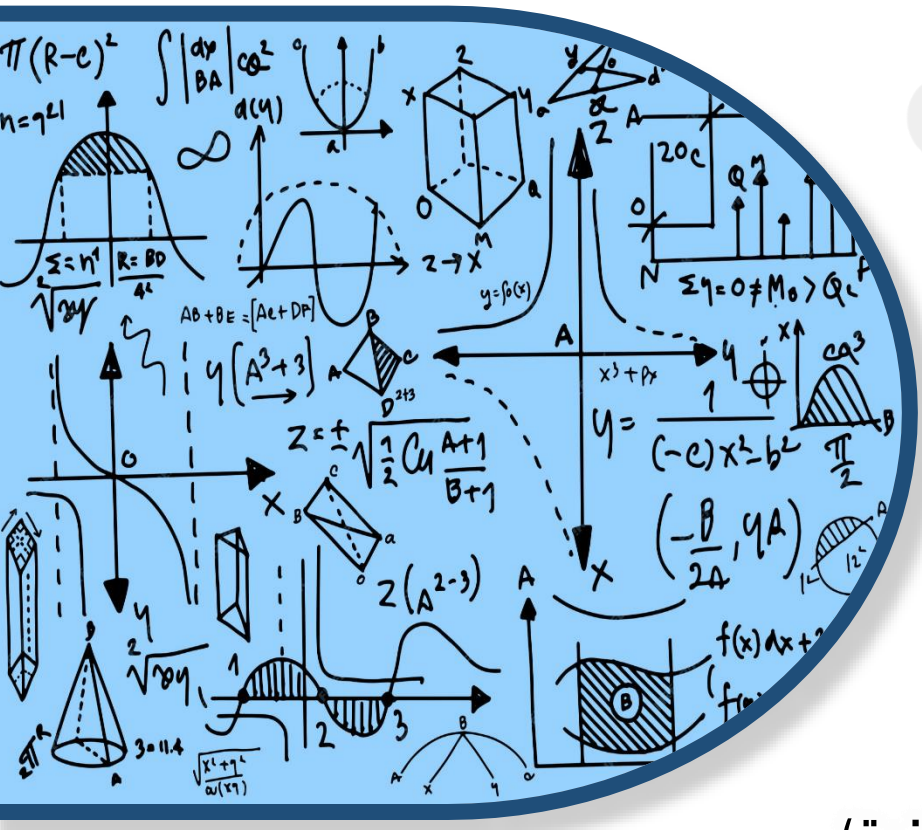
ITALIAN AGENCY FOR DEVELOPMENT COOPERATION

التكنولوجيا التطبيقية
APPLIED TECHNOLOGY
وزارة التربية والتعليم والتعليم الفني

MINISTRY OF EDUCATION AND TECHNICAL EDUCATION



ميكانيكا

الصف الثالث

التخصصات الصناعية (**لغة انجليزية**)

MECHANICS

Third Grade

For industrial (**English**)

3rd.

2023 - 2024

First Unit
STATICS

Unit (1)
Statics

Dear student by the end of studying that unit you must have the following abilities and knowledge:

1) Recognize and find moment of a force according to a point in the space.

2) Find the norm and the vector of a force according to a point.

3) Find moment of coplanar forces according to a point in its plane.

4) Recognize the general theorem of the moments.

5) Solve many applications on the moments.

6) Recognize the coplanar parallel forces.

7) Determine the line of action of the resultant of two parallel forces if they are in the same direction or in two opposite directions

8) Find moments of a set of parallel forces about a point.

9) Find the resultant of a set of parallel forces.

10) Deduce that the sum of moments of several parallel forces bout a point equals the moment of the resultant bout the same point.

11) Deduce that the sum of moments of several parallel forces bout a point equals zero if the resultant passes by the same point.

12) Deduce that the sum of moments of several parallel forces bout a point equals zero if their resultant vanishes.

13) Recognize the concept of the couple ,find the moment of the couple and deduce that the moment of the couple is a constant vector .

14) Recognize the equivalent and the equilibrium of two couples.

15) Find the resultant of several couples .

16) Prove that a set of forces equivalent to a couple by different methods

Lesson (1)

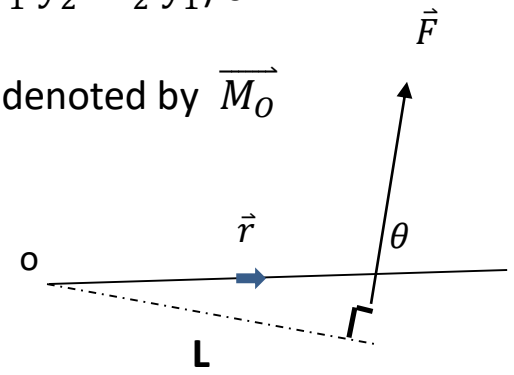
Moment of a force about a point in 2-D system

DEFINATION : The vector product of two vectors : $\vec{A} = (x_1, y_1)$, $\vec{B} = (x_2, y_2)$ denoted by

$\vec{A}$ x $\vec{B}$ and $\vec{A}$ x $\vec{B}$ =$(x_1, y_1) \times (x_2, y_2) = (x_1\ y_2 - x_2\ y_1)\ \vec{e}$

Moment of a force ($\vec{f}$) about a point (o) and denoted by $\overrightarrow{M_O}$

$$\overrightarrow{M_O} = \vec{r} \times F$$



Notes :

1- The algebraic measure of the moment :

@ If $\vec{F}$ makes the rotation about (o) anticlockwise then the algebraic measure is positive .

@ If $\vec{F}$ makes the rotation about (o) clockwise then the algebraic measure is negative .

2- Norm of the moment:

$\left\|\overrightarrow{M_O}\right\| = F$ L $\longrightarrow$ L = $\frac{\left\|\overrightarrow{M_O}\right\|}{F}$

Where : L is the length of perpendicular from (O) to the line of action of $\vec{F}$

3- Moment of a force about a point on its line of action = zero

Example (1) :

The force $\vec{F} = \vec{\imath} - 3\vec{j}$ acts at the point A (-1 ,2) find moment of $\vec{F}$ about the point B (2 , 0), then find the length of perpendicular from B to the line of action of $\vec{F}$

Solution

$\vec{r} = \overrightarrow{BA} = \vec{A} - \vec{B}$ = (-1 ,2)- (2 , 0) = (- 3 , 2)

$\overrightarrow{M_B} = \vec{r} \times \vec{F}$ = (- 3 , 2) x (1 , -3) = $7\vec{k}$

L = $\frac{\|\overrightarrow{M_B}\|}{f} = \frac{7}{\sqrt{10}} = \frac{7\sqrt{10}}{10}$ length unit

Example (2):

the force $\vec{F} = -\overrightarrow{3\imath} + k\vec{j}$ acts at the point A (3 ,1) if the moment of $\vec{f}$ about the point B(-2 , 4) is $7\vec{k}$ find value of m

Solution

$\overrightarrow{M_B} = \vec{r} \times \vec{F}$ = (5 , -3) x (-3 , k) = $6\overrightarrow{k}$,

∴ 5k – 9 = 6 ⟶ 5k= 15 ⟶ k = 3

Example (3):

the force $\vec{F} = \vec{\imath} + \vec{j}$ acts at the point A (2 ,2) prove that the moment vector of $\vec{F}$ about the origin point is the zero vector ,explain the answer ?

Solution

$\overrightarrow{M_o} = \vec{r} \times \vec{F}$ = (2 , 2) x (1 , 1)= (2-2)$\vec{k}$ = 0 $\overrightarrow{k}$

∴ the line of action of $\vec{F}$ passes by the origin point

Example (4):

The two forces $\overrightarrow{F_1} = \vec{\imath} - \vec{\jmath}$, $\overrightarrow{F_2} = m\vec{\imath} + 2\vec{\jmath}$ act at the two points A(3 ,0), B (0 ,3) respectively, determine the value of the constant m such that the sum of moments of the two forces about the origin point vanishes.?

Solution

$\overrightarrow{r_1} \times \overrightarrow{F_2}$ + $\overrightarrow{r_1} \times \overrightarrow{F_2} = \overrightarrow{0}$

(3 , 0) x (1 ,- 1) + (0 , 3) x (m ,2) $= 0\overrightarrow{k}$

$-3 - 3m = 0$

$-3m = 3$

$\therefore m = -1$

Exercise (1)

1) The force $\vec{F} = 3\vec{\imath} + \overrightarrow{4j}$ act at the point A (3,0) find the moment vector of the force with respect to the point B (-2,4) then find the length of perpendicular from B to the line of action of the force $\vec{F}$?

