1 Introduction

The concept of networks provides a basic, powerful framework for analyzing complex systems by representing real-life entities as nodes and their interactions as edges. This report aims to conduct a detailed comparative analysis of two distinct network datasets: IG56 (1), representing decomposable subspaces at degree d of the invariant rings of G_5, the cyclic groups of order 5, and Voles (3), a biological network showing interactions between voles caught in communal traps.

In the analysis, we aim to identify the differences and similarities in their structural properties, community structures, centrality distributions, and clustering behaviors, as well as a comparison with random graphs. Such findings can offer a deeper understanding of how structural dynamics vary between mathematical and biological networks.

2 Data Description

The IG56 network in Figure 1 comprises 70 nodes and 251 edges, where each node represents a decomposable subspace and edges denote linear dependencies at degree d.

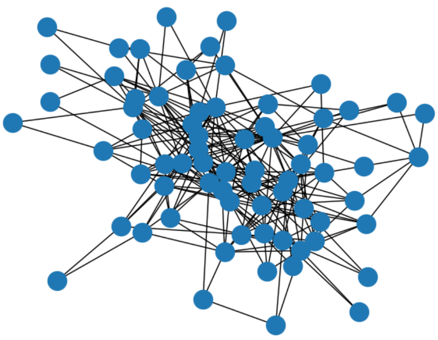

Figure 1: IG56 Network

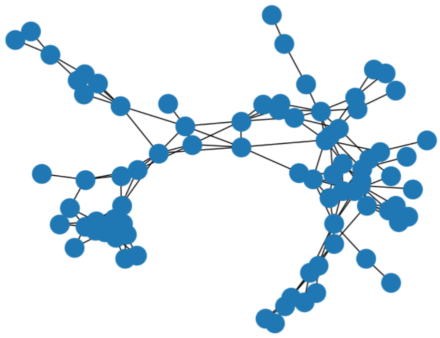

Figure 2: Voles Network

while the vole network in Figure 2 contains 82 nodes and 187 edges, where each node represents an individual vole, and edges indicate the connected pair of voles captured in the same trap, thus showing interactions and possible social structures within the vole population.

3 Methods

Both datasets were converted into undirected, unweighted graphs using NetworkX. All computations and visualizations were performed using Python and Matplotlib. We applied spectral bisection for community detection, using the sign of the second-largest positive eigenvector of the adjacency matrices of the two networks to partition nodes. To assess local structure, we computed the Watts–Strogatz clustering coefficient, counted triangles (C_3) and four-cycles (C_4). This was followed by a measure of the degree, closeness, and betweenness centrality, and finally a comparison of the real-data networks to Erdős–Rényi random graphs generated with matching node and edge counts.

4 Comparison of Results

1. **Community Structure**: Spectral bisection of the **IG56** network yielded a division into two distinct communities (Figure 3), characterized by a high density of edges within communities, with about 76 edges within the first community, and a higher interaction of about 110 edges within the second community. The inter-community relationship is low, with about 65 edges.

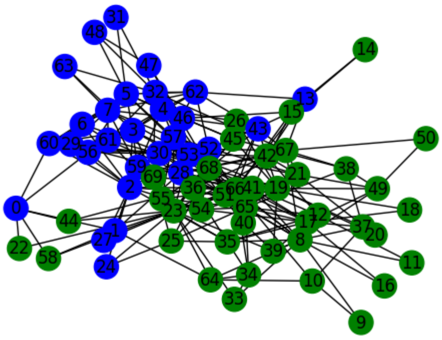

Figure 3: Community Structure of IG56 Network

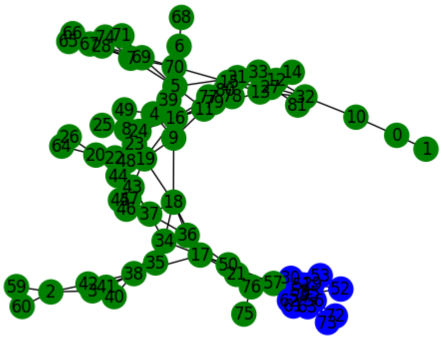

Figure 4: Community Structure of Voles Network

While the Voles network (Figure 4) was less clear than in **IG56**, with relatively fewer interactions within communities, the first community had a low intra-relationship of about 39 edges compared to 138 in the second community, resulting in less pronounced community boundaries. The inter-community relationship here is less than **IG56**, counting just 10 edges.

2. **Clustering Coefficient**: The average clustering coefficient in **IG56** (Figure 5) was 0.0175, considerably lower than the random baseline 0.098 (approximately 0.1), suggesting weak local clustering and limited internal cohesion.

