Research Topic (2)

DIET and MIXING Model

Introduction

What's OR (Operation Research) ? (Ref. 1)

Operation Research is a systematic strategy for problem-solving and decision-making that is valuable in the management of organizations. In operations research, problems are separated into fundamental parts and then solved in defined steps by mathematical analysis. What's more, It's an interdisciplinary part of Applied Mathematics and formal Sciences that utilizes methods such as mathematical modeling, Statistics and Algorithm to show up optimal or near optimal solutions to complex problems. It's typically concerned with optimizing the maximum (benefit, profit, assembly line performance, crop yield, production, band width , etc.) or minimum (loss, cost, risk, etc.) of some objective function operations research helps management achieve its goals using strategies and methods.

What're the phases of Operation Research ? (Ref. 2)

1. Defining the problem and collecting data
2. Creating a mathematical model
3. Selecting appropriate data input
4. Checking the model and its solutions
5. Preparing to the execute the model (Model Validity)
6. Final Execution or Implementation

1- <u>Defining the problem:</u>

Identify the principle that mentioned before ; Decision variables, Objectives, Constraints.

2- Creating a mathematical model :

Translate the problem definition into mathematical relationships. If the resulting model fits one of the standard mathematical models, such as LP. We can usually reach a solution by using available algorithm.

3- Selecting appropriate data input

No model will work appropriately if data input isn't proper. The purpose behind this step is to have adequate contribution to work and test the model.

4- Checking the model and its solutions

It is far the simplest of all OR phases. We should find a solution. If the model doesn't behave properly, then updating and modification is considered at this stage is a must.

5- Preparing to the execute the model (Model Validity)

checks whether or not the proposed model does what it purport to do. A model is supposed to be valid if it can give a solid and reliable prediction of the system's performance. A model must be appropriate for a longer time and can be updated every now and then taking into consideration the past, present and future aspects of the problem.

6- Final Execution or Implementation

The translation of the results into understandable operating instructions to be issued to the people who will administer the recommended system.

Formulate the Mathematical Linear Programming for the problem?

[1] Decision Variables:

Let x1,x2,x3 and x4 denote the number of units of type 1,2,3 and 4 respectively.

X1 the number of units of food of type **1**
X2 the number of units of food of type **2**
X3 the number of units of food of type **3**
X4 the number of units of food of type **4**

[2] The Objective:

is to minimize the cost i.e., Minimize Z=Rs $(45x1+40x2+85x3+65x4)$

[3] Constraints:

Constraints are on the fulfillment of the daily requirements of the various constituents i.e.,

proteins	**$3x1+4x2+8x3+6x4 \geq 800$**
fats	**$2x1+2x2+7x3+5x4 \geq 200$**
carbohydrates	**$6x1+4x2+7x3+4x4 \geq 700$**

where $x1$, $x2$, $x3$, $x4 \geq 0$

What's the meaning of basic feasible solution of a general Linear Programming Problem ?

In linear programming theory, a basic feasible solution (BFS) is a solution with an insignificant arrangement of non-zero variables. Geometrically, each BFS corresponds to a corner of the polyhedron of feasible and attainable solutions. If there exists an optimal solution, then there exists an optimal BFS. Hence, If you want to find an optimal solution, it is sufficient to consider the BFS-s. This fact is utilized by the simplex algorithm, which basically goes from some BFS to another until an optimal and ideal one is found.

What's the solution to the problem using Regular Simplex method ?

Modeling : Z=45 x1+40 x2+85 x3+65 x4. Subject to
$3x1+4x2+8x3+6x4 \geq 800$, for proteins
$2x1+2x2+7x3+5x4 \geq 200$, for fats
$6x1+4x2+7x3+4x4 \geq 700$, for carbohydrates
where $x1,x2,x3,x4 \geq 0$ hidden. Add 3 surplus and 3 artificial.

1. As the constraint -1 is of type '≥' we should subtract surplus variable S1 and add artificial variable A1.
2. As the constraint -2 is of type '≥' we should subtract surplus variable S2 and add artificial variable A2.
3. As the constraint -3 is of type '≥' we should subtract surplus variable S3 and add artificial variable A3.