2) The two forces $\overrightarrow{F_1} = \vec{\imath} + 6\vec{j}$, $\overrightarrow{F_2} = 3\vec{\imath} - 2\vec{j}$ act at the point A(-1,2)

find the moment vector of the two force with respect to the point B(0,7)

3) The two forces $\overrightarrow{F_1} = \overrightarrow{5\imath} + \vec{j}$, $\overrightarrow{F_2} = -\vec{\imath} + 2\vec{j}$ act at the point A(2,5) find the moment vector of the two force with respect to the point B(1,1)

4) The force $\vec{F} = m\vec{\imath} + \overrightarrow{nj}$ at the point A (4,2) if its moment about the origin point equals $-15\vec{k}$ and its moment about B (0 ,5) equals $15\vec{k}$

Find values of the constants m , n ?

5) The force $\vec{F} = 6\vec{\imath} + \overrightarrow{8j}$ at the point A (-1,2) find the moment vector of the force with respect to the point B (2,5)

Lesson (2)
Theorem of moments

The sum of algebraic measures of moments of a set of forces about a point equals the moment of the resultant about the same point.

Notes :

1-if $\overrightarrow{M_O} = \overrightarrow{M_H}$, then the line of action of $\overrightarrow{F}$ // $\overline{OH}$

2-if $\overrightarrow{M_O} = -\overrightarrow{M_H}$ then the line of action of $\overrightarrow{F}$ bisects $\overline{OH}$

Example (1):

The forces $\overrightarrow{F_1} = 2\vec{\imath} + 3\vec{\jmath}$, $\overrightarrow{F_2} = -\vec{\imath} + 4\vec{\jmath}$ and $\overrightarrow{F_2} = \vec{\imath} - 4\vec{\jmath}$ act at the point A (-2,5) find sum of the moments of the force about the origin point , then length of perpendicular from the origin point on the line of action of the resultant ?

Solution

sum of moments of set of forces about a point = the moment of the resultant about the same point

$\vec{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3}$ = (2,3) R = $\sqrt{4+9} = \sqrt{13}$

$\overrightarrow{M_o}$ = (-2 , 2) x (2 , 3)= -16 $\overrightarrow{k}$

Length of perpendicular from the origin point to the line of action of the resultant = $\frac{16}{\sqrt{13}}$

Example (2):

The forces $\overrightarrow{F_1} = (-4, 2)$, $\overrightarrow{F_2} = (-6, -3)$, , $\overrightarrow{F_3} = (2,1)$, $\overrightarrow{F_4} = (3, -1)$ prove that the line of action of the resultant passes through the point A (-10 ,- 2)

Solution

$\vec{R}$ = (-5,-1)

$\overrightarrow{M_A}$ = $\vec{r}$ x $\vec{F}$= (-10 , -2) x (-5 , -1) = (10 -10) $\vec{k}$ = $\vec{O}$

∴ the line of action of the resultant passes through the point A

Example (3):

The force $\vec{F} = -2\vec{\imath} - 3\vec{\jmath}$ act at the point A (1,1) prove using the moments :

the line of action of $\vec{F}$ // $\overleftrightarrow{BC}$

the line of action of $\vec{F}$ bisects $\overline{CD}$

Knowing that B (2,4) , C (4 ,7) , D (2 ,1)

Solution

$\overrightarrow{M_B}$ = $\vec{r}$ x $\vec{F}$= (-1 , -3) x (-2 , -3) = $-3\ \vec{k}$

$\overrightarrow{M_C}$ = $\vec{r}$ x $\vec{F}$= (-3 , -6) x (-2 , -3) = $-3\ \vec{k}$

∴ $\overrightarrow{M_B} = \overrightarrow{M_C}$ ∴ the line of action of $\vec{F}$ // $\overleftrightarrow{BC}$

$\overrightarrow{M_D}$ = $\vec{r}$ x $\vec{F}$= (-1 , 0) x (-2 , -3) = $3\ \vec{k}$

∴ $\overrightarrow{M_D} = -\overrightarrow{M_C}$ ∴ the line of action of $\vec{F}$ bisects $\overline{DC}$

Example (4):

ABCD is a rectangle in which AB = 8cm, BC = 12 cm the forces 16 ,14 ,F ,K kg.wt. act at

$\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{CD}$, $\overrightarrow{AD}$ respectively if the sum of the algebraic measure of the moments of the forces about each of C and the centre of the rectangle is equal to zero .find F,K

Solution

$\overrightarrow{M_C} = \vec{0}$

-16 X 12 + kX8 =0

8K = 192 ∴K= 24 gm.wt

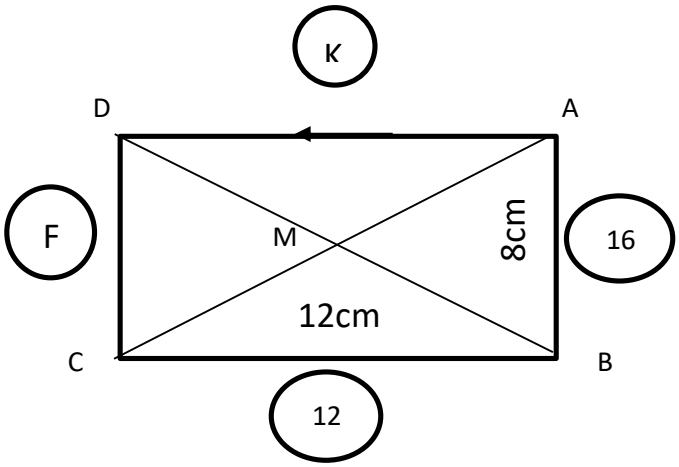
$\overrightarrow{M_M} = \vec{0}$

-16 X6 - F x 6 + 12 x4 + k x 4 = 0

-16 x 6 - f x 6 + 12 x4 + 24 x4 =0

48 – 6F = 0

∴6F = 48 ∴ F = 8 gm.wt



Example (5):

ABCD is a square of side length 6 cm , forces of magnitudes 2 ,4,6 ,8 ,$16\sqrt{2}$ newton act at $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$, $\overrightarrow{AC}$ respectively ,calculate the sum of the algebraic measure of the moments of the forces **bout** the vertex B .?

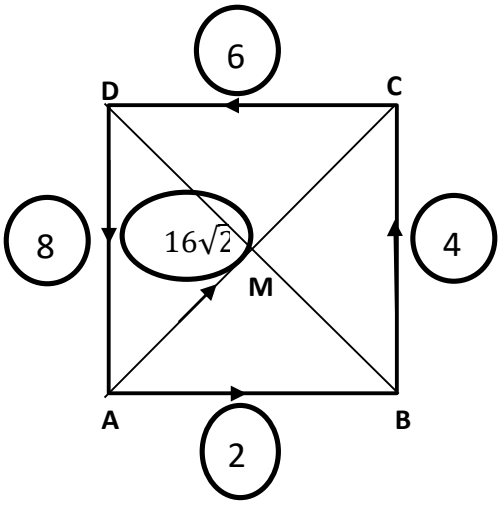
Solution

$BD=AC$ =$6\sqrt{2}$ (Pythagoras theorem)

$\overrightarrow{M_B}$ = 6 X6 + 8 X6 - 16 $\sqrt{2}$ x $3\sqrt{2}$

= 84 - 96

= -12 newton .cm



Exercise (2)

1. The two forces $\overrightarrow{F_1} = \overrightarrow{5\imath} + \vec{j}$, $\overrightarrow{F_2} = -\vec{\imath} + 2\vec{j}$ act at the point A (2,5) find the moment vector of the resultant of the two forces with respect to the point B (1,1), then find the length of perpendicular from the point B to the line of action of the resultant .
2. The two forces $\overrightarrow{F_1} = \overrightarrow{5\imath} - 4\vec{j}$, $\overrightarrow{F_2} = \vec{\imath} - 4\vec{j}$ act at the origin point prove that the line of action of the resultant passes through the point A(-3,4), then find the vector of moment of the resultant with respect to the point B (2,-5).
3. ABCD is a rectangle in which AB = 8cm, BC = 4 cm the forces of magnitudes 12 ,10 ,F ,K kg.wt act at $\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{CD}$, $\overrightarrow{AD}$ respectively ,if the sum of algebraic measure of the moments of the forces vanishes about each of A and C find F,K .
4. ABCD is a square its diagonals intersect at M , forces of magnitudes 6 , F ,K , 3 newton act at $\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{CD}$, $\overrightarrow{AD}$ respectively if the sum of the algebraic measure of the moments of the forces vanishes about each of B and M find F, K.

