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| Faculty of Computer & Information Sciences  Ain Shams University  Subject: Operations Research (SCC231)  Year: (2nd year) undergraduate  Academic year: 2nd term 2019-2020 |  |

**Research Topic (2)**

**DIET and MIXING Model**

**Introduction**

**What’s OR (Operation Research) ?**

Operation Research is an interdisciplinary branch of Applied Mathematics and formal Sciences that uses methods such as mathematical modeling, Statistics and Algorithm to arrive optimal or near optimal solutions to complex problems. It’s typically concerned with optimizing the maxima (profit, assembly line performance, crop yield, production, band width , etc.) or minimum (loss, cost, risk, etc.) of some objective function operations research helps management achieve its goals using methods.

**What’re the phases of Operation Research ? (**Ref. 1 **)**

1. Defining the problem and collecting data
2. Creating a mathematical model
3. Obtaining solutions from the model
4. Checking the model and its solutions
5. Preparing to the execute the model
6. Final Execution or Implementation

1- Defining the problem:

Identify the principle that mentioned before ; Decision variables, Objectives, Constraints.

2- Creating a mathematical model :

Translate the problem definition into mathematical relationships. If the resulting model fits one of the standard mathematical models, such as LP. We can usually reach a solution by using available algorithm.

3- Obtaining solutions from the model

It’s far the simplest of all OR phases because it entails the use of well defined optimization algorithms. We can solve most of programs graphically or mathematically. Sensitivity Analysis is an important aspect of the model solution phase.

4- Checking the model and its solutions

It Checks whether or not the proposed model does what it should do.

5- Preparing to the execute the model

The system generally is computer-based. Management Information System and

**Databases** can give up-to-date input for the model. An interactive computer based system which is called as **Decision Support System** is installed to assist the manager to use models and data to support their decision making as per requirement.

6- Final Execution or Implementation

The translation of the results into understandable operating instructions to be issued to the people who will administer the recommended system.

**Formulate the Mathematical Linear Programming for the ?**

Let x1,x2,x3 and x4 denote the number of units of type 1,2,3 and 4 respectively.

The objective is to minimize the cost i.e., Minimize *Z*=Rs.(45*x*1​+40*x*2​+85*x*3​+65*x*4​)

Constraints are on the fulfillment of the daily requirements of the various constituents i.e.,

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| **proteins** | 3x1+4x2+8x3+6x4 ​≥ 800 |
| **fats** | 2*x*1​+2*x*2​+7*x*3​+5*x*4​ ≥ 200 |
| **carbohydrates** | 6*x*1​+4*x*2​+7*x*3​+4*x*4​≥700 |

where *x*1​,*x*2​,*x*3​,*x*4​  ≥0

**What’s the meaning of basic feasible solution of a general Linear Programming Problem ? (**Ref. 2**)**

In the theory of linear programming a basic feasible solution (BFS) is a solution with a minimal set of non-zero variables. Geometrically, each BFS corresponds to a corner of the polyhedron of feasible solutions. If there exists an optimal solution, then there exists an optimal BFS. Hence, to find an optimal solution, it is sufficient to consider the BFS-s. This fact is used by the simplex algorithm, which essentially travels from some BFS to another until an optimal one is found.

