Assignment 1

Fit probability distribution on Category 4 & 5 of Atlantic Hurricane Data

Reading the Data ¶

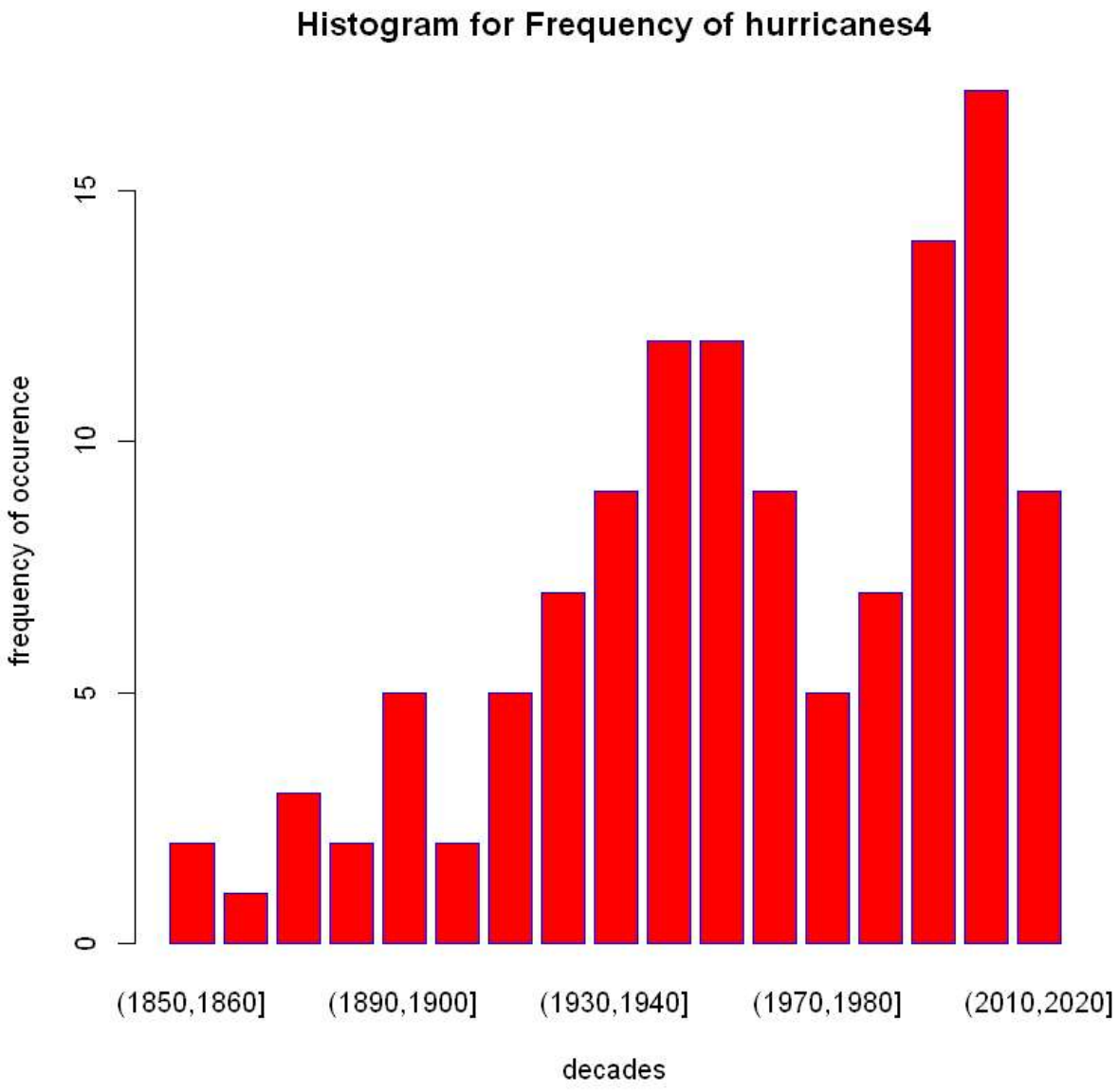
In [169]:
```
data4 <- read.csv("hurricanes4.csv", header = TRUE, sep=",")
head(data4)
```

ï..Name	Season	Month	Maximum.Sustained.Wind	Minimum.pressure
Hurricane #3	1853	August, September	150	924
1856 Last Island Hurricane	1856	August	150	934
Hurricane #6	1866	September, October	140	938
Hurricane #7	1878	September, October	140	938
Hurricane #2	1880	August	150	931
Hurricane #8	1880	September, October	140	928

Cleaning the Data

In [170]:

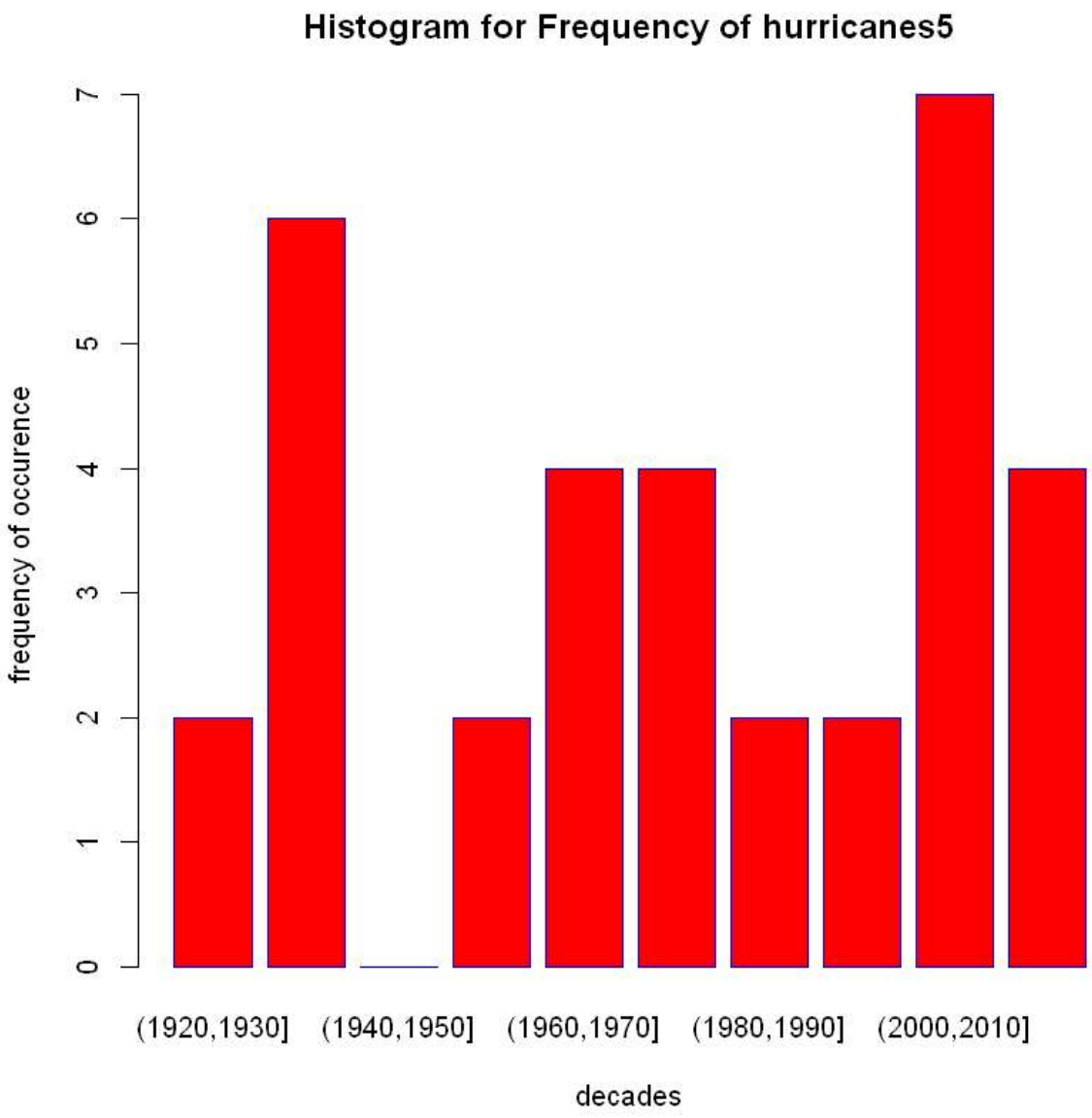
```
season_interval_4 = data4$Season

interval_4 = seq(1850,2020,by=10)

Frequency_4 = cut(season_interval_4, interval_4, dig.lab=5)

Frequencytable_4 = table(Frequency_4)

results4 <- data.frame(Frequencytable_4)

colnames(results4) <- c("decadeinterval","frequencycount")

barplot(Frequencytable_4,main="Histogram for Frequency of hurricanes4",xlab =
```



