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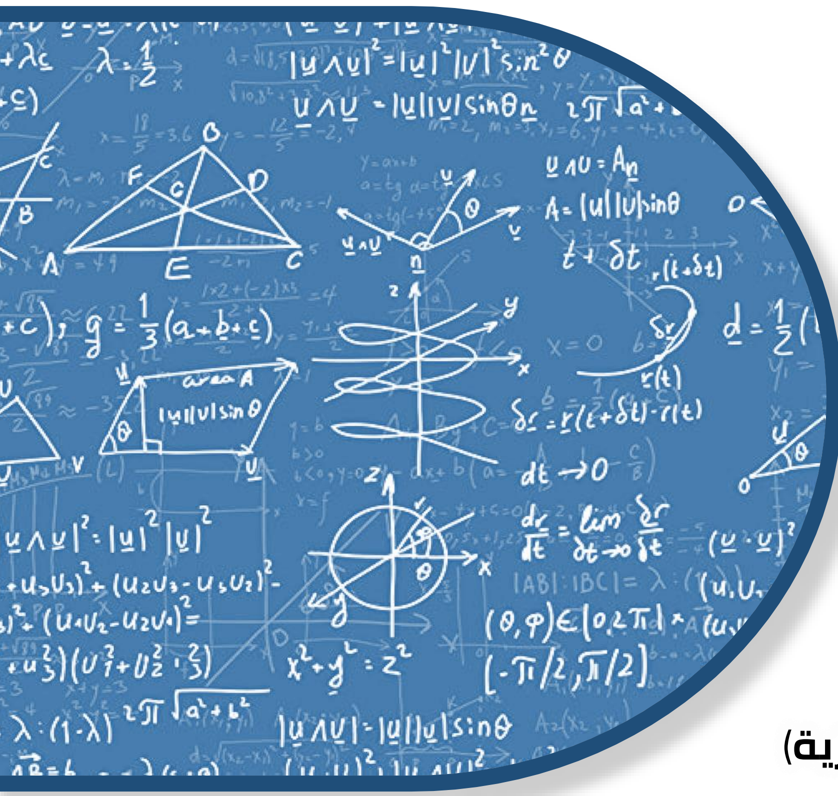

الرياضيات البحتة

الصف الثالث

طلاب مدارس التكنولوجيا التطبيقية (**لغة انجليزية**)

PURE MATHEMATICS

Third Grade

For industrial (**English**)

3rd.

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First Unit

Exponents, Logarithms, Permutations and Combinations

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Unit One

Exponents, Logarithms. Permutations and Combinations

Dear student, by the end studying of this unit you should have the following abilities and knowledge:

- To define the exponential function.
- To know the laws of exponents.
- To solve the exponential equations.
- To know the logarithmic function.
- To convert from exponential to logarithmic form.
- To know the laws of Logarithms.
- To solve the logarithmic equations.
- To solve problems on the logarithmic laws.
- To find the value of logarithmic of a number by using the calculator.
- To knew Counting Principle and application on it.
- To know Permutations and Combinations.
- To use the calculator to calculate Permutations and Combinations.

Lesson 1
Rational Exponents

We studied before Repeated Multiplication:

- $x \times x \times x \times \ldots\ldots\ldots\ldots \times x = x^n$, where $x \in R$ and $n \in Z^+$
- $x^{zero} = 1$. where $x \in R - \{0\}$
- $x^{-n} = \frac{1}{x^n}$ where $x \in R - \{0\}$,

Laws of the Exponents

(1) $x^n \times x^m = x^{n+m}$

(2) $x^n \div x^m = x^{n-m}$

(3) $(x^n)^m = x^{nm}$

(4) $(x\,y)^n = x^n \times y^n$

(5) $(x \div y)^n = x^n \div y^n$

Rational Exponents

$$\sqrt[n]{x} = x^{\frac{1}{n}}$$

Notes:

If n is an even number, then x is a real non-negative number.

