assignment 1

Due Date: 16th June 11:59 pm, 2024

Instructions

1. Read the questions carefully and provide your solutions accordingly.
2. You should upload your C++ solutions as .cpp files with appropriate file names on your Github repository.
3. Solve and upload at least 4 questions out of the given 7. Although it is recommended that you solve all the questions.
4. Do not waste your time using external resources such as Google or Chat GPT to answer the questions.
5. Allowed Languages for challenge code submission: C/C++
6. Your codes will be checked for possible plagiarism of any sort. If we find such cases, we will possibly deratify you.

Min Max Height

Problem 1.

As you know, Venkatesh likes to minimize the maximum height of his bookshelves. There are N bookshelves in his room, with the i^{th} bookshelf having a_i books. Venkatesh can do the following operation:

- Choose a bookshelf and remove the topmost book with a cost of h, where h is the bookshelf's height before removing the book.

He can do this operation any number of times, but the total cost should be less than or equal to K. Since Venkatesh doesn't like tall bookshelves, he wants to decrease their height. He has to **minimize the maximum height** of bookshelves and report this height (the height of the tallest among them after operations).

Input Format:

- The first line of the input consists of the number of test cases T.
- Each test case begins with two integers N and K, followed by N integers representing the heights of the bookshelves.

Output Format:

- The output should consist of T lines, each containing the minimum possible height of the tallest bookshelf for the corresponding test case.

Constraints

- $1 \leq T \leq 1000$
- $1 \leq N \leq 100000$
- $1 \leq a_i \leq 1000000$ for every $1 \leq i \leq N$
- $1 \leq K \leq 10^{15}$
- Sum of N over all test cases is at most 200000

Sample Input

```
3
5 23
1 3 2 4 5
5 3000000
0 1000000 2 5 99999
3 9
3 3 3
```

Sample Output

```
2
999997
2
```

Subarray Sum

Problem 2.

In the magical kingdom of Arrayland, there exists a sequence of enchanted trees, represented as an integer array nums. Each tree holds a certain amount of magical energy, which can be positive, negative, or zero. As an adventurer, you are tasked with finding the shortest path through a contiguous section of this forest that contains a sum of magical energy of at least k units.

Formally, given an integer array nums and an integer k, your objective is to **find the length of the shortest non-empty subarray** of nums such that the sum of its elements is at least k. If there is no such subarray, return -1.

Input Format:

- The first line of the input contains an integer n denoting the number of values in array and k denoting the maximum number you are told to generate.
- The second line would contain the n values inside the array.

Output Format:

- You have to give a single line output for what is the length of such subarray.

Examples

Example 1:
Input: nums = [1,2], k = 4
Output: -1

Example 2:
Input: nums = [2,-1,2], k = 3
Output: 3

Constraints

- $1 \leq \text{nums.length} \leq 10^5$
- $-10^5 \leq \text{nums}[i] \leq 10^5$
- $1 \leq k \leq 10^9$

Array Queries!!! Heaps ?

Problem 3.

You are given an array A of size N. You can perform an operation where you remove the largest and the smallest element from the array, and then add the difference between them back into the array. This operation decreases the size of the array by one. You are given Q tasks, each with an integer K.
For each task, you need to **determine the sum of all the elements in the array after K operations**.
(You might need help of C++ library container map for storing number of occurrences.)

Input format

- The first line contains two space-separated integers N and Q, denoting the number of elements in the array and the number of queries respectively.
- The second line contains N space-separated integers denoting elements of the array.
- Third line contains Q space-separated integers denoting the queries.

Output format

- The output should consist of Q space-separated integers corresponding to the output of each query.

Constraints

- $1 \leq N \leq 10^5$
- $0 \leq K < N$
- $1 \leq A_i \leq 10^9$ for every $1 \leq i \leq N$

Sample Input

```
4 4
3 4 5 3
2 1 3 0
```

Sample Output

5 9 1 15

Oreo Obsession

Problem 4.