**What’s the solution to the problem using Regular Simplex method ?**

After introducing surplus,artificial variables

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| subject to | | | | | | | | | | | | | |
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| and *x*1,*x*2,*x*3,*x*4,*S*1,*S*2,*S*3,*A*1,*A*2,*A*3≥0 | | | | | | | | | | | | | |
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| Iteration-1 |  | *Cj* | 45 | 40 | 85 | 65 | 0 | 0 | 0 | *M* | *M* | *M* |  | |
| ***B*** | ***CB*** | ***XB*** | ***x*1** | ***x*2** | ***x*3** | ***x*4** | ***S*1** | ***S*2** | ***S*3** | ***A*1** | ***A*2** | ***A*3** | **MinRatio *XBx*3** | |
| *A*1 | *M* | 800 | 3 | 4 | 8 | 6 | -1 | 0 | 0 | 1 | 0 | 0 | 8008=100 | |
| ***A*2** | *M* | 200 | 2 | 2 | **(7)** | 5 | 0 | -1 | 0 | 0 | 1 | 0 | 2007=28.5714**→** | |
| *A*3 | *M* | 700 | 6 | 4 | 7 | 4 | 0 | 0 | -1 | 0 | 0 | 1 | 7007=100 | |
| ***Z*=1700*M*** |  | ***Zj*** | **11*M*** | **10*M*** | **22*M*** | **15*M*** | **-*M*** | **-*M*** | **-*M*** | ***M*** | ***M*** | ***M*** |  | |
|  |  | *Zj*-*Cj* | 11*M*-45 | 10*M*-40 | 22*M*-85↑ | 15*M*-65 | -*M* | -*M* | -*M* | 0 | 0 | 0 |  | |

Positive maximum Zj-Cj is 22M-85 and its column index is 3. So, the entering variable is x3.  
  
Minimum ratio is 28.5714 and its row index is 2. So, the leaving basis variable is A2.  
  
∴ The pivot element is 7.  
  
Entering =x3, Departing =A2, Key Element =7  
  
R2(new)=R2(old)÷7  
  
R1(new)=R1(old) - 8R2(new)  
  
R3(new)=R3(old) - 7R2(new)

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| Iteration-2 |  | *Cj* | 45 | 40 | 85 | 65 | 0 | 0 | 0 | *M* | *M* | *M* |  |
| ***B*** | ***CB*** | ***XB*** | ***x*1** | ***x*2** | ***x*3** | ***x*4** | ***S*1** | ***S*2** | ***S*3** | ***A*1** | ***A*2** | ***A*3** | **MinRatio *XBx*1** |
| *A*1 | *M* | 571.4286 | 0.7143 | 1.7143 | 0 | 0.2857 | -1 | 1.1429 | 0 | 1 | -1.1429 | 0 | 571.42860.7143=800 |
| ***x*3** | 85 | 28.5714 | **(0.2857)** | 0.2857 | 1 | 0.7143 | 0 | -0.1429 | 0 | 0 | 0.1429 | 0 | 28.57140.2857=100**→** |
| *A*3 | *M* | 500 | 4 | 2 | 0 | -1 | 0 | 1 | -1 | 0 | -1 | 1 | 5004=125 |
| ***Z*=1071.4286*M*+2428.5714** |  | ***Zj*** | **4.7143*M*+24.2857** | **3.7143*M*+24.2857** | **85** | **-0.7143*M*+60.7143** | **-*M*** | **2.1429*M*-12.1429** | **-*M*** | ***M*** | **-2.1429*M*+12.1429** | ***M*** |  |
|  |  | *Zj*-*Cj* | 4.7143*M*-20.7143↑ | 3.7143*M*-15.7143 | 0 | -0.7143*M*-4.2857 | -*M* | 2.1429*M*-12.1429 | -*M* | 0 | -3.1429*M*+12.1429 | 0 |  |

Positive maximum *Zj*-*Cj* is 4.7143*M*-20.7143 and its column index is 1. So, the entering variable is *x*1.  
  
Minimum ratio is 100 and its row index is 2. So, the leaving basis variable is *x*3.  
  
∴ The pivot element is 0.2857.  
  
Entering =*x*1, Departing =*x*3, Key Element =0.2857  
  
*R*2(new)=*R*2(old)÷0.2857  
  
*R*1(new)=*R*1(old) - 0.7143*R*2(new)  
  
*R*3(new)=*R*3(old) - 4*R*2(new)

