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CSE 5542 HW2

① 1) Matrix A is the viewport transformation matrix. The viewport matrix maps the unit image rectangle, or cannonical view, to pixel coordinates on screen space.

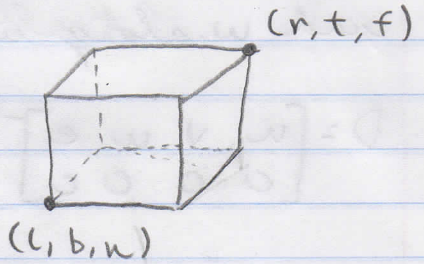
$$A = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- n_x is the number of horizontal pixels.
- n_y is the number of vertical pixels.

Matrix B is the orthographic projection matrix. This matrix transforms vertex data from camera space to the connonical view space.

$$B = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- r is the right plane
- l is the left plane
- t is the top plane
- b is the bottom plane
- n is the near plane
- f is the far plane



Additionally, this matrix gives a view with an infinite vanishing point, so all objects in this view look as if they are the same distance from each other.

Matrix C is the perspective transformation matrix. This matrix transforms vertex data from camera space to the cannonical view space. This transformation matrix utilizes the z-axis coordinates of each vertex in order to properly project the vertex onto the viewing plane, thus giving the image a sense of depth.

$$C = \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- n is the near plane distance
- f is the far plane distance

Matrix D is the camera transformation matrix. This matrix is responsible for transforming vertex data from world to camera space coordinates. The matrix is comprised of basis vectors u, v, and w along with eye position e.

$$D = \begin{bmatrix} u & v & w & e \\ 0 & 0 & 0 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix E is the <u>model</u> matrix. This matrix is responsible for transforming model vertex data from model space to world space. This matrix stores all translation, rotation, and scaling operations performed on an object.

So, $p' = A \cdot B \cdot C \cdot D \cdot E \cdot p$

$$\Updownarrow$$

$$p' = M_{vp} \cdot M_{orth} \cdot P \cdot M_{cam} \cdot M_m \cdot p$$

2) $A = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x - 1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y - 1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\vec{w} = \frac{-\overrightarrow{look}}{\|\overrightarrow{look}\|}$ $\qquad e = [x \ y \ z \ 1]$

$\vec{u} = \frac{\overrightarrow{up} \times \vec{w}}{\|\overrightarrow{up} \times \vec{w}\|}$

$\vec{v} = \vec{w} \times \vec{u}$

$$D = \begin{bmatrix} \vec{u} & \vec{v} & \vec{w} & e \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_w & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x \\ 0 & 1 & 0 & -y \\ 0 & 0 & 1 & -z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

E is the concatination of scaling, rotation, and translation operations that are used to transform the models vertex data.

Let $B \cdot C = M_{per}$, where M_{per} is the perspective projection matrix.

$$B \cdot C = M_{per} = \begin{bmatrix} \frac{c}{\alpha} & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & \frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

where, $c = \dfrac{1}{\tan\left(\frac{\Theta_h}{2}\right)}$

$\alpha = \dfrac{\text{width}}{\text{height}}$

$f = \text{far}$

$n = \text{near}$

Then, $p' = A \cdot B \cdot C \cdot D \cdot E \cdot p$

$$p' = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x - 1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y - 1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{c}{\alpha} & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & \frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cdot \begin{bmatrix} 1 & 0 & 0 & -x \\ 0 & 1 & 0 & -y \\ 0 & 0 & 1 & -z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot E \cdot p$$

Again, the value of depends on the operations performed on the object.