Udhav has a crippling Oreo addiction. He needs to buy as many oreos as possible. He only has k rupees. There are n Oreo packets. The i^{th} one costs $cost_i$ and has num_i Oreos. Since you are an irresponsible friend, help Udhav buy the set of packets with the **maximum possible number** of Oreos.

Input format

- The first line of the input contains an integer T denoting the number of test cases. The first line of each test case contains two numbers n and k. The following n lines contain two numbers $cost_i$ and num_i respectively.

Output format

- Each line contains an integer telling the maximum possible Oreos for that test case.

Constraints

- $1 \leq T \leq 100$
- $1 \leq N \leq 10$
- $1 \leq k \leq 100000000$
- $1 \leq cost, num \leq 100000000$

Sample Input

```
1
4 10
1 3
2 4
3 1
6 5
```

Sample Output

12

Bonus

Output the indices of the packets to be selected for the optimal selection. Submit the code for this part separately. If there are multiple optimal subsets, print any.

DigSum

Problem 5.

You are given n integer strings of size at most 10. Find the number of pairs (i, j) such that their concatenation string $S_i + S_j$ has a length 12 and the sum of digits in that string is exactly 50.
For example, if $S_3 =$ "87645" and $S_5 =$ "2253332", then (3,5) is a valid pair. You are expected to solve in $O(n)$ or $O(n*log(n))$ time complexity. $O(n^2)$ solution is not credited.

Input format

- The first line of the input contains an integer n denoting the number of strings. The next n lines each contain an integer string each.

Output format

- A single integer specifying the number of pairs that satisfy the condition.

Constraints

- $1 \leq n \leq 200000$
- $1 \leq \text{length}(S_i) \leq 10$

Sample Input

```
4
0123456789
55555555
1234
44
```

Sample Output

2

Hint

Read about hashing from the given resources.
Although non-hashing methods are also encouraged given that they have an acceptable time-complexity.

Grid Journey

Problem 6.

You are given an $n \times m$ grid, where the block at i^{th} row (from up) and j^{th} column (from left) is labeled as (i, j). You start at $(1, 1)$ and your goal is to reach (n, m). In one step you can move either one block up, down, right, or left, given that you are not moving out of the grid.

There is also another rule, suppose your steps are labeled as $s_1, s_2, s_3...$, for every odd i, if you make at least $i+1$ steps, then $(i+1)^{th}$ step must be a 90-degree anti-clockwise rotation of the ith step. For example, if your 3rd step is to go down, then your 4th step must go right, if your 5th step is to go right, then your 6th step must go up. If an odd-numbered step doesn't place you at (n, m) but places you at a location where you cannot follow the next step, then you are not allowed to make that choice for the odd-numbered step.
For example, in a 5×2 grid, if you reach $(2, 2)$ after 2 steps, then for the third move you can't go to (3,2), because then you won't be able to follow the 4th move.

Find the **minimum number of steps** needed to reach (n, m), or find that it is impossible.

Input format

- Two integers n and m in that order.

Output format

- Print the Minimum number of steps otherwise -1 if it is not possible to reach the end

Constraints

- $1 \leq n * m \leq 200000$

Sample Input

```
4 3
```

Sample Output

```
5
```

Thala for a Reason

Problem 7.

A k-dice is a dice which have k-faces and each face has value written from 1 to k. For a given N, you have to calculate the number of ways you can throw this dice so that we get a sum equal to N.
Since the number of ways can be large you have to calculate total ways modulo 998244353.

Input format

- The first line of the input consists of the number of test cases T.
- Each test case consists of two integers N and K

Output format

- The output should consist of T lines, each containing the number of ways modulo 998244353.

Constraints

- $1 \leq T \leq 100$
- $1 \leq N \leq 10^{18}$
- $2 \leq k \leq 20$

Sample Input

```
3
5 3
7 4
8 10
```

Sample Output

13
56
128

Explanation

For $N = 5$ and $K = 3$: We have 13 ways

1+1+1+1+1=5
1+1+1+2=5
1+1+2+1=5
1+2+1+1=5
2+1+1+1=5
2+2+1=5
2+1+2=5
1+2+2=5
3+1+1=5
1+3+1=5
1+1+3=5
3+2=5
2+3=5