LESSON (3)

MOMENT OF A FORCE ABOUT A POINT IN 3-D COORDINATE SYSTEM

IF : $\vec{F} = (F_X, F_Y, F_Z)$ acts at the point A(L ,m , n)

then : $\overrightarrow{M_A} = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{\imath} & \vec{j} & \vec{k} \\ L & m & n \\ F_X & F_Y & F_Z \end{vmatrix}$

Example (1) :

The force $\vec{F} = 2\vec{\imath} - \vec{j} + 3\vec{k}$ act at the point A (-3,1,2) find the moment of the force $\vec{F}$ about the point B (2,2,-1) ,then find the length of perpendicular from B to the line of action of the force ?

Solution

$\overrightarrow{M_B} = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{\imath} & \vec{j} & \vec{k} \\ -5 & -1 & 3 \\ 2 & -1 & 3 \end{vmatrix} = 21\,\vec{j} + 7\vec{k}$

L= $\frac{\sqrt{0^2+21^2+7^2}}{\sqrt{2^2+(-1)^2+3^2}} = \sqrt{35}$ length unit

Example (2) :

The force $\vec{F} = K\vec{\imath} + 4\vec{j} - \vec{k}$ acts at the point A its position vector with respect to the origin point is $(1\ ,2\ ,2\ )$and the component of the moment $\vec{F}$ about Y-axis is 7 moment unit ,find the value of K ,then find the length of perpendicular from (O) to the line of action of $\vec{F}$?

Solution

$$\overrightarrow{M_O} = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{\imath} & \vec{j} & \vec{k} \\ 1 & 2 & 2 \\ K & 4 & -1 \end{vmatrix} = -10\,\vec{\imath} - (-1-2K)7\vec{j} + (4-2K)\vec{k}$$

$$L = \frac{\|\overrightarrow{M_O}\|}{F} = \frac{\sqrt{(-10)^2+7^2+(-2)^2}}{\sqrt{3^2+4^2+(-1)^2}} = 2.4$$ length unit

EXERCISE (3)

1. If the force $\vec{F} = 2\vec{\imath} + 3\vec{\jmath} - \vec{k}$ acts at the point A (1,-1, 4) find :

   # the moment $\vec{F}$ about the origin point .

   # the moment $\vec{F}$ about the point B(2,-3 ,1),then deduce the length of Perpendicular from B to the line of action of the force.

2. If the force $\vec{F} = 2\vec{\imath} + L\vec{\jmath} - \vec{k}$ acts at the point A(4,-2,0) and the moment of $\vec{F}$ about the origin point equals $2\vec{\imath} + 4\vec{\jmath} + 16\vec{k}$ .What is the value of L
3. If the force $\vec{F} = 2\vec{\imath} + b\vec{\jmath} + \vec{k}$ acts at the point A(-1,3,-2) and the component of the moment of $\vec{F}$ about X-axis is 3 moment unit ,find the value of b ,then find the length of perpendicular from the origin point to the line of action of the force.
4. A force $\vec{F} = 15\vec{\imath} - 25\vec{\jmath} + 40\vec{k}$ acts at the point A(-3,-3,2),find the component of the moment of $\vec{F}$ about Y -axis

LESSON (4)

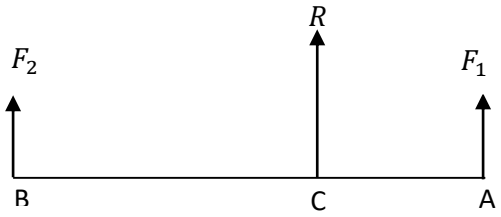
RESULTANT OF TWO PARALLEL FORCES

First resultant of two parallel forces in the same direction :

then:

$$R = F_1 + F_2$$

$$F_1 \; x \; AC = F_2 \; x \; BC$$



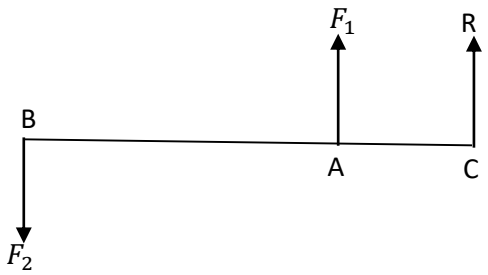
Second :resultant of two parallel forces in the two opposite directions:

Suppose that $F_1 > F_2$

$$R = F_1 - F_2$$

then:

$$F_1 \; x \; AC = F_2 \; x \; BC$$



NOTE That :

If the norm of the resultant is less than one of two forces, then two forces are in two opposite directions.

Example (1):

Two parallel forces are in the same direction of magnitudes 20 , 30 newton act at the

two points A , B respectively ,find the magnitude and the direction of the resultant of the two forces and the distance between its point of action and the point B

knowing that AB = 100 cm

Solution

R = $F_1 + F_2$ =20 + 30 =50 newton (and in the same direction of the two forces)

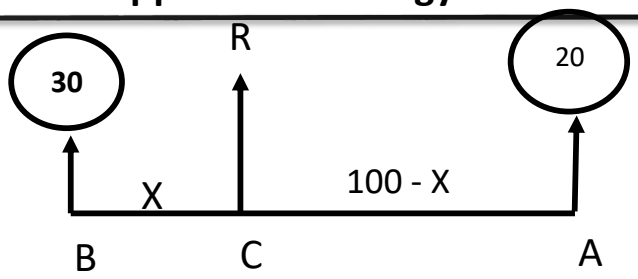
R

20 (100- x) = 30 x

2000 -20 x = 30 x

50 x = 2000

∴ x = 40 cm

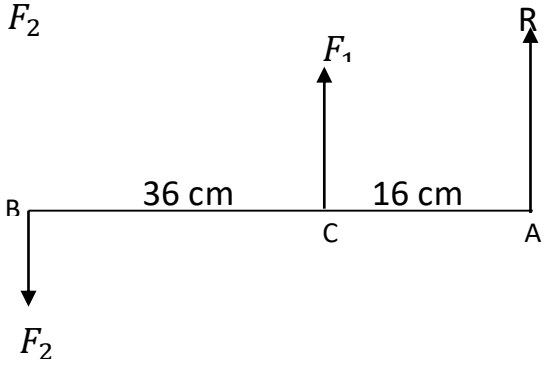


the magnitude of the resultant is 50 newtons in the direction of the two forces at 40 cm from B .

EXAMPLE (2) :

F_1 , F_2 are two parallel forces in opposite directions act at the two points A, B respectively, $F_1 > F_2$, if the magnitude of the resultant of the two forces = 90 kg.wt and acts at the point C ∈ $\overleftrightarrow{AB}$ where AB = 36 cm , AC = 16cm ,find F_1 , F_2

Solution



R = $F_1 - F_2$

∴ $F_1 - F_2$ = 90 (1)

$F_1 \times 16 = F_2 \times 52$ (2)

By solving the two equations (1) , (2)

$$\therefore F_1 = 130\ kg.wt \quad , F_2 = 40\ kg.wt$$

EXAMPLE (3):

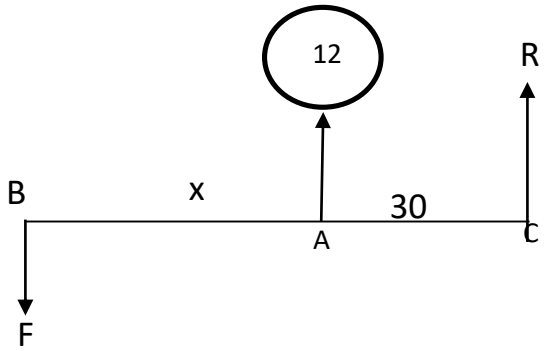
Two parallel forces of magnitudes 12, F newton and their resultant is 3 newtons and its line of action is 30 cm distant from the line of action of the first force, show that F has two values and find the distance between the lines of action of the two forces in the two cases.

Solution

The resultant is less than one of the two forces:

∴ the two forces are in opposite directions and

FIRST : 12 > F



R = 12- F

3 = 12- F F= 9 newton

9(30 +x) = 12 x 30 X = 10 cm

Then the force (F) of magnitude 9 newton and at 10 cm distant from the first force .

