1.1 Kinematics

The science of motion that treats the subject without regard to the forces that cause it. Within the science of kinematics, one studies the position, the velocity, the acceleration, and all higher order derivatives of the position variables. A robot manipulator with n joints will have n + 1 links, since each joint connects two links. The joints number from 1 to n, and number the links from 0 to n, starting from the base. By this convention, joint i connects link i − 1 to link i. It considered the location of joint i to be fixed with respect to link i − 1. When joint i is actuated, link i moves. Therefore, link 0 (the first link) is fixed, and does not move when the joints are actuated. With the i^{th} joint, we associate a joint variable, denoted by qi. In the case of a revolute joint, qi is the angle of rotation, and in the case of a prismatic joint, qi is the joint displacement: qi= {θi: joint i revolute, di: joint i prismatic}

1.1.1 D-H approach

A commonly used convention for selecting frames of reference in robotic applications is the Denavit-Hartenberg, or DH convention. In this convention, each homogeneous transformation Ai is represented as a product of four basic transformations.

Steps to procedure D-H parameter

Step l: Locate and label the joint axes $z_0 \ldots z_{n-1}$.

Step 2: Establish the base frame. Set the origin anywhere on the z_0-axis. The x_0 and y_0 axes are chosen conveniently to form a right-hand frame. For i = 1,. . , n − 1, perform Steps 3 to 5.

Step 3: Locate the origin o_i where the common normal to z_i and z_{i-1} intersects z_i. If z_i intersects z_{i-1} locate o_i at this intersection. If z_i and z_{i-1} are parallel, locate o_i in any convenient position along z_i.

Step 4: Establish x_i along the common normal between z_{i-1} and z_i through o_i, or in the direction normal to the $z_{i-1} - z_i$ plane if z_{i-1} and z_i intersect.

Step 5: Establish y_i to complete a right-hand frame.

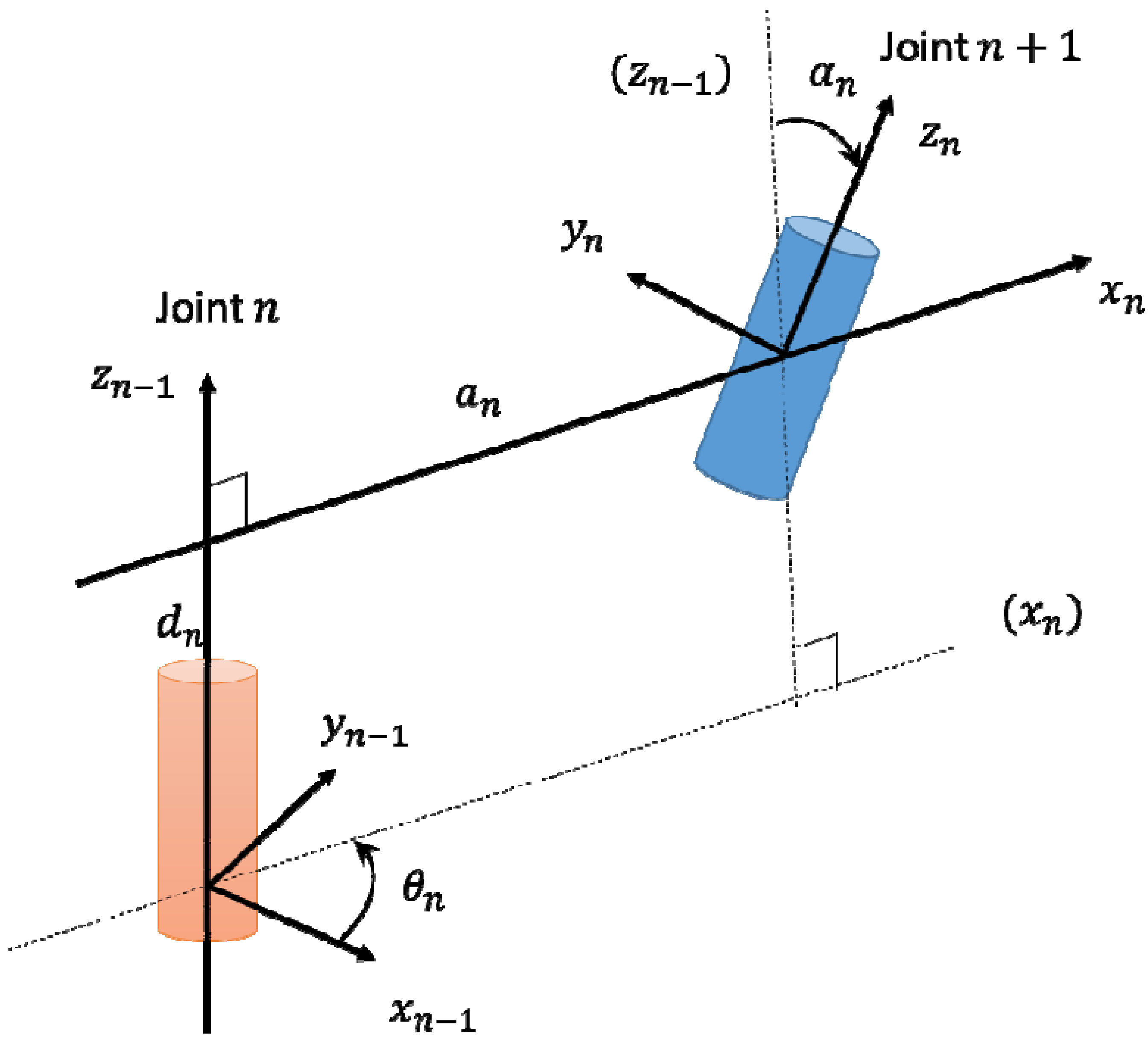
Step 6: Create a table of link parameters, $d_i\ a_i, _{\theta_i}, _{\alpha_i}$.

