JNCASR

INTERNSHIP REPORT

Aayush Sharma

Important

Warm greetings! I am honored to present my internship report to you, and I hope it provides valuable insights into my practical experiences and learning during this period.
Thank you for taking the time to review it.

S. Ganesh

Ganesh Subramanian
Professor, EMU, JNCASR

Aayush

Aayush Sharma
aayushsharma1305@gmail.com

Mass/Heat transport from drops in ambient shearing flows

Aayush Sharma[1] and Prof. Ganesh Subramanian[2]

[1]Department of Mechanical Engineering, NIT Hamirpur, 177005
[2]Engineering Mechanics Unit, JNCASR, Bengaluru, 560001

Abstract

We are investigating the issue of heat or mass transfer occurring in circulating droplets influenced by both settling and an axial electric field. The electric field can be either steady or oscillatory, generating an electrohydrodynamic flow known as the Taylor circulation. This flow adds to the circulation caused by the droplet's steady movement. To analyse this problem, we utilize four dimensionless parameters: the Peclet number (Pe), which represents the relative dominance of convection over diffusion in the transport process, the dimensionless amplitudes of the steady and unsteady electric fields, and the dimensionless frequency (W) of the modulation. In this study, we focus on the rate of heat/mass transfer in a spherical droplet within a Newtonian environment that exhibits general shear motion. The Nusselt number is used to quantify the non-dimensional transport rate. It is obtained numerically as a function of the non-dimensional Peclet number (Pe). The convection-diffusion equation governs the transport process for the scalar concentration field. To model the transport mechanism, we employ stochastic simulations based on Langevin equations for individual tracers. The density distribution associated with the tracers, which satisfy the Langevin equations, follows a convection-diffusion/Fokker-Planck equation. By using a sufficiently large number of particles (which can be considerably smaller than the actual number of molecules), we capture the physics of heat/mass transfer within the continuum framework described by the convection-diffusion equation. The density of tracer particles provides us with the normalized temperature/concentration at any given moment. The accuracy of the stochastic simulations was confirmed by comparing them to existing results for basic non-chaotic flows involving translation and axisymmetric extension. Axisymmetric extension refers to the expansion or generalization of a system, model, or concept to include symmetry around an axis. These simulations were then applied to calculate the transport rates in more complex chaotic shearing flows. The main aim of this project is to find out the relationship between Pe and Nu number and based upon that, find out transport rate relation and how can we increase the transport rate.

Keywords: diffusion, convection, transport rate.

1. Introduction

This study looks at improving heat or mass movement from droplets by using a combination of settling and an axially directed electric field to promote circulation within the droplet. The droplet is believed to be submerged in another immiscible liquid, and both liquids are leaky dielectrics. The well-known Taylor "leaky dielectric" model is used to determine the electrohydrodynamic flow field. The research looks at two scenarios: steady flows and time-modulated electric fields, both of which result in chaotic advection of passive Lagrange fluid particles.

Consider a spherical drop of Newtonian fluid having a radius in a Newtonian environment. The barrier to heat/mass transport on the drop's outside is deemed insignificant in comparison to the inside. We further assume that surface tension forces at the drop-ambient interface dominate over flow-induced viscous forces, resulting in a spherical drop that does not deform at all times. The flow is assumed to be an inertia-less Stokes flow (also known as creeping flow refers to a particular state where fluid flow is characterised by dominance of viscous forces over inertia forces and this fluid flow Re no.). We select a coordinate frame with an origin set at the drop's center. Due to the simplicity and convenience of presentation, we will use the language for mass transfer throughout the rest of the document, unless otherwise noted. The simulation-derived time-dependent concentrations field is analyzed starting with a uniform concentration as the initial condition.

At the drop surface, a zero-concentration boundary condition is imposed as the ambient phase resistance is assumed to be negligible. Since the entire computation is performed mathematically by following the movement of at first consistently conveyed tracer particles, the zero-focus limit condition is implemented by means of a sink at the drop surface which eliminates any tracer which raises a ruckus around surface and hit it.

2. Objective and Approach

Our Ultimate Objective is to find out relationship between Nusselt number and Peclet number. For this, at first, we generated Path function and Streamline plots for various values of W {0, 0.5, 1, 2, 4, ∞} (representing ratio of amplitude of two flows, Taylor and Hadamard flow). Then we perform MATLAB simulation based on Stochastic approach to find out the relation between concentration of tracer particle with time and figure out how much tracer particle left at every time step and then we find out Nusselt number by the relation

$$-\frac{2}{3}.\frac{1}{c}.\frac{dc}{dt}$$

After this, we generated plot for Nusselt vs time. Once this is plotted, the result is showing scattered value of Nusselt number for time interval of 1 to 10. So, we obtain Nud, Nusselt by averaging at this interval.