SECOND : 12 > F

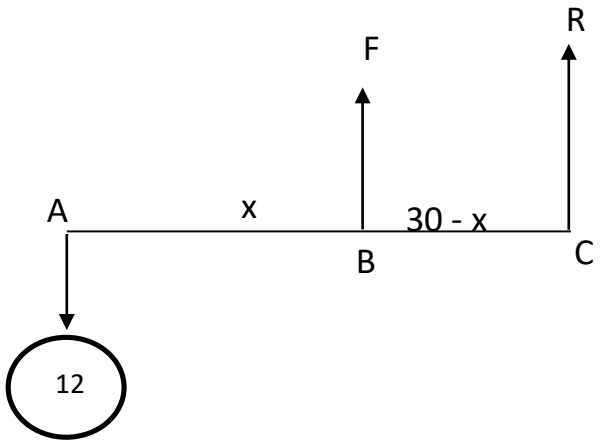
R = F - 12

3 = F - 12

F = 15 newton

15 (30 - x)= 12 x 30

x = 6 cm



then the force(F) of magnitude 15 newton and at 6 cm distant from the first force.

Exercise (4)

1- Two parallel forces of magnitudes 4,6 newton act in the same direction and act the two points A,B where :AB= 25 cm find the resultant of the two forces

2-Find the resultant of two parallel forces in opposite directions if their magnitudes are 7,12 newton and act at A, B where AB =20 cm.

3- Two parallel forces the magnitude of their resultant is 350 newton , the magnitude of one of the two forces is 500 newton and acts at a distance 51 cm from the resultant find the second force and the distance between the two lines of action of the two forces if the given force and the resultant act :

First: in the same direction Second: in two opposite directions

4- Two parallel forces and in two opposite directions of magnitudes 7 ,12 newton find the magnitude of their resultant.

5-Two parallel forces in the same directions of magnitudes 7 ,10 newton act at the two points A, B where:

AB = 51 cm if their resultant act at the point C find AC

LESSON (5)

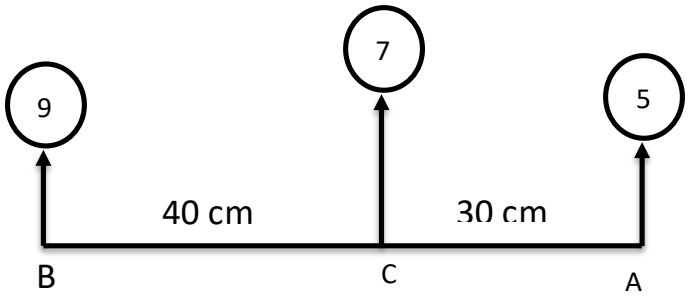
RESULTANT OF A SET OF PARALLEL FORCES

Example (1) :

In the opposite figure :

Find the magnitude and

the direction of the resultant?



solution

R= 5 + 7 +9 = 21 force unit (upwards)

Sum of moments about A = the moment of the resultant about A

-7 X 30 -9 X70 = -21 x X

∴ X = 40 cm

The magnitude of the resultant is 21 force unit upwards to the left of (A) and at 40 cm

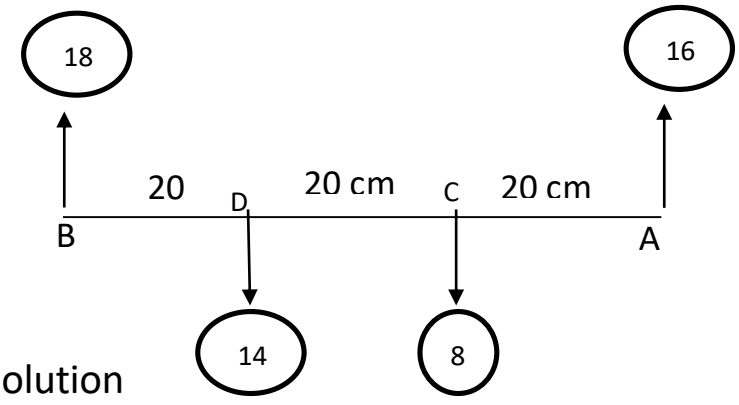
distant from it.

Example (2) :

In the opposite figure:

Find the magnitude and the direction

of the resultant?



Solution

R= 16 + 18 – 8 -14 = 12 force unit (upwards)

Sum of moments about B = moment of the resultant about B

-14 x 20 - 8 x 40 + 16 x 60 = 12 x X

∴ X = 30 cm

The magnitude of the resultant is 12 force unit upwards to the right of (B) and at 30 cm distant from it .

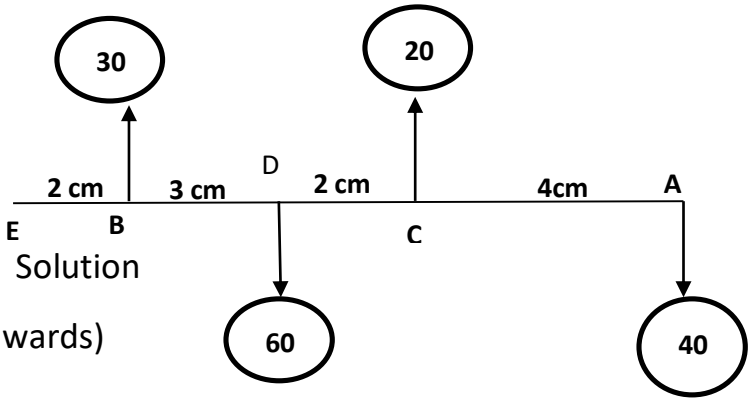
Example (3) :

A , B,C, D ,E are five points lie on a straight line where : AB = 4 cm , BC = 2 cm , CD = 3 cm, DE = 2 cm , the two forces 20 , 30 newton act vertically upwards at the two points B ,D and the two forces 40 ,60 newton act vertically downwards at the two points A ,C find the magnitude and the direction of the resultant and the point of action of the resultant .

Solution

In the opposite figure:

Find the magnitude and the direction of the resultant?



Solution

R= 40 + 60 - 20 - 30 = 50 newton (downwards)

Sum of moments of the forces about E = the moment of the resultant about E

- 40 x 11 – 60 x 5 + 20 x 7 + 30 x 2 = -50 x X

∴ X = 10.8 cm

The magnitude of the resultant is 12 newton downwards to the left of (E)

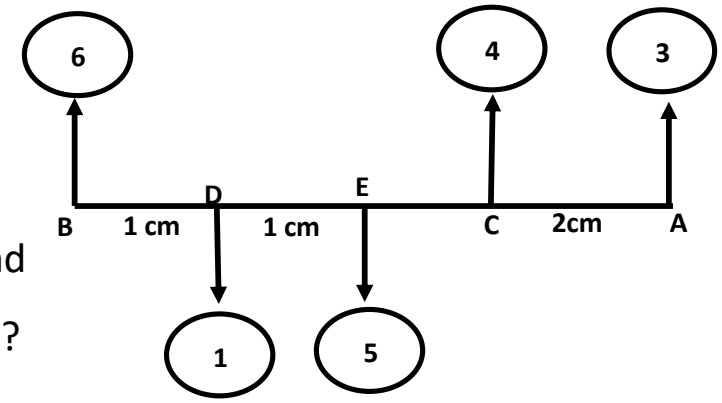
and at a distance 10.8 cm from it

Exercise (5)

1- In the opposite figure :

A set of parallel forces are perpendicular to $\overline{AB}$

find the magnitude, the direction and the point of action of the resultant?

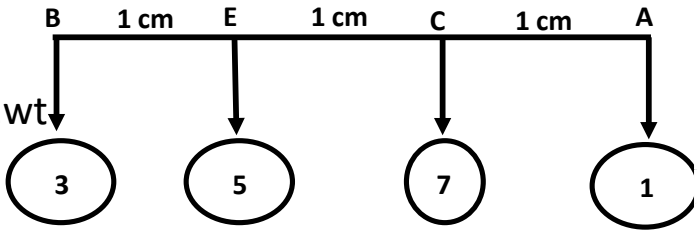


2- A , B,C, D ,E are five points lie on a straight line where : AB = 4 cm , BC = 6 cm , CD = 8 cm

and DE = 10 cm , the forces 60 , 30 , 50 , 80 , 40 gm.wt act at the points A ,B C , D ,E respectively and they are perpendicular to $\overline{AE}$, such that the first three forces in the same direction and the other two forces in the opposite direction ,determine the resultant of the set of forces ?

3-In the opposite figure:

Four weights of magnitudes 1 ,7 ,5,3 gm.wt are placed on a light rod. determine the point of suspension which keeps the rod horizontal?