a_i : $link\ length, distance\ between\ Z_{i-1} and\ Z_i (alonge\ X_i)$

$\propto_i$: $link\ twist, angle\ between\ Z_{i-1} and\ Z_i (measured\ around\ X_i)$

d_i
: $link\ offcet, distance\ between\ O_{i-1} and\ the\ inteof\ rsection\ \ Z_{i-1} and\ \ X_i (alonge\ z_{i-1})$

θ_i : $joint\ angle, angle\ between\ X_{i-1} and\ X_i (measured\ around\ Z_{i-1})$



Step 8: Form the homogeneous transformation matrices A_i by substituting the above parameters.

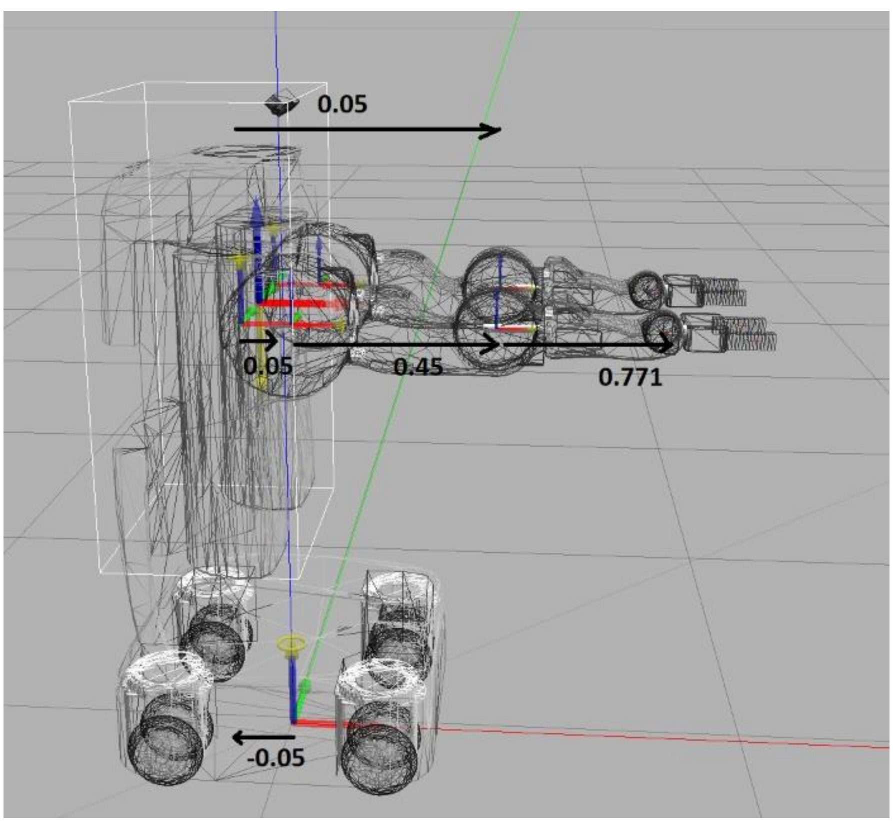
$$A_i = Rot_{z,\theta_i} * Trans_{x,\theta_i} * Trans_{x,a_i} * Rot_{x,\alpha_i}$$

$$= \begin{bmatrix} c\theta_i & -s\theta_i\, c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & -c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\alpha_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link	$\boldsymbol{a_i}$	$\boldsymbol{\alpha_i}$	$\boldsymbol{d_i}$	$\boldsymbol{\theta_i}$
1	-0.05	0	0.98	θ_1
2	0	$-\pi/2$	0	θ_2
3	0	0	0.45	θ_3
4	0	0	0.32	θ_4