In [171]:

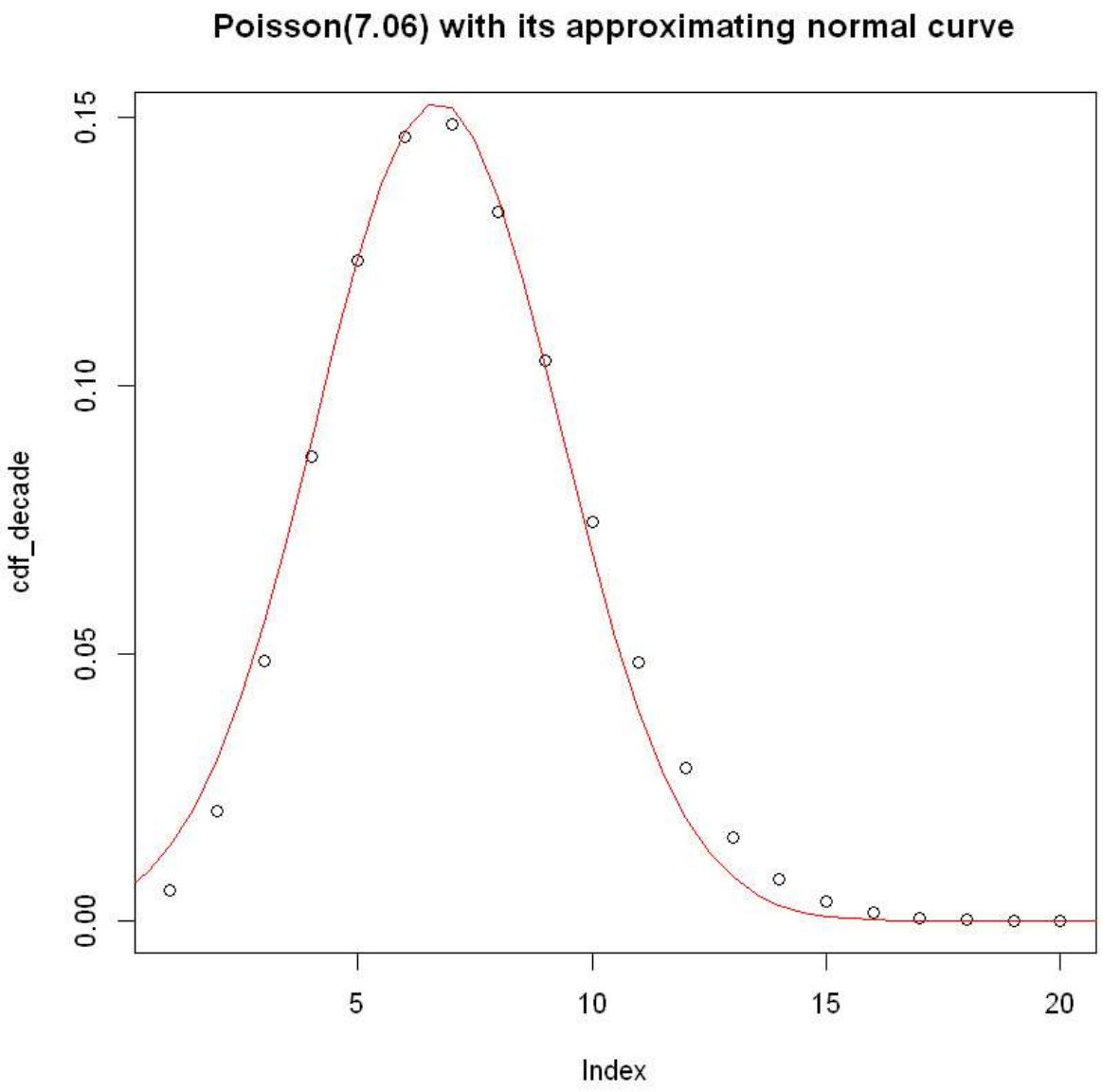
```
data5 <- read.csv("hurricanes5.csv", header = TRUE, sep=",")
head(data5)

season_interval_5 = data5$Season

interval_5 = seq(1920,2020,by=10)

Frequency_5 = cut(season_interval_5, interval_5, dig.lab=5)

Frequencytable_5 = table(Frequency_5)

results5 <- data.frame(Frequencytable_5)
colnames(results5) <- c("decadeinterval","frequencycount")

barplot(Frequencytable_5,main="Histogram for Frequency of hurricanes5",xlab =
```

Name	Season	Month	Maximum.Sustained.Wind	Minimum.pressure
Cuba	1924	october	165	910
San Felipe	1928	september	160	929
Bahamas	1932	september	160	921
Cuba	1932	november	175	915
Cuba-Brownsville	1933	august	160	930
Tampico	1933	september	160	929


Histogram for Frequency of hurricanes5
frequency of occurence
0
1
2
3
4
5
6
7
(1920,1930]
(1940,1950]
(1960,1970]
(1980,1990]
(2000,2010]
decades

