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Assignment

Parameter Estimation

Question 1: Let $(X_1, X_2, \ldots X_n)$ be a random sample of size n taken from a normal population with parameters, mean $= \theta_1$ and variance $= \theta_2$. Find maximum likelihood estimate of these two parameter.

Solution! Given that $X_1, X_2, \ldots X_n$ is a Random Sample from a normal form distribution with mean $= \theta_1$ and Variance $= \theta_2$ the likelihood function is

$$L(\theta_1, \theta_2 \mid x_1, x_2, \ldots x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_1)^2}{2\theta_2}}$$

By Taking log on both sides, we get

$$\ln L(\theta_1, \theta_2 \mid x_1, x_2 \ldots x_n) = -\frac{n}{2} \ln(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

To find MLE, we'll differentiate the log-likelihood with respect to θ_1 & θ_2, set derivative equal to zero.

(i) For θ_1 :

$$\frac{\partial}{\partial\theta_1} \ln L(\theta_1, \theta_2 \mid x_1, x_2, \ldots x_n) = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)$$

Setting this equal to zero

$$\frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \hat{\theta}_1) = 0 \Rightarrow \sum_{i=1}^{n} (x_i - \hat{\theta}_1) = 0$$

$$\therefore \hat{\theta}_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$$

So, the MLE for θ_1 is the sample mean.

(ii) For θ_2:

$$\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2 \mid x_1, x_2 \cdots x_n) = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

Setting this equal to zero:

$$\frac{-n}{2\hat{\theta}_2} + \frac{1}{2\hat{\theta}_2^2} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2 = 0$$

$$\Rightarrow \frac{n}{2\hat{\theta}_2} - \frac{1}{2\hat{\theta}_2^2} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2$$

$$\Rightarrow \hat{\theta}_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2$$

So, MLE for θ_2 is the sample variance.

Question 2: To find the MLE of θ for a random sample $X_1, X_2, \ldots X_n$, from a Bernaulli distribution with parameter θ and a known m. The likelihood for this scenario is:

$$L(\theta \mid x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} P(X_i = x_i / \theta)$$

Since X_i follows a Bernaulli distibution, $P(X_i = x_i / \theta) = \theta^{x_i}(1-\theta)^{m-x_i}$ for each i

Taking the log on both sides:

$$\ln L(\theta \mid x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} \ln\left(\theta^{x_i}(1-\theta)^{m-x_i}\right)$$

$$= \sum_{i=1}^{n} \left(x_i \ln\theta + (m - x_i)\ln(1-\theta)\right)$$

Now differentiate with respect to θ and set to zero

$$\frac{d}{d\theta}\left(\ln L(\theta \mid x_1, x_2, \ldots, x_n)\right) = 0$$

$$\sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta}\right) = 0$$

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = \frac{nm - \sum_{i=1}^{n} n_i}{1-\theta}$$

$$\therefore \theta = \sum_{i=1}^{n} \frac{x_i}{n \cdot m}$$

So, maximum likelihood estimate for θ is

$$\hat{\theta}_{MLE} = \sum_{i=1}^{n} \frac{x_i}{n \cdot m}$$