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| Iteration-3 |  | *Cj* | 45 | 40 | 85 | 65 | 0 | 0 | 0 | *M* | *M* | *M* |  |
| ***B*** | ***CB*** | ***XB*** | ***x*1** | ***x*2** | ***x*3** | ***x*4** | ***S*1** | ***S*2** | ***S*3** | ***A*1** | ***A*2** | ***A*3** | **MinRatio *XBS*2** |
| *A*1 | *M* | 500 | 0 | 1 | -2.5 | -1.5 | -1 | 1.5 | 0 | 1 | -1.5 | 0 | 5001.5=333.3333 |
| *x*1 | 45 | 100 | 1 | 1 | 3.5 | 2.5 | 0 | -0.5 | 0 | 0 | 0.5 | 0 | --- |
| ***A*3** | *M* | 100 | 0 | -2 | -14 | -11 | 0 | **(3)** | -1 | 0 | -3 | 1 | 1003=33.3333**→** |
| ***Z*=600*M*+4500** |  | ***Zj*** | **45** | **-*M*+45** | **-16.5*M*+157.5** | **-12.5*M*+112.5** | **-*M*** | **4.5*M*-22.5** | **-*M*** | ***M*** | **-4.5*M*+22.5** | ***M*** |  |
|  |  | *Zj*-*Cj* | 0 | -*M*+5 | -16.5*M*+72.5 | -12.5*M*+47.5 | -*M* | 4.5*M*-22.5↑ | -*M* | 0 | -5.5*M*+22.5 | 0 |  |

Positive maximum *Zj*-*Cj* is 4.5*M*-22.5 and its column index is 6. So, the entering variable is *S*2.  
  
Minimum ratio is 33.3333 and its row index is 3. So, the leaving basis variable is *A*3.  
  
∴ The pivot element is 3.  
  
Entering =*S*2, Departing =*A*3, Key Element =3  
  
*R*3(new)=*R*3(old)÷3  
  
*R*1(new)=*R*1(old) - 1.5*R*3(new)  
  
*R*2(new)=*R*2(old) + 0.5*R*3(new)

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| Iteration-4 |  | *Cj* | 45 | 40 | 85 | 65 | 0 | 0 | 0 | *M* | *M* | *M* |  |
| ***B*** | ***CB*** | ***XB*** | ***x*1** | ***x*2** | ***x*3** | ***x*4** | ***S*1** | ***S*2** | ***S*3** | ***A*1** | ***A*2** | ***A*3** | **MinRatio *XBx*3** |
| ***A*1** | *M* | 450 | 0 | 2 | **(4.5)** | 4 | -1 | 0 | 0.5 | 1 | 0 | -0.5 | 4504.5=100**→** |
| *x*1 | 45 | 116.6667 | 1 | 0.6667 | 1.1667 | 0.6667 | 0 | 0 | -0.1667 | 0 | 0 | 0.1667 | 116.66671.1667=100 |
| *S*2 | 0 | 33.3333 | 0 | -0.6667 | -4.6667 | -3.6667 | 0 | 1 | -0.3333 | 0 | -1 | 0.3333 | --- |
| ***Z*=450*M*+5250** |  | ***Zj*** | **45** | **2*M*+30** | **4.5*M*+52.5** | **4*M*+30** | **-*M*** | **0** | **0.5*M*-7.5** | ***M*** | **0** | **-0.5*M*+7.5** |  |
|  |  | *Zj*-*Cj* | 0 | 2*M*-10 | 4.5*M*-32.5↑ | 4*M*-35 | -*M* | 0 | 0.5*M*-7.5 | 0 | -*M* | -1.5*M*+7.5 |  |

Positive maximum *Zj*-*Cj* is 4.5*M*-32.5 and its column index is 3. So, the entering variable is *x*3.  
  
Minimum ratio is 100 and its row index is 1. So, the leaving basis variable is *A*1.  
  
∴ The pivot element is 4.5.  
  