3. Transport of Drop in a Steady Flow Condition

The initial investigation conducted by Kronig and Brink focused on the conventional outcomes of heat or mass transfer in droplets moving steadily. They specifically explored the scenario where the resistance within the droplet's internal phase dominates. In this study, they analyzed a spherical droplet undergoing settling, with an internal flow pattern known as the Hadamard-Rybczinski circulation, referred to as the "Hadamard circulation" henceforth. This circulation is characterized by a velocity U.

Consequently, the entire analysis revolves around a single dimensionless parameter known as the Peclet number, $Pe = \frac{UL^1}{D}$ where L and D are the droplet radius and the diffusion coefficient of the transported quantity, respectively. The extraction rate Λ defined through the expression for the fraction of initial mass remaining,

$$\frac{M(t)}{M_0} = \exp\left(-\frac{tD}{L * L}\right),$$

The transport in this case is diffusion controlled. On the other hand, when Pe=, Kronig and Brink presented an argument that is equivalent to assuming that the concentration is constant along the streamlines of the Hadamard circulation, which leads to the conclusion that transport is now limited by the rate of cross-streamline diffusion. Therefore, transport remains diffusion limited but with a rate that is enhanced by a factor of approximately 2.7. This enhancement is due to a reduction in the characteristic length over which diffusion must act. The study is carried out for smaller Pe number. These all estimates are done manually and upon checkup we got to know that these values and result are applicable for low Pe value. Enhanced transport from drop is observed under application of Electric Field.

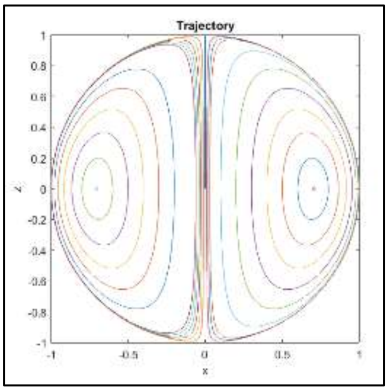
Equations correspond to this are

$$W = (W1 + W2 * \cos(Pt))$$

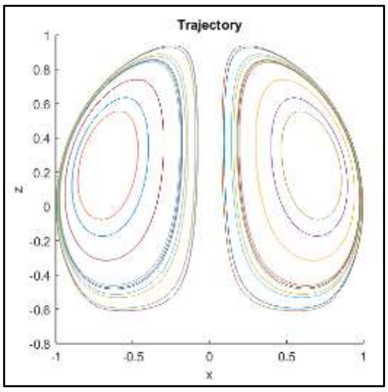
$$\frac{dx}{dt} = \left\{\frac{xz}{2(1+a)}\right\} + (W1 + W2\cos(Pt))\left\{3x\left(\frac{-1 + xx + y^2 + 3z^2}{4(1+a)}\right)\right\}$$

$$\frac{dy}{dt} = \left\{\frac{yz}{2(1+a)}\right\} + (W1 + W2\cos(Pt))\left\{3y\left(\frac{-1 + xx + y^2 + 3z^2}{4(1+a)}\right)\right\}$$

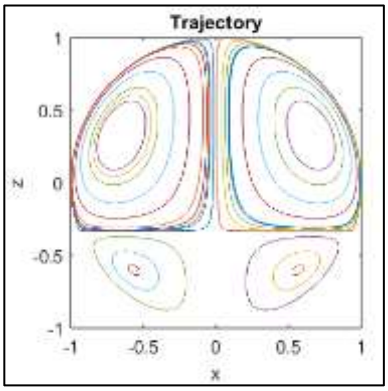
$$\frac{dz}{dt} = \left\{\frac{1 - 2xx - 2y^2 - z^2}{2(1+a)}\right\} + (W1 + W2\cos(Pt))\left\{-3z\left(\frac{-1 + 2xx + 2y^2 + z^2}{2(1+a)}\right)\right\}$$