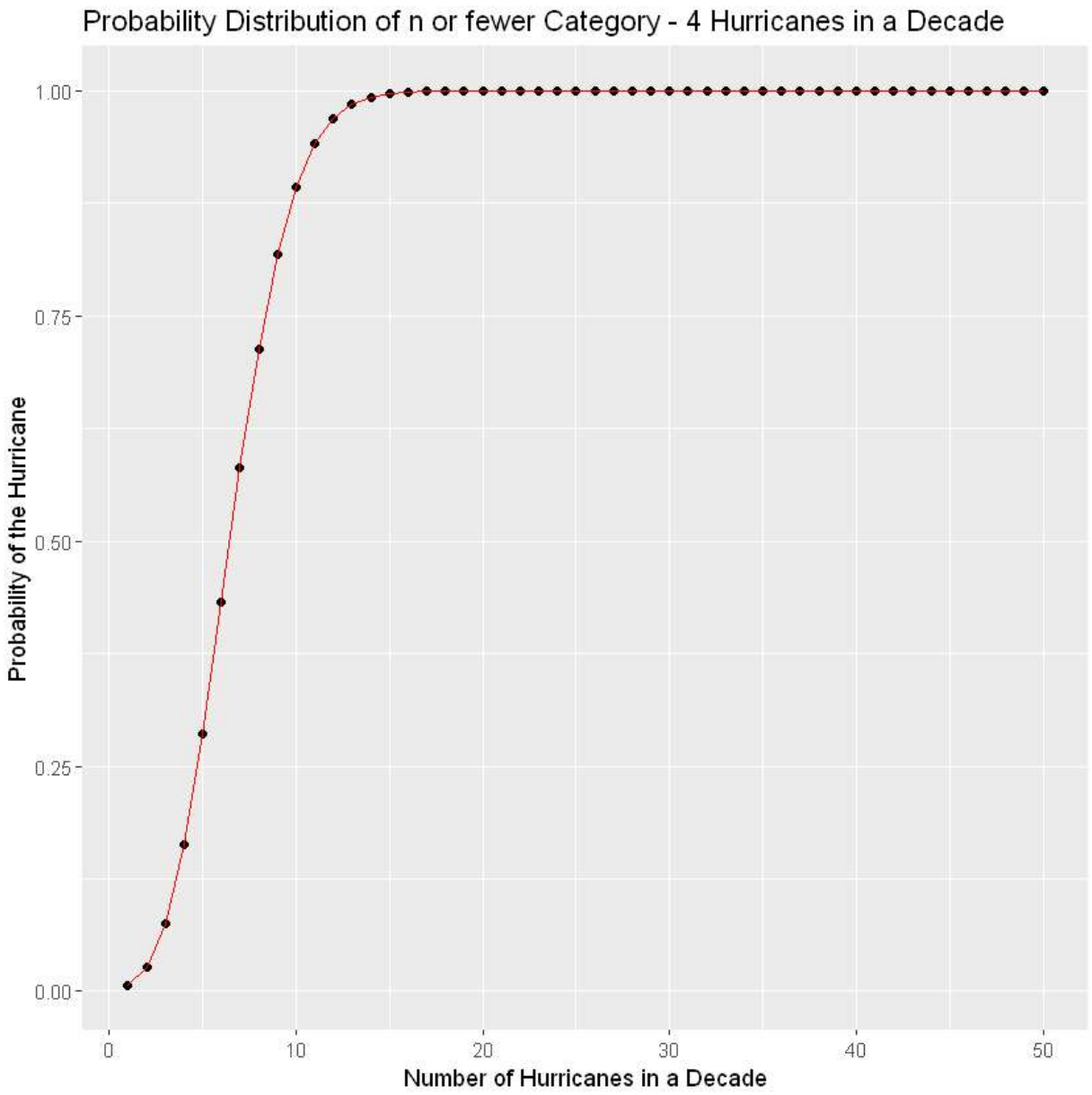
In [174]:

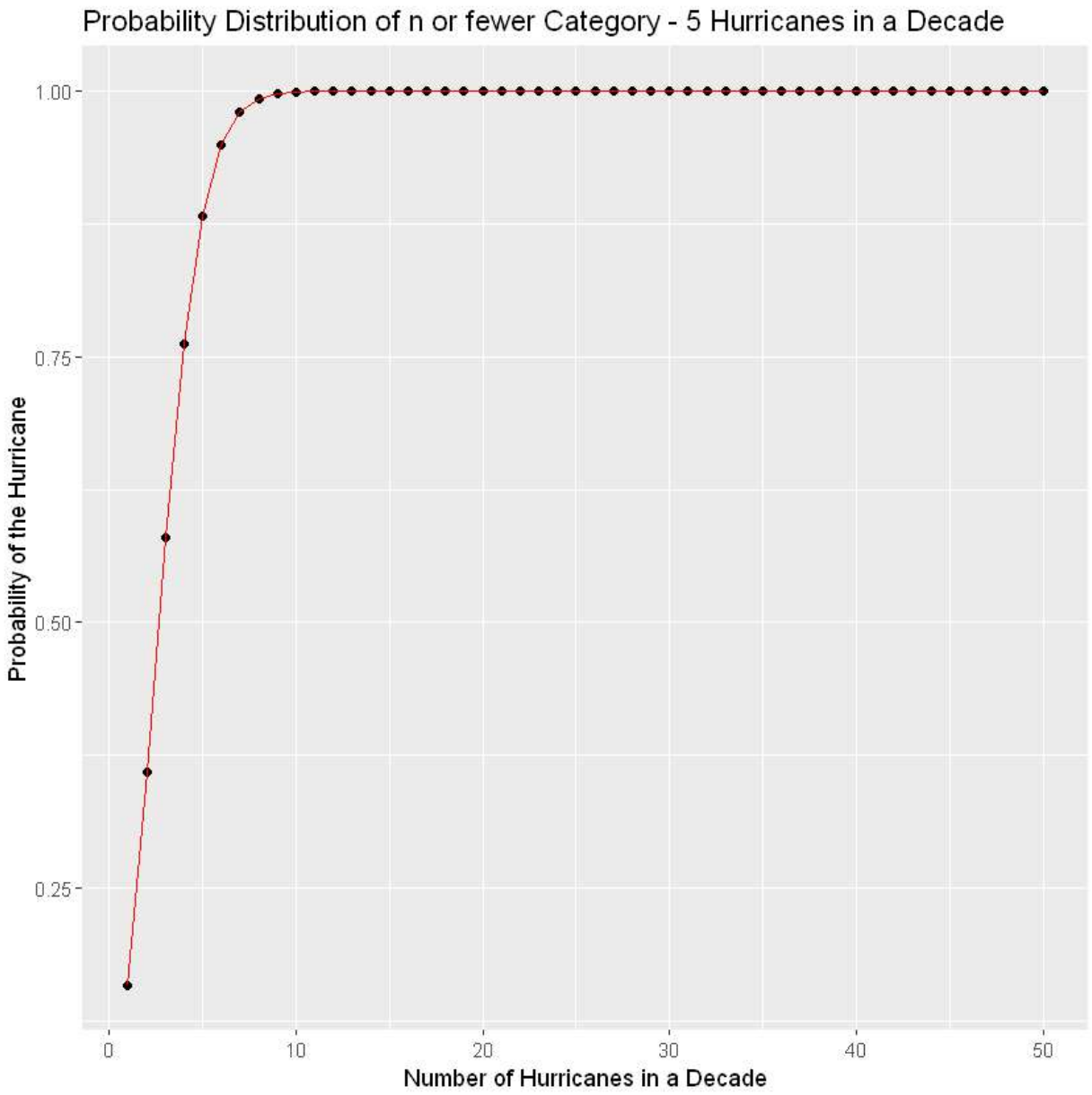
```
sd_frequency4 = sd(Frequencytable_4)

mean_frequency4 =mean(Frequencytable_4)

number_of_decades4 = nrow(Frequencytable_4)

variance4 = var(Frequencytable_4)

mean_frequency5 =mean(Frequencytable_5)

number_of_decades5 = nrow(Frequencytable_5)

variance5 = var(Frequencytable_5)

cdf_decade <- c()
for (i in 1:20){
  cdf_decade[i] <- dpois(i, mean_frequency4)
}

plot(cdf_decade, main = "Poisson(7.06) with its approximating normal curve")

normden <- function(x){
dnorm(x, mean = 6.7, sd = sqrt(6.8))
}

curve(normden, from = 0, to = 50, add=TRUE, col="red")

```


Poisson(7.06) with its approximating normal curve
cdf_decade
0.00
0.05
0.10
0.15
5
10
15
20
Index

In [106]:

```
get_ppois_poisson_probabilities <- function(lambda){

    poisson_probabilities <- c()
    for (i in 1:50){
      # Probability of *i* or less hurricanes occuring in a decade
      poisson_probabilities[i] <- ppois(i, lambda = lambda)
    }

    poisson_probabilities_df <- as.data.frame(poisson_probabilities)
    poisson_probabilities_df$num_hurricanes_in_the_decade <- c(1:50)
    colnames(poisson_probabilities_df) <- c("poisson_probabilities", "number_

    return(poisson_probabilities_df)
}

category_4_ppois_poisson_probabilities_df <- get_ppois_poisson_probabilities(

category_5_ppois_poisson_probabilities_df <- get_ppois_poisson_probabilities(

category_4_ppois_poisson_probabilities_df %>%
ggplot(aes(x = number_of_hurricanes, y = poisson_probabilities))+
geom_point()+
geom_line(color = "red")+
labs(x = "Number of Hurricanes in a Decade", y = "Probability of the Hurrican

category_5_ppois_poisson_probabilities_df %>%
ggplot(aes(x = number_of_hurricanes, y = poisson_probabilities))+
geom_point()+
geom_line(color = "red")+
labs(x = "Number of Hurricanes in a Decade", y = "Probability of the Hurrican
```


Probability Distribution of n or fewer Category - 4 Hurricanes in a Decade
Probability of the Hurricane
1.00
0.75
0.50
0.25
0.00
0
10
20
30
40
50
Number of Hurricanes in a Decade



Is it an exact Poisson distribution?