After adding surplus and artificial variables

Min $Z = 45x1 + 40x2 + 85\,x3 + 65x4 + 0\,S1 + 0\,S2 + 0\,S3 + M\,A1 + M\,A2 + M\,A3$

subject to

$3x1 + 4x2 + 8x3 + 6x4 - S1 \qquad + A1 \qquad = 800$

$2x1 + 2x2 + 7x3 + 5x4 \qquad - S2 \qquad + A2 \qquad = 200$

$6x1 + 4x2 + 7x3 + 4x4 \qquad - S3 \qquad + A3 = 700$

and $x1, x2, x3, x4, S1, S2, S3, A1, A2, A3 \geq 0$

Table 1													
Iteration-1 n-1		C_j	45	40	85	65	0	0	0	M	M	M	
B	**C_B**	**X_B**	**$x1$**	**$x2$**	**$x3$**	**$x4$**	**S_1**	**S_2**	**S_3**	**A_1**	**A_2**	**A_3**	**Min Ratio** **$X_B x_3$**
A_1	M	800	3	4	8	6	-1	0	0	1	0	0	8008=100
A_2	M	200	2	2	**(7)**	5	0	-1	0	0	1	0	2007=28.5714→
A_3	M	700	6	4	7	4	0	0	-1	0	0	1	7007=100
Z=1700M		**Z_j**	**11M**	**10M**	**22M**	**15M**	**-M**	**-M**	**-M**	**M**	**M**	**M**	
		Z_j-C_j	11M-45	10M-40	22M-85↑	15M-65	-M	-M	-M	0	0	0	

Enter =$x3$, Drop =$A2$, Key Element =7.

Positive maximum Z j-C j is 22M-85 and its column index is 3. So, the entering variable is x3.

Minimum ratio is 28.5714 and its row index is 2. So, the leaving basis variable is A2.

∴ The pivot element is 7.

R2(new)=R2(old)/7

R1(new)=R1(old) - 8R2(new)

R3(new)=R3(old) - 7R2(new)

Table 2													
Iteration-2		C_j	45	40	85	65	0	0	0	M	M	M	
B	C_B	X_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3	A_1	A_2	A_3	**Min Ratio** $\frac{X_B}{x_1}$
A_1	M	571.4286	0.7143	1.7143	0	0.2857	-1	1.1429	0	1	-1.1429	0	571.42860.7143=800
x_3	85	28.5714	**(0.2857)**	0.2857	1	0.7143	0	-0.1429	0	0	0.1429	0	28.57140.2857=100 →
A_3	M	500	4	2	0	-1	0	1	-1	0	-1	1	5004=125
Z=1071.4286M+2428.5714		Z_j	**4.7143M+24.2857**	**3.7143M+24.2857**	**85**	**-0.7143M+60.7143**	**-M**	**2.1429M-12.1429**	**-M**	**M**	**-2.1429M+12.1429**	**M**	
		Z_j-C_j	4.7143M-20.7143 ↑	3.7143M-15.7143	0	-0.7143M-4.2857	-M	2.1429M-12.1429	-M	0	-3.1429M+12.1429	0	

Enter = x_1, Drop =x3, Key Element =0.2857 .

Positive maximum Z j-C j is 4.7143M-20.7143 and its column index is 1. So, the entering variable is x.

Minimum ratio is 100 and its row index is 2. So, the leaving basis variable is x3.

∴ The pivot element is 0.2857.