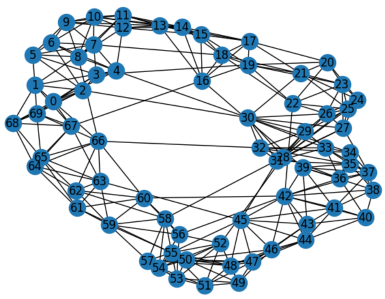

Figure 5: Clustering Coefficient of IG56 Network

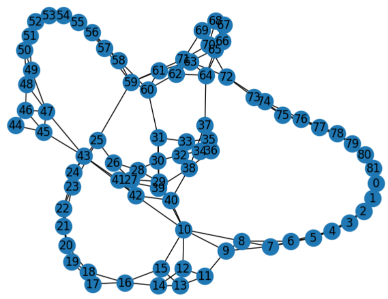

Figure 6: Clustering Coefficient of Voles Network

Voles in Figure 6 showed a moderate average clustering coefficient of 0.508, still notably higher than the random baseline of 0.055, suggesting strong local clustering and a tendency of animals to form social groups, as we would expect in a biological network.

3. **Cycle Analysis**:

The **IG56** network has a total of 9 triangles and 2,560 squares compared to the random graph with about 57 triangles and 1145 squares. The difference in triangles shows that invariants lack local social groupings, meaning their structure is more hierarchical, sparse, or procedurally constructed, and their higher number of squares suggests repetitive, symmetric, or rule-based connections, thus reinforcing that**IG56** is not a social/biological network but a mathematically constructed system, where patterns arise from algebraic constraints rather than social behaviors.

On the other hand, the Voles network has 159 triangles and 802 squares, compared to the random graph's 16 triangles and 454 squares. This high triangle count reflects strong local social groupings, while the elevated square count suggests overlapping interactions or modular social structures, confirming that the Voles network captures real-world social behavior. The natural, overlapping social interactions and real-world social behavior explain the triangle and square differences between **IG56** and the Voles network.

4. **Centrality Analysis**:

   - **Degree Centrality: IG56** (Figure 7) has degree centrality on the five nodes $23, 19, 66, 68$ and 2 (descending order) while Voles (Figure 8) has degree centrality on nodes $57, 55, 58, 5$ and 16 in descending order. This shows the algebraically significant elements and socially central or active animals.

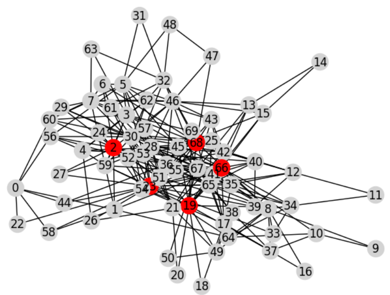

Figure 7: Degree Centrality of IG56 Network

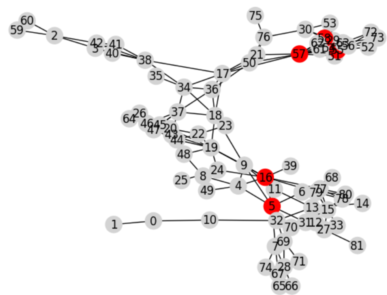

Figure 8: Degree Centrality of Voles Network

   - **Closeness Centrality: IG56** (Figure 9) has closeness centrality on the five nodes $23, 66, 68, 65$ and 19 (descending order). These are structurally central elements, linking many parts of the algebraic system.

     While Voles (Figure 10) has closeness centrality on nodes $18, 9, 19,$ and 16 in descending order, showing socially central, quickly interacting across groups.

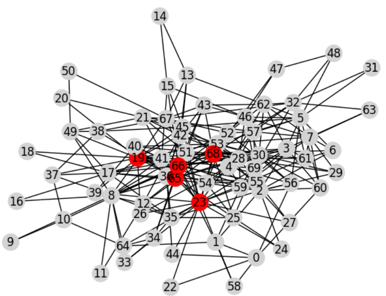

Figure 9: Closeness Centrality of IG56 Network

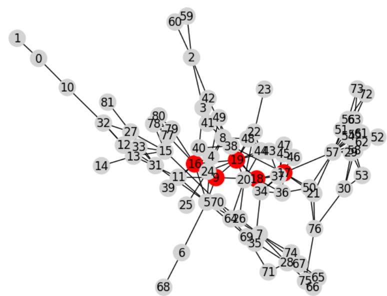

Figure 10: Closeness Centrality of Voles Network

- **Betweenness Centrality: IG56** (Figure 11) has betweenness centrality on the five nodes $23, 66, 68, 8$ and 19 (descending order) while Voles (Figure 12) has degree centrality on nodes $18, 17, 9, 6$ and 57 in descending order.