Table 1 D-H Parameters



$$
{}^{0}_{1}T = \begin{bmatrix} C_1 & -S_1 & 0 & -.05 * C_1 \\ S_1 & C_1 & 0 & -.05 * S1 \\ 0 & 0 & 1 & .98 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{1}_{2}T = \begin{bmatrix} C_2 & 0 & -S_2 & 0 \\ S_1 & 0 & C_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
, {}^{2}_{3}T = \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & .45 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{3}_{4}T = \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & .32 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}_{4}T = {}^{0}_{1}T * {}^{1}_{2}T * {}^{2}_{3}T * {}^{3}_{4}T
$$

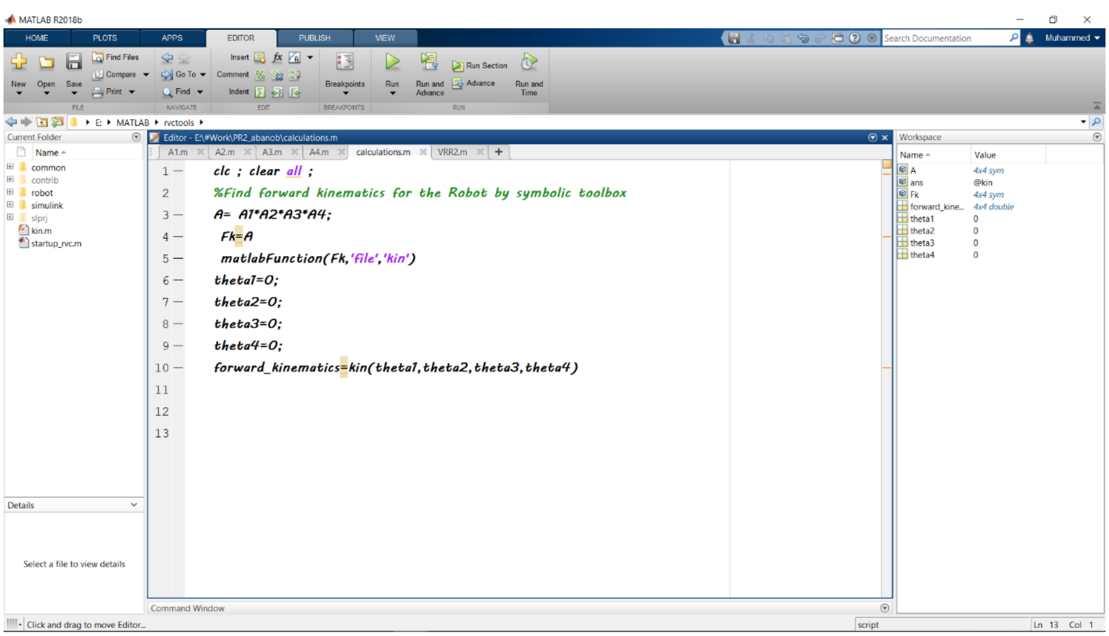
$$
{}^{0}_{4}T = \begin{bmatrix} C_{12} * C_{34} & -C_{12} * S_{34} & S_{12} & -.05 * C_1 - .77 * S_{12} \\ S_{12} * C_{34} & -S_{12} * S_{34} & C_{12} & -.05 * S_1 + .77C_{12} \\ -S_{34} & -S_{34} & 0 & .98 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
x = -.05 * C_1 - .77 * S_{12}
$$