Although the plots look similar to a Poisson distribution, we cannot assume that the data is indeed a Poisson distribution by looking at the plots. Poisson distribution has a property that the mean and variance are equal and we use this property to test the fit of our data. Wecheck if this property is satisfied or almost satisfied by our data. We compute the ratio of mean and variance of our data:

In [107]:
```
#variance_of_cat_4_hurricanes <- var(results$decadeinterval)
#mean_of_cat_4_hurricanes <- mean(results$decadeinterval)

variance_mean_ratio_for_category_4_hurricanes <- round(variance4/mean_frequen
variance_mean_ratio_for_category_4_hurricanes
```

3.07

Monte Carlo Simulation

We see that the variance of our data is 3.07 times larger than the mean of the data. We check if such a behaviour is normal or not for a poisson distribution. We check if this is just a one time anomaly or is it recurring or not. We will perform a "Monte Carlo Simulation" and repeatedly sample values from the poisson distribution with the mean/lambda equal to the mean of the hurricane data. Our main aim of the simulation is to check -

Assuming that the category 4 hurricane data is a perfect Poisson distribution, how likely it is for us to generate samples with a variance-to-mean ratio equal to 3.07

In [164]:
```
set.seed(420)
variance_mean_ratio_from_samples <- c()
num_of_monte_carlo_simulations <- 1000
num_of_points_in_sample <- nrow(Frequencytable_4)

for (monte_carlo_experiment_index in  1:num_of_monte_carlo_simulations){
    poisson_sample <- rpois(n = num_of_points_in_sample, lambda = mean_freque
    head(poisson_sample)
    variance_mean_ratio_from_samples[[monte_carlo_experiment_index]] <- var(p
}

percentage_of_samples_with_variance_mean_ratio_greater_than_3.07 <- sum(varia
percentage_of_samples_with_variance_mean_ratio_greater_than_3.07
```

0

This shows that only 0% of the monte carlo samples have a variance-mean ratio greater than 2.82. Hence, the hurricane count in a decade does not exhibit an exact Poisson process and the variability in hurricane counts is higher than a Poisson distribution with constant rate. This means that for a distribution of hurricane counts in a decade, the lambda/rate is not constant but keeps changing.

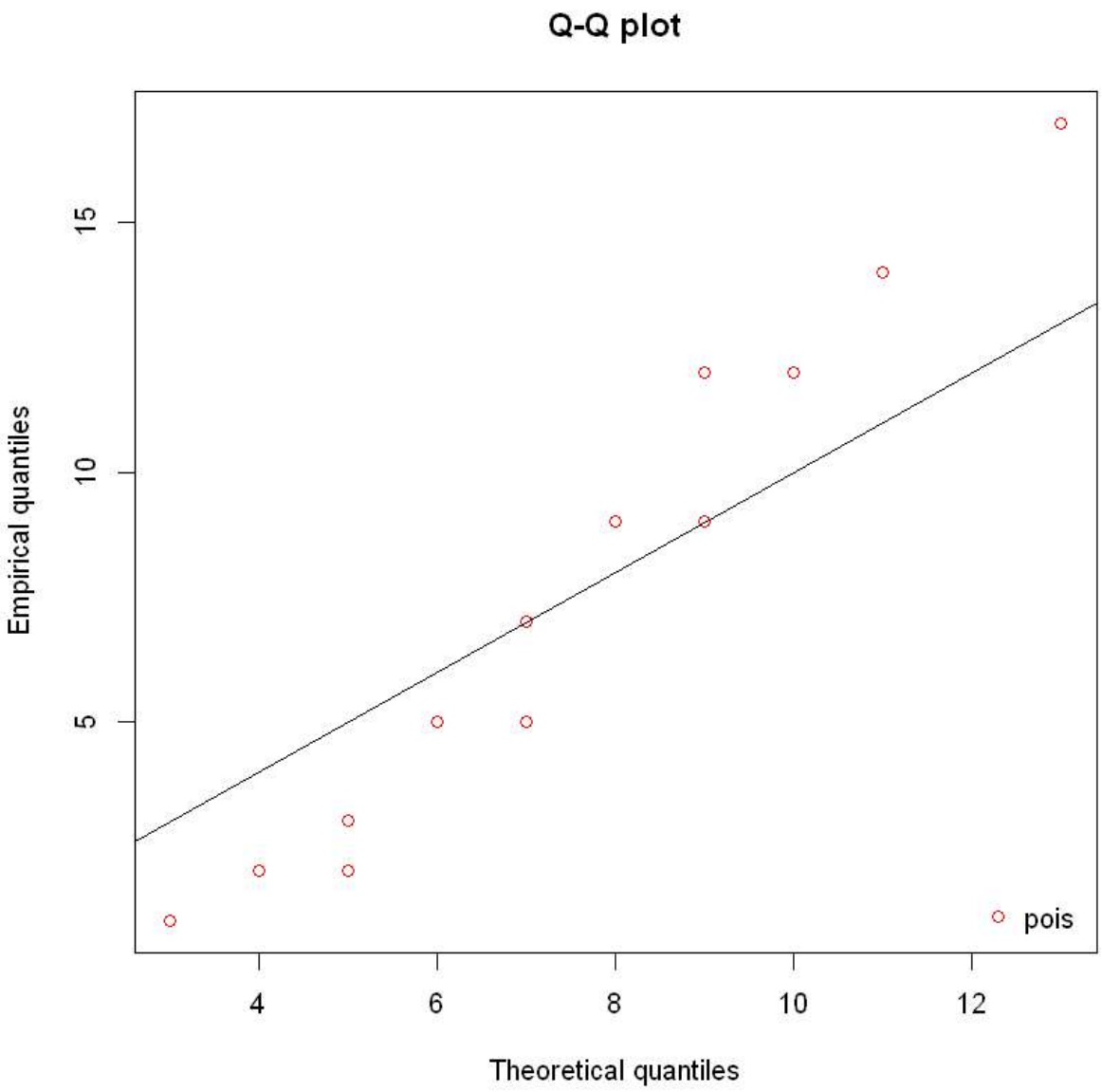
Reasons for the varying lambda/rate in hurricane data

The reasons for the non-constant rate/lambda in our hurricane data is because external climatic conditions affect the occurence of hurricane and ultimately change the lambda. These external factors could be changes in pressure, wind speeds, El Nino etc... This leads to hurricane data being a varying Poisson distribution or an inhomogeneous Poisson distribution which can be described as a Poisson distribution with a variable rate/lambda.