If n is an odd number, then x is a real.

Definition:

$$\boxed{(x)^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m}$$

Example 1:

Put in the simplest form: $\frac{6^{-3}\times 6^{5}}{6^{2}}$

Solution

$\frac{6^{-3}\times 6^{5}}{6^{2}} = (6)^{-3+5-2} = 6^{zero} = 1$

Example 2:

Put in the simplest form: $(4^{-3})^5 \times (4^6)^3$

Solution

$(4^{-3})^5 \times (4^6)^3 = (4)^{-15} \times (4)^{18} = 4^3 = 64$

Example 3:

Find the value of: $(16)^{\frac{1}{4}}$

Solution

$(16)^{\frac{1}{4}} = \sqrt[4]{16} = 2$

Example 4:

Find the value of: $-(27)^{\frac{1}{3}}$

Solution:

$-(27)^{\frac{1}{3}} = -\sqrt[3]{27} = -3$

Example 5:

Find the value of: $(16)^{\frac{3}{2}}$

Solution:

$(16)^{\frac{3}{2}} = (\sqrt{16})^3 = (4)^3 = 64$

Example 6:

Find the value of: $(27)^{\frac{4}{3}}$

Solution:

$(27)^{\frac{4}{3}} = (\sqrt[3]{27})^4 = (3)^4 = 81$

Example 7:

Reduce to the simplest form: $\frac{10^{2x+1}\times 2^{2x}}{4^x\times 25^{x+\frac{1}{2}}}$ **then find the value of the result at x =** $\frac{1}{2}$

Solution

$$\frac{10^{2x+1}\times 2^{2x}}{4^x\times 25^{x+\frac{1}{2}}} = \frac{2^{2x+1}\times 5^{2x+1}\times 2^{2x}}{2^{2x}\times 5^{2x+1}} = 2^{2x+1}$$

$$= 2^{2\times\frac{1}{2}+1} = 4$$

Exercise 1

By using the laws of the exponents, reduce the following to the simplest form:

(1) $\frac{2^{-4}\times 2^{7}}{2^{3}}$

(2) $(3^{-2})^{2}\times(3^{3})^{-1}$

(3) $\frac{3^{-6}\times 3^{3}}{3^{-5}}$

(4) $(-81)^{\frac{3}{4}}$

(5) $(128)^{\frac{-2}{7}}$

(6) $-(343)^{\frac{2}{3}}$

(7) Prove that: $\frac{9^{4x+1}\times 4^{2-2x}}{4^{9x+1}\times 48^{1-x}}=1$

(8) Prove that: $\frac{343^{2x-\frac{1}{3}}\times 4^{3x+1}}{196^{3x}\times 4}=\frac{1}{7}$

(9) Simplify to the simplest form: $\frac{5^{2x}\times 4^{x}}{10^{2x-1}}$

Lesson 2
Solving exponential equations

Rules of solving exponential equations:

- If $x^n = x^m$, then $n = m$.
- If $x^n = y^n$, then $x = y$ if n is an odd number.
  Or $x = \pm y$ if n is an even number.
  Or $n = zero$ if $x \neq y$.

Example 1:

If $3^{x-5} = 9$, find the value of x

Solution:

$3^{x-5} = 3^2$

$\therefore x - 5 = 2$

$\therefore x = 7$

Example 2:

If $3^{x+7} = 1$, find the value of x

Solution:

$3^{x+7} = 3^{zero}$

$\therefore x + 7 = zero$

$\therefore x = -7$

Example 3:

If $4^{x-1} = 5^{x-1}$, find the value of x

Solution:

$x - 1 = \text{zero}$

$\therefore x = 1$

Example 4:

If $5^{2x-1} = \frac{1}{125}$, find the value of x

Solution:

$5^{2x-1} = 5^{-3}$

$\therefore 2x - 1 = -3$

$\therefore 2x = -2$ $\therefore x = -1$

Example 5:

If $(\frac{3}{5})^x = (\frac{27}{125})^{-1}$. then find the value of x

Solution:

$(\frac{3}{5})^x = (\frac{3}{5})^{-3}$

$\therefore x = -3$

Example 6:

Solve the equation: $\sqrt[3]{9} = 27^{x+2}$

Solution:

$9 = 27^{3(x+2)}$

$3^2 = 3^{9(x+2)}$

$\therefore 9(x+2) = 2$

$\therefore x = \frac{2}{9} - 2$ $\therefore x = -\frac{16}{9}$

Example 7:

Find the value of x which satisfies the equation:

$$3^{x+1} + 3^{x-1} = 90.$$

Solution:

$3^x (3 + 3^{-1}) = 90$

$3^x \times \frac{10}{3} = 90$ (By dividing both of sides by $\frac{10}{3}$)

$3^x = 27$

$3^x = 3^3$

$\therefore x = 3$

Exercise 2

(1) Find the Solution Set of the following equations in R:

(a) $x^{\frac{7}{2}} = 128$ (b) $(2x+3)^{\frac{4}{3}} = 81$ (c) $x^{\frac{3}{2}} = 64$

(2) If $5^x = 2$ then $25^x =$ ………..

(3) Find the Solution Set of the following equations:

(a) $x^{\frac{5}{2}} = \frac{1}{32}$ (b) $\sqrt[3]{(x-1)^5} = 32$ (c) $3^{x+1} = \frac{1}{27}$

(4) Find in R the Solution Set of the following equations:

(a) $2^{x-3} = 5^{x-3}$ (b) $7^{x+1} = 3^{2x+2}$

(5) If $3^{x+1} - 3^{x-1} = 72$, find the value of x

Lesson 3
Applications on solving exponential equations

Example 1:

Find the Solution Set of the following equation:

$49^x - 50 \times 7^x + 49 = 0$

Solution:

$7^{2x} - 50 \times 7^x + 49 = 0$

$(7^x - 1)(7^x - 49) = 0$

$7^x - 1 = 0$ | $7^x - 49 = 0$

$7^x = 1$ | $7^x = 7^2$

$\therefore x = 0$ | $\therefore x = 2$

S.S. = { 0, 2}

Example 2:

If $f(x) = 5^x$, then find the value of x if $f(x) + f(3 - x) = 30$

Solution:

$5^x + 5^{3-x} = 30$ (multiply by 5^x)

$5^{2x} + 5^3 = 30 \times 5^x$

$5^{2x} - 30 \times 5^x + 125 = 0$

$(5^x - 25)(5^x - 5) = 0$

$5^x = 25 = 5^2$ | $5^x = 5$

$\therefore x = 2$ | $\therefore x = 1$

Example 3:

Solve the equation:

$x^{\frac{4}{5}} - 3\,x^{\frac{2}{5}} - 4 = 0$

Solution:

$(\,x^{\frac{2}{5}} - 4)(\,x^{\frac{2}{5}} + 1\,) = 0$

$x^{\frac{2}{5}} = 4$

$x = 4^{\frac{5}{2}}$

$x = \pm\, 32$

$x^{\frac{2}{5}} = -1$

$x = (-1)^{\frac{5}{2}}$ **(refused)**

S.S. = { 32, -32}

Exercise 3

(1) Find the Solution Set of the equation:

$$4^x + 2\times 2^x - 8 = 0$$

(2) If $f(x) = 7^x$, find the value of x which satisfies:

$$f(x) + f(2 - x) = 50$$

(3) If $f(x) = 3^x$, find the value of x which satisfies:

$$f(x) + f(2 - x) = 6$$

(4) Solve the equation:

$$2^{2x} - 6 \times 2^x + 8 = 0$$

(5) Solve the equation:

$$x^{\frac{4}{3}} - 3x^{\frac{2}{3}} - 4 = 0$$

Lesson 4
The logarithmic function and its relationship

The logarithmic function:

If a $\in$ $R^+ - \{1\}$ where: $y = \log_a x$ (logarithmic form)

can be converted to the exponential form $x = a^y$

Notice that:

- There is no logarithm for a negative number.
- There is no logarithm for Zero.