$R2(\text{new})=R2(\text{old})\div 0.2857$

$R1(\text{new})=R1(\text{old}) - 0.7143R2(\text{new})$

$R3(\text{new})=R3(\text{old}) - 4R2(\text{new})$

Table 3

Iteration-3		C_j	45	40	85	65	0	0	0	M	M	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**x_4**	**S_1**	**S_2**	**S_3**	**A_1**	**A_2**	**A_3**	**Min Ratio $\frac{X_B}{S_2}$**
A_1	M	500	0	1	-2.5	-1.5	-1	1.5	0	1	-1.5	0	5001.5=333.3333
x_1	45	100	1	1	3.5	2.5	0	-0.5	0	0	0.5	0	---
A_3	M	100	0	-2	-14	-11	0	**(3)**	-1	0	-3	1	1003=33.3333→
$Z=600M+4500$		**Z_j**	**45**	**$-M+45$**	**$-16.5M+157.5$**	**$-12.5M+112.5$**	**$-M$**	**$4.5M-22.5$**	**$-M$**	**M**	**$-4.5M+22.5$**	**M**	
		Z_j-C_j	0	$-M+5$	$-16.5M+72.5$	$-12.5M+47.5$	$-M$	$4.5M-22.5$↑	$-M$	0	$-5.5M+22.5$	0	

Enter = *S2*, Drop =A3, Key Element =3 .

Positive maximum Z j-C j is $4.5M-22.5$ and its column index is 6. So, the entering variable is S2.

Minimum ratio is 33.3333 and its row index is 3. So, the leaving basis variable is A3.

∴The pivot element is 3.

$R3(\text{new})=R3(\text{old})/3$

$R1(\text{new})=R1(\text{old}) - 1.5R3(\text{new})$

$R2(\text{new})=R2(\text{old}) + 0.5R3(\text{new})$

Table 4													
Iteration-4		C_j	45	40	85	65	0	0	0	M	M	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**x_4**	**S_1**	**S_2**	**S_3**	**A_1**	**A_2**	**A_3**	**Min Ratio $\frac{X_B}{x_3}$**
A_1	M	450	0	2	**(4.5)**	4	-1	0	0.5	1	0	-0.5	4504.5=100→
x_1	45	116.6667	1	0.6667	1.1667	0.6667	0	0	-0.1667	0	0	0.1667	116.66671.1667=100
S_2	0	33.3333	0	-0.6667	-4.6667	-3.6667	0	1	-0.3333	0	-1	0.3333	---
$Z=450M+5250$		**Z_j**	**45**	**$2M+30$**	**$4.5M+52.5$**	**$4M+30$**	**$-M$**	**0**	**$0.5M-7.5$**	**M**	**0**	**$-0.5M+7.5$**	
		Z_j-C_j	0	$2M-10$	$4.5M-32.5$↑	$4M-35$	$-M$	0	$0.5M-7.5$	0	$-M$	$-1.5M+7.5$	

Enter = *X3*, Drop =A1, Key Element =4.5 .

Positive maximum Z_j-C_j is $4.5M-32.5$ and its column index is 3. So, the entering variable is x_3.

Minimum ratio is 100 and its row index is 1. So, the leaving basis variable is A_1.

∴The pivot element is 4.5.

$R_1(\text{new})=R_1(\text{old})\div4.5$

$R_2(\text{new})=R_2(\text{old}) - 1.1667R_1(\text{new})$

$R_3(\text{new})=R_3(\text{old}) + 4.6667R_1(\text{new})$

Table 5													
Iteration-5		C_j	45	40	85	65	0	0	0	M	M	M	
B	C_B	X_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3	A_1	A_2	A_3	**Min Ratio** $\frac{X_B}{x_2}$
x_3	85	100	0	0.4444	1	0.8889	-0.2222	0	0.1111	0.2222	0	-0.1111	1000.4444=225
x_1	45	0	1	**(0.1481)**	0	-0.3704	0.2593	0	-0.2963	-0.2593	0	0.2963	00.1481=0→
S_2	0	500	0	1.4074	0	0.4815	-1.037	1	0.1852	1.037	-1	-0.1852	5001.4074=355.2632
Z=8500		Z_j	**45**	**44.4444**	**85**	**58.8889**	**-7.2222**	**0**	**-3.8889**	**7.2222**	**0**	**3.8889**	
		Z_j-C_j	0	4.4444↑	0	-6.1111	-7.2222	0	-3.8889	$-M$+7.2222	$-M$	$-M$+3.8889	

Enter = X2, Drop = X1, Key Element =4.5 .