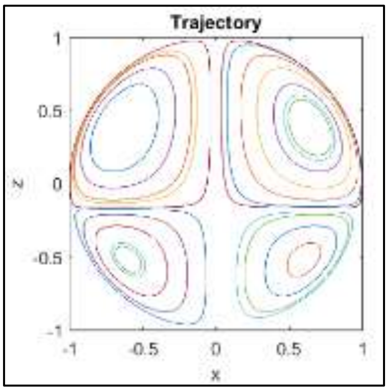
W=0



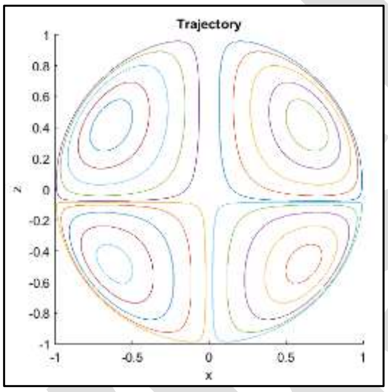
W=0.5



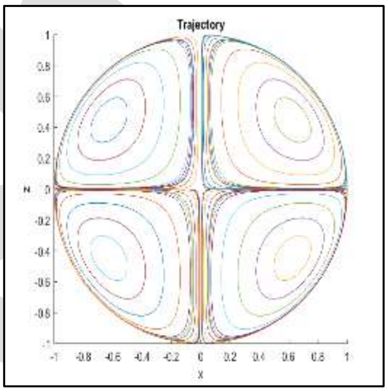
W=1



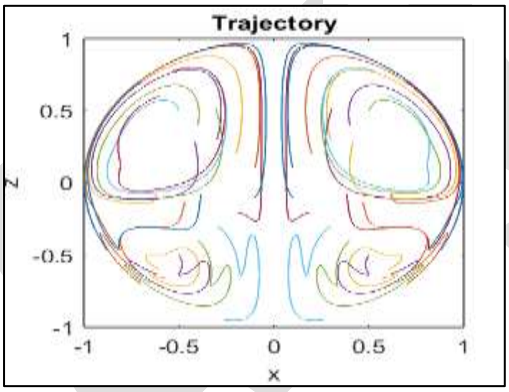
W=2

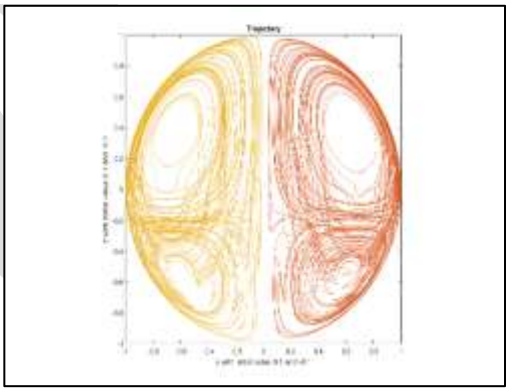


W=4



W=∞





Flow with parameter "t=1000"

$$Streamlines\ of\ the\ Hadamard\text{–}Taylor\ flow\ for\ various\ W\ (0\ to\ \infty)$$

4. Transport of Drop in an Unsteady Flow Condition

Unsteady flows create the opportunity for chaotic advection, a phenomenon characterized by the folding and stretching of fluid filaments. This chaotic advection is commonly believed to enhance transport rates by reducing the length scales over which molecular diffusion acts. Rom-Kedar and Poje were among the first to address the complexities of transport in chaotic flows, focusing on an idealized time-periodic open flow. Although their findings were specifically applicable to this type of flow, many of their concepts extend to closed flows as well. Through the use of asymptotic methods, they demonstrated that transport is a nonmonotonic function of the dimensionless frequency, exhibiting a linear relationship with frequency at low values and decaying exponentially for high frequencies. These results were subsequently confirmed by Rom-Kedar, who employed less restrictive assumptions in their analysis.

Recent studies by Solomon and Mezić and Solomon et al. have investigated chaotic mixing in spatially periodic time-modulated flows. They proposed that resonant enhancement of mixing can occur when the oscillation periods correspond to typical circulation times of weakly perturbed three-dimensional vortices. Computational and experimental evidence supported this idea. Solomon et al. further explored this concept and observed multiple local maxima in the effective dispersion coefficient as a function of frequency in such flows. When the circulation time aligns with the forcing period, tracers can travel long distances in only a few periods through repeated separatrix crossings. However, analysing the dynamics over a single period using lobe dynamics and Melnikov functions was found to be inadequate in explaining these effects.

While the aforementioned works focus on open and/or spatially periodic flows, we are more interested in results related to convective diffusion of a passive scalar in time-periodic closed flows that exhibit chaotic advection. Pierrehumbert conducted a notable study on convective diffusion in a time-periodic model flow. He observed that after an initial transient, the concentration field consists of periodically repeating underlying spatial structures that decay over time. These structures, referred to as "strange eigenmodes," have also been experimentally observed Haller and Liu analyzed these underlying structures and showed that they arise from the concentration field being attracted to a time-dependent inertial manifold spanned by a finite number of strange eigenmodes. They established critical straining rates (Peclet numbers) below which such structures cannot exist using standard energy estimates. Several examples of model flows were presented in their analysis.

Saatdjian and Leprevost conducted a related study, considering transport in the annular region between confocal ellipses with different temperature surfaces that glide without changing their orientation relative to fixed coordinates. They aimed to solve the transport problem on a fixed domain by focusing on the case of time-modulated surface velocity. They attempted to correlate the transport rates with the mixing zone approach proposed by Kaper and Wiggins. Although their parametric study had limitations, the authors noted the existence of an optimal frequency of modulation that maximizes transport.