Analysing using QQ plot

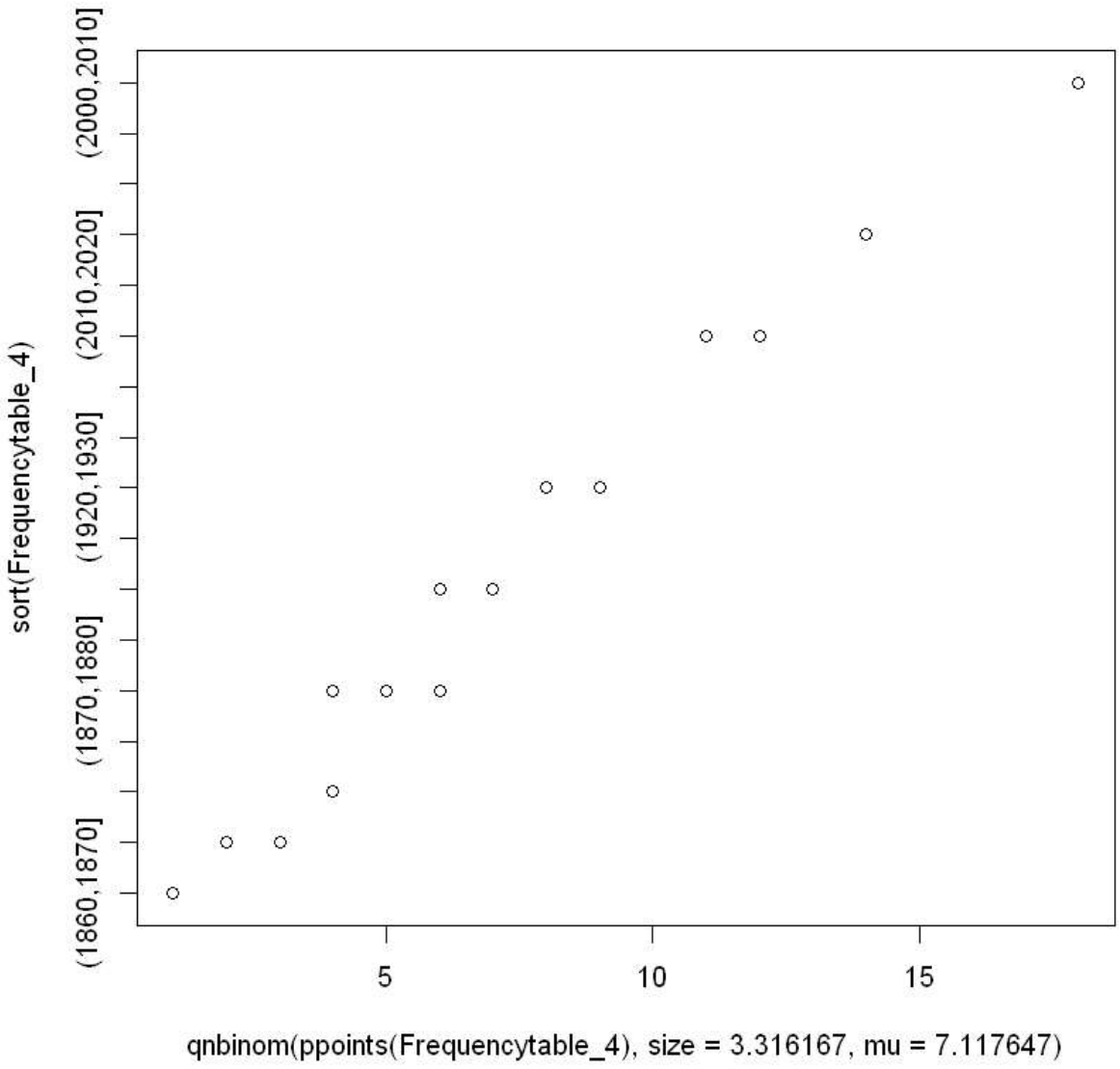
#Quantile plots are a good way to look at what distribution a data might belong to. Here, we plot a quantile plot of our #hurricane data (using the category-4 data again) and quantiles drawn from a theoretical Poisson distribution.

In [173]:
```
1 qqcomp(fitdist(results4$frequencycount, "pois"))
```

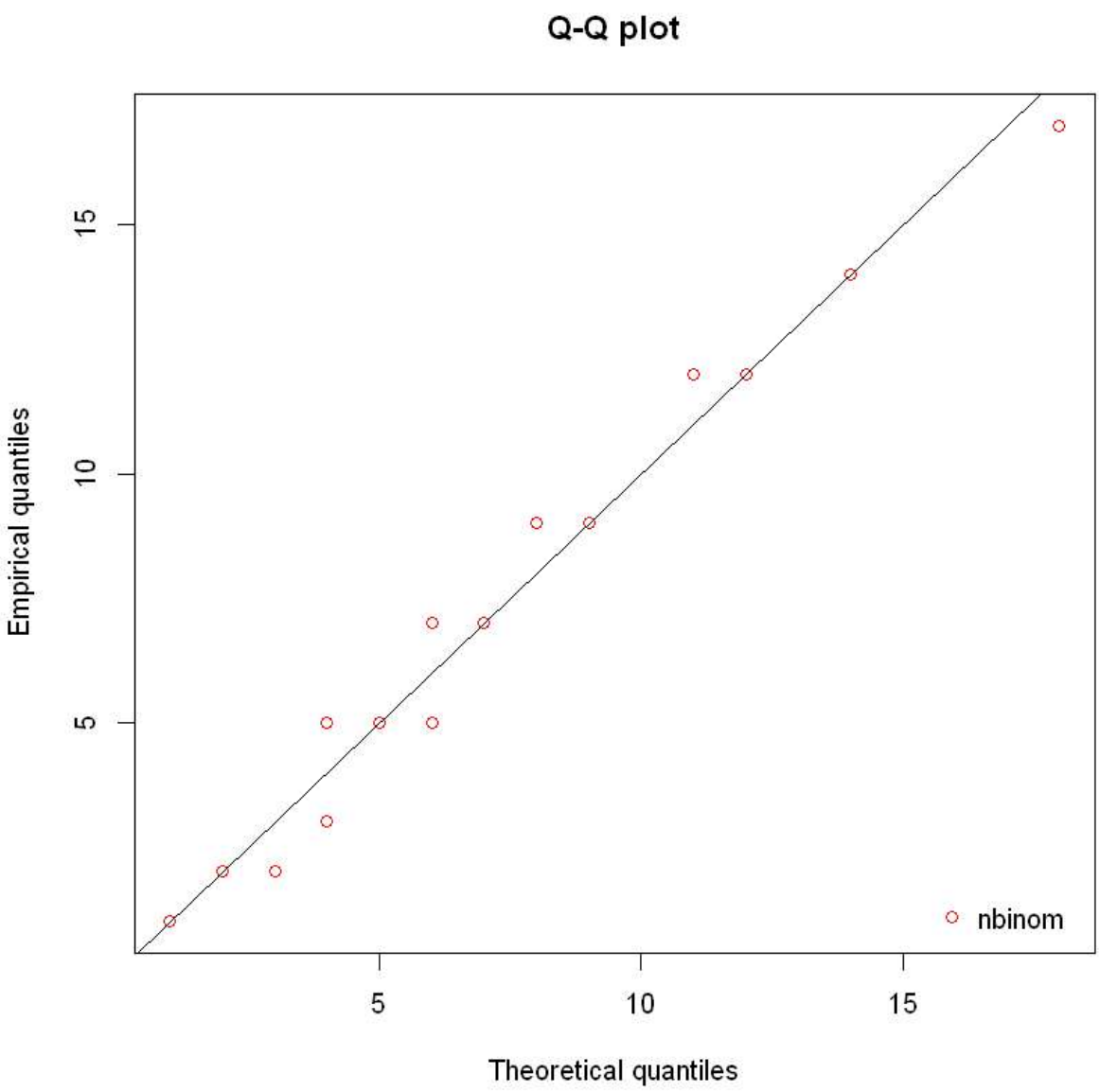


The quantile plot strengthens our conclusion that our hurricane data is not entirely a Poisson distribution with a constant rate. What if we use a negative binomial distribution for this?

In [175]:
```
1 params = fitdistr(Frequencytable_4, "Negative Binomial")
2 plot(qnbinom(ppoints(Frequencytable_4), size=3.316167, mu=7.117647), sort(Fre
```



In [176]:
```
1 qqcomp(fitdist(results4$frequencycount, "nbinom"))
```



This shows that the hurricane count data infact is similar to a negative binomial distribution. Poisson distributions are special cases of negative binomial distributions and our above distribution is a case of overdispersed Poisson distribution. In an overdispersed Poisson distribution, the observations are overdispersed in comparison to a theoretical Poisson distribution where variance is equal to the mean. This overdispersion causes the variance of the data to be greater than the mean - which is the case for our hurricane data. Such overdispersion can be reduced by varying the variance and keeping the mean constant. Since negative binomial distributions have one more parameter than a Poisson distribution, we can vary the parameter to adjust the variance keeping the mean constant.