Positive maximum Z j-C j is 4.4444 and its column index is 2. So, the entering variable is x2.

Minimum ratio is 0 and its row index is 2. So, the leaving basis variable is X1.

∴The pivot element is 0.1481.

$R2$(new)=$R2$(old)÷0.1481

$R1$(new)=$R1$(old) - 0.4444$R2$(new)

$R3$(new)=$R3$(old) - 1.4074$R2$(new)

Table 6													
Iteration-6		C_j	45	40	85	65	0	0	0	M	M	M	
B	C_B	X_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3	A_1	A_2	A_3	**Min Ratio** $\frac{X_B}{x_4}$
x_3	85	100	-3	0	1	**(2)**	-1	0	1	1	0	-1	1002=50→
x_2	40	0	6.75	1	0	-2.5	1.75	0	-2	-1.75	0	2	---
S_2	0	500	-9.5	0	0	4	-3.5	1	3	3.5	-1	-3	5004=125
$Z=8500$		Z_j	**15**	**40**	**85**	**70**	**-15**	**0**	**5**	**15**	**0**	**-5**	
		Z_j-C_j	-30	0	0	5↑	-15	0	5	$-M+15$	$-M$	$-M-5$	

Enter = X4, Drop = X3, Key Element = 2 .

Positive maximum Z j-C j is 5 and its column index is 4 . So, the entering variable is X4.

Minimum ratio is 50 and its row index is 1. So, the leaving basis variable is X3.

∴The pivot element is 2.

$R1(\text{new})=R1(\text{old})/2$
$R2(\text{new})=R2(\text{old}) + 2.5R1(\text{new})$
$R3(\text{new})=R3(\text{old}) - 4R1(\text{new})$

Table 7													
Iteration-7		C_j	45	40	85	65	0	0	0	M	M	M	
B	C_B	X_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3	A_1	A_2	A_3	**Min Ratio** $\frac{X_B}{S_3}$
x_4	65	50	-1.5	0	0.5	1	-0.5	0	**(0.5)**	0.5	0	-0.5	500.5=100→
x_2	40	125	3	1	1.25	0	0.5	0	-0.75	-0.5	0	0.75	---
S_2	0	300	-3.5	0	-2	0	-1.5	1	1	1.5	-1	-1	3001=300
$Z=8250$		Z_j	**22.5**	**40**	**82.5**	**65**	**-12.5**	**0**	**2.5**	**12.5**	**0**	**-2.5**	
		Z_j-C_j	-22.5	0	-2.5	0	-12.5	0	2.5↑	$-M+12.5$	$-M$	$-M-2.5$	

Enter = S3, Drop = X4, Key Element = 0.5 .

Positive maximum Z j-C j is 2.5 and its column index is 7 . So, the entering variable is S3.

Minimum ratio is 100 and its row index is 1. So, the leaving basis variable is X4.

∴The pivot element is 0.5 .

$R1(\text{new})=R1(\text{old})/0.5$

$R2(\text{new})=R2(\text{old}) + 0.75R1(\text{new})$

$R3(\text{new})=R3(\text{old}) - R1(\text{new})$

Table 8													
Iteration-8		C_j	45	40	85	65	0	0	0	M	M	M	
B	C_B	X_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3	A_1	A_2	A_3	**Min Ratio**
S_3	0	100	-3	0	1	2	-1	0	1	1	0	-1	
x_2	40	200	0.75	1	2	1.5	-0.25	0	0	0.25	0	0	
S_2	0	200	-0.5	0	-3	-2	-0.5	1	0	0.5	-1	0	
***Z*=8000**		Z_j	**30**	**40**	**80**	**60**	**-10**	**0**	**0**	**10**	**0**	**0**	
		Z_j-C_j	-15	0	-5	-5	-10	0	0	$-M+10$	$-M$	$-M$	

Since all $Zj\text{-}Cj \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x1=0$, $x2=200$,$x3=0$, $x4=0$

then Min Z= 40*200 = **8000.**

Find the Dual Problem, and determine which is computationally advantageous to solve either the Primal or the Dual, then solve both problems .