Entering =*x*3, Departing =*A*1, Key Element =4.5  
  
*R*1(new)=*R*1(old)÷4.5  
  
*R*2(new)=*R*2(old) - 1.1667*R*1(new)  
  
*R*3(new)=*R*3(old) + 4.6667*R*1(new)

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| Iteration-5 |  | *Cj* | 45 | 40 | 85 | 65 | 0 | 0 | 0 | *M* | *M* | *M* |  |
| ***B*** | ***CB*** | ***XB*** | ***x*1** | ***x*2** | ***x*3** | ***x*4** | ***S*1** | ***S*2** | ***S*3** | ***A*1** | ***A*2** | ***A*3** | **MinRatio *XBx*2** |
| *x*3 | 85 | 100 | 0 | 0.4444 | 1 | 0.8889 | -0.2222 | 0 | 0.1111 | 0.2222 | 0 | -0.1111 | 1000.4444=225 |
| ***x*1** | 45 | 0 | 1 | **(0.1481)** | 0 | -0.3704 | 0.2593 | 0 | -0.2963 | -0.2593 | 0 | 0.2963 | 00.1481=0**→** |
| *S*2 | 0 | 500 | 0 | 1.4074 | 0 | 0.4815 | -1.037 | 1 | 0.1852 | 1.037 | -1 | -0.1852 | 5001.4074=355.2632 |
| ***Z*=8500** |  | ***Zj*** | **45** | **44.4444** | **85** | **58.8889** | **-7.2222** | **0** | **-3.8889** | **7.2222** | **0** | **3.8889** |  |
|  |  | *Zj*-*Cj* | 0 | 4.4444↑ | 0 | -6.1111 | -7.2222 | 0 | -3.8889 | -*M*+7.2222 | -*M* | -*M*+3.8889 |  |

Positive maximum *Zj*-*Cj* is 4.4444 and its column index is 2. So, the entering variable is *x*2.  
  
Minimum ratio is 0 and its row index is 2. So, the leaving basis variable is *x*1.  
  
∴ The pivot element is 0.1481.  
  
Entering =*x*2, Departing =*x*1, Key Element =0.1481  
  
 *R*2(new)=*R*2(old)÷0.1481  
  
*R*1(new)=*R*1(old) - 0.4444*R*2(new)  
  
*R*3(new)=*R*3(old) - 1.4074*R*2(new)

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| Iteration-6 |  | *Cj* | 45 | 40 | 85 | 65 | 0 | 0 | 0 | *M* | *M* | *M* |  |
| ***B*** | ***CB*** | ***XB*** | ***x*1** | ***x*2** | ***x*3** | ***x*4** | ***S*1** | ***S*2** | ***S*3** | ***A*1** | ***A*2** | ***A*3** | **MinRatio *XBx*4** |
| ***x*3** | 85 | 100 | -3 | 0 | 1 | **(2)** | -1 | 0 | 1 | 1 | 0 | -1 | 1002=50**→** |
| *x*2 | 40 | 0 | 6.75 | 1 | 0 | -2.5 | 1.75 | 0 | -2 | -1.75 | 0 | 2 | --- |
| *S*2 | 0 | 500 | -9.5 | 0 | 0 | 4 | -3.5 | 1 | 3 | 3.5 | -1 | -3 | 5004=125 |
| ***Z*=8500** |  | ***Zj*** | **15** | **40** | **85** | **70** | **-15** | **0** | **5** | **15** | **0** | **-5** |  |
|  |  | *Zj*-*Cj* | -30 | 0 | 0 | 5↑ | -15 | 0 | 5 | -*M*+15 | -*M* | -*M*-5 |  |

Positive maximum *Zj*-*Cj* is 5 and its column index is 4. So, the entering variable is *x*4.  
  
Minimum ratio is 50 and its row index is 1. So, the leaving basis variable is *x*3.  
  
∴ The pivot element is 2.  
  
Entering =*x*4, Departing =*x*3, Key Element =2  
  
*R*1(new)=*R*1(old)÷2  
  
*R*2(new)=*R*2(old) + 2.5*R*1(new)  
  
*R*3(new)=*R*3(old) - 4*R*1(new)