LESSON (6)

Equilibrium a set of coplanar parallel forces

If a body is in equilibrium under the action of set of coplanar parallel forces then :

- The sum of algebraic measures of the forces = zero
- The sum of algebraic measures of the moments of the forces about any point in its plane

equals zero

Expressions guide to the equilibrium:

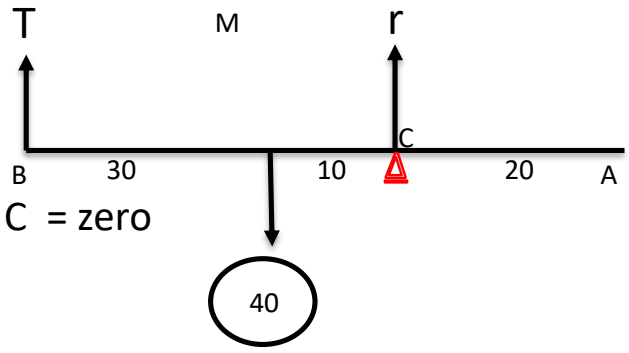
About to move – about to rotate – without upsetting the equilibrium –

the least weight can be hanged - the maximum weight can be hanged

Example (1) :

$\overline{AB}$ is a uniform rod of length 60 cm and weight 40 gm .wt rests on a support at 20 cm distant from A ,the rod is kept in equilibrium horizontally by a light vertical string connects at its end B ,find the magnitude of the tension in the string and the reaction of the support ?

Solution



r + T = 40

the sum of moments of the forces about C = zero

40 x 10 -T x 40 = zero

∴ T = 10 gm.wt

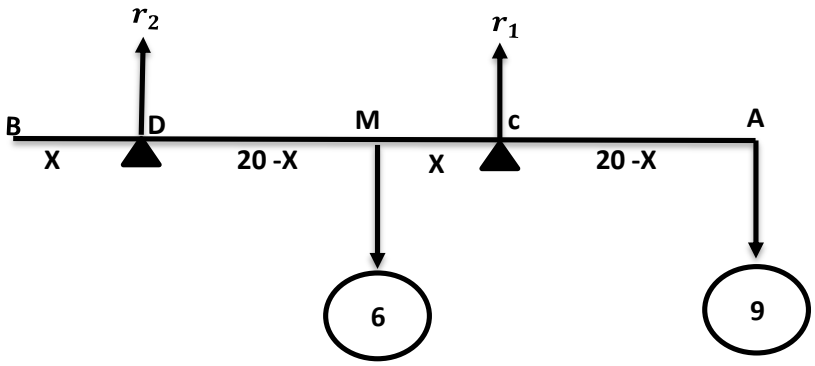
∴ r= 30 gm.wt

Example (2) :

$\overline{AB}$ is a uniform rod of length 40 cm and weight 6 kg .wt rests horizontally on two supports at C and D , the distance between the two supports is 20 cm, it is found that the rod is about to rotate if a weight 9 kg.wt is suspended from A.

find the distance between the two supports.

Solution



when a weight 9 kg.wt hanged from A, then the rod about to rotate

$\therefore$ r_2 = 0

the sum of moments of forces about C = 0

6 X - 9 (20 - X) = 0

6X -180 + 9X = 0 $\therefore$ X = 12 cm

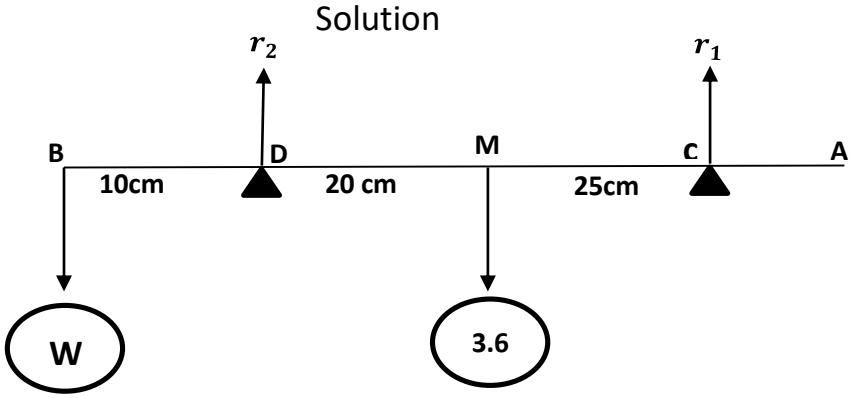
$\therefore$ the distance between the support C and the point A = 8 cm

the distance between the support D and the point B = 12 cm

Example (3) :

$\overline{AB}$ is a uniform rod its length 60 cm and weight 3.6 kg .wt. rests horizontally on two supports one of them is 5 cm distant from A and the other is 10 cm distant from B, find the pressure on the two supports, then the maximum weight can be hanged from the end B without upsetting the equilibrium of the rod?

Solution



$r_1 + r_2$ =3.6

The sum of moments of the forces about c =0

3.6 x 25 -r_2 x 45 =0

∴ r_2 = 2 kg.wt

r_1 = 1.6 kg.w

Note that :

When hanging the weight at B

r_1 = 0

The sum of moments of the forces about D = 0

W x 10 – 3.6 x 20 =0

∴W = 7.2 kg.wt

Exercise (6)

1-Two supports at A,B at a distance 20 cm from the two ends of a wooden board of length 200 cm and weight 16 kg.wt. acts at its midpoint and carries a box of weight 24kg.wt.at 60cm from the support (A) .

Find the pressure on the two supports?

2-AB is a uniform wooden board of mass 10 kg. and length 4 metre rests in a horizontal position on two supports one of them at A and the other at a point

1 metre distant from B , show the distance at which a child of weight 50 kg.wt stands on the board to make the reactions on the two supports equal.?

3-A rod AB its length 90 cm and weight 50 newton and acts at its midpoint rests in a horizontal position on two supports one of them at A and the other at C

30 cm distant from B and carries a weight 20 newton at a point 15 cm distant from B, determine the values of the pressure on each support, also find the magnitude of the weight should be hanged from the end B to make the rod about to rotate.

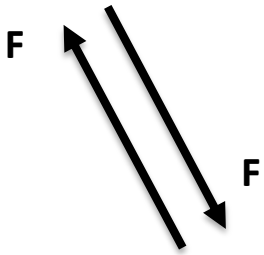
4-A uniform rod of length 2 metres and mass 75 kg.wt. rests in a horizontal position on two supports at its ends, a weight 15 kg.wt.is suspended from a point on the rod at 50 cm distant from one of its ends, find the reaction on each support

Lesson 7
Couple

Definition:

Couple: a system of two forces of equal magnitudes

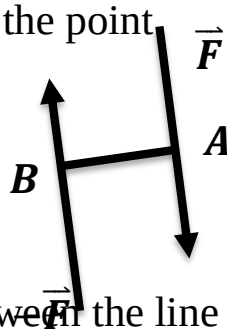
and in opposite directions and acting in different lines of action.



Theorem:

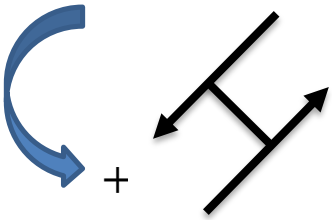
The moment of a couple is a constant vector does not dependent on the point about which we take the moments of the two forces

$$\vec{M} = \overrightarrow{BA} \times \vec{F} = \overrightarrow{AB} \times -\vec{F}$$

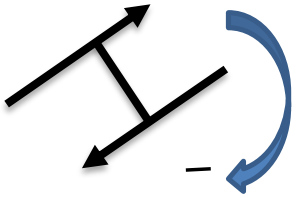


- Norm of the moment of the couple =

magnitude of one of the two forces x the perpendicular distance between the line of action of two forces



M = F x r



- The sign of the algebraic moment of the couple is negative if the rotation is clockwise.
- The sign of the algebraic moment of the couple is positive if the rotation is anticlockwise.

Definition:

- If the two couples are in equilibrium, then the sum of their moments = zero
  i.e. $M_1 + M_2 = 0$
- If the two couples are equivalent, then $M_1 = M_2$

Example 1:

Two forces $\vec{F}_1 = 2\vec{\imath} - b\vec{\jmath}, \vec{F}_2 = a\vec{\imath} - 5\vec{\jmath}$ are forming a couple. Find a and b

Solution

∵ The two forces form a couple ∴ $\vec{F_1} = -\vec{F_2}$

∴ a = -2 b = 5

Example 2:

If the magnitude of the moment of a couple is 350 newton.m and the magnitude of one of the two forces is 70 newton, then: find the length of

the arm of the moment.