Lee et al. were among the first researchers to explore the impact of electric field modulation on transport rates in unsteady flows. While their main focus was on steady nonuniform electric fields and the computation of dielectrophoretic velocity, they acknowledged the potential for chaotic advection with time-varying electric fields. They provided Poincare sections (A Poincare section is a tool used to analyse the dynamics of a system by observing its behaviour on a lower-dimensional surface. It involves taking cross-sections of the system's trajectory at regular intervals or when specific conditions are met. This allows for the visualization and characterization of recurring patterns, such as periodic or chaotic behaviour, without examining the full trajectory in the entire phase space. Poincare sections provide valuable insights into the long-term behaviour and transport properties of dynamical systems) as evidence and presented solutions to the convective diffusion equation for limited parameter ranges. Their assessment of transport enhancement was conducted using complex and nonstandard methods, and the conditions under which their results are applicable remain unclear. Nevertheless, they observed a nonmonotonic relationship between enhancement and frequency.

Ward and Homsy conducted a series of two papers that examined chaotic advection in uniform time-modulated electric fields, taking into account the significant effects of steady translation velocity. They demonstrated how modulation of the electric field leads to deviations in the location of the stagnation disk. The cyclic variation of flow fields among these patterns resulted in trajectory splitting and chaotic advection. Mixing was quantified by determining the percentage of drop volume containing chaotic trajectories. Their findings revealed that the mixed volume increases linearly with frequency at low frequencies, decays exponentially at high frequencies, and exhibits a large broadband plateau over an intermediate frequency range spanning 1-2 decades. Moreover, it was possible to identify parameter values where nearly the entire drop volume experienced mixing. The first two features align with the general considerations but do not provide insights into the rate of heat and mass transfer in such chaotically stirred droplets.

5. Translational and Linear Flow inside Spherical drop

When dealing with extremely slow flows (Re = 0), solving the internal flow field becomes straightforward. If there is no movement or displacement (U = 0), the solution in a stable state can be determined as follow

$$\boldsymbol{u}_{\boldsymbol{G}}(\boldsymbol{x}) = \left(\frac{1}{2(1 + \lambda)}\right)[(5r^2 - 3)\boldsymbol{E} \cdot \boldsymbol{x} - 2\boldsymbol{x}\boldsymbol{x} \cdot \boldsymbol{E} \cdot \boldsymbol{x}] + (½ * w \times \boldsymbol{x})$$

If the distant flow is purely translational (with u1 = U), the internal flow field can be described as follow

$$uT(x) = \left(\frac{U}{2(1 + \lambda)}\right).(xx - (2r^2 - 1)I)$$

Here, $r^2 = xx$ and λ is the internal fluid to external fluid viscosity ratio. It is to be noted that the above equations have been non-dimensionalised with the length scale a. Due to the chaotic nature, the pathlines of the particles go all over the drop and cause mixing due to convection. This phenomenon is called chaotic advection. Due to this chaotic mixing in the drop, one expects a huge enhancement in the transport rate at large Pe.

6. Stochastic Description of Transport: The Langevin Equation

We are considering time scale diffusion Convective-Diffusion equation and corresponding Langevin equation instead of considering convection time scale Convective-Diffusion equation. Stochastic simulations based on Langevin equations are employed to calculate the transport rate in our study. The Langevin equations describe the 3-dimensional Brownian motion of individual tracer particles.

The Convective-Diffusion equation is

$$\frac{dc}{dt} + Pe.(u.\nabla c) = \Delta c$$

And corresponding Langevin equation is

$$dx = Pe.u.dt + \sqrt{2dt}.(wt)$$

In these equations, the term $Pe.u.dt$ represents the particle's convection due to the flow field, which indirectly depends on time through the position vector x(t). The second term, $\sqrt{2dt}.(wt)$, represents Gaussian white noise that drives diffusive motion of the tracer particles at long times. The white noise is implemented using a Wiener process, which is characterized by specific statistical properties.

The Langevin equation and the Fokker-Planck equation are interconnected, where each trajectory obtained from integrating the Langevin equation represents a specific instance of a stochastic process. This process follows the probability density governed by the Fokker-Planck equation. The Fokker-Planck equation describes the evolution of the average local density of tracer particles. While the solution of the Langevin equation is a random function with a defined average, the Fokker-Planck equation provides a deterministic solution in terms of concentration.

To solve the Langevin equations, we utilize a numerical scheme like the Runge-Kutta method. This method enables us to calculate the equations for each tracer particle at every time step. Initially, the tracers are evenly distributed within the droplet. The absorption boundary condition is enforced by removing tracers that come into contact with the droplet surface. The normalized concentration is determined by comparing the number of particles inside the droplet at a specific time to the initial number of particles. Now, after this we are going to use the imposed velocity concept in steady and unsteady state condition. Velocity is given as

$$U = (d3)(E.X) + (d4)(\Omega.X) - \frac{2}{21}(d2)(X)(X.E.X) + \frac{5}{21}(d2)(r2)(E.X)$$

