2 Scotch whisky selection design

A grocery store that has traditionally sold beer and wine is interested in expanding its liquor selection to include Scotch whisky. It has identified 86 different whiskies, each originating from a distinct distillery in Scotland.

The file `whiskies-attributes-prices.csv` contains information about each whisky, including its unit price and its rating on 12 different dimensions (how smoky it is, how much body it has, how floral it is, and so on), which range from 0 to 4. The file `whiskies-utilities.csv` contains the utility for each of the 86 whiskies for 100 representative customer types. The utilities for all of the customers are calibrated so that the no-purchase utility of each customer is exactly 27. A customer of a particular type is assumed to follow a first-choice model of behavior, i.e., they will evaluate the utility of each of the available whiskies and choose to purchase the one with the highest utility, or choose to not purchase any whisky at all. Assume that each customer type is equally probable, i.e., the weight λ_k of each customer type is $1/100$.

The grocery store would like to choose a selection of whiskies to offer so as to maximize the expected per-customer revenue. In Part 1 and 2, we will formulate and solve the problem without any constraints on the set of whiskies that is offered; in Part 3 and 4, we will consider incorporating additional constraints.

Part 1: Formulating the problem

We will first formulate the problem.

a) Write down an abstract formulation of the problem as an integer program. Define the decision variables, constraints and the objective function, and any parameters that your formulation requires. Explain why your formulation is correct. (Do not formulate any constraints with actual numbers).

Part 2: Solving the problem

Implement your formulation and solve the problem.

a) What is the expected revenue of the optimal selection of whiskies?

b) Which whiskies are offered in the optimal solution?

c) What are the choice probabilities of the whiskies in the optimal solution?

Part 3: Balancing the whisky selection

Let us now try to understand how the objective value of the formulation will change if we impose certain constraints. For each of the following, explain whether you expect the optimal objective value from Part 2 to increase, stay the same or decrease, *without* solving any integer program with Gurobi. You may wish to analyze the solution you obtained in Part 2. Answer each part independently of the others.

a) The grocery store imposes the requirement that there at most 20 whiskies in the selection.

b) The grocery store imposes the requirement that there at most 5 whiskies in the selection.

c) The grocery store imposes the requirement that there is at least one whisky that rates 4 on smokiness.

Part 4: Extending the formulation

The grocery store is interested in enriching the formulation by incorporating additional requirements on the selection of whiskies. Explain how you would modify the formulation to accommodate each of the requirements below. Any modification should ensure the problem is still a valid integer program (i.e., any changes should still result in linear constraints and a linear objective function). Your modifications should be in the form of mathematical constraints, e.g.:

- "Add the constraint $x_1 + x_2 + y_{1,5} \leq 3$", or
- "Add the constraint $\sum_{i \in I} x_i + \sum_{j \in J} y_{k,j} = 4$, where I is the set of objects with property ... and J is the set of objects with property ... ."

You may define additional parameters or sets to define your mathematical statements. You do not need to implement any of these in Python/Gurobi.

a) The fraction of customers choosing Ardbeg is at most 0.15.

b) The fraction of customers choosing Ardbeg, if it is offered, is at least 0.30.

c) If Bowmore is offered, then we must offer (at least one of) either Oban or Talisker.

d) The average rating on the "smoky" dimension (in `whiskies-attributes-prices.csv`) of the selection of whiskies is at least 3.0.