Solution

M = F x r ∴ $350 = 70r$ ∴ $r = 5\ metre$

Example 3:

Two forces $\vec{F_1} = 5\vec{i} + a\vec{y}\vec{j} + 3\vec{k}$, $\vec{F_2} = b\vec{i} - 9\vec{j} + c\vec{k}$ **are forming a couple. Find a + b + c**

Solution

b = -5 , a = 9 , c = - 3 ∴ $a + b + c = 1$

Example 4:

If $\vec{F_1}, \vec{F_2}$ are two forces form a couple where $\vec{F_1} = -3\vec{i} + 2\vec{j}$ acts at the point A (1, 1), $\vec{F_2}$ acts at the point B (-1, -2) find the moment of the couple and the length of perpendicular drawing from A on the line of action of $\vec{F_2}$

Solution

$\vec{M} = \overrightarrow{AB} \times \vec{F_2}$ =(-2,-3) x (3 ,-2) = (4+ 9) $\vec{k}$ = 13 $\vec{k}$

M = F x r ∴ $13 = \sqrt{13}\, r$ ∴ $\sqrt{13}$ length unit

Exercise (7)

1. Two forces $\vec{F}_1 = 3\vec{\imath} - b\vec{j}, \vec{F}_2 = a\vec{\imath} - 5\vec{j}$ are forming a couple. Find (a , b)

2. A couple consists of two forces each of magnitude 12 newton and the length of perpendicular between them is 8 cm is equivalent to a couple of two forces the perpendicular distance between them 6 cm. Find the magnitude of each its two forces

3. If $\vec{F}_1 = 4\vec{\imath} - a\vec{j}$, $\vec{F}_2 = 2b\vec{\imath} - 5\vec{j}$ are forming a couple. Find 2a + b

4. If $\overrightarrow{M_1}, \overrightarrow{M_2}$ are two equilibrium couples and $\overrightarrow{M_1} = 20\,\overrightarrow{k}$, then: find $\overrightarrow{M_1} - \overrightarrow{M_2}$

5. If $\vec{F}_1, \vec{F}_2$ are two forces act at the two points A(1, 3), B (0, 5) respectively and form a couple its moment vector is $20\,\vec{k}$, find $\vec{F}_1$

Lesson (8)
Resultant couple (First case)

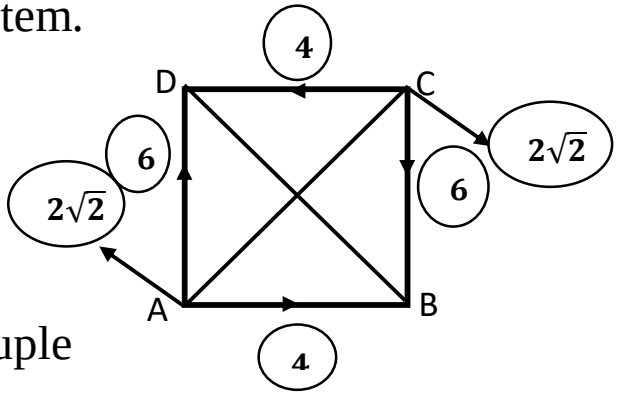
First case:

If a set of forces form many couples,
then the set is equivalent a resultant couple.

Its moment = the sum of algebraic measure of the moment of these couples

Example (1)

ABCD is a square of side length 30 cm, forces of magnitude 6, 4, 6, 4 newton act at $\overrightarrow{AB}, \overrightarrow{CB}, \overrightarrow{CD}, \overrightarrow{AD}$ respectively and two forces each of magnitude $2\sqrt{2}$ newton act at the two points A , B in the direction of $\overrightarrow{BD}, \overrightarrow{DB}$ respectively . Find the algebraic measure of the moment of the couple which equivalent to the system.



- The two forces (4, 4) form a couple
  $\therefore M_1 = 4 \times 30 = 129$ newton.cm
- The two forces (6, 6) form a couple
  $\therefore M_2 = -6 \times 30 = -180$ newton.cm
- The two forces $(2\sqrt{2}, 2\sqrt{2})$ form a couple
  $\therefore M_2 = -2\sqrt{2} \times 30\sqrt{2} = -120$ newton.cm

$\therefore$ the system is equivalent to a couple the algebraic measure of its moment = 120 – 180 – 120 = -180 newton.cm

Remember that:

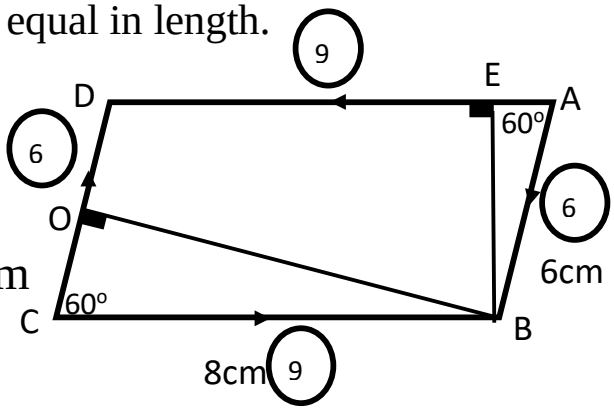
- The two diagonals of the square are perpendicular, equal in length and bisect each other
- The length of the diagonal = $\sqrt{2}$ L where L is the side length

Example (2)

ABCD is a parallelogram in which AB = 6cm, BC= 8cm, m(∡A)= 60°. forces of magnitude 6, 9, 6, 9 gm. wt act at $\overrightarrow{AB}, \overrightarrow{CB}, \overrightarrow{CD}, \overrightarrow{AD}$ respectively prove that the system equivalent to a couple and find its moment.

Solution

- Two diagonals are not equal , not perpendicular and bisect each other.
- Each two opposite sides are parallel and equal in length.



BE= 6 sin 60^{O} = 3 $\sqrt{3}$ cm

BO = 8 sin 60^{O}=3 $\sqrt{3}$ cm

- The two forces (6, 6) form a couple

  ∴M_1= - 6 x 4 $\sqrt{3}$ = -24 $\sqrt{3}$ hg.wt.cm
- The two forces (9, 9) form a couple

  ∴M_2= 9 x 3 $\sqrt{3}$ = 27 $\sqrt{3}$ hg.wt.cm

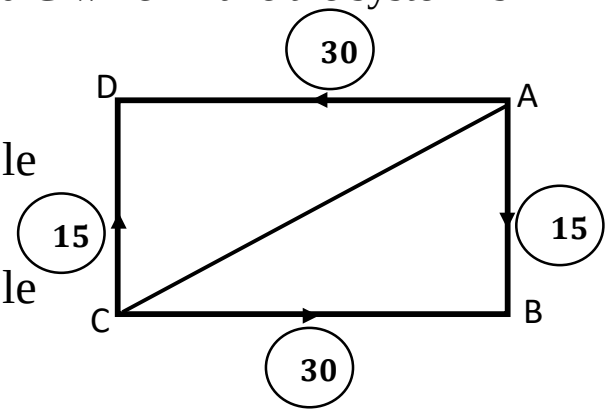
- ∴ the system is equivalent to a couple,

  the algebraic measure of its moment = 3 $\sqrt{3}$ gm.wt.cm

Example (4):

ABCD is a rectangle in which AB =30 cm,BC = 40 cm . forces of magnitude 15, 30, 15, 30 newton act at $\overrightarrow{BA}, \overrightarrow{BC}, \overrightarrow{DC}, \overrightarrow{DA}$ respectively, prove that the system equivalent to a couple and find its moment and then find two forces act perpendicular to $\overline{AC}$ at the two points A and C which make the system is in equilibrium.

Solution



- The two forces (15, 15) form a couple

  ∴M_1= 15 x 40 = 600 newton.cm
- The two forces (30, 30) form a couple

$\therefore M_2 = -30 \times 30 = -900$ newton.cm

The system is equivalent to a couple,

the algebraic measure of its moment = -300 newton.cm

- To make the system in equilibrium ⟶ F x AC – 300 = zero
- ∴ F = 6 newton ⟶ the two forces are 6, 6 newton

Remember that:

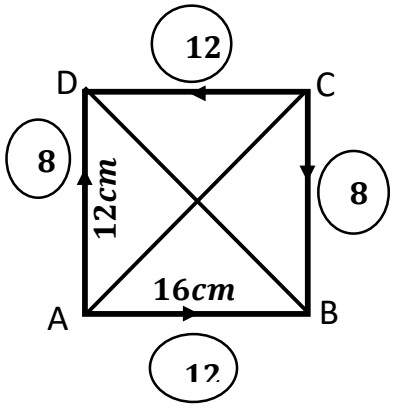
- The two diagonals of the rectangle are equal in length, are not perpendicular and bisect each other
- Each two opposite sides are parallel and equal in length.
- All its angels are equal in measure, each of measure 90^o

Exercise (8)

1. In the opposite figure :

ABCDI is a square

Find the algebraic measure of the moment of the couple of the system of forces shown in the figure.