Example 1:

Find the value of x if: $\log_x 81 = 4$

Solution:

$\log_x 81 = 4$ (convert to the exponential form)

$\therefore x^4 = 81$

$\therefore x = (81)^{\frac{1}{4}}$

$\therefore x = 3$ (and the negative value is refused)

Example 2:

Find the value of x if: $\log_5 125 = x$

Solution:

$\log_5 125 = x$ (convert to the exponential form)

$\therefore 5^x = 125$

$\therefore 5^x = 5^3$

$\therefore x = 3$

Example 3:

Solve the equation: $\log_2(x^2 + 2x) = 3$

Solution:

$\log_2(x^2 + 2x) = 3$ **(We can convert to exponential form)**

$\therefore (x^2 + 2x) = 2^3$

$\therefore x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$\therefore x = -4$ **or** $x = 2$

$\therefore$ **S.S.** $= \{-4, 2\}$

Exercise 4

(1) Solve the equation: $\log_3(2x - 5) = 1$

(2) Solve the equation: $\log_x(x + 2) = 2$

(3) Solve the equation: $\log_{81} x = \frac{3}{4}$

(4) Solve the equation: $\log_x 5x = 2$

(5) Solve the equation: $\log_3 27 = x + 2$

(6) Solve the equation: $\log_2(4^x - 2) = x$

Lesson 5
Properties of logarithms

(1) $\log x + \log y = \log xy$.

(2) $\log x - \log y = \log \frac{x}{y}$.

(3) $\log x^n = n \log x$

(4) $\log_x x = 1$

(5) $\log_x 1 =$ zero

(6) Base changing property: $\log_y x = \frac{\log_n x}{\log_n y}$

(7) The multiplicative inverse property: $\log_y x = \frac{1}{\log_x y}$

Note:

If the base is not mentioned, then it is 10 and is called the common logarithm

$\log 10 = 1$	, $\log 100 = 2$	, $\log 1000 = 3$
$\log 0.1 = -1$	, $\log 0.01 = -2$	, $\log 0.001 = -3$

Example 1:

Simplify: $\log 2 + 2 \log 3 + \log 1 - \log 18$

Solution:

The expression $= \log 2 + \log 3^2 + \log 1 - \log 18$

$$= \log \frac{2 \times 9 \times 1}{18} = \log 1 = \text{zero}$$

Example 2:

Prove that: $\log\frac{170}{7} - \log\frac{18}{35} + \log\frac{36}{17} = 2$

Solution:

L.H.S. $= \log\dfrac{\frac{170}{7}\times\frac{36}{17}}{\frac{18}{35}} = \log 100 = 2 =$ R.H.S.

Example 2:

Prove that: $\log 125 - 2\log 27 + \frac{3}{2}\log 100 = 3\log 4.5$

Solution:

L.H.S. $= \log 125 - \log 27^2 + \log 100^{\frac{3}{2}}$

$= \log\dfrac{125\times 27^2}{100^{\frac{3}{2}}} = \log\dfrac{729}{8}$

R.H.S. $= 3\log 4.5 = \log (4.5)^3 = \log\dfrac{729}{8}$

$\therefore$ The two sides are equal.