where E is defined as symmetric component of velocity and is given as

$$E = \frac{\Gamma + (\Gamma)t}{2}$$

and Ω is defined as antisymmetric component of velocity and is given as

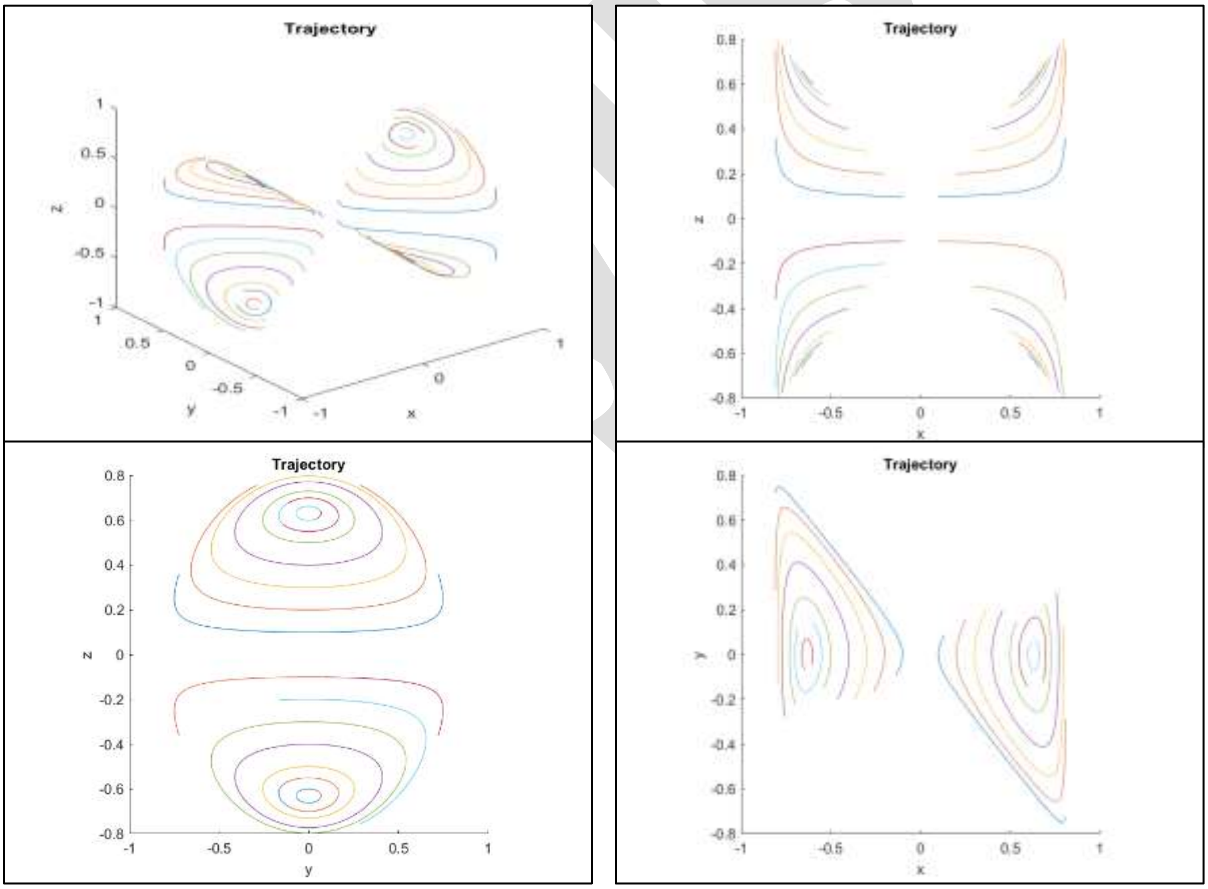
$$\Omega = \frac{\Gamma - \Gamma t}{2}$$

$$\Gamma = \nabla * uimp$$

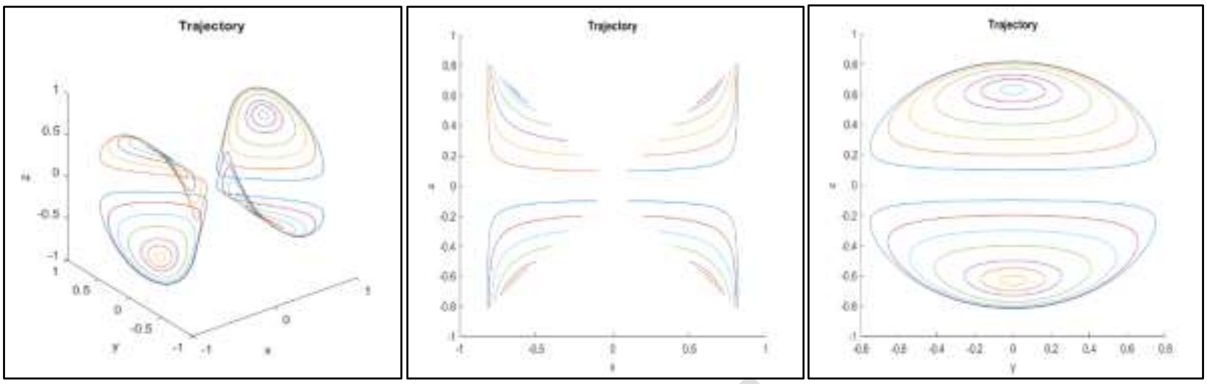
and $uimp$ is imposed velocity and is given by

$$uimp = \Big(\big((2\beta \sin(wt) + 2\cos(wt)) * x\big) + \big((2\sin wt + 2\beta \cos(wt)) * y\big) + (0 * z)\Big)$$