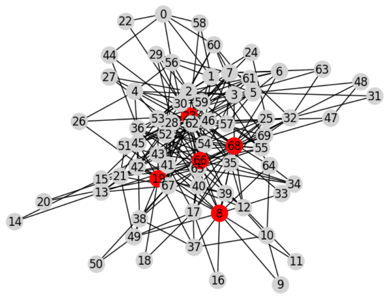

Figure 11: Betweenness Centrality of IG56 Network

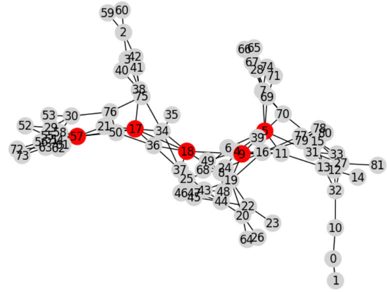

Figure 12: Betweenness Centrality of Voles Network

Do the same nodes appear as the most central for each measure in each network?

Partially.

In **IG56**, nodes **23, 66, 68,** and **19** appear across all three centrality measures, indicating they are consistently central both in connectivity and structural importance.

In **Voles**, there is **some overlap**: nodes like **18, 9,** and **57** appear in multiple measures, but not all. This suggests that different animals play different roles, some are highly connected (degree), while others act more as bridges (betweenness) or are centrally located (closeness).

Is the clustering coefficient particularly large?

IG56: No, the clustering coefficient is low (0.0175), much lower than the random baseline (0.098). This suggests very weak local clustering, typical of a rule-based or hierarchical structure.

Voles: Yes, the clustering coefficient is high (0.508), much higher than random (0.055), indicating strong local clustering, which is typical of real-world social or biological networks. In summary, the consistency of central nodes across metrics in **IG56** contrasted with the variable centrality patterns seen in the Voles network. This suggests clearly defined hierarchical roles in **IG56**, whereas the biological interactions within Voles produce a more dynamic and distributed centrality landscape.

Is the community structure convincing?

IG56: Likely not convincing, because the network shows few triangles, low clustering, and high regularity, which points to mathematical modularity rather than organic community formation.

Voles: Yes, because the combination of high clustering, many triangles, and varying centralities suggests natural community structures, likely reflecting real social or spatial groupings among the animals.

4.1 Alignment with Lectures

The results and analysis align with key concepts discussed during lectures. The high clustering coefficient and community structure in the **Voles** network are evident of a social network, as covered in real-world examples of animal and human interactions in offices and companies as discussed in class.

Conversely, the **IG56** network, with low clustering, sparse triangles, and regular patterns, reflects properties of artificial or mathematically defined networks, reinforcing the idea that such systems lack the community features seen in naturally evolving networks. Also, the differences in centrality roles across animals in the vole's network mirror the diverse social functions discussed under centrality analysis, while the repetition of central nodes in **IG56** matches expectations for algebraic mathematical systems with structured dependencies.

5 Conclusions

In conclusion, the analysis of the **IG56** and **Voles** networks highlights the contrast between mathematically constructed systems and real-world social networks. The **IG56** network, with its low clustering coefficient, sparse triangles, and regular centrality patterns, exemplifies an algebraic structure where relationships are driven by rigid rules and algebraic constraints.

In contrast, the **Voles** network exhibits characteristics typical of biological and social systems, including strong local clustering, numerous triangles, and varying centralities, which reflect dynamic and natural community structures. These findings demonstrate how network analysis techniques can be applied to both mathematical and real-world datasets, providing insights into the structural properties and functional roles within different types of systems.

This project was a collaborative effort by all group members. Each member actively participated in the calculations, reasoning, and discussions that shaped the analysis. The report writing and typesetting were handled by **Kizito**, while **Seyi** was responsible for the coding and implementation. **Toyeeb** analyzed the results obtained from the Colab notebook, and **Sandile** documented the outputs from the code to facilitate clear interpretation and analysis.

References

[1] . Dumas, J.-G. (2017). *SIMC: Sparse Integer Matrix Collection* [Data set]. Persiceto. https://doi.org/10.18709/PERSCIDO.2017.11.DS185

[2] . AIMS Networks Lecture Notes, 2025.

[3] . Davis, S., Abbasi, B., Shah, S., Telfer, S., & Begon, M. (2015). Spatial analyses of wildlife contact networks. Journal of the Royal Society Interface, 12(102), 20141004. https://doi.org/10.1098/rsif.2014.1004