Exercise 5

(1) Simplify to the simplest form: log 2 + log 5

(2) Simplify to the simplest form: $\log_5 15 - \log_5 3$

(3) Simplify to the simplest form: log 54 – 3 log 3 – log 2

(4) Simplify to the simplest form: $\log_{abc} a + \log_{abc} b + \log_{abc} c$

(5) Prove that: $\log_4 38 - \log_4 42 + \log_4 56 - \log_4 19 + \log_4 24 = 3$

(6) without using calculator, Prove that: $\frac{2\log 9 \times \log 8}{\log 3 \times 3\log 2} = 4$

Lesson 6
Solving logarithmic equations

Example 1:

Solve the equation: $\log_3 x = \log_x 3$

Solution:

$$\frac{\log x}{\log 3} = \frac{\log 3}{\log x}$$

$(\log x)^2 = (\log 3)^2$

$\text{Log } x = \pm \log 3$

$\log x = \log 3$ Or $\log x = -\log 3$

$\therefore x = 3$ $\log x = \log (3)^{-1}$

$\therefore x = \frac{1}{3}$

$\therefore$ S.S.= $\{3, \frac{1}{3}\}$

Example 2:

Solve the equation: $\log (x^2 + 9x) = 1$

Solution:

Convert from logarithmic form to exponential form

(note: the base is 10)

$x^2 + 9x = 10$

$x^2 + 9x - 10 = 0$

$(x + 10)(x - 1) = 0$

$\therefore x = -10$ or $x = 1$

$\therefore$ S.S. $= \{-10, 1\}$

Example 3:

Solve the equation: $\log_2(x^2 + 6x + 9) - \log_2(x - 1) = \log_5 625$

Solution:

$\log_2 \frac{x^2 + 6x + 9}{x-1} = 4 \log_5 5 = 4$

$\frac{x^2 + 6x + 9}{x-1} = 2^4 = 16$

$x^2 + 6x + 9 = 16x - 16$

$x^2 - 10x + 25 = 0$

$(x - 5)^2 = 0$

$\therefore x = 5$

$\therefore$ S.S. = { 5 }

Exercise 6

(1) Solve the equation: $\log_2 x + \log_x 2 = 2$

(2) Solve the equation: $\log_2 x + \log_2(x + 12) = 3$

(3) Find the Solution Set of the following equation:

$$\log (x - 1)^3 - 3 \log (x - 3) = \log 8$$

(4) Solve the equation: $\log_2 2x + \log_2 x = 3$

(5) Solve the equation: $\log (x + 8) - \log (x - 1) = 1$

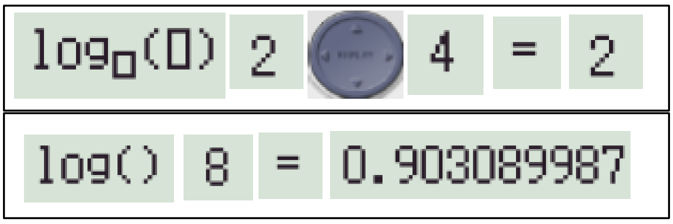
(6) Solve the equation: $\log_3(x + 6) = 2 \log_3 x$

Lesson 7
Solving exponential equations by using

We can use calculator to find the value of logarithm as follow:

(1) find the value of: $\log_2 4$

(2) find the value of: log 8



Practice:

Use the calculator to find the value of:

- $\log_3 12$ = ….
- $\log_3 24$ = ….
- log 128 = …
- $\log_5 125$ = ….
- log 100 = …
- log 1000 = …
- log 500 = …

Example 1:

Find the value of x if: $3^{5x-2} = 7^{x+1}$

Solution:

(by taking logarithm for both sides)

$(5x - 2) \log 3 = (x + 1) \log 7$

$5x \log 3 - 2\log 3 = x \log 7 + \log 7$

$5x \log 3 - x \log 7 = 2\log 3 + \log 7$

$x(5 \log 3 - \log 7) = 2\log 3 + \log 7$

$\therefore x = \frac{2 \log 3 + \log 7}{5 \log 3 - \log 7} \simeq 1.17$

Example 2:

If: $5^{x+2} = 3^{2x-5}$, then Find the value of x to the nearest two decimals places

Solution:

$(x + 2) \log 5 = (2x - 5) \log 3$

$x \log 5 + 2\log 5 = 2x \log 3 - 5\log 3$

$x \log 5 - 2x \log 3 = -2\log 5 - 5 \log 3$

$x(\log 5 - 2 \log 3) = -2\log 5 - 5 \log 3$

$\therefore x = \frac{-2 \log 5 - 5\log 5}{\log 5 - 2\log 3} \simeq 14.82$

Exercise 7

(1) If $(x+4)^{\frac{2}{3}}$ = 6.123,find the value of x to the nearest two decimal places.