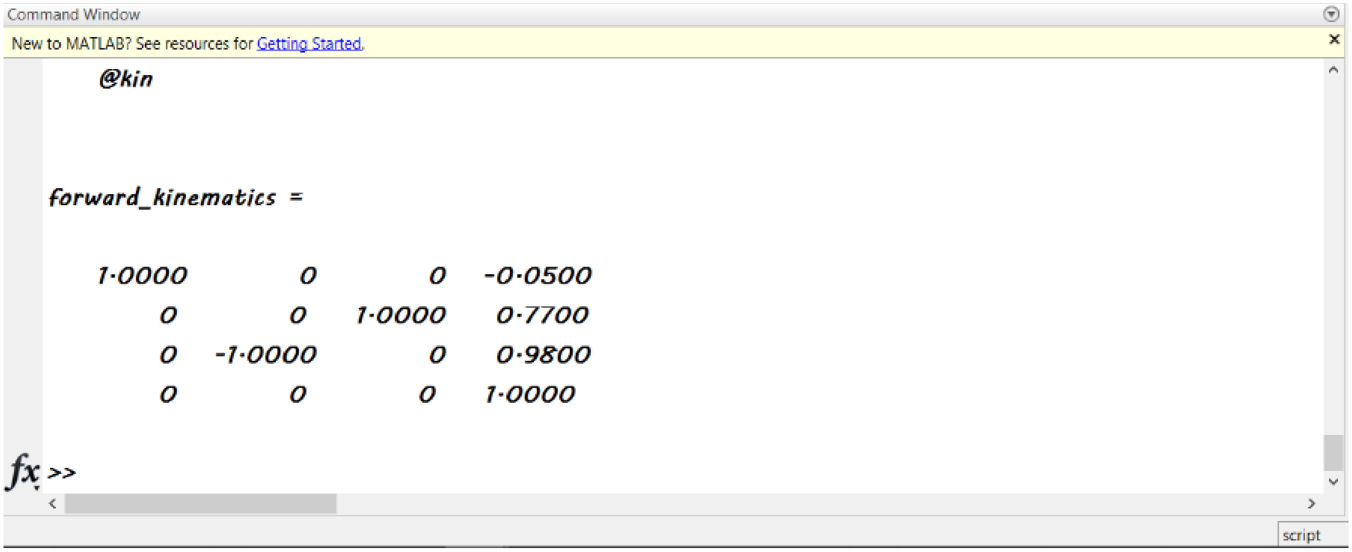
$$
y = -.05 * S_1 + .77C_{12}
$$

$$
z = 0.98
$$

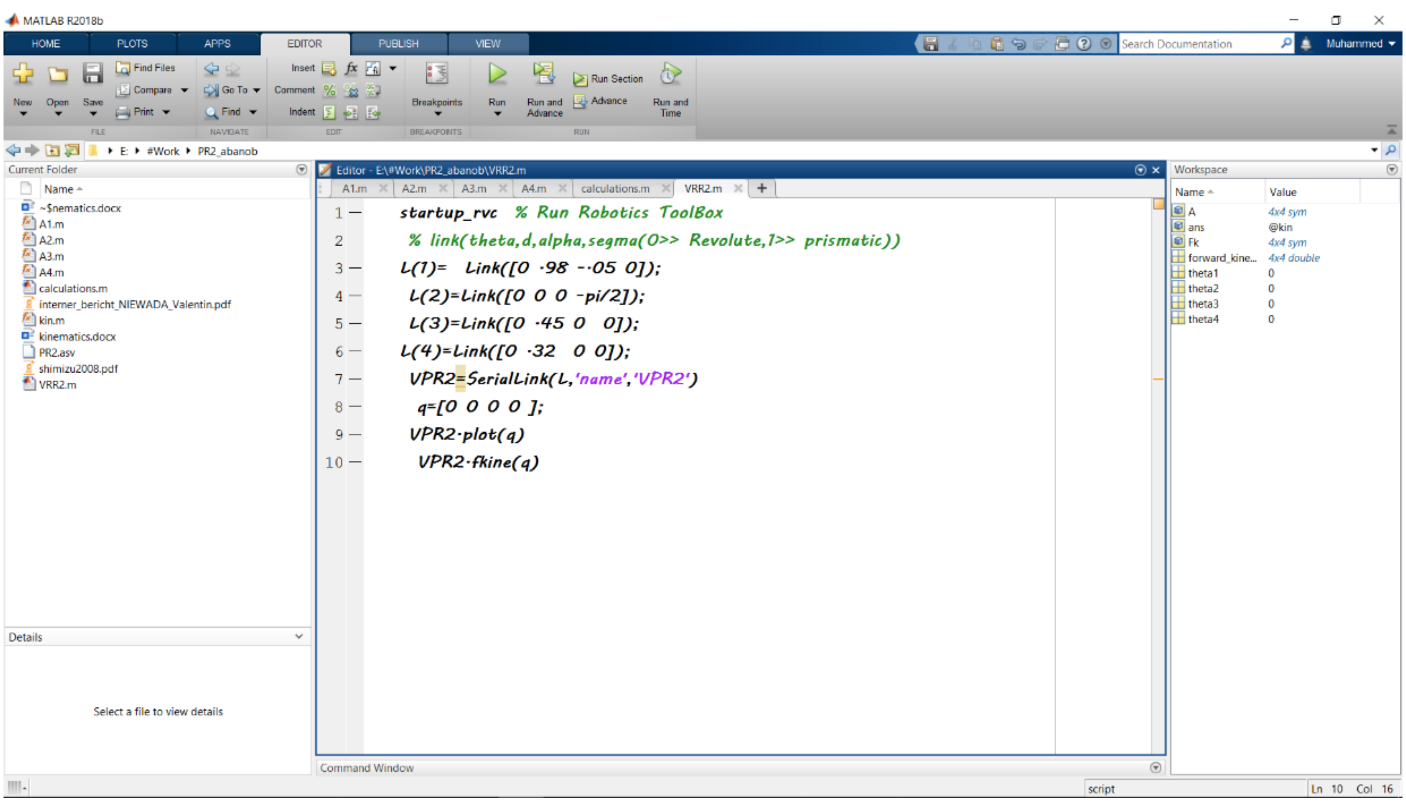
Forward kinematics using symbolic toolbox at MATLAB