2. ABCD is a rectangle in which AB =12 cm, AD = 5 cm two forces each of magnitude 39 newton act at the two points A,C and their line of action in direction of $\overrightarrow{BD}, \overrightarrow{DB}$ .Find the norm of the moment of the couple

3. ABCD is a rectangle in which AB =30 cm, BC = 40 cm . forces of magnitude 15, 30, 15, 30 newton act at $\overrightarrow{BA}, \overrightarrow{BC}, \overrightarrow{DC}, \overrightarrow{DA}$ respectively find the algebraic measure of the moment of the given forces. If two forces act at the two points A and C perpendicular to $\overline{AC}$ which make the system in equilibrium

find the two forces.

4. ABCD is a parallelogram in which CD = 6cm, AD= 8cm, m(∠C) = 60°. forces of magnitude 8, 10, 8, 10 gm. wt act at $\overrightarrow{AB}, \overrightarrow{CB}, \overrightarrow{CD}, \overrightarrow{AD}$ respectively

prove that the system equivalent to a couple and find its moment.

Lesson (9)

Resultant couple (second and third)

Second case:

A system of coplanar forces is said to be equivalent to a couple if the following two conditions are satisfied together:

1. The sum of the algebraic components of the forces in any direction = zero.
2. The sum of the moment of the forces about any point $\neq 0$

Third case:

If the sum of the algebraic measure of the moment of a system of coplanar forces with respect to three non collinear point in its plane is constant and is not equal to zero, this system equivalent to a couple the algebraic measure of its moment equals this constant.

Example (1):

The forces $\vec{F}_1 = 2\vec{\imath} + 3\vec{\jmath}$, $\vec{F}_2 = \vec{\imath} + 2\vec{\jmath}$, $\vec{F}_3 = -6\vec{\imath} + \vec{\jmath}$, $\vec{F}_4 = 3\vec{\imath} - 6\vec{\jmath}$ act at the points A (2 , 4), B (3 , 2), C (1 , 2), D (1 , 1).

Prove that the system equivalent to a couple and find its moment

Solution

$\vec{R} = (2,3) + (1,2) + (-6,1) + (3,-6) = \vec{0}$

$\overrightarrow{M_O} = (2,4) \times (2,3) + (3,2) \times (1,2) + (1,2) \times (-6,1) + (1,1) \times (3,-6)$

$= -2\,\vec{k} + 4\,\vec{k} + 13\,k - 9\vec{k} = 6\,\vec{k}$

∴ The system is equivalent to a couple

the algebraic measure of its moment = 6 moment unit

Example(2):

The two forces $\vec{F}_1 = 3\vec{\imath} - 4\vec{\jmath}, \vec{F}_2 = -3\vec{\imath} - 4\vec{\jmath}$ act at A(1, 2), B (-3 ,4) respectively. Prove that they form a couple and find the norm of its moment.

Solution

$$\vec{R} = (3,-4) + (-3,4) = \vec{0}$$

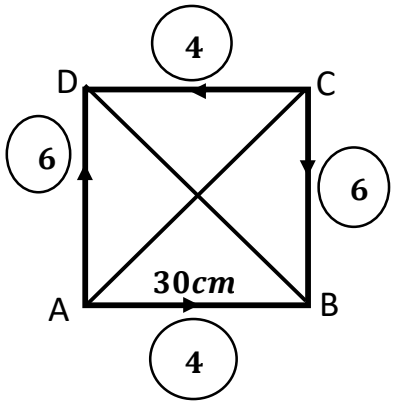
$$\overrightarrow{M_B} = \overrightarrow{BA} \times \vec{F} = (4,-2) \times (3,-4) = -10\,\vec{k}$$

∴The system is equivalent to a couple the norm of its moment = 10 moment unit

Example (3):

ABCD is a square of side length 30 cm , forces of magnitude 8, 10, 8, 10 newton act at $\overrightarrow{AB}, \overrightarrow{CB}, \overrightarrow{CD}, \overrightarrow{AD}$ respectively . Find the algebraic measure of the moment of the couple which equivalent to the system.

Solution



$$\overrightarrow{M_A} = -6 \times 30 + 4 \times 30 = -60\ newton.cm$$

$$\overrightarrow{M_B} = -6 \times 30 + 4 \times 30 = -60\ newton.cm$$

$$\overrightarrow{M_C} = -6 \times 30 + 4 \times 30 = -60\ newton.cm$$

The system equivalent to a couple,

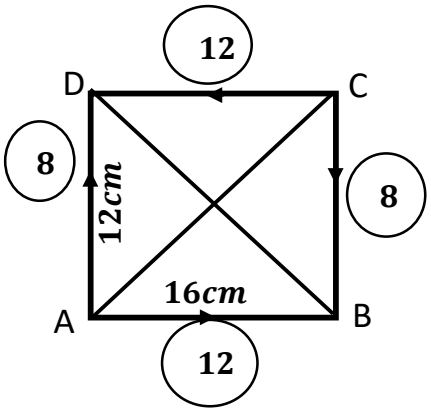
the algebraic measure of its moment = -60 newton.cm

Notice that:

This example can be solved by using the first case.

Exercise (9)

1. In the opposite figure:

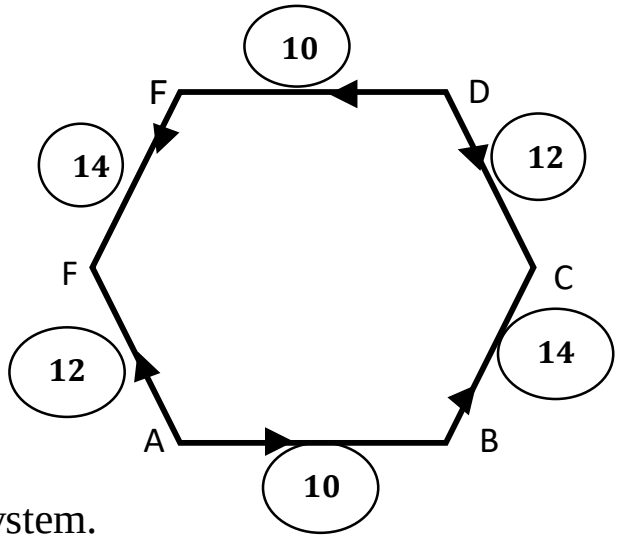


By using the third case

find the algebraic measure of the moment

of the couple equivalent to the given system.

2. In the opposite figure:



ABCDEF is a regular hexagon whose

side length is10 cm.

By using the third case

find the algebraic measure of the moment

of the couple that equivalent to the given system.

Notice that:

The perpendicular distance between any two parallel sides in the regular hexagon of side length L is $\sqrt{3}\,L$

3. The forces $\vec{F}_1 = 2\vec{\imath} - 4\vec{\jmath}$, $\vec{F}_2 = \vec{\imath} - 3\vec{\jmath}$, $\overrightarrow{F_3} = -3\vec{\imath} + 7\vec{\jmath}$ act at the points A(-1 , 1), B (-2 , 3), C (0 , 1) respectively .

Prove that the system is equivalent a couple and find the norm of its moment

Lesson (10)

The fourth case:

If three coplanar forces act on a rigid body and are completely represented by the sides of a triangle taken the same way round, then this system is equivalent to a couple.

The magnitude of its moment = 2 × the area of the triangle × the magnitude of the force represented the unit length

- This rule is generalized for more than three forces.

Example (1):

ABC is an equilateral triangle of side length 10 cm, equal forces of magnitude 2 newton act at $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$, $\overrightarrow{AB}$ respectively. Find the magnitude of the moment of the equivalent couple to the system.