(2) Find the value of x to the nearest two decimal places where:

$$7^{3x-2} = 5$$

(3) Find the value of x to the nearest two decimal places where:

$$7^{x+1} = 5^{x-3}$$

(4) Find the value of x to the nearest two decimal places where:

$$7^{x-2} = 4^{x+3}$$

(5) Find the value of x to the nearest two decimal places where:

$$7^{7-2x} = 13.4$$

Lesson 8
Counting Principle

Fundamental Counting Principle:

If the number of ways to perform a certain task = m,
the number of ways to perform another certain task = l,
the number of ways to perform a third certain task = n and so on ….
then: the number of ways to perform these tasks together = $m \times l \times n \times$……

Example 1:

By how many ways it is possible to choose a boy from a group of three boys and a girl from a group of two girls?

Solution:

Number of ways = $3 \times 2 = 6$ ways.

Example 2:

By how many ways it is possible to choose a uniform which consists of a shirt and a trousers from 5 shirts and 3 trousers?

Solution:

Number of ways = $5 \times 3 = 15$ ways.

Example 3:

By how many ways it is possible to forma 3-digit number from the set { 1, 2, 3, 4}?

Solution:

Number of ways = $4 \times 4 \times 4 = 64$ ways.

units	tens	Hundreds
4	4	4

Example 4:

By how many ways it is possible to form a 3-digit from the set { 0, 1, 2, 3, 4}?

Solution:

Number of ways = $5 \times 5 \times 4 = 100$ ways.

Example 5: (conditional counting principal)

By how many ways it is possible to form different 3-digit number from the set {1, 2, 3, 4}?

Solution:

units	tens	Hundreds
4	3	2

Number of ways = $4 \times 3 \times 2 = 24$ ways.

One of the digits takes units place and is not repeated in the other places

One of the 3 remainder digits takes tens place and isn't used again.

The remaining 2 digits takes hundreds place.

Example 6: (conditional counting principal)

By how many ways it is possible to form a 3-digit different number from the set {0, 1, 2, 3, 4}?

Solution:

units	tens	Hundreds
3	4	4

Number of ways = $3 \times 4 \times 4 = 48$ ways.

We cannot put the zero in the hundreds place.

Number of ways to choose a digit in the hundreds place = 4

Number of ways to choose a digit in the tens place = 4

Number of ways to choose a digit in the units place = 3

Exercise 8

(1) By how many ways it is possible to form a 2 different -digit number from the set {0, 1, 2, 3, 4}?

(2) By how many ways can four students sit down on four desks in a raw?

(3) How many 3- digit odd numbers can be formed from the set {2, 3, 6, 8}?

(4) By how many ways it is possible to form a 4 different -digit numbers from the set {2, 3, 4, 5} such that tens digit is an even?

(5) How many 3- digit numbers can formed from the set { 2, 3, 5}

(6) How many 4 different - digit numbers can form from the set {2, 3, 6, 8} such that units digit is 6?

