

1- No of days in week = 7 = n

$$P(\text{success}) = 0.6 = p$$

$$P(\text{failure}) = 1 - 0.6 = 0.4 = q$$

By Bernoulli Distribution formula:-

$$P(x=r) = {}^nC_r \times p^r \times q^{n-r}$$

$$P(x=3) = {}^7C_3 \times (0.6)^3 \times (0.4)^4$$
$$= 0.19$$

2- $P(R) = 0.3$

$P(D/R)$ = Probab. of stock market will go down after it rains

$$= 0.7$$

$P(D \cap R)$ = Prob. of both occurring simultaneously.

$$= P(D/R) \times P(R)$$

$$= 0.21$$

3- $\mu = 0.05$

$$\sigma = 0.02$$

$$z\text{-score for } 0.03 = \frac{x - \mu}{\sigma} = \frac{0.03 - 0.05}{0.02} = -1.00$$

$$z\text{-score for } 0.07 = \frac{0.07 - 0.05}{0.02} = 1.00$$

~~Prob. value for (-2) = 0.0540~~

$$\text{Prob. value for } (-2) = 0.15866$$

$$\text{Prob. value for } (1) = 0.84134$$

$$\Rightarrow \text{Req. Prob.} = 0.84134 - 0.15866$$

$$= 0.68268$$

4- In Poisson distribution, the mean represent average no of events happening within a specific interval.

Mean sales per day = 10 units

Expected no. of units sold over 5-day period
 $= 5 \times 10$
 $= 50$ units.

5- Independent & Identically distributed refers to a specific property of random variables.

→ All the random values in the set share the same probability distribution

→ Outcome of one has no effect on other.

Daily Return → iid

↓
has variance of 0.0004

⇒ Since daily returns are ~~separate~~ independent, sum of variances over multiple days is simply sum of individual variances.

⇒ Sum of the returns over 10 days is essentially the avg. return multiplied by 10.

Total variance = No. of Days \times Daily variance
 $= 0.004$

6-

Variance of time series = 1
Autocorrelation at lag k :-

$$\rho_k = \frac{\gamma_k}{\gamma_0}$$

Autocovariance at lag k :- γ_k

$\gamma_0 \rightarrow$ variance of time series
 $\rho_k \rightarrow$ autocorrelation at lag k .
 $\gamma_k \rightarrow$ autocovariance " "

$$\rho_1 = 0.8$$

$$\rho_2 = 0.6$$

$$\gamma_0 = 1$$

$$\gamma_2 = \rho_2 \times \gamma_0 = 0.6$$

Autocovariance at lag 2 = 0.6

7-

$$\text{Avg}[4, 8, 6] = \frac{4+8+6}{3} = 6$$

$$\text{Avg}[8, 6, 5] = 6.33$$

$$\text{Avg}[6, 5, 9] = 6.67$$

$$\text{Avg}[5, 9, 7] = \frac{5+9+7}{3} = 7$$

$$\text{Avg}[9, 7, 10] = \frac{9+7+10}{3} = 8.67$$

3-day moving avg $\rightarrow [6, 6.33, 6.67, 7, 8.67]$

Q8-) For ARIMA model:-

$$(1 - \phi_1 B)(1 - B) Y_t = (1 + \theta_1 B) \varepsilon_t$$

$$\phi_1 = 0.7$$

$$\theta_1 = -0.4$$

$$(1 - 0.7B)(1 - B) Y_t = (1 - 0.4B) \varepsilon_t$$

After Expanding, apply differencing op.

$$(1 - B) Y_t = Y_t - Y_{t-1}$$

Now, model eqⁿ:-

$$(1 - 0.7B)(Y_t - Y_{t-1}) = (1 - 0.4B) \varepsilon_t$$

$$Y_t - 1.7Y_{t-1} + 0.7Y_{t-2} = \varepsilon_t - 0.4\varepsilon_{t-1}$$

Interpretation:-

$$\phi_1 = 0.7$$

→ First lag of differenced series is +ve correlated with a coefficient of 0.7. Suggests a strong +ve relationship b/w Y_{t-1} & Y_t after differencing.

$$\theta_1 = -0.4$$

:- First lag of error term (ε_t) has a neg. effect on Y_t with a coeff of -0.4. That is positive error term at time $t-1$ will result in a decrease in Y_t .

9-

Sample mean (\bar{x}) = 0.0015

Sample std. dev. (s) = 0.0005

Sample size (n) = 30

Pop. mean (μ_0) = 0.002

t-statistic:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$t \approx -5.48$$

degrees of freedom

$$= 30 - 1 = 29$$

Using t-table, critical value = ± 2.045

Since absolute value of calculated t-statistic is greater than t-value, reject null hypothesis

10-

$$\text{Confidence interval} = \bar{x} \pm \left(\frac{t_{\alpha/2} \times s}{\sqrt{n}} \right)$$

$$\alpha = 1 - 0.95 = 0.05$$

$$\alpha/2 = 0.025$$

$$df = n - 1 = 29$$

$$\text{Conf. Int.} = (0.001263; 0.001737)$$

11-

Example of Non-stationary Time Series:-

Stock price series.



Have trends over time & don't have constant mean / variance.

A method to transform a non-stationary time series is differencing.

12-

Additive model assumes:-

$$Y_t = T_t + S_t + R_t$$

↓ ↓ ↘
Obs. value Trend component Seasonal component Residual component

Steps to Decompose :-

1- Identify the Trend Component (T_t) by using method such as moving averages / LOESS (Locally estimated scatterplot smoothing).

2- Extract the Seasonal Component (S_t) :-

~~Calculate~~ → Deselect seasonality
→ Averaging method.

3- Calculate the Residual Component (R_t)

Subtract both the trend & seasonal components from orig. series to obtain the residual component.

13-

MAE (Mean Absolute Error) :-

$$MAE = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i|$$

RMSE (Root mean Squared Error)

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2}$$

Comparing MAE & RMSE :-

- MAE has linear score, that is all individual diff. are weighted equally.
- RMSE is quadratic, that is larger errors have a disproportionately large impact on the score.

Model:-

Model A has MAE of 5
Model B has RMSE of 6.

a. ERROR :-

MAE of 5 means avg. absolute error of Model A is 5 units.

a. RMSE of 6 suggests that the errors are somewhat higher on average for model B.

∴ Based on MAE of 5 for model A & RMSE of 6 for model B, Model A is considered to perform better due to low avg. error.