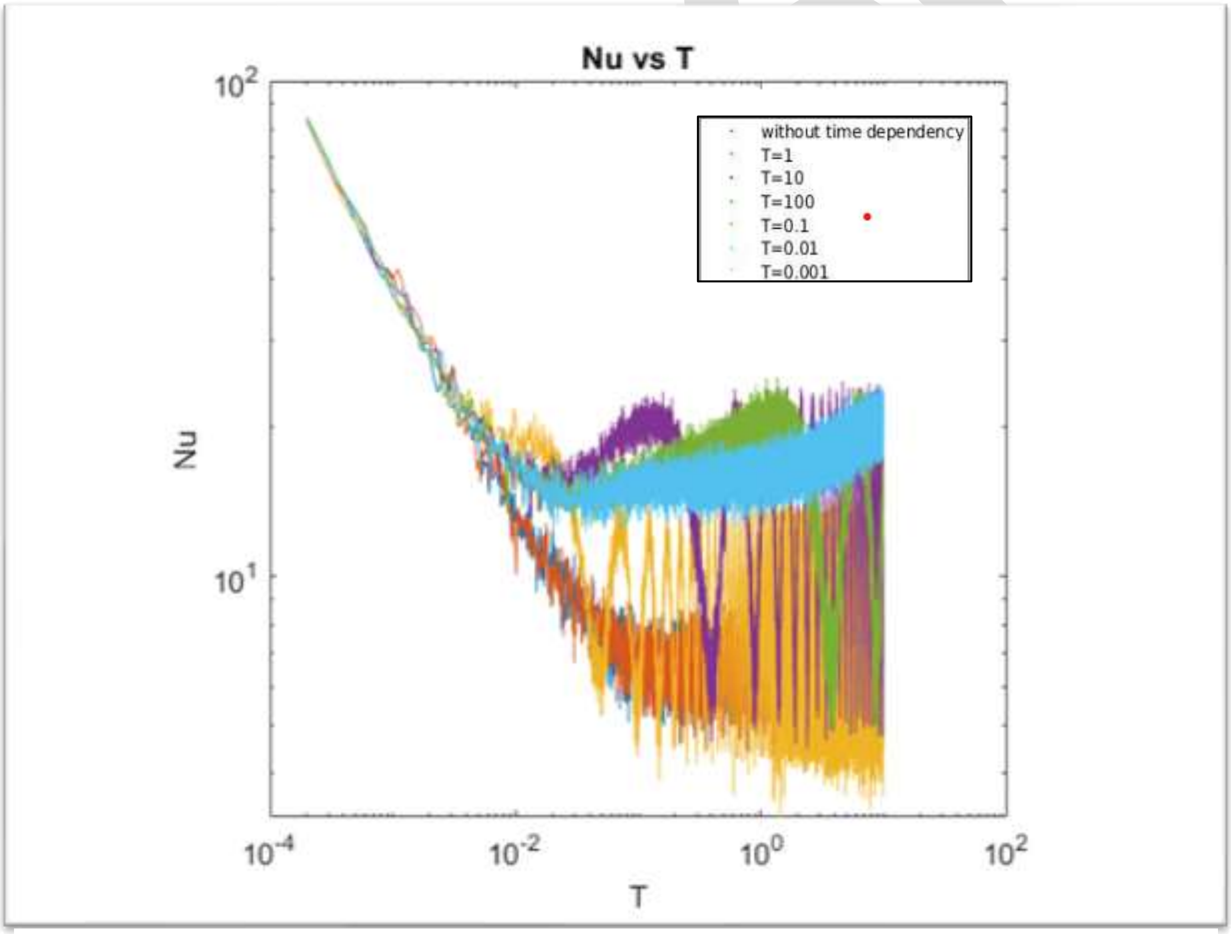
These expressions are defining the functions with added Unsteadiness. We generated path lines based upon these functions which is shown as