Lesson 9
Factorial of a number, Permutation

Factorial of a positive integer number (n) is written as $\lfloor\!\underline{\,n\,}$ where:

$\lfloor\!\underline{\,n\,}$ = n (n – 1)(n – 2)................$\times$ 3$\times$ 2 $\times$ 1

Notes:

$\lfloor\!\underline{\,0\,} = 1$, $\lfloor\!\underline{\,1\,} = 1$, $\lfloor\!\underline{\,n\,} = n\lfloor\!\underline{\,n-1\,}$

Number of sitting down of (n) people in a one row = $\lfloor\!\underline{\,n\,}$

Number of sitting down of (n) people in a circle = $\lfloor\!\underline{\,n-1\,}$

Example 1:

Find

(a) $\dfrac{\lfloor\!\underline{\,10\,}}{\lfloor\!\underline{\,8\,}}$ (b) if $\lfloor\!\underline{\,n\,}$ = 120 then find the value of n

Solution:

(a) $\dfrac{\lfloor\!\underline{\,10\,}}{\lfloor\!\underline{\,8\,}} = \dfrac{10 \times 9 \times \lfloor\!\underline{\,8\,}}{\lfloor\!\underline{\,8\,}} = 90$

(b) $\lfloor\!\underline{\,n\,} = 120 \Longrightarrow \lfloor\!\underline{\,n\,} = \lfloor\!\underline{\,5\,} \Longrightarrow n = 5$

Example 2:

Find the solution set of: $\dfrac{\lfloor\!\underline{\,n\,}}{\lfloor\!\underline{\,n-2\,}} = 30$

Solution:

$\dfrac{n\,(n-1)\,\lfloor\!\underline{\,n-2\,}}{\lfloor\!\underline{\,n-2\,}} = 3$ n (n – 1) = 6 $\times$ 5 $\Longrightarrow$ n = 6

Permutations:

How many 3-digit numbers can be formed from the set { 2, 3, 5}

Number of numbers = $3 \times 2 \times 1 = 6$ numbers.

Every number of those numbers is called permutation and is written as ${}^{3}P_{3}$

Definition

the number of permutations of (n) different objects taking (r) at a time is denoted by ${}^{n}P_{r}$ where:

$${}^{n}P_{r} = n\,(n-1)\,(n-2)\ \ldots\ldots\ (n-r+1) \qquad r \le n,\ r \in N \text{ and } n \in Z^{+}$$

Notes:

(a) ${}^{n}P_{zero} = 1$ (b) ${}^{n}P_{r} = \frac{|\underline{n}}{|\underline{n-r}}$

Example 1:

- ${}^{6}P_{3} = 6 \times 5 \times 4 = 120$
- ${}^{7}P_{5} = 7 \times 6 \times 5 \times 4 \times 3 = 2520$
- ${}^{4}P_{4} = 4 \times 3 \times 2 \times 1 = 24$

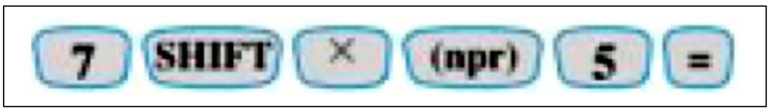
Example 2:

Find the number of ways to sit 5 students on 7 seats in one row.

Solution:

Number of ways = ${}^{7}P_{5} = 7 \times 6 \times 5 \times 4 \times 3 = 2520$

We can use the calculator:



Example 3:

By how many ways can 7 persons be arranged to sit on 7 seats in the form of a circle ?

Solution:

Number of ways = $^7P_7 = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

Example 4:

If $^7P_r = 840$ **then find the value of** $\lfloor\underline{r-4}$

Solution:

$^7P_r = 840 = {}^7P_4$

$\therefore r = 4$

$\lfloor\underline{r-4} = \lfloor\underline{\;0\;} = 1$

Exercise 9

(1) By how many ways it is possible to form a 2 different -digit number from the set {3, 4, 5, 6}?

(2)By how many ways can 7 children be arranged in a circle ?

(3) How many ways can a president and vice president be selected from a 12-member committee?