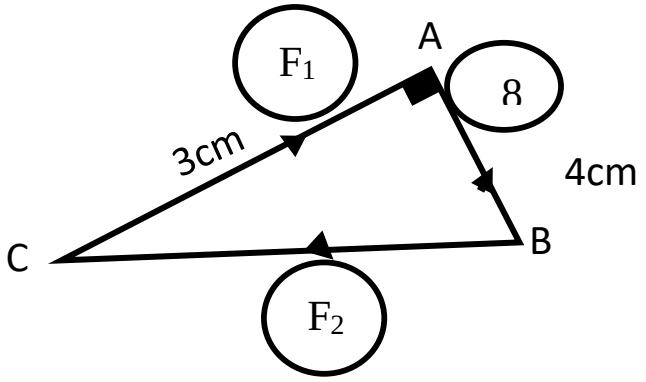
Solution

The forces in one cyclic order $\frac{force}{side\ legnth} = \frac{2}{10} = \frac{1}{5}$

The magnitude of its moment = $2 \times \frac{1}{2} \times 10 \times 10 \sin 60° \times \frac{1}{5} = 10\sqrt{3}$newton.cm

Example (2):

ABC is a right angled triangle at A, AB = 4 cm, AC = 3 cm the shown forces measured by newton are represented completely by the sides of the triangle. Find $F_1 + F_2$



Solution

BC = 5cm (Pythagoras theorem)

The forces in one cyclic order and represented completely by the sides of the triangle.

$$\therefore \frac{8}{4} = \frac{F_1}{3} = \frac{F_2}{5} \qquad \therefore F_1 = 6\ newton, F_2 = 10\ newton$$

$$\therefore F_1 + F_2 = 16 newton$$

Example (3):

ABCD is a rectangle in which AB = 9, BC = 24 cm, O is the midpoint of $\overline{AD}$, the forces of magnitude 18, 48, 30, 24 gm.wt act at $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CO}, \overrightarrow{OA}$ respectively. Prove that the system equivalent a couple and find the magnitude of its moment.

Solution

CO = 15 cm (Pythagoras theorem)

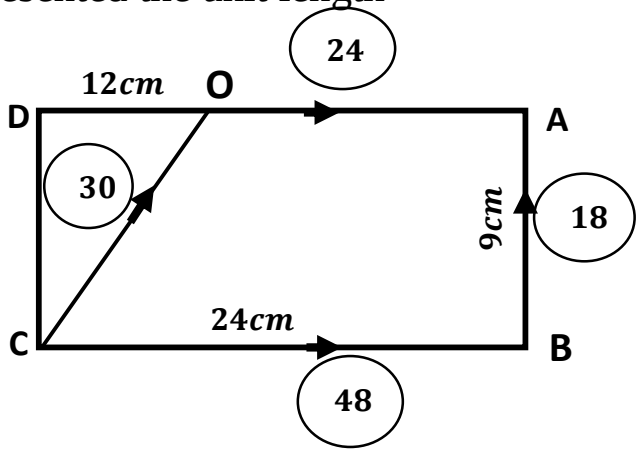
The forces in one cyclic order and represented completely by the sides of the triangle.

$\therefore \frac{18}{9} = \frac{48}{24} = \frac{24}{12} = 2$ $\therefore$ the system is equivalent to a couple

The magnitude of its moment = 2 × the area of the triangle × the magnitude of the force represented the unit length

The magnitude of its moment =

$$2 \times \frac{1}{2}(24+12) \times 9 \times 2 = 324\ gm.wt.cm$$



Exercise (10)

1. ABC is a triangle in which AB = AC = 13 cm, BC = 24 cm, the forces of magnitude 39, 72, 39 newton act at the $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$ respectively. Prove that the system equivalent a couple and find the magnitude of its moment.
2. ABCD is a rectangle in which AB = 9cm, BC= 24cm,X is the midpoint of $\overline{BC}$ forces of magnitudes 27, 36, 45 newton act at $\overrightarrow{AB}, \overrightarrow{BX}, \overrightarrow{XA}$ respectively. Prove that the system equivalent to a couple and find the magnitude of its moment.
3. ABC is a triangle in which AB = 7 cm, BC = 8 cm, m (∠ABC) = 120^{O} forces of magnitude 17.5, 20, 32.5 newton act at $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$ respectively if the system equivalent to a couple find the magnitude of its moment.
4. Three forces of magnitude 10.5, 12, 19.5 newton represented completely by the directed line segment $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$ respectively on the triangle ABC where AC= 13 cm, then
5. ABCD is a trapezium in which $\overline{AD}$ // $\overline{BC}$, m(∠ A) = 90º , AD = 10 cm , AB = 12 cm , BC = 15 cm forces of magnitude F_1 , F_2, 26, F_3 newton act on $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}, \overrightarrow{DA}$ respectively. If the system equivalent to a couple find the value of $F_1 + F_2 + F_3$

Test

First question

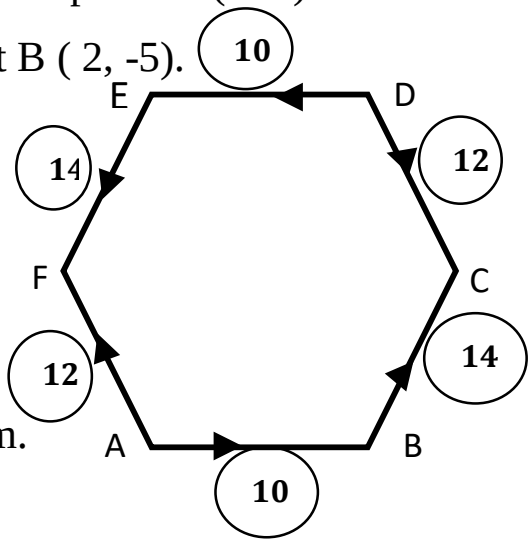
1. ABC is a triangle in which AB = 7 cm, BC = 8 cm, m(∡ABC) = 120^{O} forces of magnitude 17.5 , 20, 32.5 newton act at $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$ respectively if the system. equivalent to a couple find the magnitude of its moment.
2. The force $\overrightarrow{F} = 6\vec{\imath} + 8\vec{\jmath}$ act at a point A(-1,2) find the moment of $\overrightarrow{F}$ about the point B (2, 5)

Second question:

1. Two forces $\vec{F}_1 = 5\vec{\imath} - 4\vec{\jmath}$, $\vec{F}_2 = \vec{\imath} - 4\vec{\jmath}$ act at the origin point, prove that the line of action of their resultant passes through the point A (-3.4). Find the moment vector of the resultant about the point B (2, -5).
2. In the opposite figure:

   ABCDEF is a regular hexagon whose side length 10 cm.

   find the algebraic measure of the moment of the couple that equivalent to the given system.



Third question:

1. If the force $\overrightarrow{F} = 2\vec{\imath} - l\vec{\jmath} - \vec{k}$ acts at the point A (4 , -2 , 0), the moment of $\overrightarrow{F}$ about the origin point equals $2\vec{\imath} + 4\vec{\jmath} + 16\,\vec{k}$, then find the value of l

2. ABCD is a parallelogram in which CD = 6cm, AD= 8cm, m(∠C)= 60°.

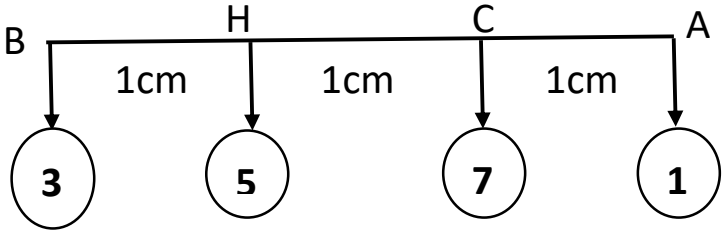
   forces of magnitude 8, 10, 8, 10 gm. wt act at $\overrightarrow{AB}, \overrightarrow{CB}, \overrightarrow{CD}, \overrightarrow{AD}$ respectively

   prove that the system equivalent to a couple and find its moment.

Fourth question:

1. Two parallel forces of magnitude 4, 6 newton act at the points A and B in the same direction, AB= 25 cm find the resultant of the two forces.
2. A couple consists of two forces each of magnitude 12newton and the perpendicular distance between them equals 8 cm, is equivalent to another couple consists of two forces and the perpendicular distance between them 6 cm find the magnitude of the two forces.

Fifth question:



1. In the opposite figure :

   Four weights of magnitudes 1, 7, 5, 3 kg.wt are placed on a light rod. Determine the point of suspension that makes the rod in horizontal position.
2. AB is a regular rod of length 40 cm and weight 6 kg.wt rests on two supports at C and D in horizontal position , the distance between the two supports is 20 cm .If a weight of magnitude 9 kg.wt is suspended at the point A, it makes the rod about to rotate. Find the position of the two supports

Evaluation