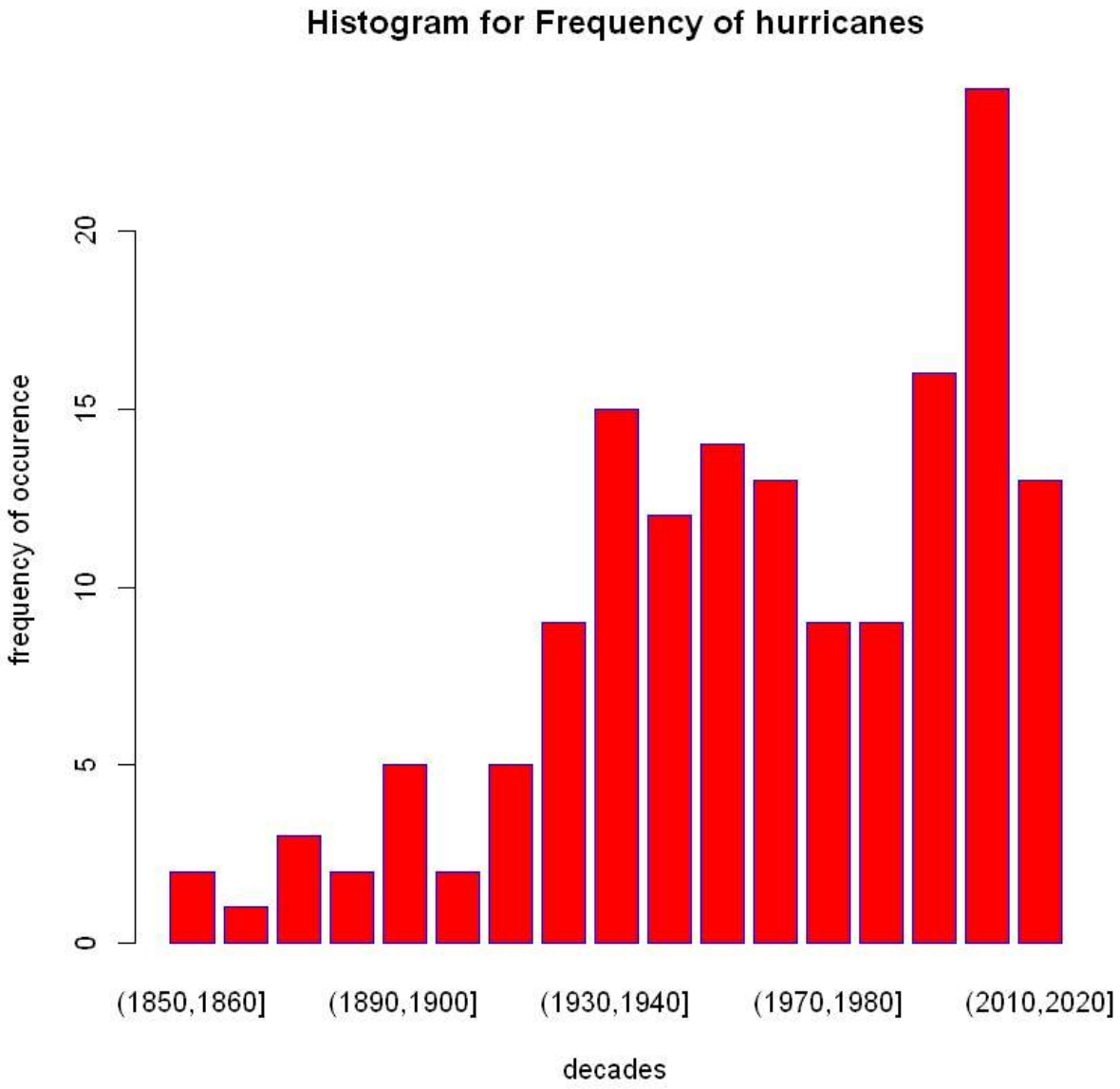
Let do some analysis on the combine data of Hurricane category 4 and 5

In [81]:
```
data <- read.csv("Combinehurricane4and5.csv", header = TRUE, sep=",")
head(data)
```

ï..Name	Season	Month	Maximum.Sustained.Wind	Minimum.pressure
Cuba	1924	october	165	910
San Felipe	1928	september	160	929
Bahamas	1932	september	160	921
Cuba	1932	november	175	915
Cuba-Brownsville	1933	august	160	930
Tampico	1933	september	160	929

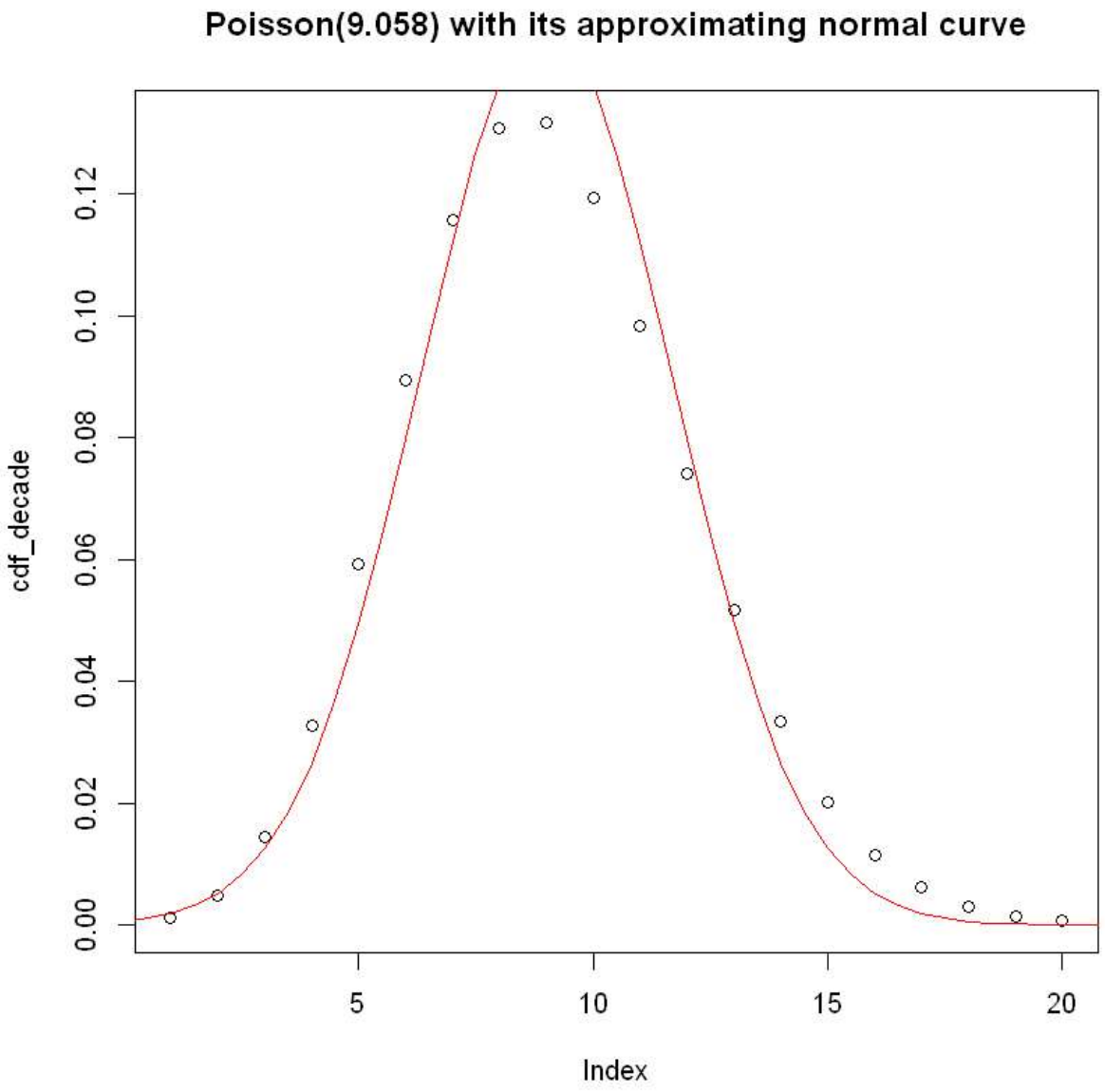
In [177]:

```
season_interval = data$Season

interval = seq(1850,2020,by=10)

Frequency = cut(season_interval, interval, dig.lab=5)

Frequencytable = table(Frequency)

results <- data.frame(Frequencytable)

colnames(results) <- c("decadeinterval","frequencycount45")

barplot(Frequencytable,main="Histogram for Frequency of hurricanes",xlab = "d
```