(4) Find the value of n which satisfies:

(a) $\underline{|n} = 24$

(b) $\dfrac{\underline{|n+1}}{\underline{|n-1}} = 42$

(c) ${}^{15}P_n = 2730$

(5) If ${}^{9}P_{r-1} = 504$, find the value of: $\underline{|r+1}$

Lesson 10
Combinations

Definition:

We denote to number of combinations formed from (r) objects chosen from (n) elements by ${}^nC_r$ where $r \le n$, $r \in N$, $n \in Z^+$

The rules of combinations:

For all $r \le n$, $r \in Z^+$, $n \in Z^+$

- ${}^nC_r = \dfrac{\lfloor n}{\lfloor r \; \lfloor n-r}$
- If ${}^nC_x = {}^nC_y$, then: $x = y$ or $x + y = n$
- ${}^nC_r = {}^nC_{n-r}$ (reducing law)
- ${}^nC_n = {}^nC_{zero} = 1$
- $\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n-r+1}{n}$ (ratio law)

Example 1:

If ${}^nC_{n-2} = 36$, then: find the value of n

Solution:

${}^nC_{n-2} = {}^nC_2 = 36 = {}^9C_2$

$\therefore n = 9$

Example 2:

If ${}^{21}C_{4n-7} = {}^{21}C_{3n}$ then find the value of n

Solution:

$4n - 7 = 3n$

$\therefore n = 7$

$4n - 7 + 3n = 21$

$\therefore n = 4$

Example 3:

If ${}^{7}C_{r} = {}^{7}C_{3r-5}$, ${}^{n}C_{r} : {}^{n}C_{r-1} = 8 : 3$, find the value of n and r

Solution:

$3r - 5 = r$

$\therefore 2r = 5$ (refused)

$3r - 5 + r = 7$

$\therefore r = 3$

${}^{n}C_{r} : {}^{n}C_{r-1} = 8 : 3$

$\frac{n-3+1}{3} = \frac{8}{3}$ $\quad \therefore n - 2 = 8$ $\quad \therefore n = 10$

Example 4:

By using the calculator, find the value of: ${}^{5}C_{4} + {}^{7}C_{2}$

Solution:

5	shift	÷	4	+	7	shift	÷	2	=

The result = 26

Exercise 10

(1) If ${}^{13}C_r : {}^{13}C_{r+1} = 9 : 5$, find the value of r

(2) Find the value of (n) if ${}^{n}C_{n-3} = 120$

(3) Find the value of (n) if ${}^{25}C_{3n-5} = {}^{25}C_{2n}$

(4) By using the calculator, find the value of: ${}^{17}C_9 - {}^{17}C_{14}$

(5) By how many ways can a 4 members team be selected from 9 persons?

(6) 7 persons subscribe in a competition so that one match is held between each two find the number of matches of this competition.

(7) A class contains 10 boys and 8 girls, by how many ways can we form an activity committee of five people so that it consists of three boys and two girls?

Unit Test

First Question:

(1) Find the value of x which satisfies the equation:

$$3^{x+1} + 3^{x-1} = 90.$$

(2) Solve the equation: $\log_2(x^2 + 2x) = 3$

Second Question:

(1) If $^{28}C_r = {}^{28}C_{2r-5}$, find the value of r

(2) Simplify: log 2 + 2 log 3 + log 1 – log 18

Third Question:

(1) Simplify to the simplest form: $\dfrac{5^{2x} \times 4^{x}}{10^{2x-1}}$

(2) Find the Solution Set of the equation:

$$\log (x-1)^3 - 3 \log (x-3) = \log 8$$

Fourth Question:

(1) Solve the equation: $\log_2 x + \log_x 2 = 2$

(2) If: $^{9}P_{r-1} = 504$, then find the value of: $\lfloor\!\underline{r + 3}$

Fifth Question:

(1) Solve the equation:

$$x^{\frac{4}{3}} - 3x^{\frac{2}{3}} - 4 = 0$$

(2) Find the value of x to the nearest two decimal places where:

$$7^{3x-2} = 5$$