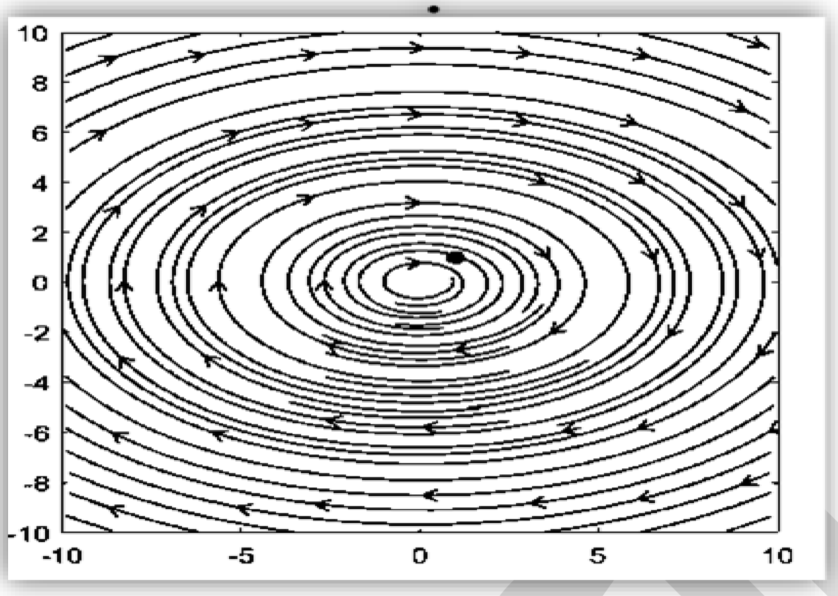
$These\ figures\ represent\ steady\ state\ path\ lines\ (no\ un-steadiness\ is\ added)$



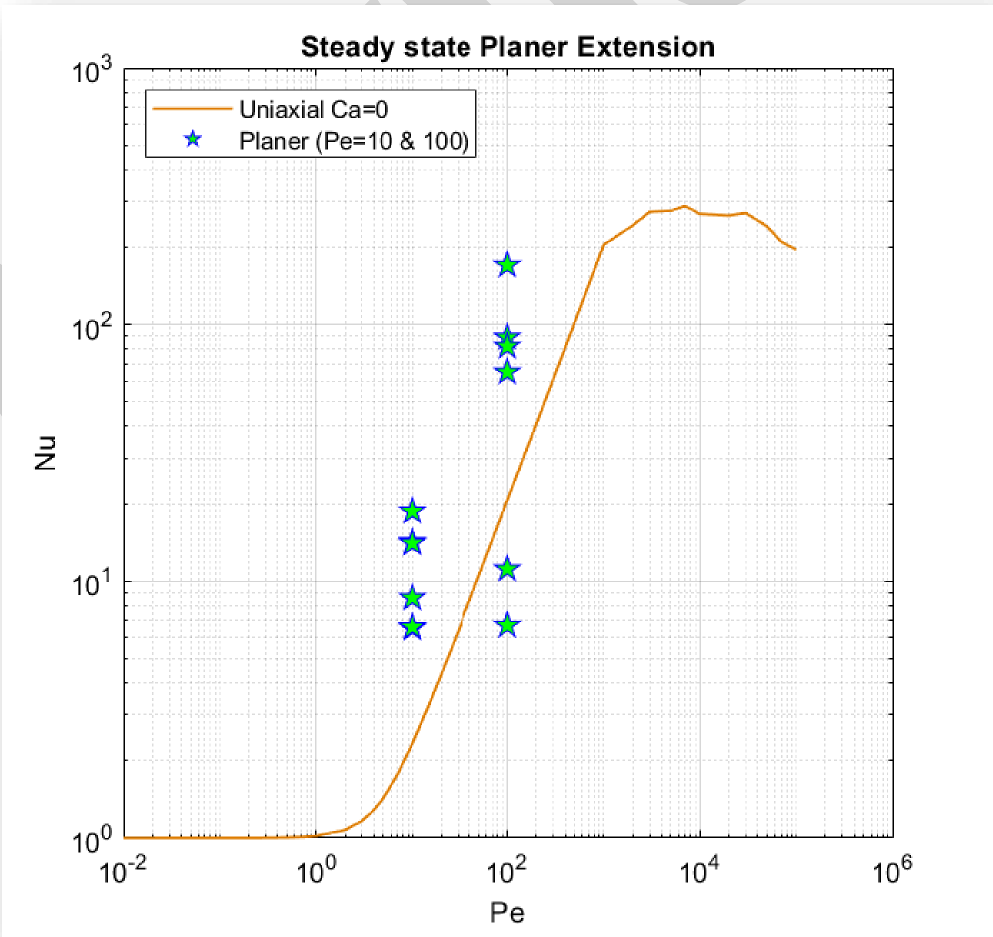
These figures represent unsteady state path lines