This code was intended to Find the kinematics at home position



Forward kinematics using Robotics toolbox at MATLAB



Results:

```
Command Window
New to MATLAB? See resources for Getting Started.

VPR2 =

VPR2 (4 axis, RRRR, stdDH, fastRNE)

+---+-----------+-----------+-----------+-----------+-----------+
| j |     theta |         d |         a |     alpha |    offset |
+---+-----------+-----------+-----------+-----------+-----------+
|  1|         q1|       0.98|      -0.05|          0|          0|
|  2|         q2|          0|          0|     -1.571|          0|
|  3|         q3|       0.45|          0|          0|          0|
|  4|         q4|       0.32|          0|          0|          0|
+---+-----------+-----------+-----------+-----------+-----------+

grav =    0  base = 1  0  0  0   tool =  1  0  0  0
          0         0  1  0  0           0  1  0  0
       9.81         0  0  1  0           0  0  1  0
                    0  0  0  1           0  0  0  1

ans =

    1.0000         0         0   -0.0500
         0    0.0000    1.0000    0.7700
         0   -1.0000    0.0000    0.9800
         0         0         0    1.0000

fx >>
```

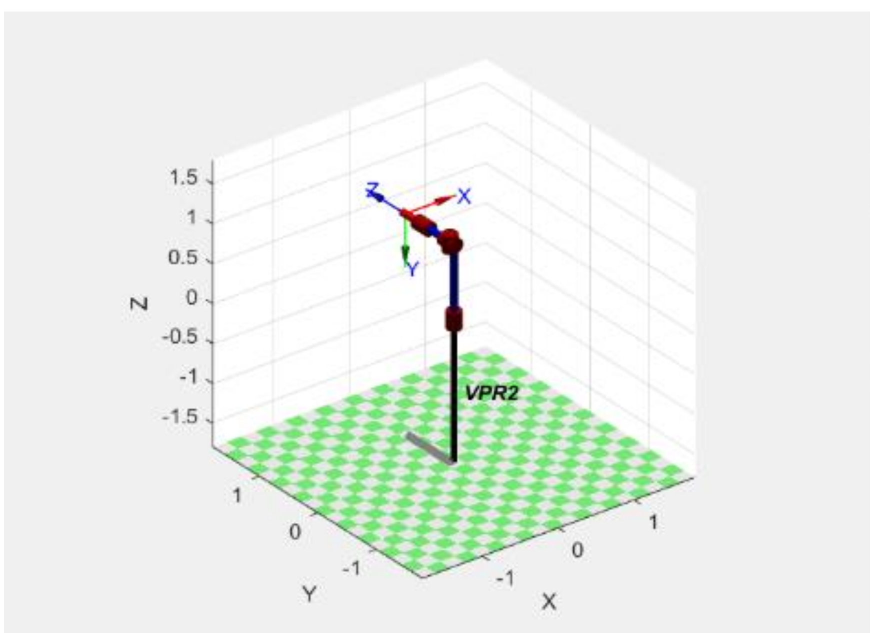


Fig. 1 Simplified 4R robot arm using MATLAB