<u>1-Dual problem :</u>

Max= b^t y
a^ty <= c y >= 0
max z= 800 y1+200 y2+700 y3.
Subject to
3 y1+2 y2+6 y3<=45,
4 y1+2 y2+4 y3<=40,
8 y1+7 y+7 y3<=85,
6 y1+5 y2+4 y3<=65.

<u>2-Determine which is computationally advantageous to solve either the Primal or the Dual:</u>

CE=(constrains+1)(no of variable +constrains + artificial+1)(artificial+1).
CEP=(3+1)(4+3+3+1)(3+1)=176.
CED=(4+1)(3+4+0+1)(0+1)=40
Dual problem is better than primal problem .

<u>3- Solve the Dual problem:</u>

Z= 800 y1+200 y2+700 y3+0s1+0s2+0s3+0s4.
3 y1+2 y2+6 y3+s1=45,
4 y1+2 y2+4 y3+s2=40,
8 y1+7 y+7 y3+s3=85,
6 y1+5 y2+4 y3+s4=65.

	Y1 800	Y2 200	Y3 700	S10	S20	S30	S40	Y1 800	
S1	0	3	2	6	1	0	0	0	45
S2	0	4	2	4	0	1	0	0	40
S3	0	8	7	7	0	0	1	0	85
S4	0	6	5	4	0	0	0	1	68
	-800	-200	-700	0	0	0	0	-800	

Enter = y1, Drop = S2 .

R2=R2/4
R1=R1-3R2
R3=R3-8R2
R4=R4-6R2

	Y1 800	Y2 200	Y3 700	S10	S20	S30	S40	Y1 800	
S1	0	0	1/2	3	1	-3/4	0	0	15
Y1	800	1	1/2	1	0	1/4	0	0	10
S3	0	0	3	-1	0	-2	1	0	5
S4	0	0	2	2	0	-3/2	0	1	5
	0	200	100	0	200	0	0	0	

So
S1=x1=0,s2=x2=200,s3=x3=0,s4=x4=0;

Then Z=200*40=**8000**.

4-solve primal with regular simplex methods (in number 4)

I solved this problem previously in this research , to open click
Here.

Solve the Problem Using Dual Simplex Method, if possible.

The condition is met min+ so we can solve it but Need to adjust the constrains

1- constrains hit *-1 to <=

Then add slacks
Min Z =45 x1+40 x2+85 x3+65 x4+0s1+0s2+0s3
-3*x*1-4*x*2-8*x*3-6*x*4+s1=-800, for proteins
-2*x*1-2*x*2-7*x*3-5*x*4-s2=-200, for fats
-6*x*1-4*x*2-7*x*3-4*x*4+s3=-700, for carbohydrates
*x*1,*x*2,*x*3,*x*4 ,s1,s2,s3 ≥0 hidden

		X1	X2	X3	X4	S1	S2	S3	
		45	40	85	65	0	0	0	
S1	0	-3	-4	-8	-6	1	0	0	-800
S2	0	-2	-2	-7	-5	0	1	0	-200
S3	0	-6	-4	-7	-4	0	0	1	-700
		45	40	85	65	0	0	0	

Enter x2,Drop s1
R1=R1*-1THEN R1=R1/4
R2=R2-2R1
R3=R3-4R1

		X1	X2	X3	X4	S1	S2	S3	
		45	40	85	65	0	0	0	
X2	40	3/4	1	2	3/2	-1/4	0	0	200
S2	0	-1/2	0	-3	-2	-1/2	1	0	200
S3	0	-3	0	1	2	-1	0	1	100
		15	0	5	5	10	0	0	

X2=200,x1=0,x3=0,x4=0
Then z=40*200=8000.