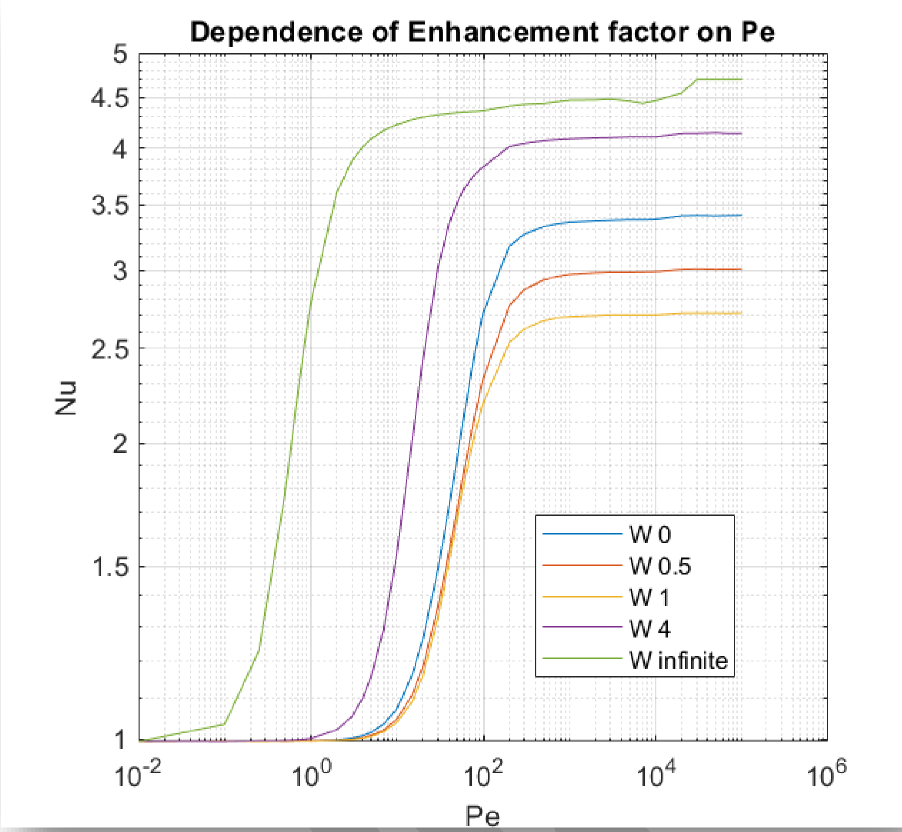
The time dependence of the transport rate at different P e values



$$Effect\ of\ \frac{t}{T} = 0\ on\ streamlines$$



$$Nu\ Vs\ Pe\ for\ different\ T\ and\ Pe\ 10\ and\ 100$$



Nusselt number plotted as a function of Peclet

Conclusions

In this study, we conduct stochastic simulations to investigate the steady-state mass/heat transfer occurring outside a liquid sphere and the transient transport taking place within a liquid sphere under simple shear creeping flow conditions, specifically focusing on the known Stokes velocity field at small Peclet numbers. To validate our approach, we compare our results with well-established outcomes for cases involving pure diffusion, pure translation, and pure extensional flow. The enhancement of the Nusselt number (Nu) is attributed to convection; however, it is important to note that convection alone is insufficient to fully account for the observed Nu values. We find that the Nu number increases with a higher velocity of tracer particles, signifying that the movement of these particles plays a crucial role in the augmentation of Nu.

Acknowledgement

I would like to express my deepest appreciation to the Jawaharlal Nehru Centre for Advanced Scientific Research (JNCASR) for granting me the incredible opportunity to participate in the prestigious Summer Research Fellowship Programme 2023. My heart is filled with gratitude towards the exceptional guidance and mentorship provided by Prof. Ganesh Subhramanian, whose expertise and support have been invaluable throughout this project. I am also indebted to Mr. Pavan Singeetham for generously dedicating his precious time to assist me in this endeavor. I am heartly thankful to Mr. Sangamesh Gudda for constantly helping me in theory behind logics. Finally, I want to express my heartfelt thanks to my parents, whose unwavering support and unwavering care have been an unwavering source of strength and motivation.

References

[1] *Michelle D. Bryden and Howard Brenner. Mass-transfer enhancement via chaotic laminar flow within a droplet. Journal of Fluid Mechanics, 379:319331, 1999.*

[2] *C. I. Christov and G. M. Homsy. Enhancement of transport from drops by steady and modulated electric fields. Physics of Fluids, 21(8):083102, 2009.*

[3] *C. W. Gardiner. Handbook of stochastic methods for physics, chemistry and the natural sciences, volume 13 of Springer Series in Synergetics. Springer-Verlag, Berlin, third edition, 2004.*

[4] *Gheorghe Juncu. A numerical study of the unsteady heat/mass transfer inside a circulating sphere. International Journal of Heat and Mass Transfer, 53(15):3006 {3012, 2010.*

[5] *R Kronig and J C. Brink. On the theory of extraction from falling drops. 2:142{154, 07 1951.*

[6] *L. Gary Leal. Advanced Transport Phenomena: Fluid Mechanics and Convective Transport Processes. Cambridge Series in Chemical Engineering. Cambridge University Press, 2007.*

[7] *H. A. Stone, Ali Nadim, and Steven H. Strogatz. Chaotic streamlines inside drops immersed in steady stoke flows. Journal of Fluid Mechanics, 232:629646, 1991.*