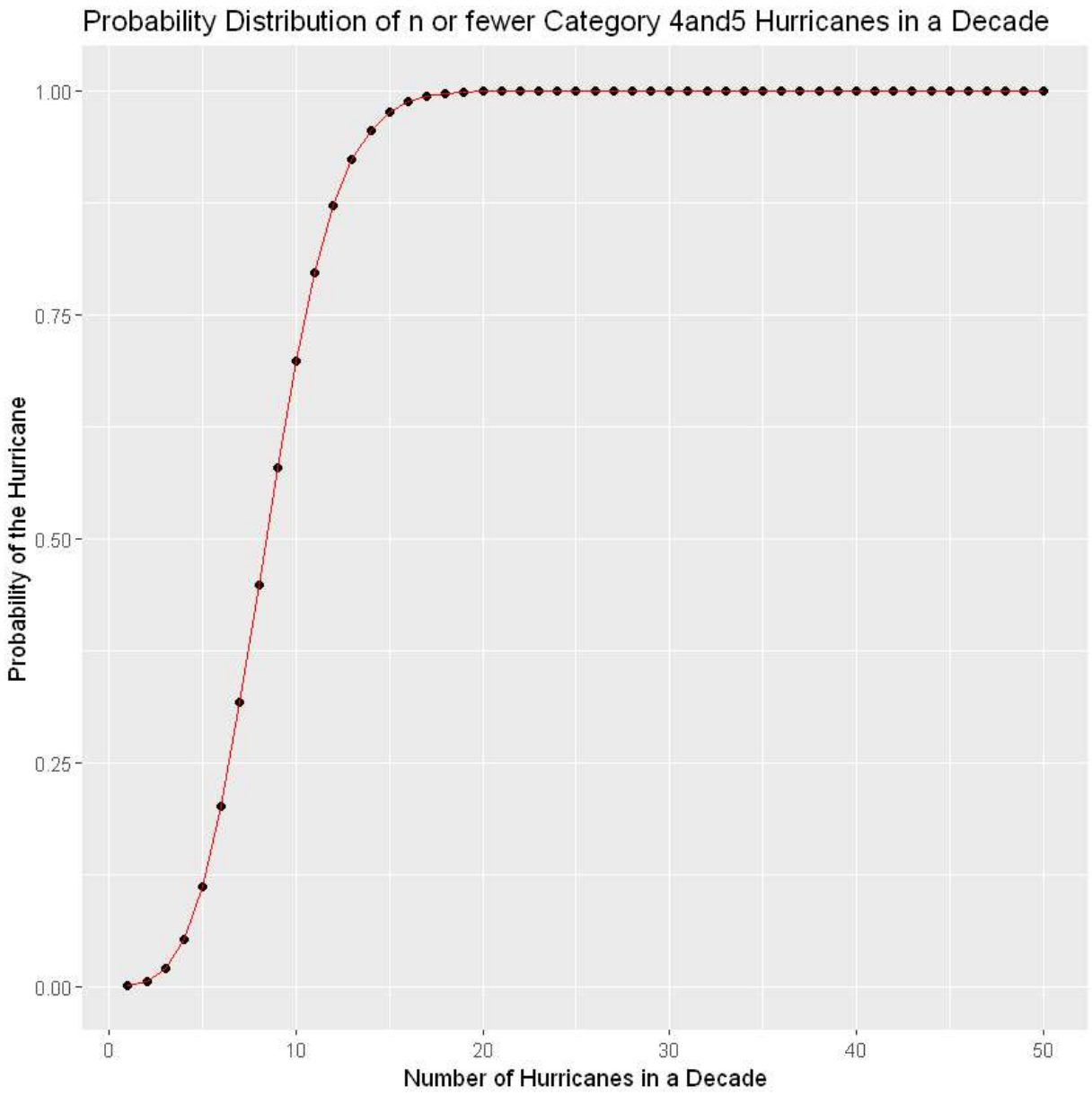
In [178]:

```
sd_frequency = sd(Frequencytable)

mean_frequency =mean(Frequencytable)

number_of_decades = nrow(Frequencytable)

variance = var(Frequencytable)

cdf_decade <- c()
for (i in 1:20){
  cdf_decade[i] <- dpois(i, mean_frequency)
}

plot(cdf_decade, main = "Poisson(9.058) with its approximating normal curve")

normden <- function(x){
  dnorm(x, mean = 9, sd = sqrt(7.3))
}

curve(normden, from = 0, to = 50, add=TRUE, col="red")

```


Poisson(9.058) with its approximating normal curve
cdf_decade
0.00
0.02
0.04
0.06
0.08
0.10
0.12
5
10
15
20
Index

In [104]:

```
get_ppois_poisson_probabilities <- function(lambda){

    poisson_probabilities <- c()
    for (i in 1:50){
      # Probability of *i* or less hurricanes occuring in a decade
      poisson_probabilities[i] <- ppois(i, lambda = lambda)
    }

    poisson_probabilities_df <- as.data.frame(poisson_probabilities)
    poisson_probabilities_df$num_hurricanes_in_the_decade <- c(1:50)
    colnames(poisson_probabilities_df) <- c("poisson_probabilities", "number_

    return(poisson_probabilities_df)
}

category_45_ppois_poisson_probabilities_df <- get_ppois_poisson_probabilities

category_45_ppois_poisson_probabilities_df %>%
ggplot(aes(x = number_of_hurricanes, y = poisson_probabilities))+
geom_point()+
geom_line(color = "red")+
labs(x = "Number of Hurricanes in a Decade", y = "Probability of the Hurrican