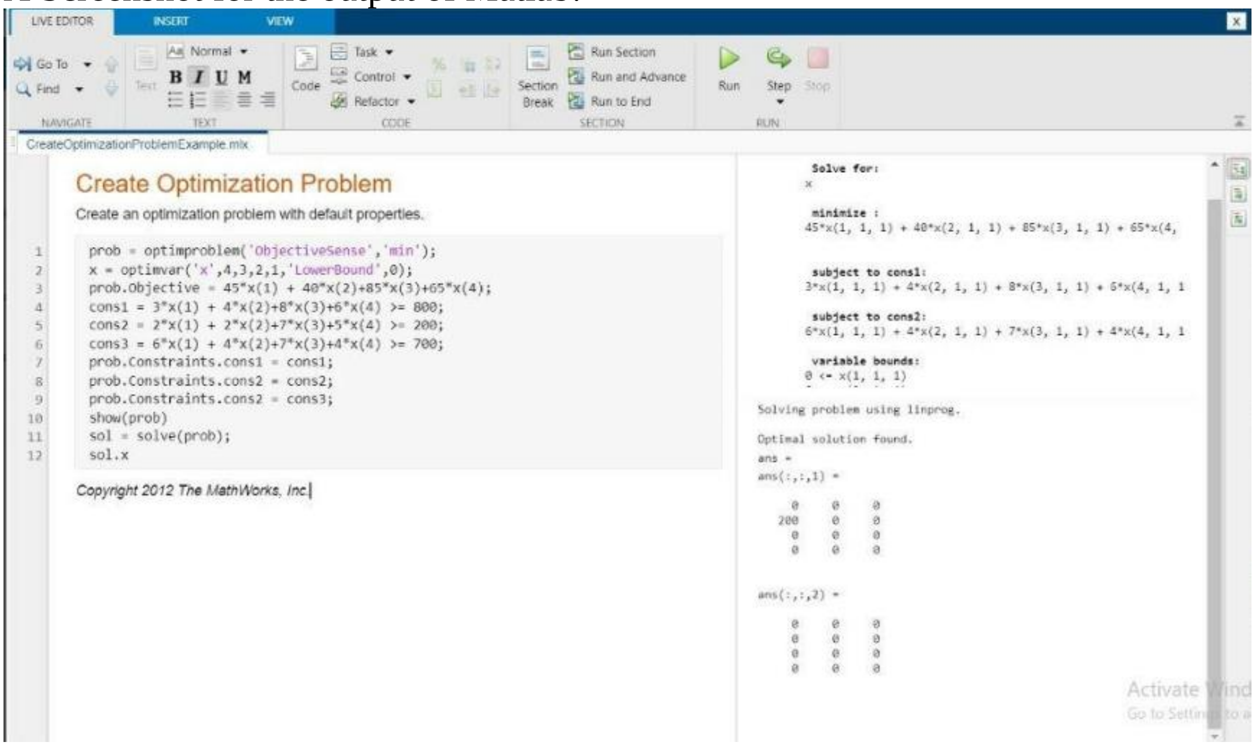
By using any computer software (e.g. TORA, MATLAB, …) solve this optimization problem, or write down a program that solve the Problem using any programming language you know (program one method ONLY; Regular method, Dual Simplex method, or Dual Problem) .

The Code of Solving the problem using **Regular method** with **MatLab**:

```
prob = optimproblem('ObjectiveSense','min');
x = optimvar('x',4,3,2,1,'LowerBound',0);
prob.Objective = 45*x(1) + 40*x(2)+85*x(3)+65*x(4);
cons1 = 3*x(1) + 4*x(2)+8*x(3)+6*x(4) >= 800;
cons2 = 2*x(1) + 2*x(2)+7*x(3)+5*x(4) >= 200;
cons3 = 6*x(1) + 4*x(2)+7*x(3)+4*x(4) >= 700;
```

```
prob.Constraints.cons1 = cons1;
prob.Constraints.cons2 = cons2;
prob.Constraints.cons2 = cons3;
show(prob)
sol = solve(prob);
sol.x
```

A Screenshot for the output of Matlab:



Reference:

[1] **Tech Target** (Website): [Link]

[2] **Tutors Globe** (Website): [Link]

[2] **“ Schaum's Outline of Operations Research 2nd Edition “** (Book): [Link]

[4] **Article shared by Nikhila C** (Article): [Link]

[5] **MatLab (Website): [Link]**