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| Iteration-7 |  | *Cj* | 45 | 40 | 85 | 65 | 0 | 0 | 0 | *M* | *M* | *M* |  |
| ***B*** | ***CB*** | ***XB*** | ***x*1** | ***x*2** | ***x*3** | ***x*4** | ***S*1** | ***S*2** | ***S*3** | ***A*1** | ***A*2** | ***A*3** | **MinRatio *XBS*3** |
| ***x*4** | 65 | 50 | -1.5 | 0 | 0.5 | 1 | -0.5 | 0 | **(0.5)** | 0.5 | 0 | -0.5 | 500.5=100**→** |
| *x*2 | 40 | 125 | 3 | 1 | 1.25 | 0 | 0.5 | 0 | -0.75 | -0.5 | 0 | 0.75 | --- |
| *S*2 | 0 | 300 | -3.5 | 0 | -2 | 0 | -1.5 | 1 | 1 | 1.5 | -1 | -1 | 3001=300 |
| ***Z*=8250** |  | ***Zj*** | **22.5** | **40** | **82.5** | **65** | **-12.5** | **0** | **2.5** | **12.5** | **0** | **-2.5** |  |
|  |  | *Zj*-*Cj* | -22.5 | 0 | -2.5 | 0 | -12.5 | 0 | 2.5↑ | -*M*+12.5 | -*M* | -*M*-2.5 |  |

Positive maximum *Zj*-*Cj* is 2.5 and its column index is 7. So, the entering variable is *S*3.  
  
Minimum ratio is 100 and its row index is 1. So, the leaving basis variable is *x*4.  
  
∴ The pivot element is 0.5.  
  
Entering =*S*3, Departing =*x*4, Key Element =0.5  
  
*R*1(new)=*R*1(old)÷0.5  
  
*R*2(new)=*R*2(old) + 0.75*R*1(new)  
  
*R*3(new)=*R*3(old) - *R*1(new)

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| Iteration-8 |  | *Cj* | 45 | 40 | 85 | 65 | 0 | 0 | 0 | *M* | *M* | *M* |  |
| ***B*** | ***CB*** | ***XB*** | ***x*1** | ***x*2** | ***x*3** | ***x*4** | ***S*1** | ***S*2** | ***S*3** | ***A*1** | ***A*2** | ***A*3** | **MinRatio** |
| *S*3 | 0 | 100 | -3 | 0 | 1 | 2 | -1 | 0 | 1 | 1 | 0 | -1 |  |
| *x*2 | 40 | 200 | 0.75 | 1 | 2 | 1.5 | -0.25 | 0 | 0 | 0.25 | 0 | 0 |  |
| *S*2 | 0 | 200 | -0.5 | 0 | -3 | -2 | -0.5 | 1 | 0 | 0.5 | -1 | 0 |  |
| ***Z*=8000** |  | ***Zj*** | **30** | **40** | **80** | **60** | **-10** | **0** | **0** | **10** | **0** | **0** |  |
|  |  | *Zj*-*Cj* | -15 | 0 | -5 | -5 | -10 | 0 | 0 | -*M*+10 | -*M* | -*M* |  |

Since all *Zj*-*Cj*≤0  
  
Hence, optimal solution is arrived with value of variables as :  
*x*1=0,*x*2=200,*x*3=0,*x*4=0  
  
Min *Z*=8000

**By using any computer software (e.g. TORA, MATLAB, …) solve this optimization problem, or write down a program that solve the Problem using any programming language you know (program one method ONLY; Regular method, Dual Simplex method, or Dual Problem) .**

Code :

prob = optimproblem('ObjectiveSense','min');

x = optimvar('x',4,3,2,1,'LowerBound',0);

prob.Objective = 45\*x(1) + 40\*x(2)+85\*x(3)+65\*x(4);

cons1 = 3\*x(1) + 4\*x(2)+8\*x(3)+6\*x(4) >= 800;

cons2 = 2\*x(1) + 2\*x(2)+7\*x(3)+5\*x(4) >= 200;

cons3 = 6\*x(1) + 4\*x(2)+7\*x(3)+4\*x(4) >= 700;

prob.Constraints.cons1 = cons1;

prob.Constraints.cons2 = cons2;

prob.Constraints.cons2 = cons3;

show(prob)

sol = solve(prob);

sol.x

A Screenshot for the output of Matlab

A screenshot of a social media post

Description automatically generated