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YOUVA

Name: Sayushi Puri Roll No.: 102103676 Group: 3COE24 Assignment Parameter Estimation Question! Let (X, X2, -- X2) be a semple of taken from a normal population with parameter, mean = 9, and variance = 92. Find maximum likelihood estimate of these luss parameter. Solution! Given that X, X, , --- Xn is a Random Sample from 9 normal form distribution with mean = 0, and Variance = 0, the likelihood function is  $L(9, 0, 1, x_1, --x_n) = \prod_{i=1}^{n} L e^{-2Q_2}$ By Taking log on both sides, we get  $\ln \left( \left( \frac{\partial}{\partial t} \right) \frac{\partial}{\partial t} \right) \frac{\partial}{\partial t} \frac{\partial}{\partial t}$ To find MLE, nei'll differentiate the log-likelihood with supert to 0, & 02, set derivative equal to zero.  $\frac{\partial}{\partial \theta_1} \ln L(\theta_1, \theta_2 | \chi_1, \chi_2, \dots, \chi_n) = \frac{1}{\theta_2} \sum_{i=1}^n (\chi_i - \theta_1)$ Sitting this equal to get on  $(n_i - \hat{0}_i) = 0$   $\Rightarrow \sum_{i=1}^{\infty} (n_i - \hat{0}_i) = 0$ : . ê, = 1 £ n; Co, the MIE for a is the sample mean.

$$\frac{\partial}{\partial \theta_{2}} \ln L(\theta_{1}, \theta_{2} \mid \mathcal{H}_{1}, \mathcal{H}_{2} - - \mathcal{H}_{n}) = -\frac{n}{2\theta_{2}} + \frac{1}{2\theta_{2}} \frac{\mathcal{H}_{i} - \theta_{1}}{2\theta_{2}}$$

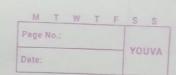
Setting this equal to zero:

$$\frac{-n+1}{2\hat{\theta}_{2}} = \frac{n}{2\hat{\theta}_{2}^{2}} (x_{i} - \hat{0}_{i})^{2} = 0$$

$$= \frac{1}{20} \frac{m}{20} - \frac{m}{20} (x_i - 0_i)^2$$

$$\Rightarrow \hat{Q} = \frac{1}{n} \sum_{i=1}^{\infty} (n_i - \hat{Q}_i)^2$$

So, MIE for O, is the sample starionce.



 $L(\theta|\chi_1,\chi_2,\ldots,\chi_n) = \prod_{i=1}^n P(\chi_i = \chi_i/\theta)$ 

Since Xi follows a Bernaulli distribution,  $P(X_i = \chi_i/0)$ =  $\theta^{n_i} (1-\theta)^{m-n_i}$  for each i

Taking the log on both sides:

 $ln L(\theta | \chi_1, \chi_2, \dots, \chi_n) = \sum_{i=1}^n ln (\theta^{\chi i} (1-\theta)^{m-\chi_i})$ 

 $= \sum_{i=1}^{m} \left( \chi_{i} \ln \theta + \left( m - \chi_{i} \right) \ln \left( 1 - \theta \right) \right)$ 

Noue differentiate with suspect to 0 and set to zero

 $\frac{d}{d\theta} \left( \ln L \left( \frac{\partial (\chi_1, \chi_2, \dots, \chi_n)}{\partial x_1, \chi_2, \dots, \chi_n} \right) = 0$   $\frac{E}{i^{-1}} \left( \frac{\pi_i}{\theta} - \frac{m - \pi_i}{1 - \theta} \right) = 0$ 

 $\frac{\sum_{i=1}^{n} n_{i}}{n} = nm - \sum_{i=1}^{n} n_{i}$ 

1 0 = 2 x1'
i=1 n-m

So, marinum likelihood estimate for 0 is

PALE i=1 n.m