```



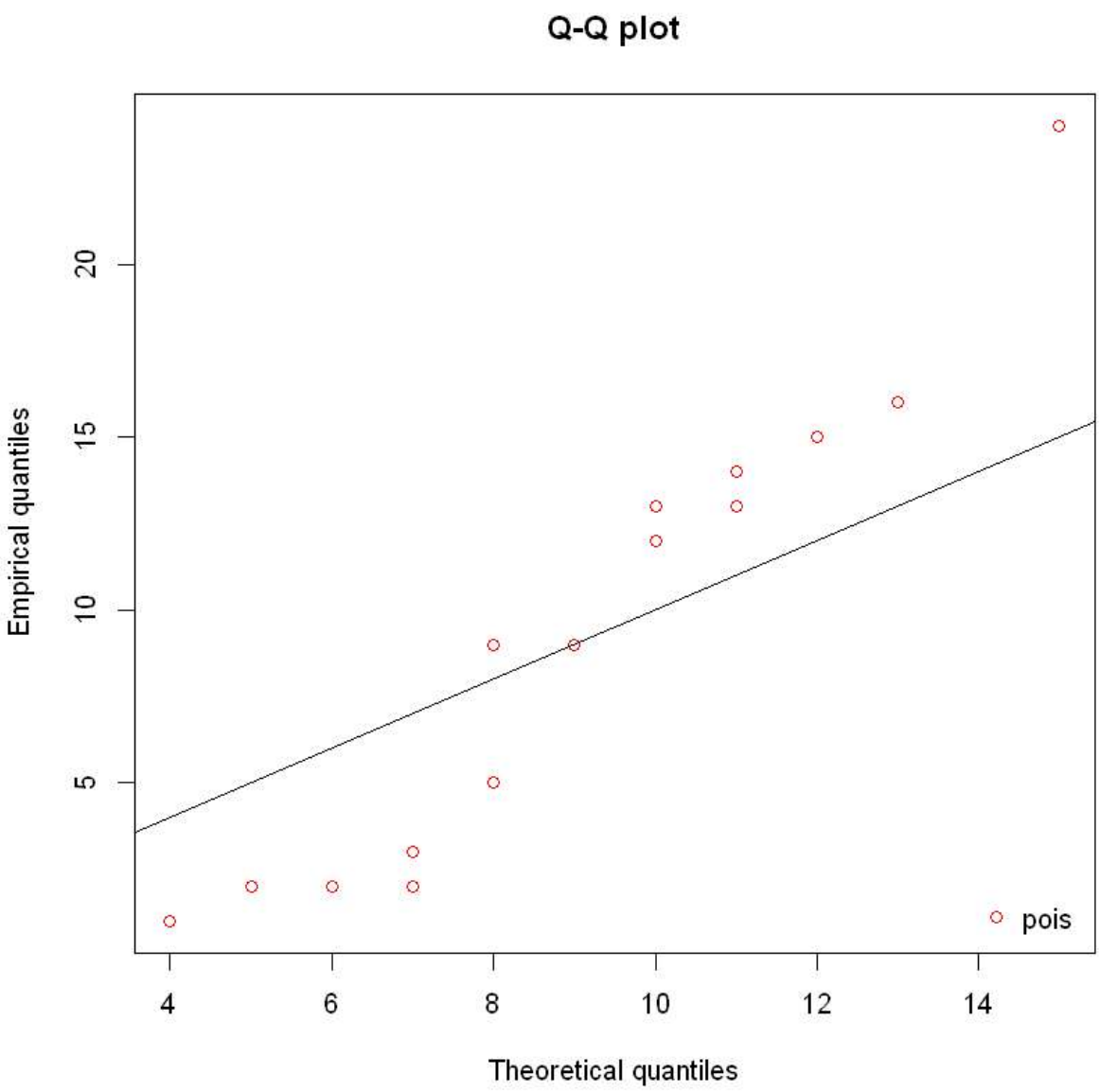
Is it an exact Poisson distribution? Although the plots look similar to a Poisson distribution, we cannot assume that the data is indeed a Poisson distribution by looking at the plots. Poisson distribution has a property that the mean and variance are equal and we use this property to test the fit of our data. Wecheck if this property is satisfied or almost satisfied by our data. We compute the ratio of mean and variance of our data:

In [165]:
```
1 variance_mean_ratio_for_category_45_hurricanes <- round(variance/mean_frequen
2 variance_mean_ratio_for_category_45_hurricanes
```

4.52

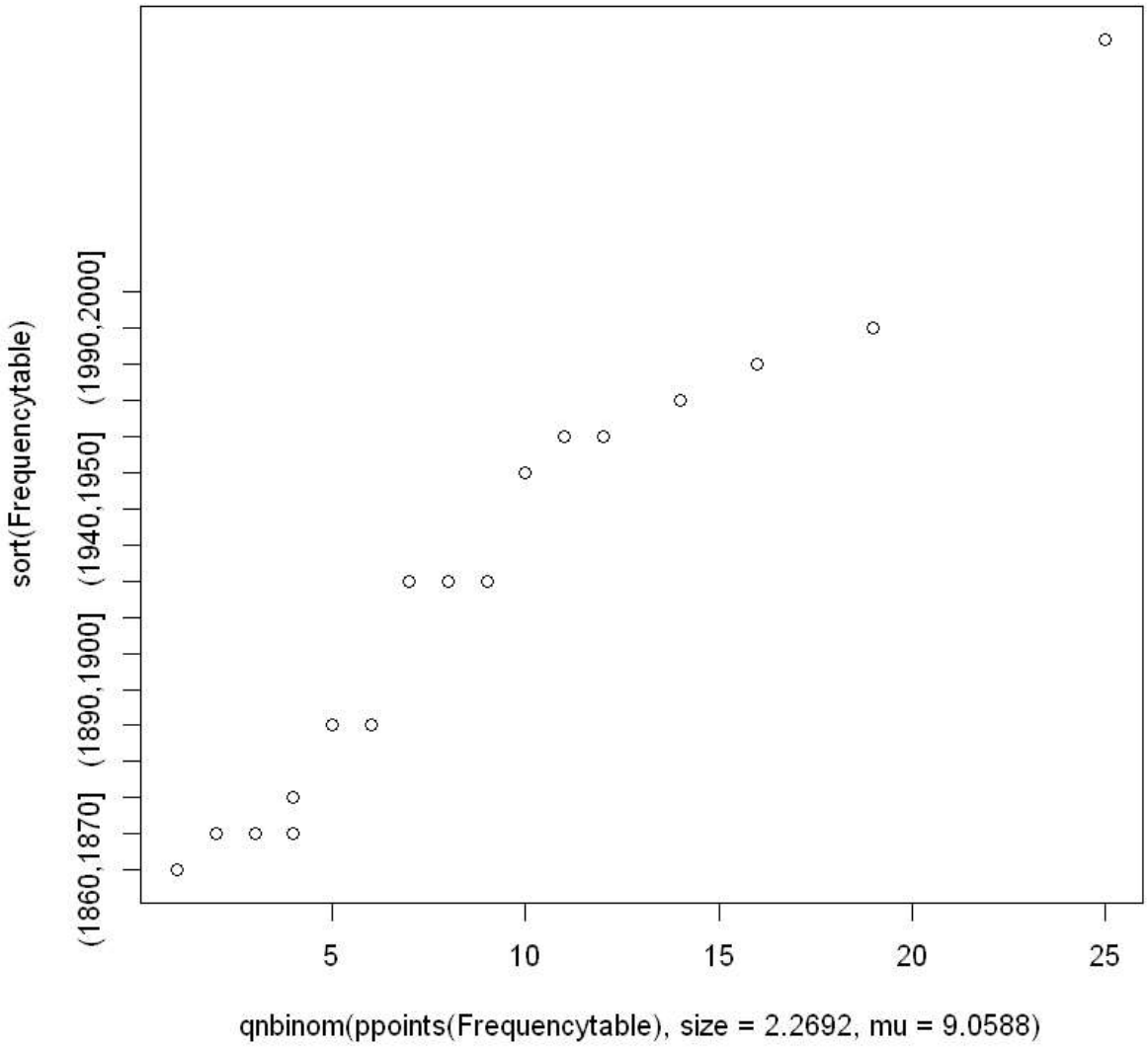
We see that the variance of our data is 4.52 times larger than the mean of the data. We check if such a behaviour is normal or not for a poisson distribution

In [167]:
```
1 qqcomp(fitdist(results$frequencycount45, "pois"))
```



The quantile plot strengthens our conclusion that our hurricane data is not entirely a Poisson distribution with a constant rate. What if we use a negative binomial distribution for this?

In [137]:
```
1 params = fitdistr(Frequencytable, "Negative Binomial")
2 plot(qnbinom(ppoints(Frequencytable), size=2.2692, mu=9.0588), sort(Frequency
```



In [138]:
```
1 qqcomp(fitdist(results$frequencycount45, "nbinom"))
```